Theory of recombination and experimental results

Igor D. Kaganovich
Edward A. Startsev, Adam Sefkow
and Ronald C. Davidson

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey, 08543

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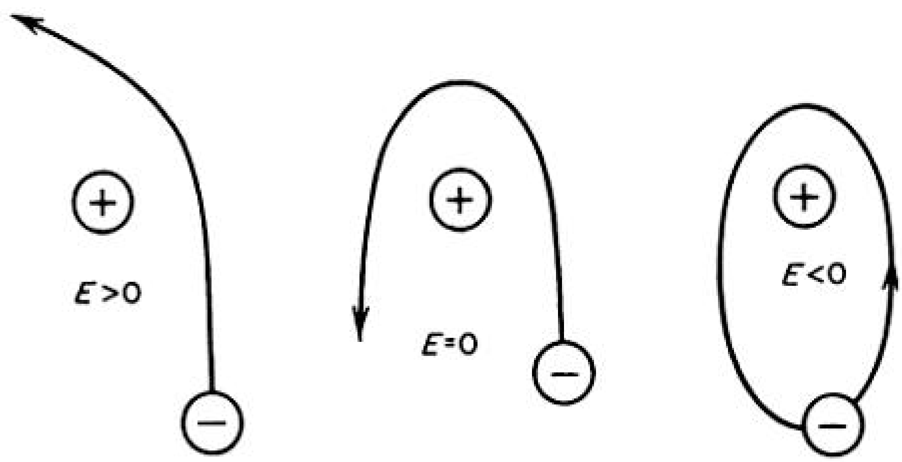
Outline of Presentation

- **Introduction**
- **Classical theory of radiative recombination**
- **Quantum and relativistic corrections**
- **Estimates for upper excited Rydberg states.**
- **Dielectronic recombination**
- **Experimental evidence of enhanced recombination**
- **Possible explanations**

Introduction to radiative recombination

- To recombine electron has to lose its energy due to radiation.

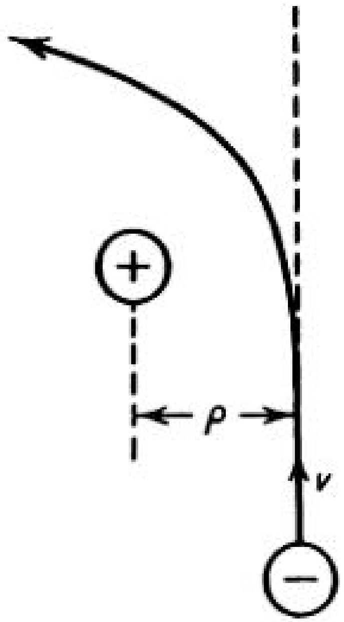
Figures from Zeldovich Raizer "Physics of Shock waves …"



Process can be described in classical mechanics, if

$$v << \frac{Ze^2}{\hbar} = 2.2Z \times 10^8 cm/s$$ or E < 13.6Z^2 eV.

Estimate of radiation



$$S = \frac{2}{3}\frac{e^2}{c^3}\mathbf{w}^2.$$

The radiant energy emitted due to an electron with acceleration **w.**

$$dq_\nu = d\nu \int_0^\infty S_\nu 2\pi\rho \, d\rho$$

The energy emitted in the frequency interval dν per ion and per unit electron flux.

$$dq_\nu = \frac{32\pi^2}{3\sqrt{3}} \frac{Z^2 e^6}{m^2 c^3 v^2} d\nu$$

Integration taken over hyperbolic orbit about an ion in the limit of small velocity.

$$\nu \gg \frac{mv^3}{2\pi Z e^2}$$

Estimate of cross section

$$\sigma_{cn} = \frac{\Delta\sigma_\nu}{\Delta n} = \frac{1}{h\nu}\frac{\Delta q_\nu}{\Delta n} = \frac{1}{h\nu}\left(\frac{dq_\nu}{d\nu}\right)\frac{\Delta\nu}{\Delta n}. \qquad h\nu = E + |E_n| = \frac{mv^2}{2} + \frac{I_H Z^2}{n^2}.$$

$$\sigma_{cn} = \frac{128\pi^4}{3\sqrt{3}}\frac{Z^4 e^{10}}{mc^3 h^4 v^2 \nu}\frac{1}{n^3} = \frac{2.1\cdot 10^{-22}}{n^3}\frac{I_H Z^2}{E}\frac{I_H Z^2}{h\nu}\,\text{cm}^2.$$

$$\sigma_c = 2.1\cdot 10^{-22}\frac{I_H Z^2}{E}\sum_{n=1}^{\infty}\frac{1}{n^3}\frac{1}{(E/I_H Z^2 + 1/n^2)} = \frac{2.8\cdot 10^{-21} Z^2}{E_{ev}}\varphi\left(\frac{I_H Z^2}{E}\right).$$

$E \ll I_H Z^2$, $\varphi \approx (\sum_{n=1}^{n^*} 1/n) + \frac{1}{2}$, where $n^* \approx (I_H Z^2/E)^{1/2}$.

Discussion of cross section

$$\sigma_c = 5.4\ 10^{-16}\frac{1}{(137)^3}\frac{I_H Z^2}{E}\sum_{n=1}^{\infty}\frac{1}{n(1+n^2E/I_H Z^2)}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(1+n^2E/I_H Z^2)} \approx \ln n_{\max} + 0.5 \qquad n_{\max} = \sqrt{I_H Z^2/E}$$

For E~1eV n_{max} ~300 and Ln(n_{max})~6

$I_n = I_H Z^2/n^2 \sim E$ $\qquad$ $Ze^2/r \sim E$ $r \sim Za_0 I_H/E \sim 1000a_0$

Most recombined electrons are very loosely bounded.

For E~1eV such loosely bound electrons can be stripped, so n* can be different.

$\sigma \sim 10^{-16}/E(eV)\ cm^2$

The upper limit of highly excited Rydberg states is limited by experimental conditions

- The field emission due to magnetic force in the steering dipole will result in the ionization of loosely bound electrons with

$$n_{\max} = \left(\frac{6.2\,10^{8} Z^{3}}{B_{dipole} 300} \right)^{1/4}$$

Here, B is the magnetic field in dipole. For B=480G

n_{max} =215 < n*=291 !

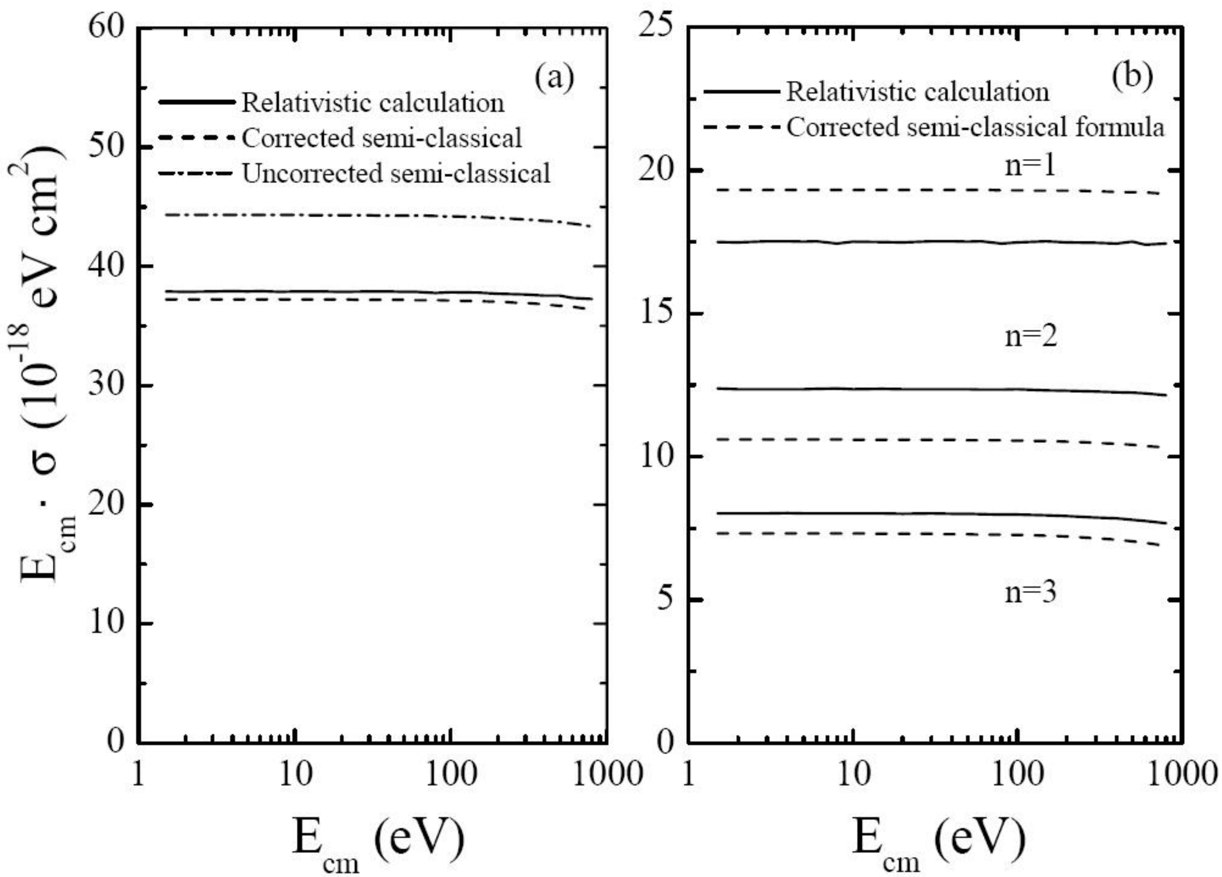
So a small reduction of the effective recombination should be expected.

Quantum corrections

- Approximations used in (Kramers) derivation:
  - Quasi-classical treatment
  - Parabolic electron trajectory
  - Continuous spectrum
- $E<13.6Z^2$ eV and $n>>1$.
- Corrections are largest at n~1, about 20%.
- Corrections are easy to account, known (Gaunt) factors.

Relativistic corrections

- $Z/137 \ll 1$ corrections are expected ~ $(Z/137)^2$.
- for $Z \cong 90$ 10% corrections for n~1.



RR of bare U^{92+}

(a) Comparison sum of 3 fully relativistic, semi-classical Gaunt-factor corrected calculation (dashed line) and uncorrected (dash-dotted line) calculations.

(b) *n*-selective comparison.

W. Shi, et al., Eur. Phys. J. D 15, 145 (2001).

Quantum and Relativistic corrections to total Radiative Recombination

- Contribution from n~1 states into total cross sections is small ~1/6.
- The total correction ~ 20/6% < 4%.

$$\sigma \sim \sum_{n=1}^{\infty} \frac{1}{n(1+n^2 E / I_H Z^2)} \approx \ln n_{\max} + 0.5 \sim 6$$

Any recombination process without radiation can be more effective than radiative recombination

$$\sigma_c = 5.4\ 10^{-16} \frac{1}{(137)^3} \frac{I_H Z^2}{E} \ln n_{\max} \sim 1/\alpha^3$$

Any processes without radiation does not have small factor $(1/137)^3$.

Its cross section is $\sim 10^6$ times larger!

Possible candidates:

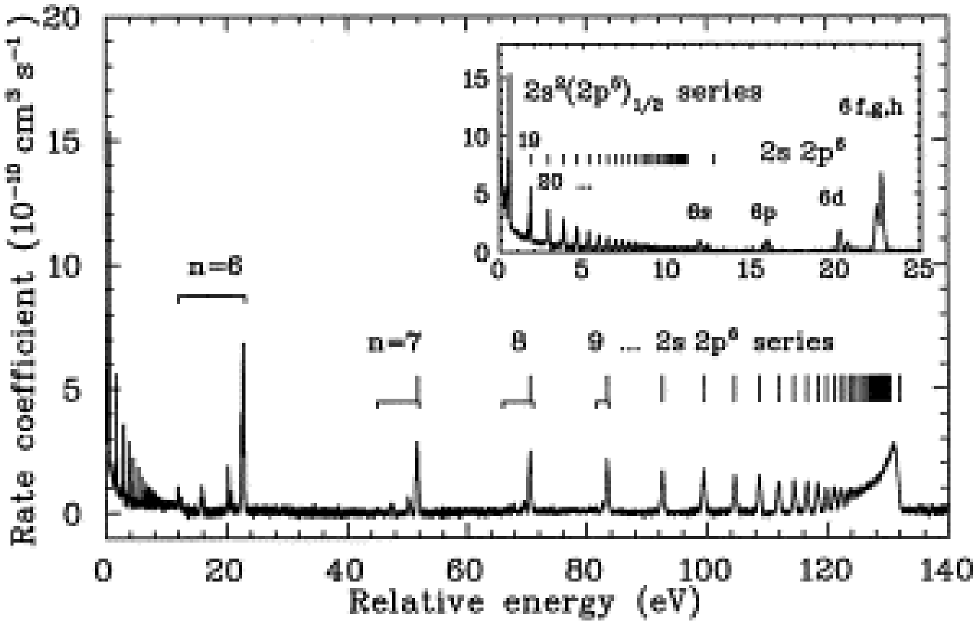
Dielectronic recombination
Charge exchange

Dielectronic recombination

- Dielectronic recombination occurs if there are other bounded electrons.
- Au^{+Z} +e => $Au^{+(Z-1)**}$
  - $E=I_n + \Delta I_{n0}$ exact resonance between electron kinetic energy and change in energy of bounded electron and potential energy of captured electron.
- $Au^{+(Z-1)**}$ => $Au^{+(Z-1)*}$ $+h\nu$

Example of Dielectronic recombination

- Recombination rate coefficient of Fe17+ ions stored in the TSR at 5.6 MeV/*u*, as a function of the relative energy Ê.
- On top of the continuous RR signal the series of DR resonances 2s2p6 *nl* for *n*>= 6 (with resolved fine structure for *n*=6–8) is seen; levels with up to *nc*≈124 are estimated to contribute to the Rydberg peak with a series limit at 132 eV.



A. Wolf et al, NIMPR A **441**, 183 (2000).

Experimental Results 1990

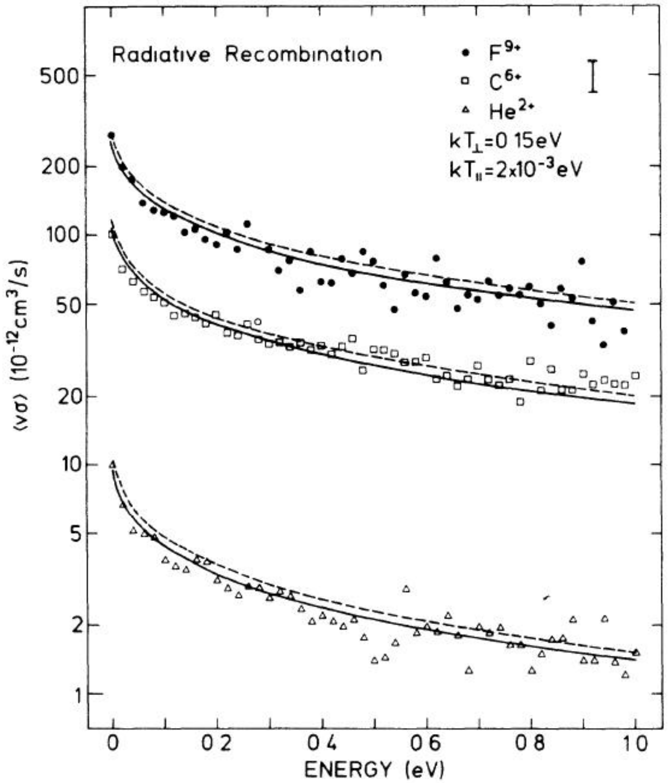
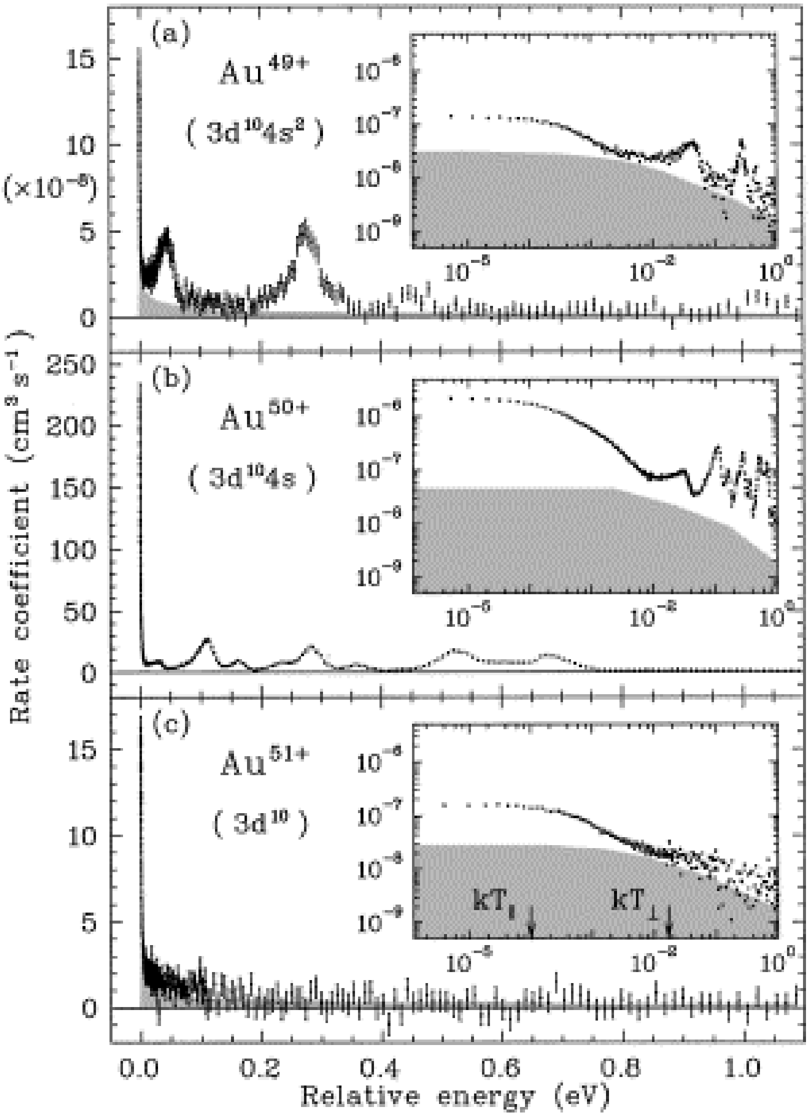


FIG. 6. Experimental rate coefficient $\langle v\sigma \rangle$ as a function of relative energy for He^{2+}, C^{6+}, and F^{9+}. The dashed curve is the rate coefficient obtained with the Bethe and Salpeter cross section, and the full curve is the same calculation corrected for $n=1$, 2, and 3 according to Stobbe. The error bar shows the uncertainty from the determination of the target length, which was put equal to 85±15 cm.

- L.H. Andersen and J. Bolko, PRA **42**, 1184 (1990).
- Tandem accelerator, Aarhus, Denmark
- Good agreement between theory and experiment for small Z<10.

Experimental Results 2000



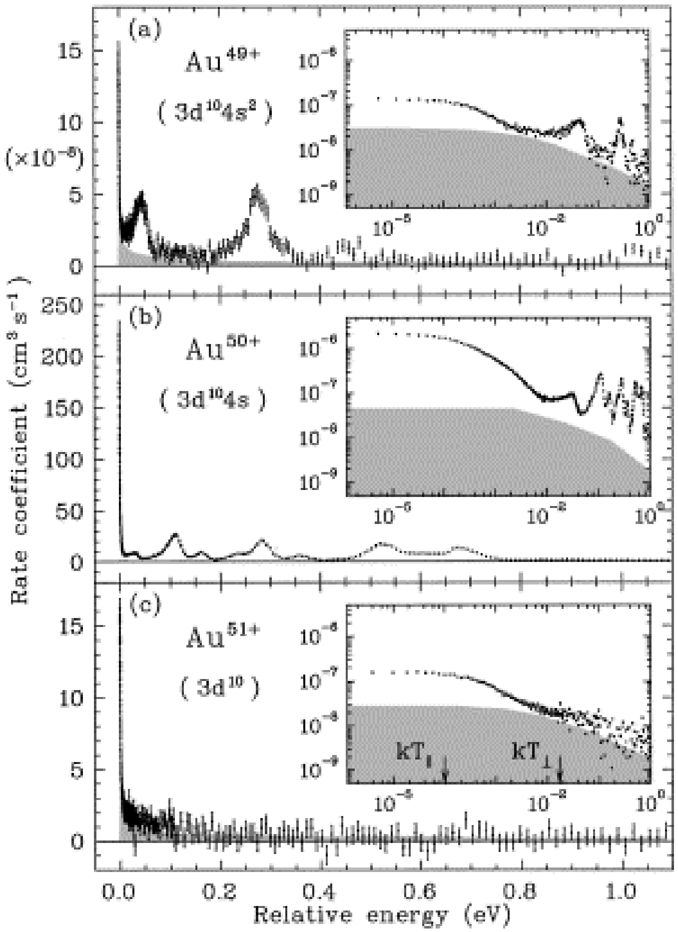
A. Wolf *et al*, NIMPR A **441**, 183 (2000).

- TSR, Heidelberg

Recombination rate coefficient of highly charged Au ions with the given charge states and configurations stored in the TSR at 3.6 MeV/*u*, measured as a function of the relative energy Ê. No detailed assignment of the DR resonances for Au49+ and Au50+ has been obtained. The theoretical RR rate coefficients are shaded areas (Rydberg-state detection limit n_{max} =360).

Enhancement occurs for $E \sim 3T_{||} < T_{\perp}$.

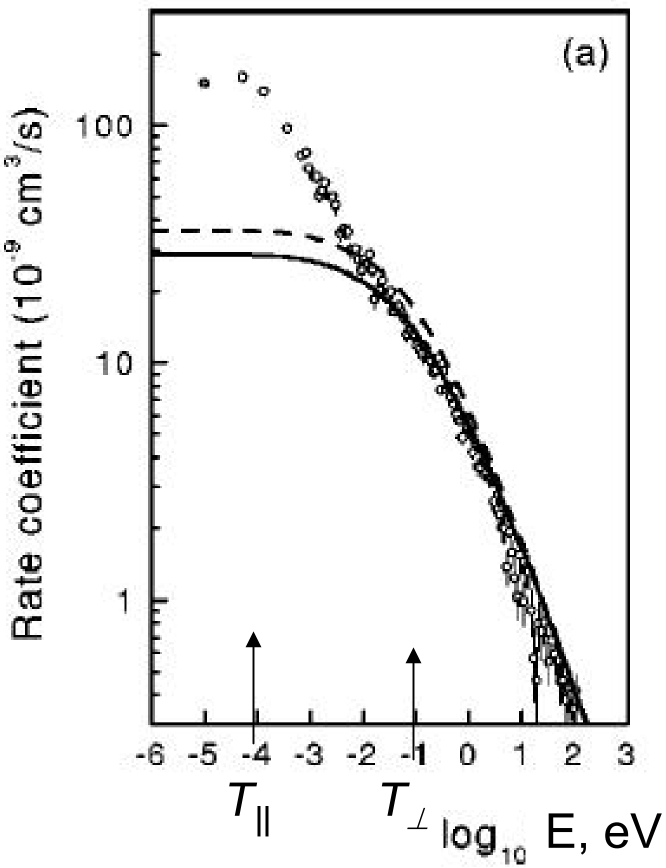
Experimental Results 2000



"Moderate enhancement factors of 5 as compared to the calculated RR are found for Au49+ and Au51+, while the enhancement factor for Au50+ with respect to RR theory amounts to 60."

"In summary, all experimental evidence indicates that the strong enhancement of recombination rates during electron cooling found for certain complex highly charged ions is caused by low-energy dielectronic resonances. A recent experiment in particular shows that such resonances can have a very large natural width, which can bring a considerable part of their strength to zero collision energy without sharp requirements on the exact resonance position."

Experimental Results 2001



A. Hoffknecht, *et al.,* PRA 63, 012702 (2001).

ESR, GSI, Darmstadt

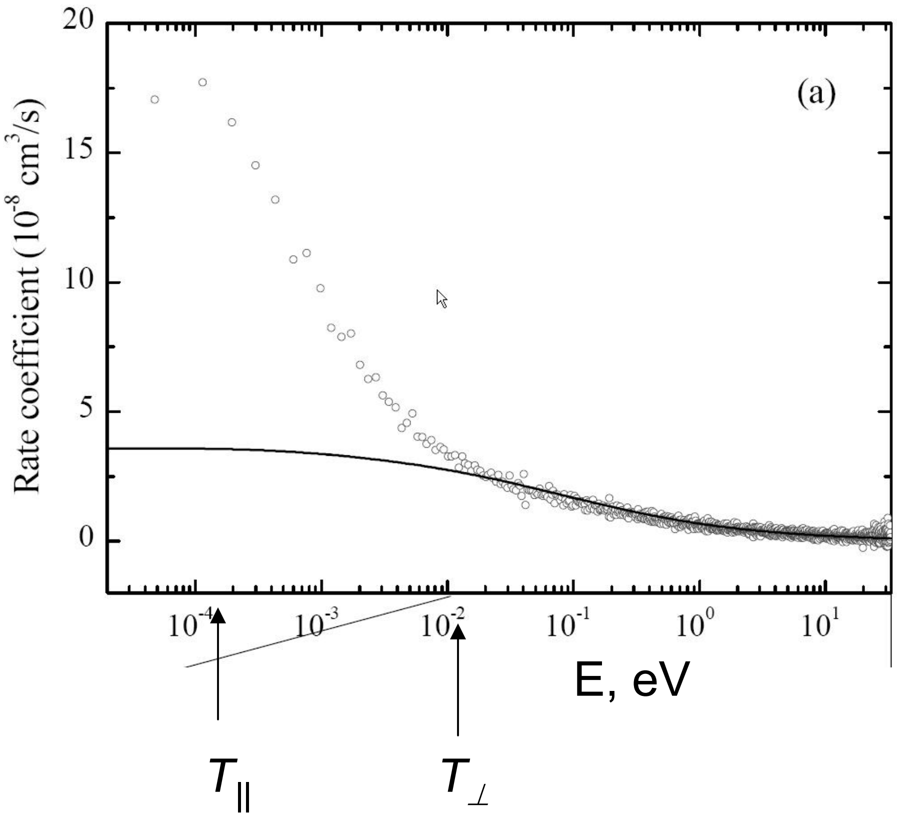
Absolute recombination rate of Bi^{83+} plotted against relative energy between the electron and the ion. Open circles represent measured results; lines the calculated results. The solid line is a calculation with n_{max} = 116, and 442 dashed line.

$T_\perp$ = 120meV and $T_{||}$ =0.1 meV.

The magnetic guiding field strength was 104 mT.

The enhancement rate does not depend on electron density.

Experimental Results 2001



W. Shi, *et al.*, Eur. Phys. J. D 15, 145 (2001).

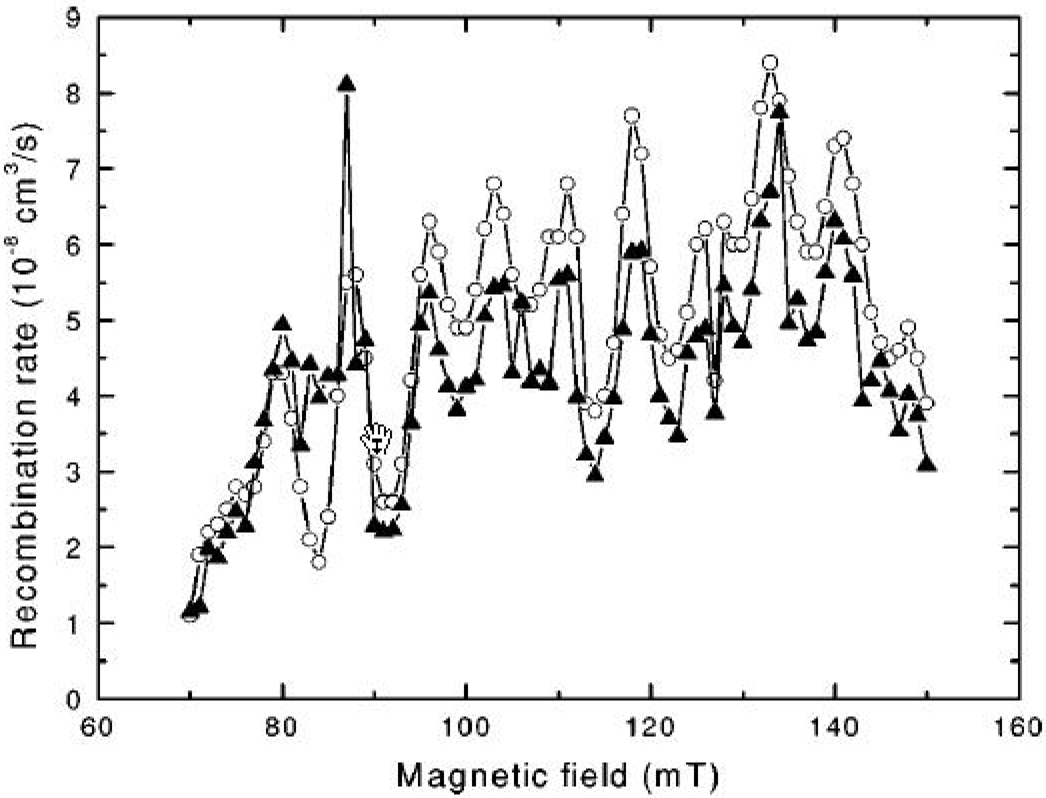
ESR, GSI, Darmstadt

Absolute recombination rate of U^{92+} plotted against relative energy between the electron and the ion. Open circles represent measured results; lines the calculated results. The solid line is a calculation with n_{max} = 130,

$T_{\perp}$ = 120meV and $T_{\|}$ =0.1 meV.

The magnetic guiding field strength was 104 mT.

The enhancement may depend on guiding B.



Comparison of recombination-rate coefficients at *E*= 50 eV obtained via two different methods. The open circles show the recombination rate calculated from the counting rate of recombined particles. These data do not include the background subtraction and corrections due to the potential and angle distribution inside the cooler. The full triangles represent rate coefficients obtained via the storage lifetime of the Bi831 beam in the ring.

- Though it maybe systematic effect due to change in $T_\perp$ or merging angle.

Theoretical description of recombination enhancement

- Three-body recombination
- Density enhancement due to plasma screening effects
- Effect of magnetic field on cross section
- Transient electric Field-Induced Recombination (FIR) during merging of electron and ion beams
- FIR with radiative stabilization
  - M. Horndl, et al., PRL 95, 243201 (2005).

Simulation of merging section*

- Movies of densities, electric field and current.

BNL (Au 20TeV) (e- 55MeV).mov

BNL (densities).mov

BNL (E contours).mov

BNL (J contours).mov

*By Adam Sefkow

Conclusions

Moderate enhancement factors of 5 as compared to the calculated radiative recombination are typically found in the range of relative energy E< $T_{\perp}$, 10$T_{||}$.

The enhancement factor for Au50+ with respect to RR theory amounts to 60.

This is probably due to dielectronic recombination.

Most probable course: Transient electric Field-Induced recombination during merging of electron and ion beams with radiative stabilization.

Back-up Vugraphs