2.5 The Fluid model

In this section we will describe the distribution of the electrons in the bunch. We have coupled Maxwell-Vlasov equation from the previous section.

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}+\frac{1}{2ik_s}\nabla_\perp{}^2\right)E=\frac{m_0 ec^2K[JJ]}{2\gamma_0}\sum_j e^{-i\theta_j}\,\delta(\bar{r}-\bar{r}_j) \qquad \text{Eqn 2.5-1}$$

Let's concentrate on the right hand side of the Eqn 2.5-1. If we average over a small volume ΔV

$$\frac{1}{\Delta V}\int_{\Delta V} d\bar{r}\sum_j e^{-i\theta_j}\,\delta(\bar{r}-\bar{r}_j)=\frac{1}{\Delta V}\sum_{j=1}^{\Delta N}e^{-i\theta_j}=\frac{\Delta N}{\Delta V}\left(\frac{1}{\Delta N}\sum_{j=1}^{\Delta N}e^{-i\theta_j}\right)=n(\bar{r},t)<e^{-i\theta_j}> \qquad \text{Eqn 2.5-2}$$

where $n(\bar{r},t)$ is the particle density.

Therefore $$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}+\frac{1}{2ik_s}\nabla_\perp{}^2\right)E=\frac{m_0 ec^2K[JJ]}{2\gamma_0}n(\bar{r},t)<e^{-i\theta_j}> \qquad \text{Eqn 2.5-3}$$

Let n_0 be the peak density and $n_0 f(z,t,\gamma,\bar{r}_\perp)$ be the distribution function such that

$n(\bar{r},t)=\int n_0 f(z,t,\gamma,\bar{r}_\perp)d\gamma$ is the particle density.

Then Eqn 2.5-3 becomes

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}+\frac{1}{2ik_s}\nabla_\perp{}^2\right)E=\frac{k_w D_1}{\gamma_0}<e^{-i\theta_j}>\int f d\gamma \qquad \text{Eqn 2.5-4}$$

Where D_1 is

$$D_1=\frac{m_0 n_0 ec^2K\left[JJ\right]}{2k_w} \qquad \text{Eqn 2.5-5}$$

It is more convenient to use the independent variables (z,θ) instead of (z,t). We change the variables using the definition of phase from Eqn 2.3-16. We obtain

$$\left(\frac{\partial}{\partial z}+k_w\frac{\partial}{\partial\theta}+\frac{1}{2ik_s}\nabla_\perp{}^2\right)E=\frac{k_w D_1}{\gamma_0}\int\left(f<e^{-i\theta_j}>\right)d\gamma \qquad \text{Eqn 2.5-6}$$

It is clear that in $f(z,\theta,\gamma)$ the only components, which will contribute to the growth significantly, are the ones near $e^{i\theta}$. Let F be the slow varying amplitude.

$f = Fe^{iq} + c.c. + f_0$ Eqn 2.5-7

In this definition f_0 represent the smooth distribution and F represent the microbunching. In Eqn 2.5-6 the term

$\int (f < e^{-iq_j} >) d\,g = \int Fd\,g +$ Oscillating term

So we write Eqn 2.5-6 as

$$(\frac{\P}{\P t} + \frac{\P}{\P q} + \frac{1}{2ik_s k_w}\nabla_{\perp}{}^{2})E = \frac{D_1}{g_0}\int Fd\,g \qquad \text{Eqn 2.5-8}$$

and get envelope Maxwell equation. ($t \equiv k_w z$)

The Coupled Maxwell-Vlasov Equation

Remembering Eqn 2.4-10 and Eqn 2.4-11 we had

$$\frac{dq_i}{dt} = 2\frac{g_j - g_0}{g_0} \text{ and } \frac{d\,g_j}{dt} = -\frac{D_2}{g_0}(Ee^{iqj} + c.c.)$$

This is a Hamiltonian system. Thus the distribution function obeys the Liouville's theorem. $\frac{\P f}{dt} + \dot{q}\frac{\P f}{\P q} + \dot{g}\frac{\P f}{\P g} + (\bar{v}_{\perp}.\nabla_{\perp})f = 0$

Using Liouville's theorem and arranging the slow and fast parts of the distribution function we can obtain the Coupled Maxwell-Vlasov equations as:

$$(\frac{\P}{dt} + \frac{\P}{\P q} + \frac{1}{2ik_s k_w}\nabla^{2}{}_{\perp})E = D_1\frac{1}{g_0}\int Fd\,g \qquad \text{Eqn 2.5-9}$$

$$(\frac{\P}{dt} + 2\frac{g - g_0}{g_0}i)F = D_2\frac{1}{g_0}\frac{\partial f_0}{\partial g}E \qquad \text{Eqn 2.5-10}$$