11 THE ELECTRON BEAM MEASUREMENTS

11.1 Emittance Measurement Using Quadrupole Scan

In order to measure the emittance we need to measure the elements of the beam matrix, which was described in Section 2.9, and then calculate its determinant. For this purpose we use a quadrupole and a screen (or camera) separated by a known distance as shown in Figure 11-1.

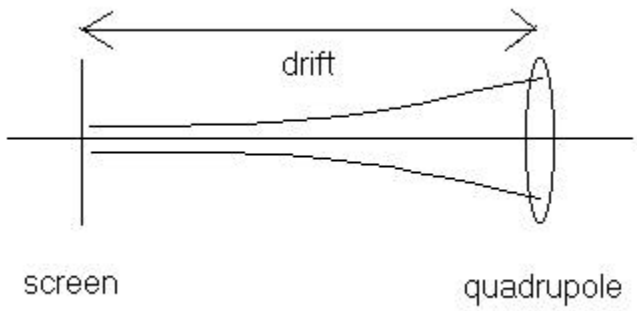

Figure 11-1 The sketch of quadrupole scan technique

Let s_q be the sigma matrix at the quadrupole.

For the beam vector X_q at the quadrupole, we apply the Eqn 2.9-21 as

$$X_q^{\,t} s_q^{\,-1} X_q = 1 \qquad \text{Eqn 11-1}$$

Similarly for the beam vector X_s on the screen it can written as

$$X_s^{\,t} s_s^{\,-1} X_s = 1 \qquad \text{Eqn 11-2}$$

where s_s is the sigma matrix on the screen. Assuming the transport matrix from the quadrupole to the screen is $R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$ X_s can be written as

$$X_s = R * X_q \qquad \text{Eqn 11-3}$$

Eqn 11-2 can be written as

$$X_s^{\,t} s_s^{\,-1} X_s = (R * X_q)^t * s_s^{\,-1} * (R * X_q) = X_q^{\,t} * R^t * s_s^{\,-1} * R * X_q = 1 \qquad \text{Eqn 11-4}$$

Comparing the Eqn 11-4 with the Eqn 11-1 tells us that the transform of the sigma matrix is

$$s_q^{\,-1} = R^t * s_s^{\,-1} * R \qquad \text{Eqn 11-5}$$

By inverting we obtain $s_q = R^{-1} * s_s * (R^t)^{-1}$ and thus

$$s_s = R * s_q * R^t \qquad \text{Eqn 11-6}$$

There are three unknowns in s_q matrix. The only parameter that we can measure is the beam size on the screen, which is $(s_s)_{11}$ element of the sigma matrix. The R matrix is a multiplication of quadrupole matrix and a drift space matrix. The quadrupole matrix is calculated as a function of quadrupole current and the drift matrix is a function of the distance, which is constant in this case.

Eqn 11-6 can be expanded as $(s_s)_{11} = R_{11}^{\,2}.(s_q)_{11} + 2.R_{11}.R_{12}.(s_q)_{12} + R_{12}^{\,2}.(s_q)_{22}$ [10]. By measuring the rms beam size ($\sqrt{(s_s)_{11}}$) on the screen for a number of quadrupole current settings we obtain a set of equations and solve for the three elements of the s_q matrix. Thus we can calculate the determinant of the sigma matrix and emittance. This is the geometrical emittance. The normalized emittance, which is a conserved quantity under acceleration, is obtained by multiplying the geometrical emittance by $b\,g$

The typical normalized emittance at the ATF is about 3-6 μrad.

11.2 Electron Beam Pulse Length Measurement

The pulse length of the electron beam is another important parameter. The FEL gain depends critically on the peak current, which in turn depends on the charge and the pulse length of the electron bunch. As we mentioned in Chapter 3, charge is measured by the Faraday cup. Thus we need to measure the pulse shape of the electron beam to determine the peak current.

The electrons are initially accelerated by the RF gun up to 4.5 MeV energy. Then the two sections of the Linac accelerate the electrons to the design energies of the various experiments. In the normal operating mode of the Linac, the electrons are accelerated on the crest of the accelerating field for minimum energy spread as shown in Figure 11-2.

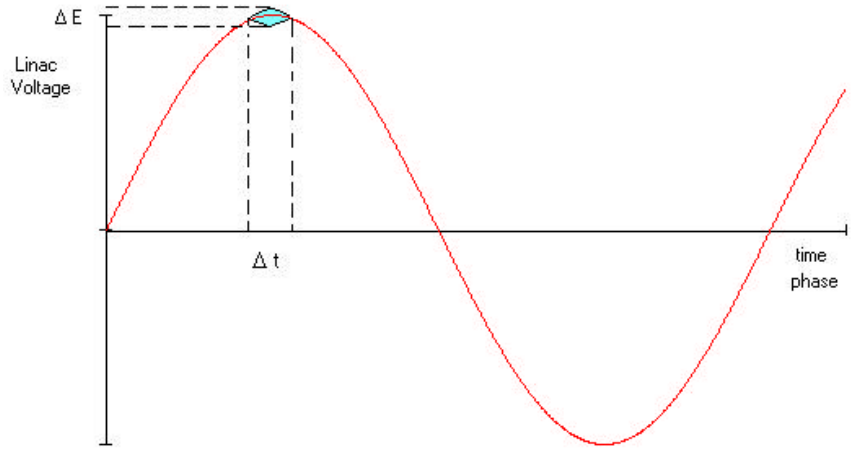

Figure 11-2 On-crest accelerating field waveform of the electron beam

In this phase there is no linear relation between the pulse duration and the energy spread. However if we accelerate the electrons on the rising edge of the accelerating field as shown in Figure 11-3, then the pulse duration is almost linearly proportional to the energy spread. Since we want to avoid space charge effects we do not want to reduce the accelerator field at the beginning of the Linac. Here we use the virtue of having two sections of the Linac. The first section is tuned for the minimum energy spread (on the crest of the field or at a phase of 90 ^{o}) and the second section is tuned to the rising edge, at about 60°, using a mechanical phase shifter. Thus by measuring the energy spread we can measure the pulse duration of the electron beam.

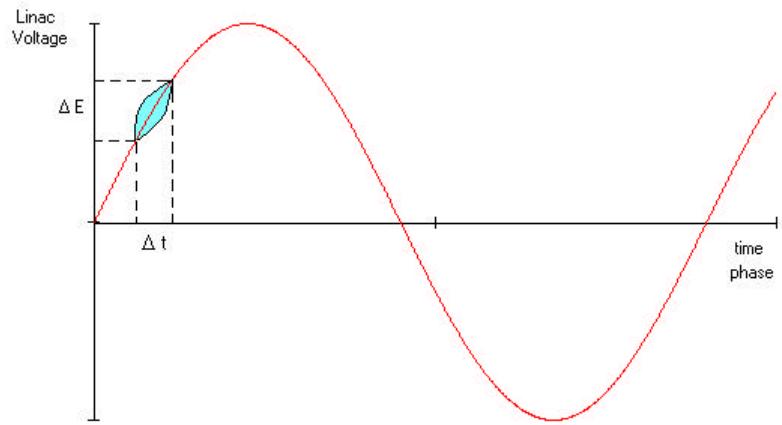

Figure 11-3 Off-crest accelerating field waveform of the electron beam

The energy of the electrons is

$$E = E_g + E_1 + E_2 \qquad \text{Eqn 11-7}$$

where E_g is the energy gain in the RF gun and the E_1 and E_2 are the energy gain the first and second section respectively. The energy gain in the second section can be expressed as

$$E_2 = E_2{}^0 \sin(wt - \Delta f + f_0) \qquad \text{Eqn 11-8}$$

where $wt = \Delta f$ and $f_0 = 60$ ^{o}. In this region energy spread can be expanded as

$$\Delta E = \frac{dE}{dt}_{t=\frac{\Delta f}{w}} \Delta t \qquad \text{Eqn 11-9}$$

and the higher order terms are neglected. Thus

$$\Delta E = \frac{dE}{dt}_{t=\frac{\Delta f}{w}} \Delta t \qquad \text{Eqn 11-10}$$

$$\Delta E = E_2{}^0 w \cos(wt - \Delta f + f_0)_{t=\frac{\Delta f}{w}} \Delta t = E_2{}^0 w \cos(60) \Delta t = \frac{E_2{}^0 w \Delta t}{2} \qquad \text{Eqn 11-11}$$

We measure the energy spread with a spectrometer consisting of the high energy slit (HES) and a Faraday cup. The HES is positioned downstream of the first dipole as described in the Chapter 3. It is located where the betatron oscillation is minimized, therefore the horizontal beam size is dominated by the energy spread. Since the overall energy is reduced we reduce the dipole current by 20 % thus we can see the beam on the HES. We close the opening of the HES to 2.6 mm, which is equivalent to 0.5 % of the

energy (~1.1 ps) [10]. We measure the charge passing through the slit with the Faraday cup and scan the mechanical phase shifter, letting different parts of the beam pass through. This scan gives us the longitudinal current distribution of the electron beam (see Figure 11-4)

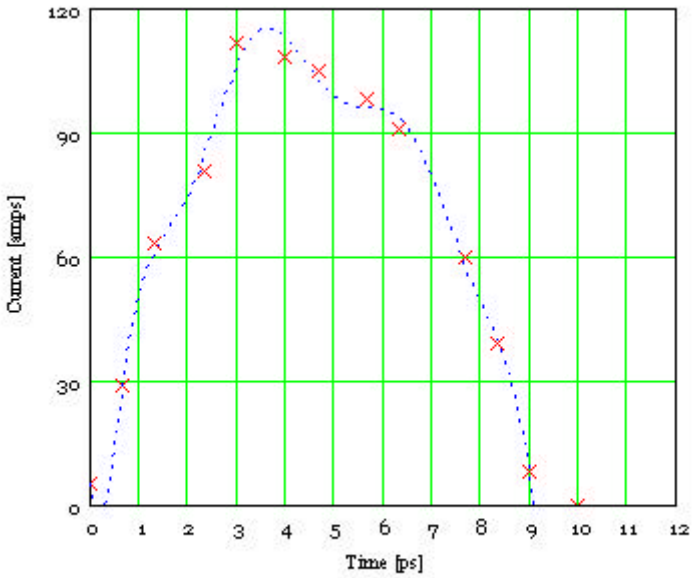

Figure 11-4 Longitudinal pulse shape of the electron beam

Thus the pulse length is determined by FWHM of the curve. In the case of Figure 11-4, the result is approximately 6 ps.