Quantum and classical computing for self-interacting dark matter cosmology

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Abstract

An issue of fundamental importance in particle physics is the nature of dark matter (DM) as it accounts for 85% of all matter in the universe and yet it is still eludes our understanding. Cold Dark Matter (CDM) models have received the most time and attention as dark matter candidates, however simulations based on these models predict overly dense galactic centers as compared to observation for small scale structure, known as the core cusp problem. Self-interacting dark matter (SIDM) provides a compelling alternative as they solve the core cusp problem while maintaining the accuracy of CDM models on large scale structure. I will explore the thermodynamics of SU(2) gauge field theory focusing on the early Universe as it moves from radiation to matter domination with age as related to the critical temperature for conformal glueballs, a possible candidate for SIDM. I will also explore two SIDM models using a mix of classical computing via Mathematica and quantum computing via quantum algorithms created in IBM's Qiskit. .

Introduction

WHY STUDY SELF-INTERACTING DARK MATTER?

- Solves small scale structure (<1Mpc) density (core cusp problem) of cold dark matter (CDM) models while maintaining accuracy of large-scale structure (> 1Mpc) simulations of CDM

The distribution of dark matter in the cluster above is shown with blue contour lines. The dark matter clump for the galaxy at the left is displaced from the position of the galaxy itself possibly implying that dark matter self-interactions of an unknown nature are occuring.

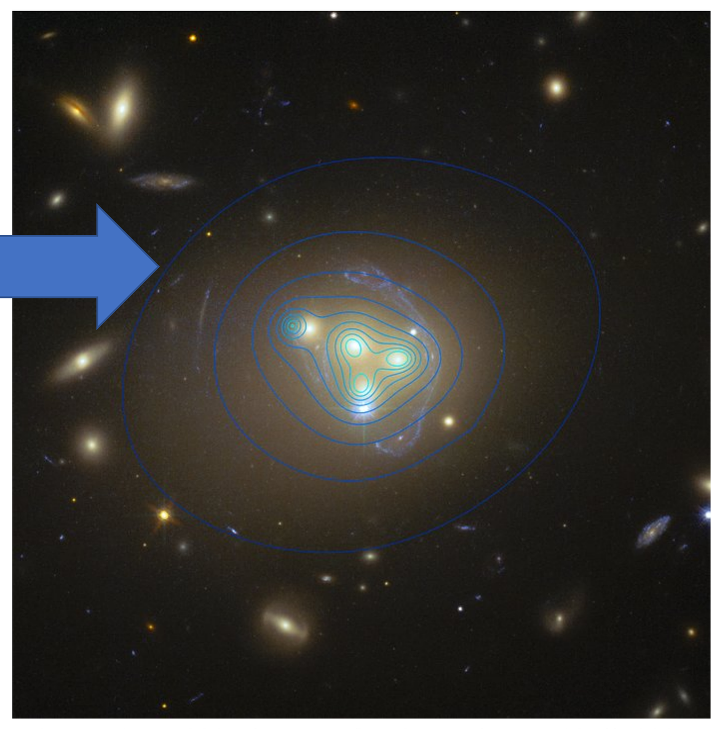


Figure 1: Hubble image of galaxy cluster Abell 3827 showing dark matter distribution **Source**: NASA/ESA Hubble Space Telescope **Credit:** ESO/R. Massey

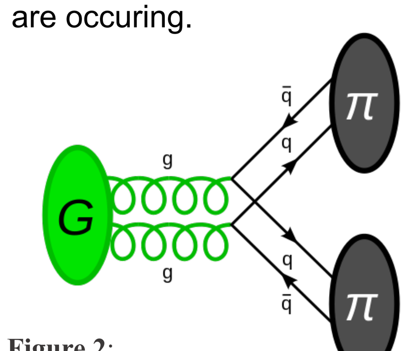


Figure 2: Feynman diagram of glueball decaying into two pions. **Credit**: Commons/Wikimedia

POSSIBLE CANDIDATE: GLUEBALLS

- Predicted bound state by Quantum Chromodynamics (QCD)
- Emergence due to strong interaction between gluons, no quarks
- Strongly interacting amongst themselves, weakly with Standard Model particles

Methods

CLASSICAL COMPUTING with Mathematica

- Used for SU(2) Quantum Chromodynamics (QCD) equation of state calculations
- Solving differential equations to find exact ground state energy
- Generating 2D contour plots and 3D models of wavefunctions
- Creating Hamiltonians to import into Qiskit
  - Hamiltonian is an operator which represents the energy of a quantum system, here represented in matrix form
    - Used Oscillator Kronecker product basis for Variational Quantum Eigensolver algorithm

QUANTUM COMPUTING with IBM's Qiskit

QISKIT

- IBM open-source software stack with Python interface, communicates with IBM quantum computer in real time

Variational Quantum Eigensolver

- Algorithm for calculating an upper bound of ground state energy of Hamiltonian
  - Choose ansatz wavefunction and optimization parameter, will compute until eigenvalue is minimized

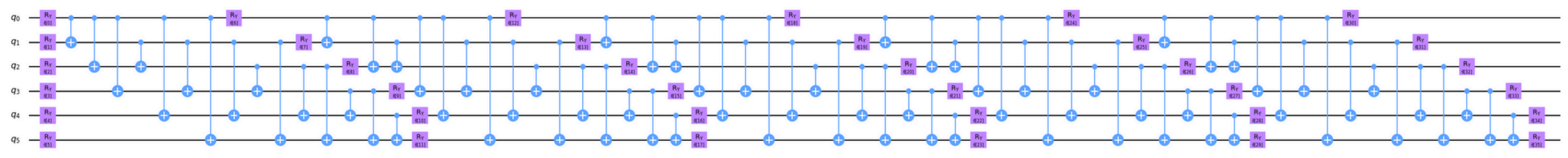


Figure 3: Image of VQE quantum circuit generated for SIDM Model One (see description below) – 6 qubits

Results

SU(2) LATTICE QCD

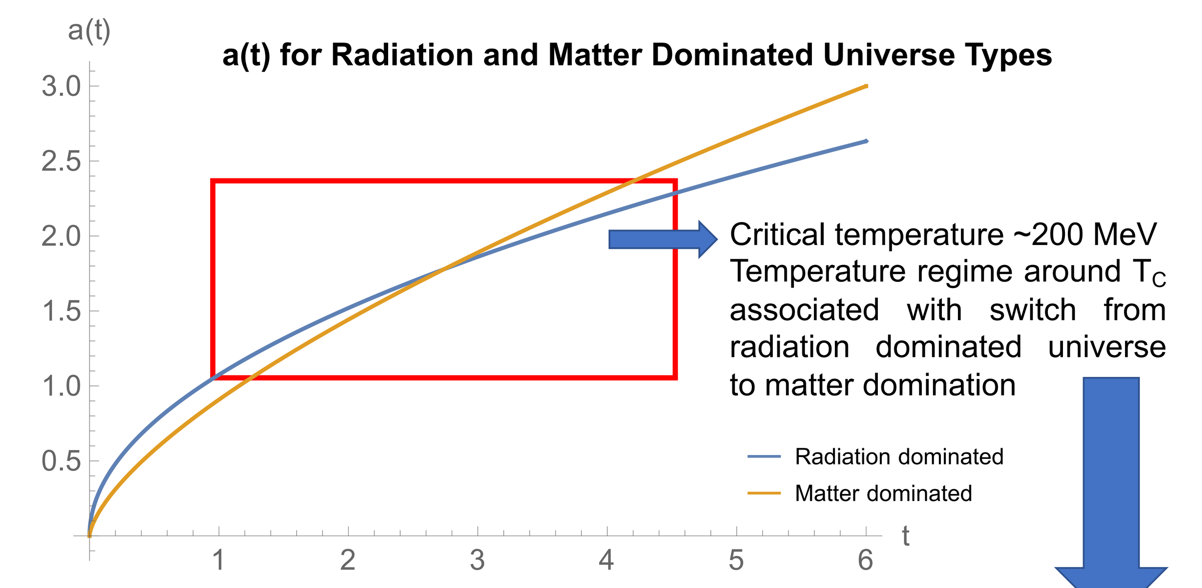
$$3M_P\left(\frac{da/dt}{a}\right)^2=\varepsilon$$

1st Friedmann equation: Describes the relationship between the time evolution of universe (a(t): radius of universe) to energy density (ε).

Solving Friedmann equation for Radiation and Matter

Radiation	Matter
$dt=\int\left(\frac{\rho_{r_0}}{3a^2}\right)^{-1/2}da$	$dt=\int\left(\frac{\rho_{m_0}}{3a}\right)^{-1/2}da$
$t=\frac{\sqrt{3}}{2\sqrt{\rho_{r_0}}}a^2$	$t=\frac{2a^{3/2}}{\sqrt{3\rho_{m_0}}}$
$a(t)=(\frac{4\rho_{r_0}}{3})^{1/4}t^{1/2}$	$a(t)=\left(\frac{3\rho_{m_0}}{4}\right)^{1/3}t^{2/3}$

a(t) for Radiation and Matter Dominated Universe Types



Critical temperature ~200 MeV Temperature regime around T_C associated with switch from radiation dominated universe to matter domination

Lattice QCD Equation of State Study

Lattice QCD study for $\mu=0$			
T: Temperature	p: Pressure	s: Entropy	ε: Energy Density
1.40	2.54	0.189	1.96
1.45	6.24	0.315	4.51
1.59	8.88	0.909	6.17
1.74	21.4	2.45	13.7
1.80	38.7	3.57	23.5
1.86	58.7	5.35	34.4
1.96	113	10.1	62.9
2.03	170	15.3	91.3
2.06	196	18.6	104
2.13	238	26.6	124
2.27	341	47.3	171
2.40	449	74.1	218

Table 1: Lattice QCD study data. N_t = 6. Temperature units (100MeV). Energy density and pressure units $(100MeV)^4$. Entropy units $(100MeV)^3$. [McGuigan]

SELF-INTERACTING DARK MATTER MODELS

Conformally coupled scalar fields

Model One

$$H=\frac{P_X^{\ 2}}{2}+\frac{X^2}{2}+\frac{P_Y^{\ 2}}{2}+\frac{Y^2}{2}+\lambda_X X^4+\lambda_Y Y^4+\frac{1}{a^4}\lambda_{mix}X^4Y^4$$

- **X**: Visible sector (baryonic matter) conformally coupled scalar field with self interactions
  - **Y**: Dark sector (dark glueballs) conformally coupled scalar field with self interactions
    - $\boldsymbol{\lambda_{mix}}$: Coupling constant (mixing) between both fields

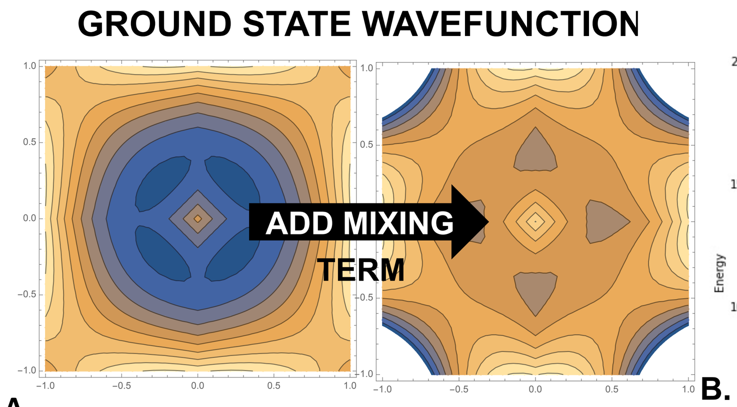
SU(2) Gauge fields model **Model Two**

$$H=\frac{P_X^{\ 2}}{2}+g_X^2X^4+\frac{1}{2}(P_Y+\theta Y^2)^2+g_Y^2Y^4+\frac{P_Y^{\ 2}}{2}+\frac{\lambda_{mix}}{a^4}(P_X+X^2)^2(P_Y+\theta_Y Y^2+Y^2)^2+\theta_Y(P_Y+\theta_Y Y^2)Y^2$$

- **X**: Visible sector (baryonic matter) SU(2) gauge field with self interactions
- **Y**: Dark sector (dark glueballs)) SU(2) gauge field with self interactions
- $\boldsymbol{\lambda_{mix}}$: Coupling constant between both fields
- $\boldsymbol{\theta_y}$: Theta term for dark sector

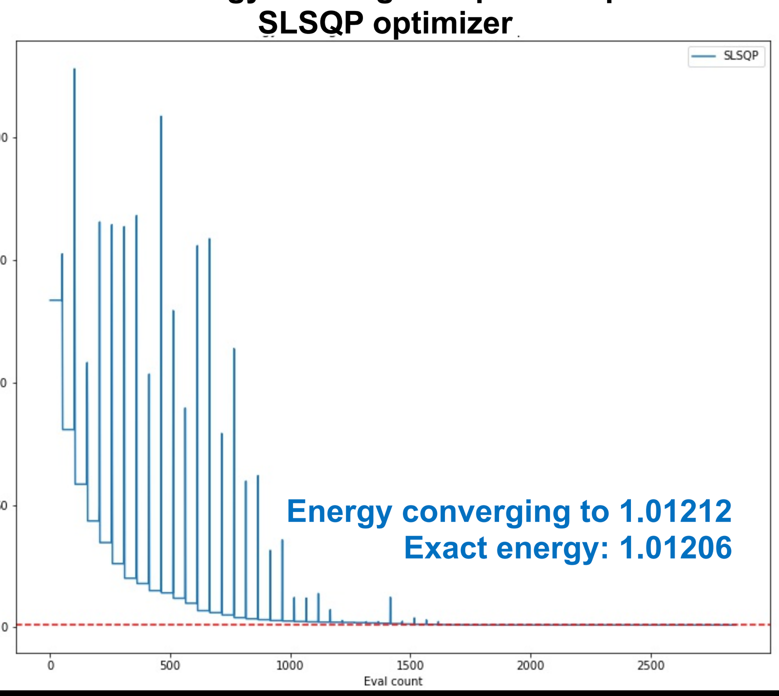
MODEL ONE: Conformally coupled scalar fields

GROUND STATE WAVEFUNCTION



A. Self-interactions. No mixing.
Contour plots
B. Self-Interactions. With mixing

VQE Energy convergence plot – 8 qubits SLSQP optimizer



VQE RESULTS

Model One VQE results						
Matrix Size of 2 Bosonic fields.	Qubits	Pauli Terms	Time (s)	Ground State Energy (VQE)	Ground State Energy (Exact)	Diff (%)
16x16	4	25	0.108	1.0120850	1.01206 (1.01208468)	0.0020
64x64	6	361	7.958	1.01220476	1.01206 (1.01205876)	0.0138
256 x 256	8	3025	54.900	1.012124134	1.01206 (1.01205913)	0.0059

Table 2: Quantum Variational Eigensolver results for SIDM model 1 for 16x16, 64x64, and 256x256 size oscillation basis Hamiltonian matrices. SLSQP optimizer. Ground state energy exact column shows EXACT Mathematica result on top and Python exact result for discretized Hamiltonian in ().

Conclusion

- **Glueballs** are an interesting candidate for self-interacting dark matter as a theory with **high naturalness**! And as many DM models are ruled out, SIDM models are predicting current observables remarkably well.
- More analysis should be done on predictions for cosmological observables.
- A lot of work in experimental frontier for detection possibly through gravitational wave background left from the first order de/confinement phase transition
- **Quantum computing** is a quickly developing technology that has the potential to be an excellent tool for studying quantum phenomena such as quantum cosmology.
- More work needs to be done to achieve accurate results with consistency.
- However, updates to Qiskit, improving choices in basis and optimizers have significantly improved results within last few months.

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