2.11 Transverse Coherence

Similar to the electron beam the radiation beam also has a beam size and the divergence. Let s_x and s_r be the electron and radiation beam sizes, s'_x and s'_r be the divergences of the electron and the radiation beams respectively at the waist positions. The emittance can be written as

$$e = s_x . s'_x \qquad \text{Eqn 2.11-1}$$

The corresponding equation for the radiation is

$$s_r . s'_r = \frac{l}{4p} \qquad \text{Eqn 2.11-2}$$

Thus the beam can be described as the convolution of a coherent radiation and the probability distribution of the electrons as

$$\Sigma_x = \sqrt{s_x^{\ 2} + s_r^{\ 2}} \text{ and } \Sigma'_x = \sqrt{s'^2_x + s'^2_r} \qquad \text{Eqn 2.11-3}$$

If the electron beam moments can be neglected compared to radiation moments then

$$\Sigma_x . \Sigma'_x \sim s_r s'_r = \frac{l}{4p} \qquad \text{Eqn 2.11-4}$$

In this case the radiation beam is transversely coherent which means the radiation would show Young interference fringes. If $\Sigma_x . \Sigma'_x >> \frac{l}{4p}$ then the radiation is transversely incoherent, and will fail to produce Young interference fringes.

2.12 Temporal Coherence

Longitudinally, a coherent wave can be represented as

$$E_o(t) = e_o.\exp(-\frac{t^2}{4s_t^{\ 2}} - iw_0 t) \qquad \text{Eqn 2.12-1}$$

We are assuming a Gaussian distribution where s_t is the coherence length in time and $c\,s_t$ is the coherence length in distance. The Fourier transform of the time domain gives us the frequency domain. The relation between the coherence length and the rms of the frequency profile $\left|e_0(w)^2\right|$ can be derived as:

$$s_{\Delta w} = \frac{1}{2s_t} \qquad \text{Eqn 2.12-2}$$

Another form of this expression is

$$c\,s_t\,s_{\frac{\Delta w}{w_0}} = \frac{l}{4p} \qquad \text{Eqn 2.12-3}$$

where $s_{\frac{\Delta w}{w_0}} \equiv \frac{s_{\Delta w}}{w_0}$

The Eqn 2.12-3 can also be written in terms of wavelength, which we measure with the spectrometer as:

$$c\,s_t\,s_{\frac{\Delta l}{l_0}} = \frac{l}{4p} \qquad \text{Eqn 2.12-4}$$

This expression is for the rms power widths of the time and frequency distributions. In the experiment we usually prefer the FWHM values thus the expression becomes

$$c\,s_t\,s_{\frac{\Delta l}{l_0}} = \frac{2\ln(2)}{p}\,l \qquad \text{Eqn 2.12-5}$$