Friction Force with Magnetic Field Errors

George Bell, *Tech-X Corp.*

Brookhaven National Lab:
Alexei Fedotov, Vladimir Litvinenko & Ilan Ben-Zvi

Tech-X Corp.:
David Bruhwiler, Dan Abell, Peter Messmer and the VORPAL team

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Tech-X Corporation
5621 Arapahoe Ave., Suite A
Boulder, Colorado 80303
http://www.txcorp.com

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Motivation

- Simulations are needed to validate theory and explore regimes where theory is weak
  - Quantify friction force with no undulator
  - Quantify friction force with a perfect undulator
  - Quantify the effect of magnetic field errors
    - Magnet misalignments
    - Higher order effects in the undulator field (sextupole)
    - Other magnetic field errors
- Numerical approach
  - Binary collision model
  - Boris push of particles in fields
  - implemented in VORPAL

The Binary Collision Model

- Solves the 2 body problem exactly for ion/electron pairs
  - handle each pair separately in center-of-mass frame
  - Sum over all particles

$$\vec{x}\,'_{e,i} = \vec{x}_{e,i} + \Delta t\,\vec{v}_{e,i} + \frac{\mu}{m_e}\sum_j \delta\vec{x}_{ij} \qquad \vec{v}\,'_{e,i} = \vec{v}_{e,i} + \frac{\mu}{m_e}\sum_j \delta\dot{\vec{x}}_{ij}$$

$$\vec{x}\,'_{Au,j} = \vec{x}_{Au,j} + \Delta t\,\vec{v}_{Au,j} - \frac{\mu}{m_{Au}}\sum_i \delta\vec{x}_{ij} \qquad \vec{v}\,'_{Au,j} = \vec{v}_{Au,j} - \frac{\mu}{m_{Au}}\sum_i \delta\dot{\vec{x}}_{ij}$$

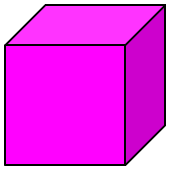
- Deviations represent differences between exact 2 body calculations and drift solutions

$$\delta\vec{x}_{ij} = \vec{x}_{ij}(\Delta t) - \left[\vec{x}_{ij}(0) + \Delta t\,\dot{\vec{x}}_{ij}(0)\right]$$

$$\delta\dot{\vec{x}}_{ij} = \dot{\vec{x}}_{ij}(\Delta t) - \left[\dot{\vec{x}}_{ij}(0)\right]$$

Baseline Parameters

- e^- Beam parameters
  - Density: 9.50E13 e^-/m^3
  - Rms e- velocity: 2.8E5, 2.8E5, 9.0E4 m/s [x,y,z]
  - $\gamma = 108$
- Undulator parameters
  - Length: 80m
  - Wavelength: 8cm
  - Sections: 10
  - Field on axis: $B_0 = 10G = .001T$
  - Interaction time: 2.47 nanoseconds (beam frame)
- Problem setup in VORPAL
  - Domain size: 0.8mm x 0.8mm x 0.8mm
  - Gold ions per domain: 8
  - Electrons per domain (actual): 4.864E4
  - Periodic domain



Field Expansion from Animesh Jain

Harmonic Expansion

- Axial variation results in a 3-D field.
- A simple harmonic expansion results under the assumption of periodicity along the Z-axis with wavelength λ:

$$B_{r,\theta,z}(r,\theta,z) \equiv B_{r,\theta,z}(r,\tilde{\theta}); \quad \tilde{\theta} = \theta - kz$$

$$k = (d\alpha / dz) = \text{rate of change of dipole field angle} = 2\pi / \lambda$$

$$B_r(r,\tilde{\theta}) = B_0 \sum_{n=1}^{\infty} \left[\frac{2^n n!}{n^n (kR_{ref})^{n-1}} \right] I_n'(nkr) \left[\tilde{b}_n \sin(n\tilde{\theta}) + \tilde{a}_n \cos(n\tilde{\theta}) \right]$$

$$B_\theta(r,\tilde{\theta}) = B_0 \sum_{n=1}^{\infty} \left[\frac{2^n n!}{n^n (kR_{ref})^{n-1}} \right] \frac{I_n(nkr)}{kr} \left[\tilde{b}_n \cos(n\tilde{\theta}) - \tilde{a}_n \sin(n\tilde{\theta}) \right]$$

$$B_z(r,\tilde{\theta}) = -(kr) B_\theta(r,\tilde{\theta}) \qquad n = 1 \text{ is Dipole term, etc.}$$

Transforming fields to the beam frame

- A pure magnetic field in the lab frame

$$\vec{B} = \left(B_x(x,y,z), B_y(x,y,z), B_z(x,y,z)\right)$$

in the beam frame becomes:

$$\vec{E}' = \gamma\vec{v} \times \vec{B} = \gamma\beta c\left(-B_y, B_x, 0\right)$$
$$\vec{B}' = \left(\gamma B_x, \gamma B_y, B_z\right)$$

  – Primed variables represent beam frame variables
- If velocities in the beam frame are non-relativistic, we can ignore B'.
  – 10G field gives a transverse velocity in the beam frame of .006c

Undulator Field in the Beam Frame

- The main undulator field in the beam frame

$$\vec{E}' = E_1\left(1 + O(x^2)\right)\begin{pmatrix} -\sin\Omega t' \\ \cos\Omega t' \\ 0 \end{pmatrix}$$

$$\vec{B}' = \gamma B_0\left(1 + O(x^2)\right)\begin{pmatrix} \cos\Omega t' \\ \sin\Omega t' \\ 0 \end{pmatrix}$$

$$E_1 = \gamma\beta c B_0 \qquad \Omega = \frac{2\pi\gamma\beta c}{\lambda} = \gamma\beta c k$$

Numerical simulation difficulties

- Electron cooling requires matching the mean velocity of electrons and ions
  – In numerical simulations including E and B, we need to check that this speed match occurs
- Lack of fringe field effects
  – If we start the simulation with the basic undulator field, the average electron velocity may not be zero.
  – This numerical problem can be corrected by
    - Adding a velocity bump at the start
    - Ramping the fields up from zero
  – Either results in zero electron average velocity
  – The magnetic field (in the beam frame) can also induce a drift

Reduction of friction from the undulator

- According to theoretical work, the undulator should reduce the friction by an amount

$$\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \Big/ \ln\left(\frac{\rho_{\max}}{r_0}\right)$$

  – Where

$\rho_{\max} \cong 7.4E-4m$ is the maximum impact parameter, determined by time of flight through the undulator

$\rho_{\min} \cong 2.2E-7m$ is the minimum impact parameter

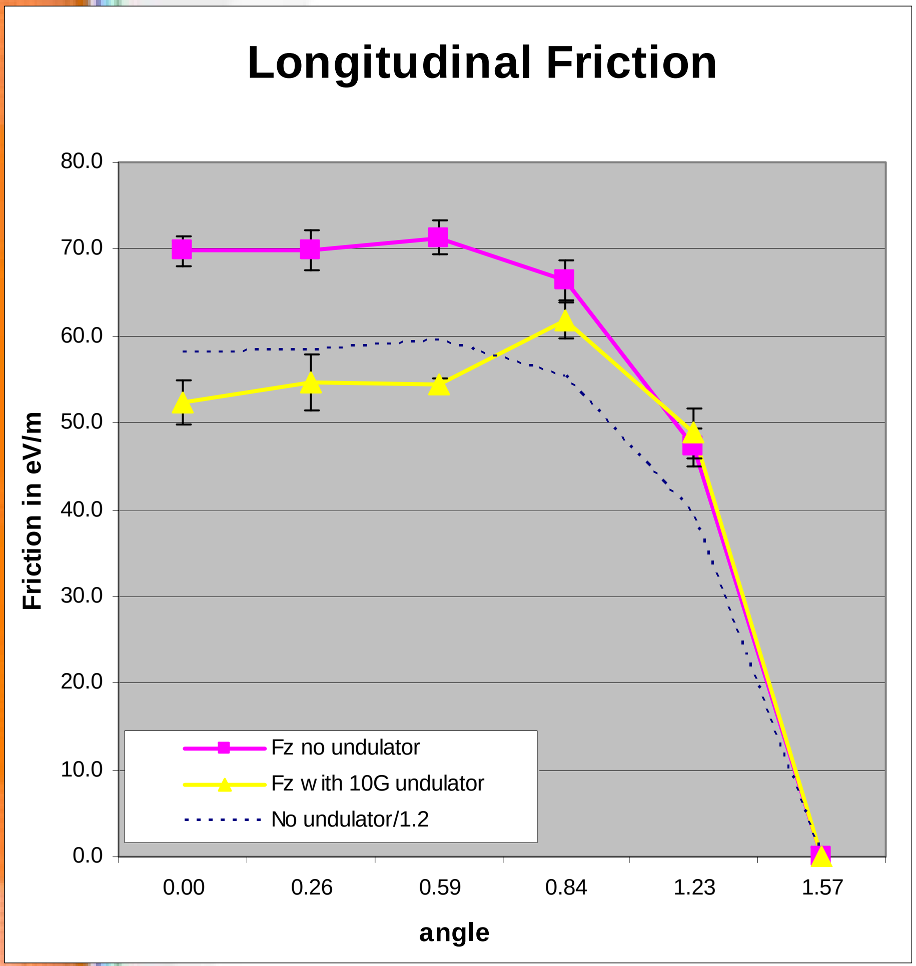
$r_0 \cong 8.8E-7m$ is the amplitude of transverse electron oscillations due to the undulator

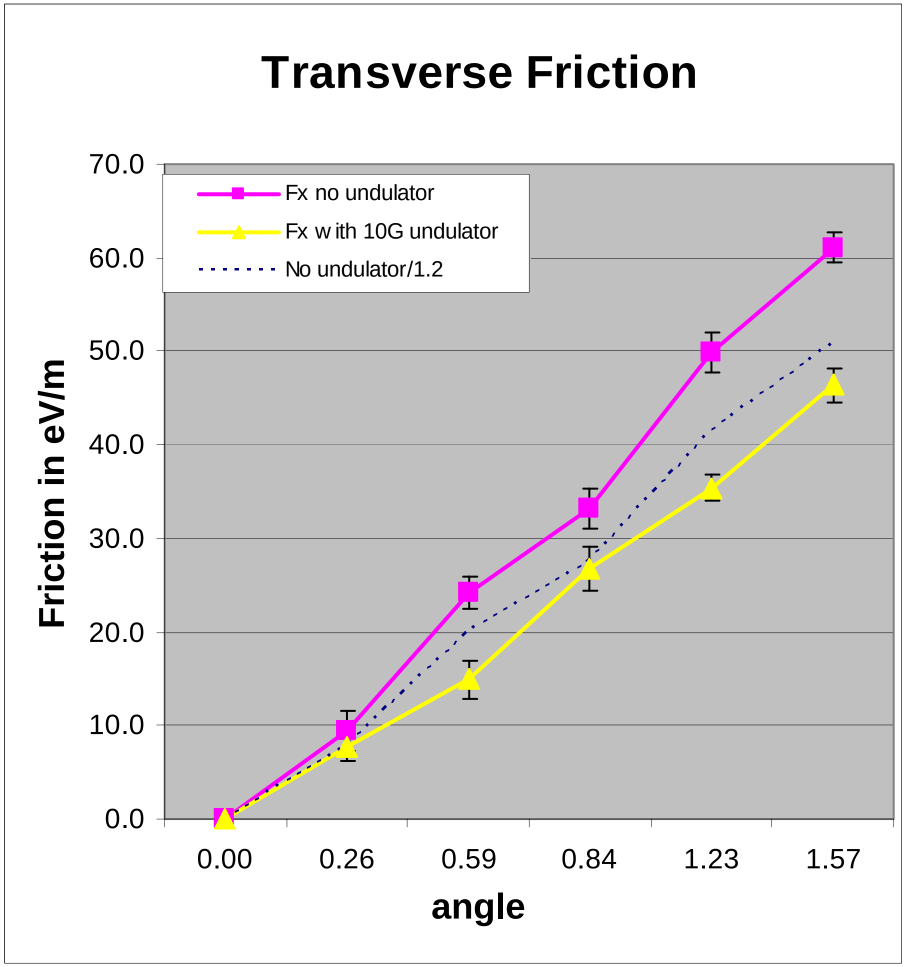
$\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \Big/ \ln\left(\frac{\rho_{\max}}{r_0}\right) \cong 1.21$ For a 10 Gauss, 8cm wavelength undulator

1.59 For a 50 Gauss, 8cm wavelength undulator

Undulator friction results (10 Gauss)

|Ion velocity| = 3e5 m/s in beam frame





Electrons are observed to drift in z (beam frame)

- Predicted by the trajectory of the electron in a helical undulator
  - Freund and Antonsen: Principles of FEL's (sec 2.1.1)

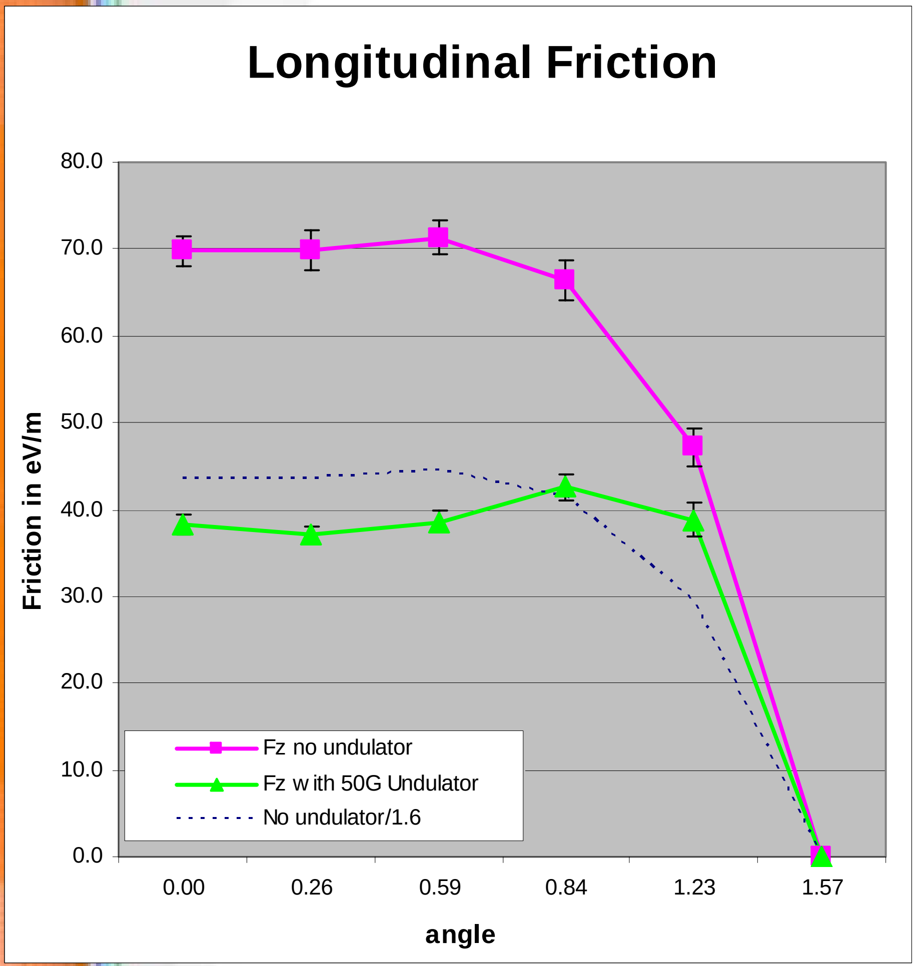
$$\frac{v_z}{c} = \beta\sqrt{1-\left(\frac{v_w}{\beta c}\right)^2} \quad \text{Lab Frame}$$

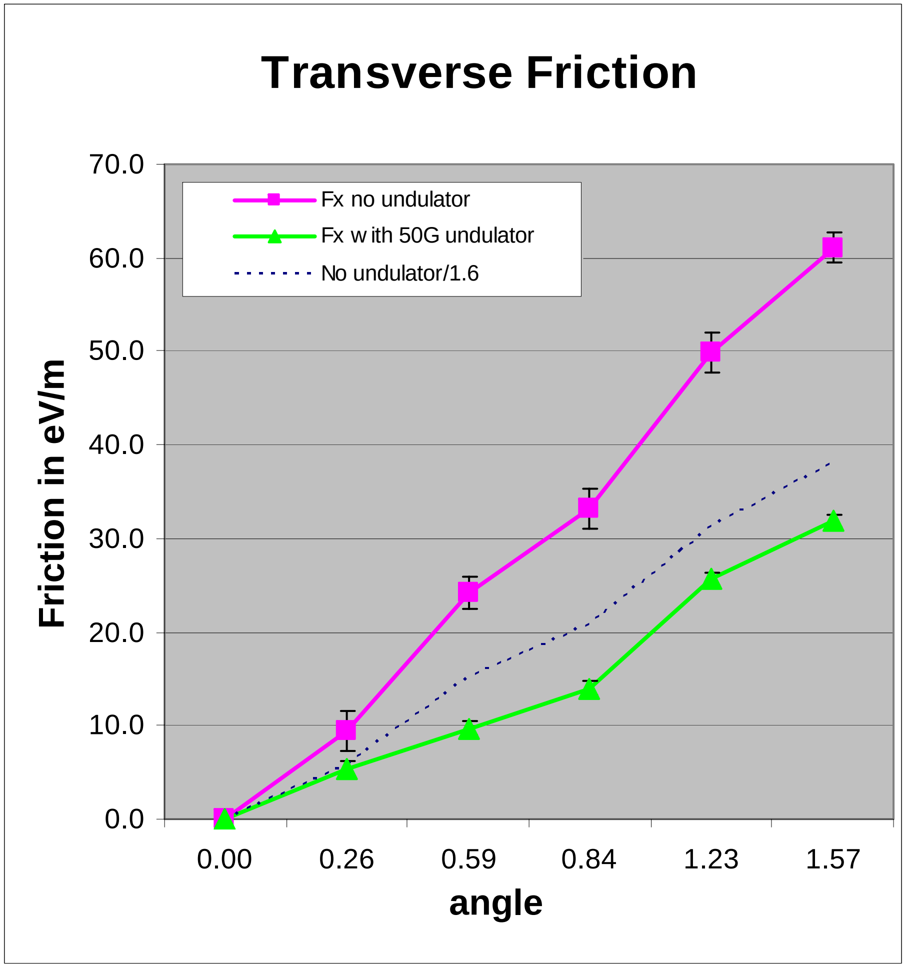
$$\delta v_z \cong -\frac{1}{2}\left(\frac{v_w}{\beta c}\right)^2 c$$

$$\delta v_z' \cong -\frac{1}{2}\left(\frac{v_w'}{\beta c}\right)^2 c \quad \text{Beam Frame}$$

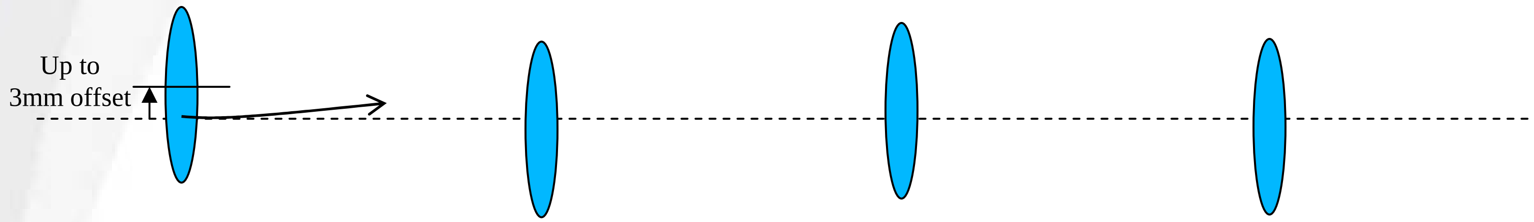
  - B_0=10G gives $\delta v_z' = -8.1E3$ m/s
  - B_0=50G gives $\delta v_z' = -2.0E5$ m/s
- Not present if we only have E in the beam frame
- This effect is easily corrected by re-adjusting the energy of the electrons
  - In VORPAL, we give them an initial kick in z

Undulator results (50 Gauss undulator)





Axial magnet misalignment model



- Each 10m section is offset axially in a random direction
- Each section acts like a thin lens to give a kick to all electrons (same direction)
  - Modeled by a constant electric field which changes direction 10 times over the the undulator length

Thin lens model details

- Each section of the undulator acts like a thin lens with strength (inverse of focal length)

$$\Delta = (1.74E5)B_0^2 L / \gamma^2 = 1.5E-4m^{-1}$$

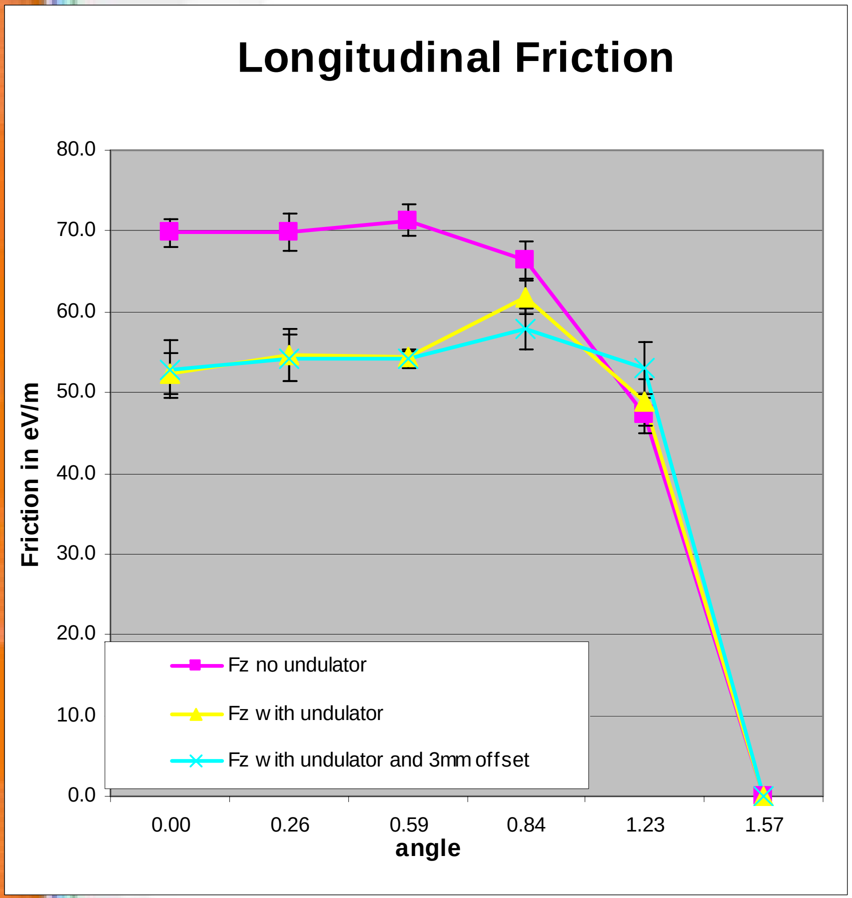
  - B_0 = field strength in Tesla = .001T
  - L = length of a section = 10m
- The resulting transverse velocity in the e$^-$ beam is

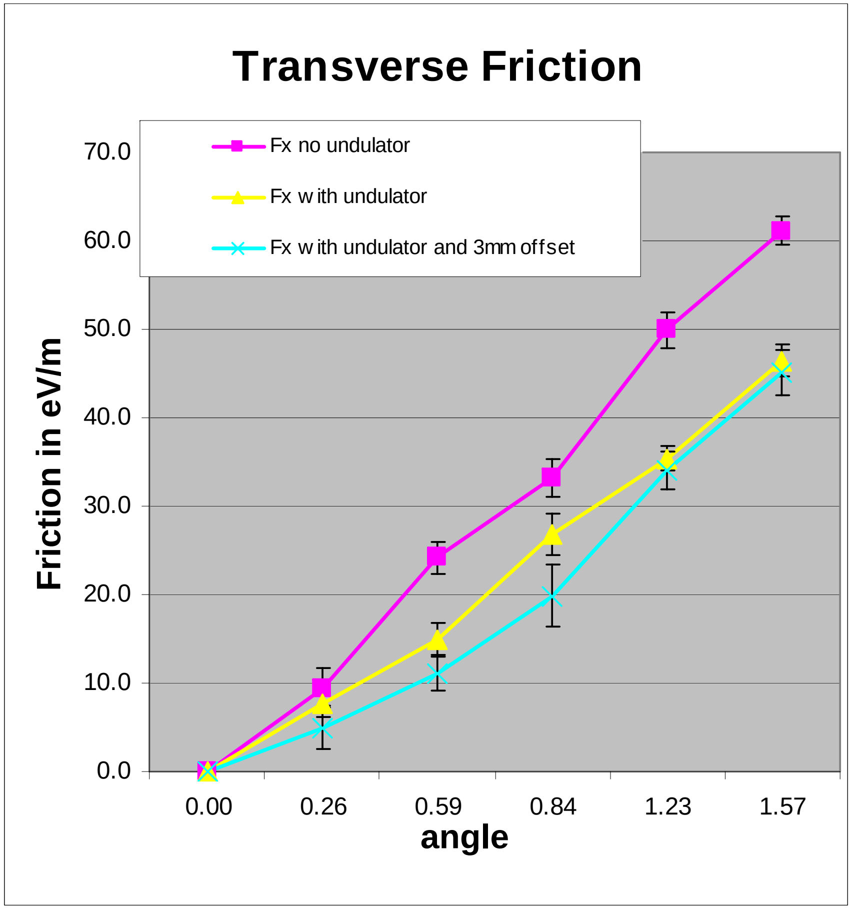
$$\Delta \cdot x_{offset} \gamma \beta c = 1.5E4 m/s$$

for $x_{offset} = 3mm$

- For modeling in VORPAL, we use a constant electric field which gives this transverse velocity increase after passage through one section

Results from the misalignment model





Sextupole term in the magnetic field

- The sextupole field in the beam frame

$$\vec{E}' = E_{3a}\begin{pmatrix} 2xy\cos 3\Omega t' - (x^2 - y^2)\sin 3\Omega t' \\ (x^2 - y^2)\cos 3\Omega t' + 2xy\sin 3\Omega t' \\ 0 \end{pmatrix}$$

$$E_{3a} = \gamma\beta c\,\frac{\tilde{a}_3 B_0}{\left(R_{ref}\right)^2} \qquad \Omega = \frac{2\pi\gamma\beta c}{\lambda} = \gamma\beta c k$$

  – Using

$$\tilde{a}_3 = 0.01$$

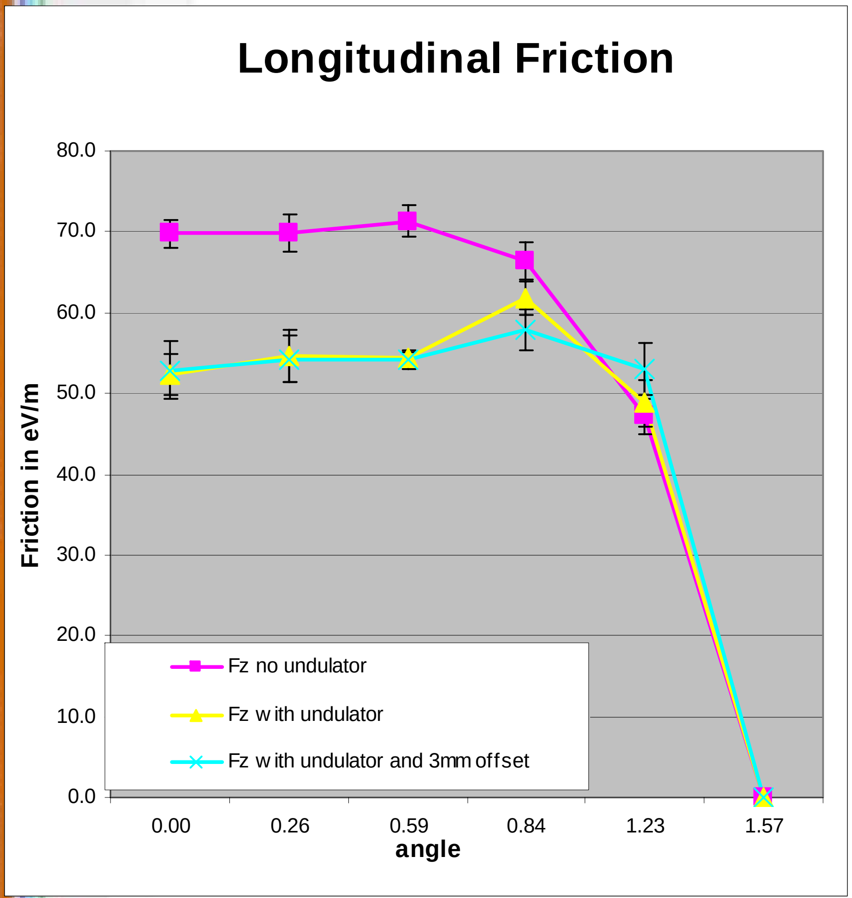
$$R_{ref} = 20mm = .02m$$

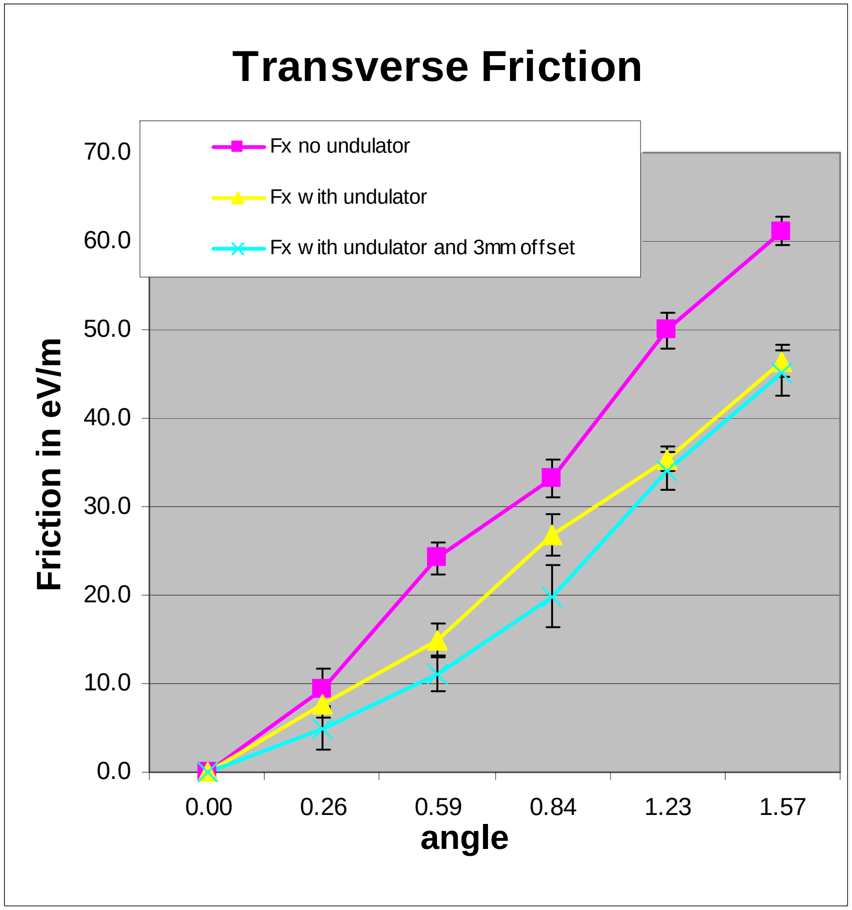
$$E_{3a} = 25\gamma\beta c B_0$$

  – Domain moved near edge of beam (x=3mm; y=1mm)

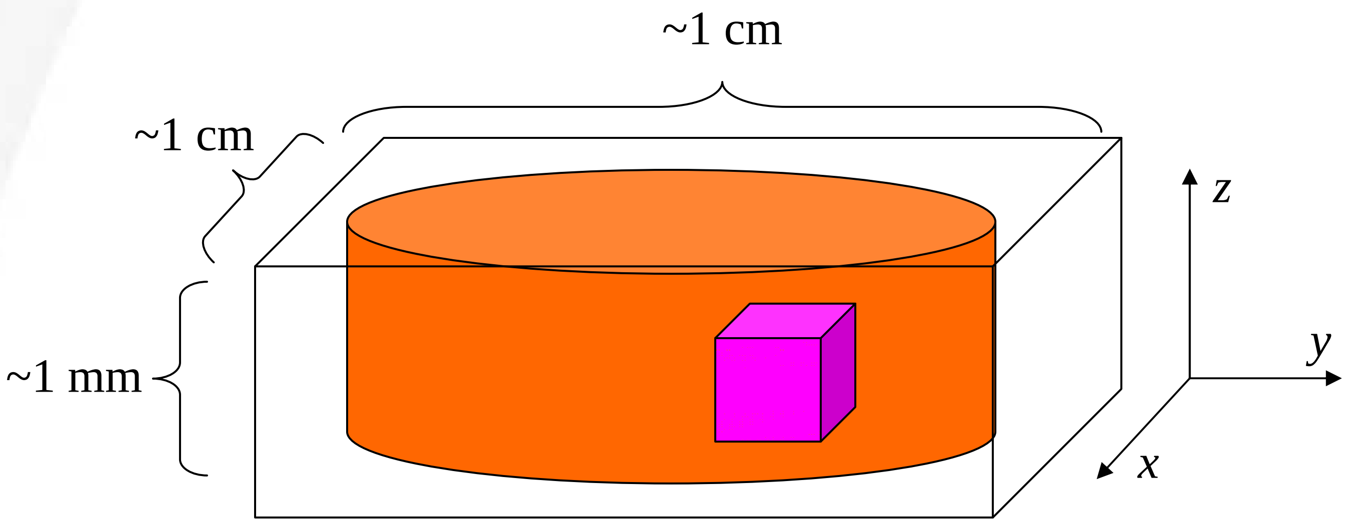
Undulator with Sextupole Simulation

- Very small time steps are needed to resolve the frequency 3Ω.





A more accurate model



- Increase domain to include the entire beam
  - Still thin in the z direction
  - Free space boundary conditions
  - Approximately 200 times more computationally intensive.

Conclusions

- For the 10G undulator, using the current baseline parameters:
  - No significant reduction in friction with a 3mm axial offset of the sections
  - No significant reduction in friction seen when sextupole terms are included
- Other error fields can be simulated upon request

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