A Nondestructive Method for Measuring the RMS Length of Charge Bunches Using the Wake Field Radiation Spectrum

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Abstract: We report progress in the development of a nondestructive technique to measure bunch rms-length in the psec range and below, and eventually in the fsec range, by measuring the high-frequency spectrum of wake field radiation which is caused by the passage of a relativistic electron bunch through a channel surrounded by a dielectric. We demonstrate both experimentally and numerically that the generated spectrum is determined by the bunch rms-length, while the choice of the axial and longitudinal charge distribution is not important. Measurement of the millimeter-wave spectrum will determine the bunch rms-length in the psec range. This has been done using a series of calibrated mesh filters and the charge bunches produced by the 50MeV rf linac system at ATF, Brookhaven. We have developed the analysis of the factors crucial for achieving good accuracy in this measurement, and find the experimental data are fully understood by the theory. We point out that this technique also may be used for measuring fsec bunch lengths, using a prepared planar wake field microstructure.

INTRODUCTION

There are a number of diagnostics which have been used to measure the axial length of relativistic electron bunches in the picosecond (psec) and sub-psec ranges, corresponding to lengths of 1000 μm and less [1]. Among these are coherent transition radiation, diffraction radiation, synchrotron radiation, energy modulation, spontaneous emission single-shot spectrum, and electro-optical detection. Various rf pickup structures (e.g. loop, stripline, resonant cavity) are used to detect the transient radiation produced by the bunch. Certain techniques (e.g. monitoring Cerenkov radiation when the bunch traverses a solid target) can result in destruction of the bunch. Each diagnostic has its advantages and disadvantages. Ideally, one would want a diagnostic that disturbs the bunch as little as possible, and that uses instrumentation that is inexpensive, easy to adjust, and routine to calibrate. In this, we describe a non-destructive diagnostic which can measure the bunch RMS- length using an observation of the frequency spectrum of wake field radiation set up as the bunch passes through a vacuum channel in a hollow dielectric element.

Operation of the new diagnostic device is based on the following principles. A bunch or train of bunches enters a short section of beam pipe into which a hollow dielectric liner has been inserted. The dielectric can be made of alumina, which has low losses and good vacuum properties. Inside this structure, the bunch emits coherent Cerenkov radiation, which can be extracted and detected externally. The bunch passes along the axis of the structure and through a vacuum channel in the dielectric; the transverse size of this channel can be chosen to control the interaction and the energy radiated by the bunch. The smaller the width of the channel, the greater is the intensity of the Cerenkov radiation; but the greater also is the drag (or perturbation) on the bunch's motion. The radiation will follow the bunch down the vacuum pipe, and when the radiation is diverted, it can be extracted from the vacuum by passing it through a suitable window. The axisymmetric bunch excites a spectrum of co-propagating waveguide modes that constitute a periodic wake field disturbance following the bunch. Roughly speaking, the power spectrum of these modes is a function that increases with frequency for radiation whose wavelengths exceed the bunch length, decreases with frequency for shorter wavelengths, and falls to negligible amplitude beyond a cutoff frequency. This is a common feature for bunches radiating coherently, where the radiated power scales as N_b^2, with N_b being the number of electrons in the bunch. Thus, mm-size bunches exhibit a transition from a rising power spectrum to a falling power spectrum at a wavelength in the mm range. Interest within the accelerator community in a new bunch-length monitor is ever intensifying for bunches of mm and sub-mm dimensions, so the instrumentation described here is designed to operate in this wavelength region. We also point out that a bunch length diagnostic for laser accelerators, where the length is in the fsec range, would be useful too.

DIAGNOSTIC FOR MM-DIMENSION BUNCHES

In this section, we take the apparatus used to extract diagnostic information from passing charge bunches to be an annular cylinder of low-loss dielectric inserted into a close-fitting metallic drift tube forming part of an evacuated beam transport line. Our experience with such structures shows that a suitable dielectric in the range of millimeter wavelengths would be alumina [κ = 9.6] which, in addition to having good vacuum properties and low losses, is nearly free of dispersion in the microwave and mm-wave portions of the spectrum [2]. The outer radius of the alumina is ~ 1.93 cm, and a 3mm diameter axisymmetric circular cross-section channel allows propagation of the bunch through vacuum. The length of the dielectric section does not play an important role in determining the radiation spectrum, provided the length exceeds a few times the transition radiation zone length near the entrance and exit of the channel. For the dimensions given above, a dielectric length of a few 10's of cm is a good choice. This is to insure that Cerenkov wake field radiation dominates the transition radiation that is emitted as the bunch enters or leaves the structure. Furthermore, experience has shown that charge evidently does not accumulate on the surface of ceramics such as alumina, as it may on other dielectrics.

Fig 1 is a diagram of the dielectric-lined beam line element installed in the Omega-P/Yale/Columbia experiment at ATF, Brookhaven for studying wake fields. Fig 2 is a schematic of a portion of the beam line, showing transverse radiation output by use of a 45° deflecting mirror. The vacuum window is made of Z-axis cut crystal quartz for optimum transmission of the radiation over a wide range of wavelengths. A prior determination of the microwave spectrum of wake field radiation using the radial probe "SMA", permits one to determine the period of the wake field, has been reported [3].

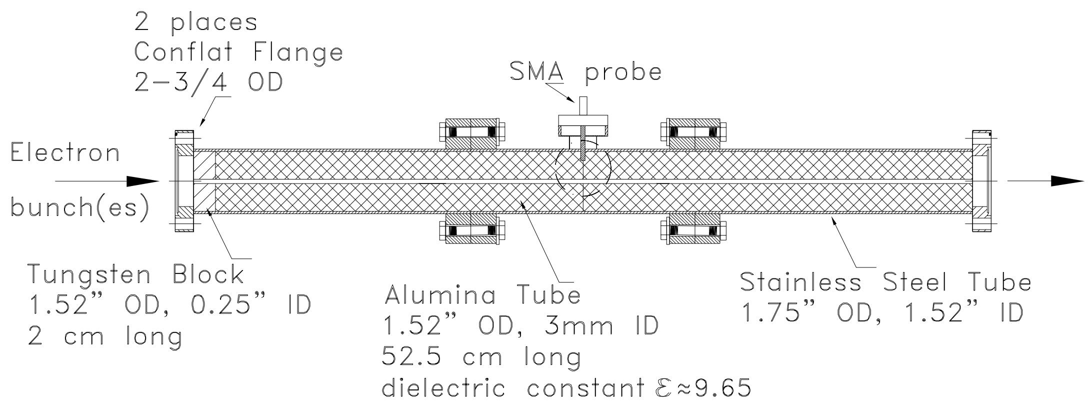

FIGURE 1. Diagram of the dielectric-lined beam line element installed in the Omega-P/Yale/Columbia experiment at ATF, Brookhaven for studying wake fields.

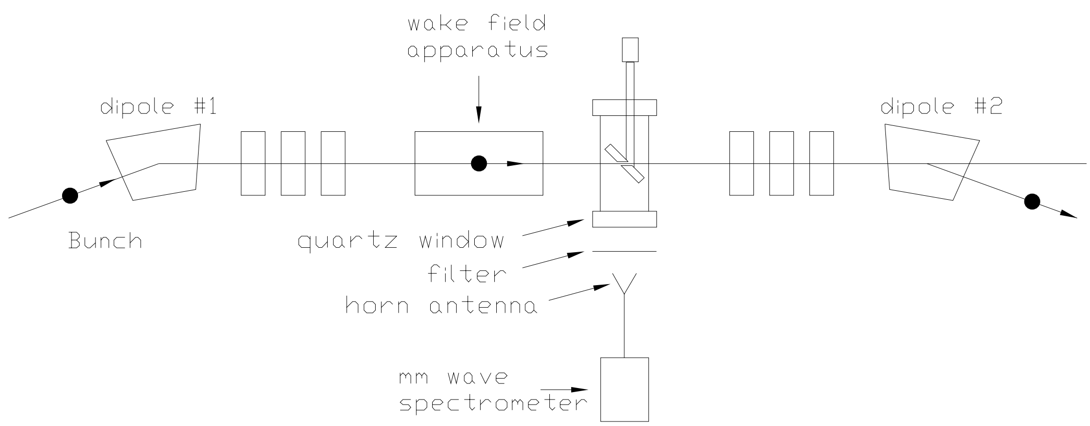

FIGURE 2. Schematic of a portion of the beam line, showing transverse radiation output by use of a 45° deflecting mirror.

We now present a brief summary of an analysis for the fields set up within a structure of the type described. The theory has been thoroughly developed in recent years [4-9]. In cylindrical geometry, orthonormal wave functions can be found for the fields which separate into TE and TM classes for axisymmetric excitations, and into hybrid modes with mixed polarization otherwise. With axisymmetric bunch distributions, only TM modes have an electric field (E_z) that is finite at the bunch location, and that consequently contributes to the radiation generated. Axisymmetric TE modes have vanishing electric fields on axis. Non axisymmetric modes that contribute to (unstable) transverse motions are excited by a bunch moving off-axis [9]. Conditions can be found where all TM modes have phase velocities equal to the speed of the electrons ($v \cong c$), and so wake fields move in synchronism with the bunches. We neglect contributions from transition radiation localized near axial boundaries of the dielectric insert [10]. Field components for the complete orthonormal TM mode set along an infinitely long waveguide can be written in the form

$$E_z(r,z,t) = \sum_m E_m \exp(-i\omega_m z_o / v)\, f_m(r)/\alpha_m \,,$$

where α_m is a normalizing constant, $z_o = z - vt$, and f_m is an expression involving Bessel functions which describes the radial dependence of the solution [7]. Wave-numbers in the vacuum channel and in the dielectric are related by $k_{1m}^2 + \omega_m^2/v^2 = \omega_m^2/c^2$ and $k_{2m}^2 + \omega_m^2/v^2 = \omega_m^2/\kappa c^2$, where the evanescent transverse wave number in the vacuum is k_{1m}, the real transverse wave number in the dielectric is k_{2m}. Thus $k_{1m} = \omega_m/c\beta\gamma = k_{2m}\gamma_\kappa/\gamma$, where $\gamma_\kappa = (\kappa\beta^2 - 1)^{-1/2}$. The dispersion relation can be obtained [11], from which it is found that in the limit of high dielectric constant $\kappa >> 1$, there is a nearly periodic spacing of eigenfrequencies where, for all but a few lowest modes, the mode separation is $\Delta\omega = \omega_{m+1} - \omega_m \approx \pi\beta c\left[(R-a)(\kappa\beta^2 - 1)^{1/2}\right]^{-1}$. The wake field is accordingly more strongly peaked and more nearly periodic in z_o as the eigenfrequencies become more nearly periodic and constructive phase interference between modes can occur. This constructive mode interference causes the wake fields to be localized at periodic intervals behind a moving bunch; the shorter the bunch, the greater the number of modes excited, and the more sharply-peaked are these localized fields.

To find wake fields induced by an electron bunch, one expands in orthonormal modes the solution of the inhomogeneous wave equation and constructs a Green's function. For a bunch containing N_b electrons distributed along a finite RMS length Δz = $2\sqrt{\langle z^2\rangle - \langle z\rangle^2}$ in a Gaussian function, one finds

$$E_z(r,z,t) = -E_o \sum_m [f_m(r)/\alpha_m]\cos(\omega_m z_o/v)\exp\left(-\omega_m^2\,\Delta z^2/8v^2\right)$$

Here, $E_o = -N_b e/2\pi\varepsilon_o a^2$, and the results are valid only behind the bunch ($z_0 < 0$), following the dictates of causality; in front of the bunch $E_z = 0$. Note that only the radius of the hole a, and the bunch charge $-eN_b$ determine the total magnitude of the wake field, while the bunch width Δz determines the distribution of amplitudes amongst the spectrum of normal modes. The energy loss dW/dz and total radiated power scale as N_b^2. Everywhere behind the Gaussian bunch

$$dW/dz = \frac{N_b^2 e^2}{4\pi\varepsilon_0 a^2}\sum_m \left(\exp(-\omega_m^2\Delta z^2/8v^2)\right)^2/\alpha_m$$

Numerical solutions for the wake fields are shown in the figures below for examples which might be typical in a diagnostic application: the parameters are from experiments carried out at ATF-Brookhaven, which uses 50 MeV electron bunches obtained from an rf linac. The dimensions of the dielectric insert are chosen to provide a fundamental wake field period in this device of 21 cm, which is twice the linac rf wavelength. This choice has to do with the method used for generating bunches, which uses time-delayed optical pulses incident upon the ATF rf photocathode gun.

An example of the power spectrum which we predict to be emitted from a typical well-focused bunch obtained at ATF, Brookhaven, is shown in Figure 3.a. Radiation from a Gaussian charge distribution having different RMS length is shown. Most of the power is contained in the mm-wave portion of the spectrum. A high frequency "cutoff" limit can be identified by fitting a straight line to the descending slope of the high frequency radiation spectrum, projecting it onto the horizontal axis. Millimeter diagnostic spectroscopy can be carried out up to ~ 500GHz, thus the technique can be adapted to bunches having length in the sub-psec region. However, we have found that the choice of axial charge distribution is not important, providing each distribution is characterized by the same bunch RMS- length. In Fig. 3.b, we show a spectrum generated by three different bunch shapes: a Gaussian, a triangular, and a rectangular charge distribution along the bunch length axis. For each example, the same RMS length is chosen for the bunch. The result is quite interesting, as it shows that as long as the bunch RMS- length is used for comparison, the spectrum generated is almost the same. This establishes the utility of the diagnostic, since the details of the bunch charge distribution do not affect the measurement of the key parameter: namely, the bunch RMS- length.

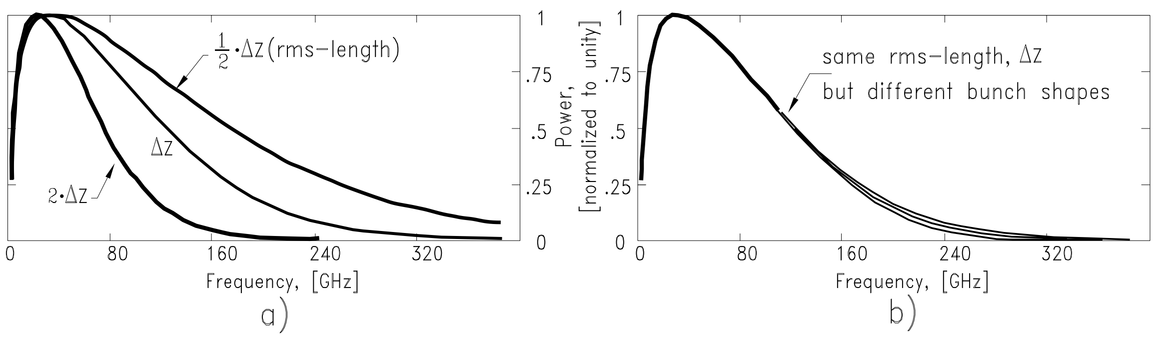

FIGURE 3. Power spectrum that we predict to be emitted from a typical well-focused bunch obtained at ATF-BNL. (a) Radiation from a Gaussian charge distribution having different RMS length; Δz = 1.8psec; (b) spectrum generated by three different bunch shapes: a Gaussian, a triangular, and a rectangular charge distribution along the bunch length axis

For a non-axisymmetric bunch charge distribution, hybrid modes will be excited in addition to the set of TM modes as described above [9]. However, the general dependence of the spectrum upon bunch length involves the same features described here for the TM modes.

The spectrum computed in Figure 3 is for just one bunch; however, we comment on the case when several bunches are actually provided. If one superimposes the radiation fields of several equally spaced bunches, then the shape of the spectrum depends markedly upon the ratio of the wake field period to the bunch spacing. If this ratio is an even number, then the high frequency spectrum (where the modes are evenly spaced) is severely attenuated; if this ratio is an odd number, then the power spectrum envelope resembles that of a single bunch, albeit enhanced in magnitude, with certain TM modes deleted. If the ratio is neither even or odd, the spectrum is so complicated that is likely to be useless for bunch length diagnostic purposes. To illustrate this odd/even behavior

(Figure 4), we have taken the parameters of the ATF wake field apparatus, where the wake field period is 21cm, to compute the spectrum for a train of bunches with spacing 1/2, 1/3, and 1/4 the wake field period. From these results, we conclude that the spectrum bunch length diagnostic wake field apparatus must have a wake field period of three times the bunch spacing; the wake field period can be adjusted by correct choice of the dielectric thickness.

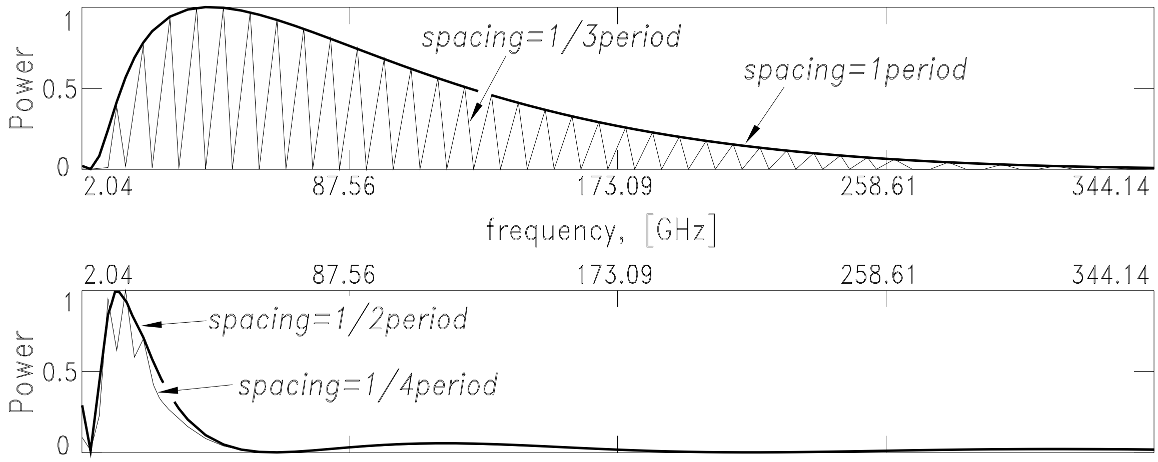

FIGURE 4. Spectrum for a train of bunches with spacing 1, 1/2, 1/3, and 1/4 the wake field period, normalized to the same maximum value.

EXPERIMENT

This experiment has been run on beam-line #2 at ATF. The radiation is deflected from the bunch by a reflector with a slot (to permit the bunch to pass), and passes through a Z-cut quartz vacuum window into the experimental area. The radiation is in the form of a single pulse for each bunch having duration of roughly 1.7nsec and kW level (Fig.5). We analyze the spectrum using a set of 5cmX5cm calibrated metal mesh filters mounted onto a turntable structure which is driven by an external motor so that meshes with different filter functions can be rotated into the beam of millimeter radiation emerging from the window, using a remote control. The detector is a Schottky-barrier diode which has been calibrated, along with the meshes, by the manufacturer Virginia Diodes Inc. Attenuation is provided by a pad of white notepaper, so that the detector operates in its linear regime. We confirmed experimentally that at the chosen attenuation the transition radiation generated by the mirror slot is of negligible contribution to the detector signal. The range of filtered frequencies is from 55 to 120 GHz. In order to calibrate the system, which includes the beam drift pipe, the reflector, the diffraction-coupled window, and any attenuating material, it is necessary to measure a bunch length using independent means at ATF. Diagnostic information, such as bunch charge, energy lost from the bunches, bunch length, is obtained routinely from the permanent ATF facility hardware available to all users. The measurements being done here are part of a larger program involving the study of the superposition of wake fields from multi-bunch trains.

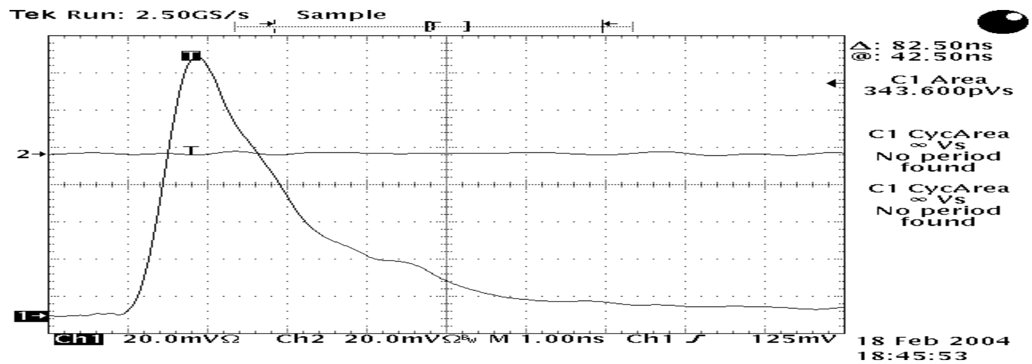

FIGURE 5. Example of the detector response (after the 68 GHz filter, and the bunch with $\Delta z \approx 5.6$ psec); the horizontal scale is 1ns/div; the vertical scale is 20mV/div

The experiment has been successfully run, having started first with the development of a detailed plan for aligning the equipment and transporting up to three bunches through the hardware. The RMS diameter of the bunch at the entry and exit to the dielectric element is in the range of 250-300μm, and thus it is well removed from the walls of the dielectric channel. Thus far, two bunches, following each other separated by 10.5, 21, or 31.5 cm, have been transported through the hardware, suffering negligible particle losses; this disposes of any reservations having to do with the charging up of the channel walls and the potentially harmful effects that might thereby result. The transport system is achromatic, accommodating an input bunch energy variation of up to 1MeV without affecting the bunch transport. Multiple bunches are used for studies of the superposition of wake fields, but in the spectrum study which follows, we use only *one* bunch to generate the radiation.

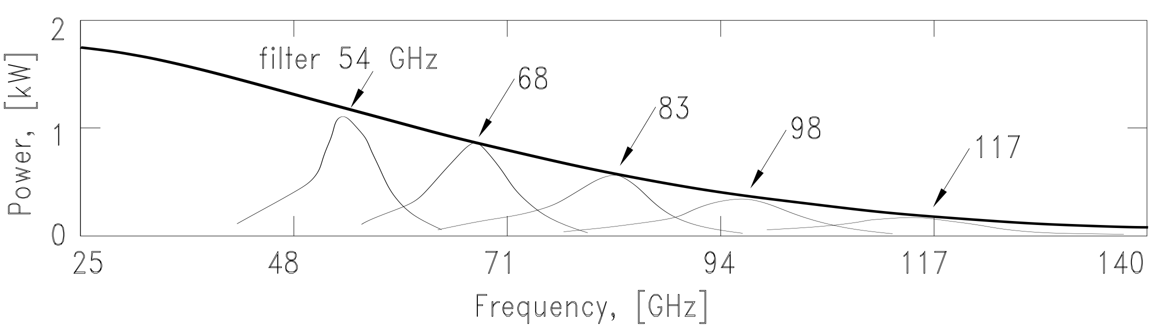

FIGURE 6. Filters break the spectrum of wake field radiation into five channels. The solid line is the theoretical emitted spectrum from a single bunch having RMS length of 4.2psec and charge 310pC.

Figure 6 shows how the filters break the spectrum of wake field radiation into five channels. The solid line is the theoretical emitted spectrum from a single bunch having rms width of 4.2psec and charge 310pC, being an envelope of the closely spaced TM_{0n} modes (separation ≈ 2.85GHz). It is seen that the half-transmission width of the filters is ~10GHz, thus ~4 TM modes are accepted by each filter. The detector responds to the square of the electric field, thus its signal depends on the sum of the squares of the amplitude coefficients of the TM modes. The integrated detector response would be proportional to the area under the transmission curve of a particular mesh.

Bunch charges and bunch lengths are measured by well-documented techniques available to users in that laboratory. To measure the bunch length, a variable phase shift is introduced for the second rf linac section. A particle located at the head of the bunch thereby gains a different energy than one located at the tail. Thus the bunch profile along the time-axis is mapped onto an energy-axis. After passing through a dipole, the bunch trajectory experiences a transverse shift for every energy shift. Using a slit followed by a monitor, one can scan the charge profile of the bunch as the linac phase is varied (about 1psec/degree),

Referring to Figure 3, we point out that the *ratio* of power emitted at any given spectrum channel wavelength depends on the ratio of the bunch lengths. We may, of course, observe the entire millimeter spectrum at frequencies centered on 54, 68, 83, 98, and 117GHz, as provided by the different meshes. The detector response depends on wavelength, as does the response of the horn it is connected to. This has been measured by the vendor. However, there may be other wavelength-dependent transmission effects associated with the radiation emerging from the dielectric, transmitted by the overmoded beam transport pipe, and reflected by the mirror. Fortunately, if we form the ratio of power emitted into each channel (fixed wavelength) for two different bunch lengths, then the wavelength dependence of the hardware response will cancel at each frequency, and the ratio will provide an accurate determination of bunch length, once a reference bunch length is determined using an independent technique. In Figure 7 we show how the ratio varies with frequency for a particular choice of bunch lengths (3.9 and 1.7psec) and charges (166 and 105pC). The two curves nearly overlaying each other describe two ways of forming that ratio: using the maximum of the filtered transmitted power, or using the total energy transmitted by the filter.

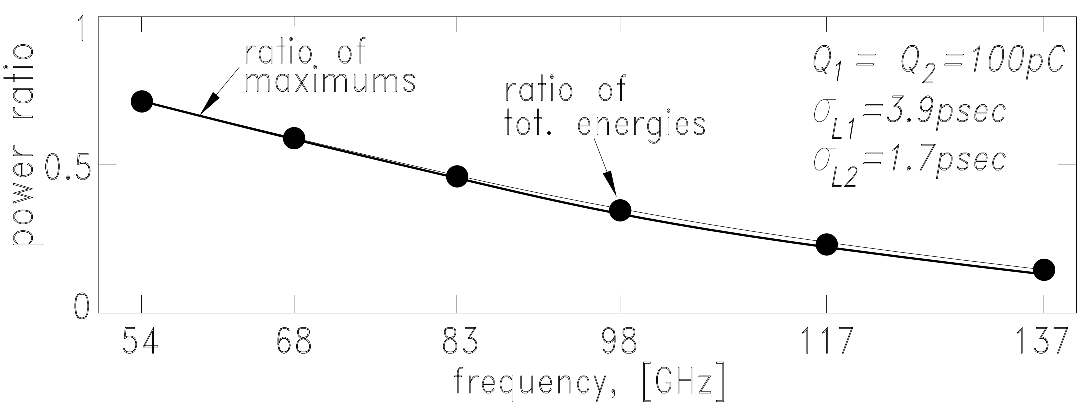

FIGURE 7. Ratio of power emitted into each channel (fixed wavelength) for two different bunch lengths (3.9 and 1.7psec) and charges (166 and 105pC). The two curves nearly overlaying each other describe two ways of forming that ratio: using the maximum of the filtered transmitted power, or using the total energy transmitted by the filter.

One can appreciate that the approach using signal ratios, in addition to being more accurate, is also simpler, in that a few fixed-frequency channels can be used, and considerable expense can be avoided since no absolute measurement of the transmission function of these components is necessary. Needless to say, ease of use and accuracy are important considerations for choosing a bunch length diagnostic.

The experiment at ATF has generated data which permit the measurement of the length of charge bunches provided by the rf linac (50MeV), and enough data have already been gathered to establish the feasibility of this technique. In Figure 8 we show data acquired for two cases: a 5.6psec long bunch having charge 430pC, and a 4.2psec long bunch having charge 310pC.

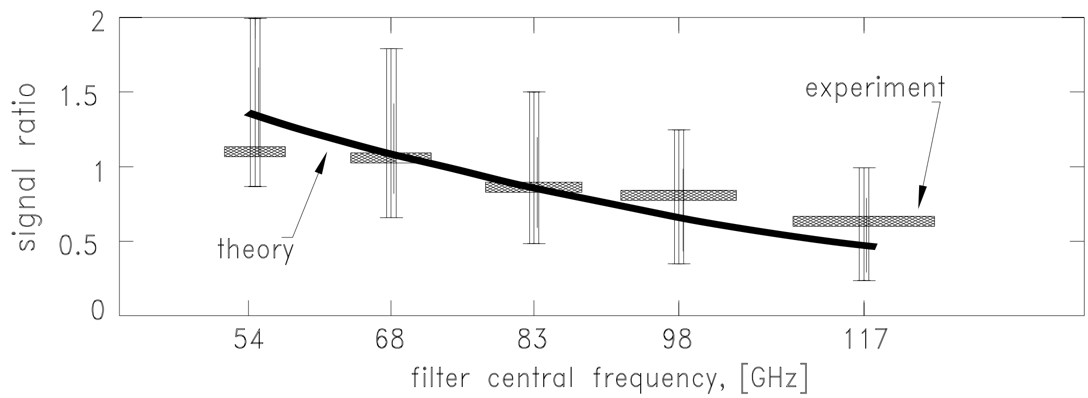

FIGURE 8. Ratio of power emitted into each channel (fixed wavelength) for two different bunch lengths (5.6 and 4.2psec) and charges (430 and 310pC). The solid black line is the prediction of the theoretical ratio of signal powers. The crosshatched horizontal lines (extending over the filter bandwidth frequency) represent the experimental ratio of the total energy received by the detector. The variation in charge from one shot to another measured by a Faraday cup result in large vertical error bars.

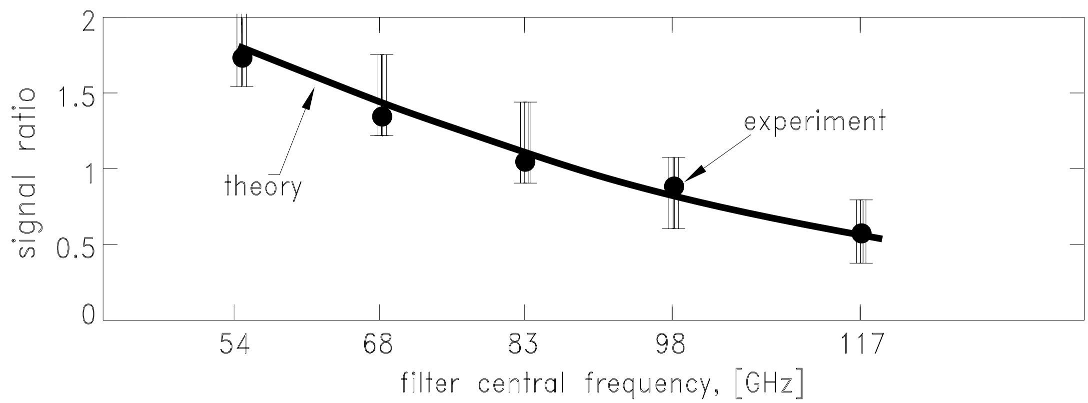

FIGURE 9. Ratio of power emitted into each channel (fixed wavelength) for two different bunch lengths (3.9 and 1.7psec) and charges (166 and 105pC). The solid black line is the prediction of the theoretical ratio of signals. The dots represent the experimental ratio. The variation in charge from one shot to another is measured by calculating the charge variations through signal variations on the detector.

The solid black line is the prediction of the theoretical ratio of signal powers for these two different bunches. The error bars on this line represent the possible range of variation (two-sigma at each point) of the black theory curve due to errors in measurement of bunch charge (±30pC by a Faraday cup) and bunch RMS- length (±0.2psec). The crosshatched horizontal lines (extending over the filter bandwidth frequency) represent the experimental ratio of the total energy received by the detector from the wake field radiation pulse at each filter frequency for the two types of bunch. It is found that the ratio measurements fall within the range predicted by theory.

There is very little variation of signal behavior from one shot to another, which suggests that charge measurements performed by a Faraday cup are noisier than the actual charge variations. Based on the detector signal one would conclude that the variation in charge is much smaller. In Figure 9 we show data acquired for two cases: a 3.9psec long bunch having charge 166pC, and a 1.7psec long bunch having charge 105pC. The solid black line is the prediction of the theoretical ratio of signals. Again, the error bars on this line represent the possible range of variation (two-sigma at each point) of the black theory curve due to errors in measurement of bunch charge (±3pC by calculating the charge variations through signal variations on the detector) and bunch RMS- length (±0.2psec).

The accuracy $\delta\Delta z / \Delta z$ of measurement of RMS length Δz is found to be

$$\delta\Delta z / \Delta z = \left(C_{\Delta z} / \Delta z^2\right) \times \left(\Delta Q / Q\right)$$

where $\Delta Q/Q$ is the charge fluctuation (two sigma). The coefficient $C_{\Delta Z}$ is determined by the structure parameters, which for the device at ATF give $C_{\Delta Z} \approx 13.8$ psec^2.

The minimum RMS length one can resolve with this technique is

$$\Delta z_{MIN} = \sqrt{C_{\Delta z} \times \left(\Delta Q / Q\right)}$$

For $\Delta Q/Q \approx 3\%$ one may expect to resolve the RMS length $\Delta z \geq 0.64$ psec. Improving to $\Delta Q/Q \approx 1.5$ % one could resolve the RMS length $\Delta z \geq 0.45$ psec.

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