Stability issues in electron coolers

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Electron cooling method uses the energy exchange between ions and electrons

Long Distance Coulomb Force

$$l \propto \frac{m\langle v^2 \rangle}{4\pi n q^2}$$

the electrostatic energy of the fluctuation is compare with kinetic energy

beam reference system

n – density of a charge particles,
q, m – charge of the particle,
<v> – typical velocity of the particle

$$l^3 n \frac{m\langle v^2 \rangle}{2} \approx \frac{(n l^3 q)^2}{l}$$

fluctuation with wave length λ

1. Laslett tune shift
2. Coherent mode of ion-electron interaction
3. Fluctuation mechanism
4. “Overcooling” problem
5. Some experimental scaling

$\lambda \gg l$ coherent wave interaction

$\lambda \approx l$ fluctuation interaction

$\lambda \ll l$ single particle interaction, friction force

Tune shift

$$M_{sc} = \begin{pmatrix} 1 & 0 \\ \frac{4\pi\xi}{\beta_{cool}} & 1 \end{pmatrix} \qquad M_{ring} = \begin{pmatrix} \cos(2\pi\nu) & \beta_{cool}\sin(2\pi\nu) \\ -\frac{1}{\beta_{cool}}\sin(2\pi\nu) & \cos(2\pi\nu) \end{pmatrix}$$

$$\Delta\nu \approx \xi$$

$$\xi = \frac{1}{\gamma^3\beta^2}\frac{N\cdot Z}{2(2\pi)^{3/2}}\frac{r_e\, l_{cool}\,\beta_{cool}}{\sigma_x\,\sigma_y\,\sigma_z}$$

$$l_{cool} = 80\ \mathrm{m}$$

$$\beta_{cool} = 500\ \mathrm{m}$$

$$\gamma = 107$$

Electron

$$N_e = 3\cdot 10^{10}$$

$$\sigma_x = \sigma_y = 0.43\ \mathrm{cm}$$

$$\sigma_s = 1.0\ \mathrm{cm}$$

Gold Ion

$$N_{Ai} = 10^9 \qquad Z = 79$$

$$\sigma_x = \sigma_y = 0.3\ \mathrm{cm}$$

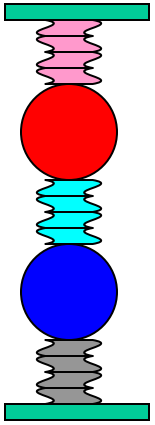
$$\sigma_s = 20\ \mathrm{cm}$$

$$\xi_{ee} = 0.47 \qquad \xi_{ei} = 0.13$$

Tune shift is not very small ?!

Coherent interaction of the electron and ion beam in the cooling section

Base model of the two interact oscillators



$$\frac{\partial^2 \xi}{\partial^2 t} + \omega_1{}^2 \xi = \omega_{11}{}^2 \nu$$

$$\frac{\partial^2 \nu}{\partial^2 t} + \omega_2{}^2 \nu = \omega_{21}{}^2 \xi$$

Initial conditions

$$\xi = \xi_0, \quad \xi' = \xi_0{}' \qquad \text{ions}$$

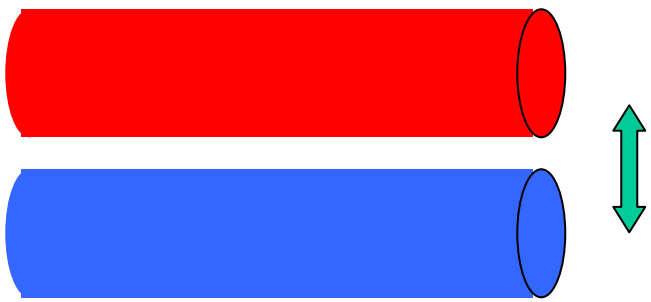
$$\nu = 0, \quad \nu' = 0 \qquad \text{electrons}$$

$$\begin{pmatrix} \xi \\ \xi' \end{pmatrix}_1 = M \begin{pmatrix} \xi \\ \xi' \end{pmatrix}_0$$

$$\det(M) = 1 + \frac{2 \cdot \omega_{12}{}^2 \cdot \omega_{21}{}^2}{(\omega_1 - \omega_2)^2 + 4 \cdot \omega_{12}{}^2 \cdot \omega_{21}{}^2} \left[\cos \omega_+ t \cos \omega_- t - 1\right] + \frac{1}{\omega_+ \omega_-} \frac{\omega_{12}{}^2 \cdot \omega_{21}{}^2 \left(\omega_1{}^2 + \omega_2{}^2\right)}{(\omega_1 - \omega_2)^2 + 4 \cdot \omega_{12}{}^2 \cdot \omega_{21}{}^2} \sin \omega_+ t \sin \omega_- t$$

$$1 \quad < 0 \quad \lesseqgtr 0$$

Simple model, no magnetic field



ions

electrons

transverse motion
with large wave length

decrement

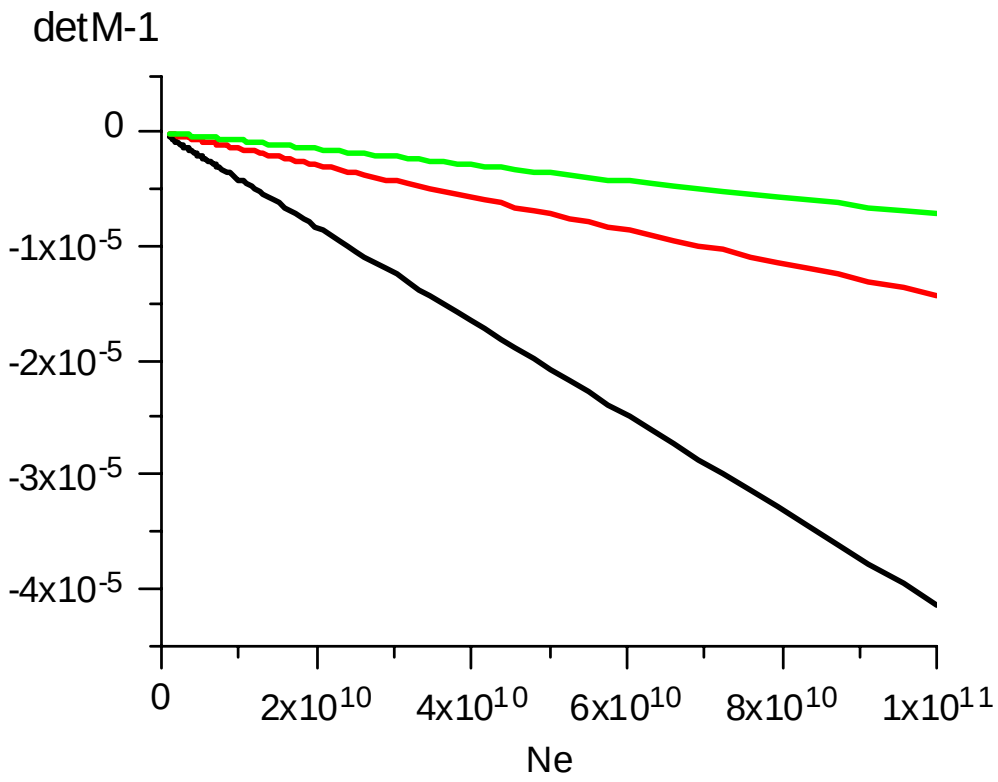
increment/ decrement

$$\det(M) = 1 + \frac{2 \cdot \omega_{ei}^{2} \cdot \omega_{ie}^{2}}{\omega_0^{4}} \left[\cos \omega_0 t - 1\right] + \frac{\omega_{ei}^{2} \cdot \omega_{ie}^{2}}{\omega_0^{4}} \omega_0 t \sin \omega_0 t$$

$$\omega_0 = \sqrt{\omega_{ei}^{2} + \omega_{ie}^{2}}$$

$$\omega_{ei}^{2} = \frac{2\pi n_i Z_i e^2}{m_e} \qquad \omega_{ie}^{2} = \frac{2\pi n_e Z_i e^2}{m_i}$$

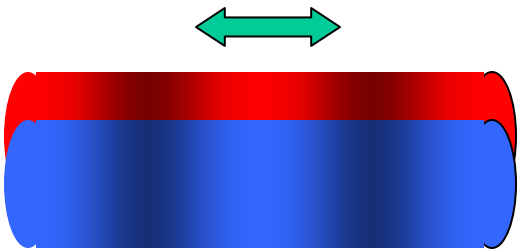
Only decrement, no increment



Matrix determinant versus number of the electron per bunch

At a smaller density, the interaction with the electron beam leads to the damping of a coherent oscillation mode.This damping rate may be larger than the rate induced by the usual single particle friction force.

Longitudinal – longitudinal coherent interaction



$$\frac{\partial^2 \xi}{\partial^2 t} + \omega_{pi}{}^2 Z_k^2 \xi = \omega_{ie}{}^2 Z_k^2 \nu$$

$$\frac{\partial^2 \nu}{\partial^2 t} + \omega_{pe}{}^2 Z_k^2 \nu = \omega_{ei}{}^2 Z_k^2 \xi$$

$$Z_k^2 = k^2 a^2 \ln\left(\frac{b}{a} + \frac{1}{2}\right)$$

$$\omega_{pi}{}^2 = \frac{2\pi n_i Z_i{}^2 e^2}{m_i}$$

$$\omega_{ei}{}^2 = \frac{2\pi n_i Z_i e^2}{m_e}$$

$$\omega_{pe}{}^2 = \frac{2\pi n_i e^2}{m_e}$$

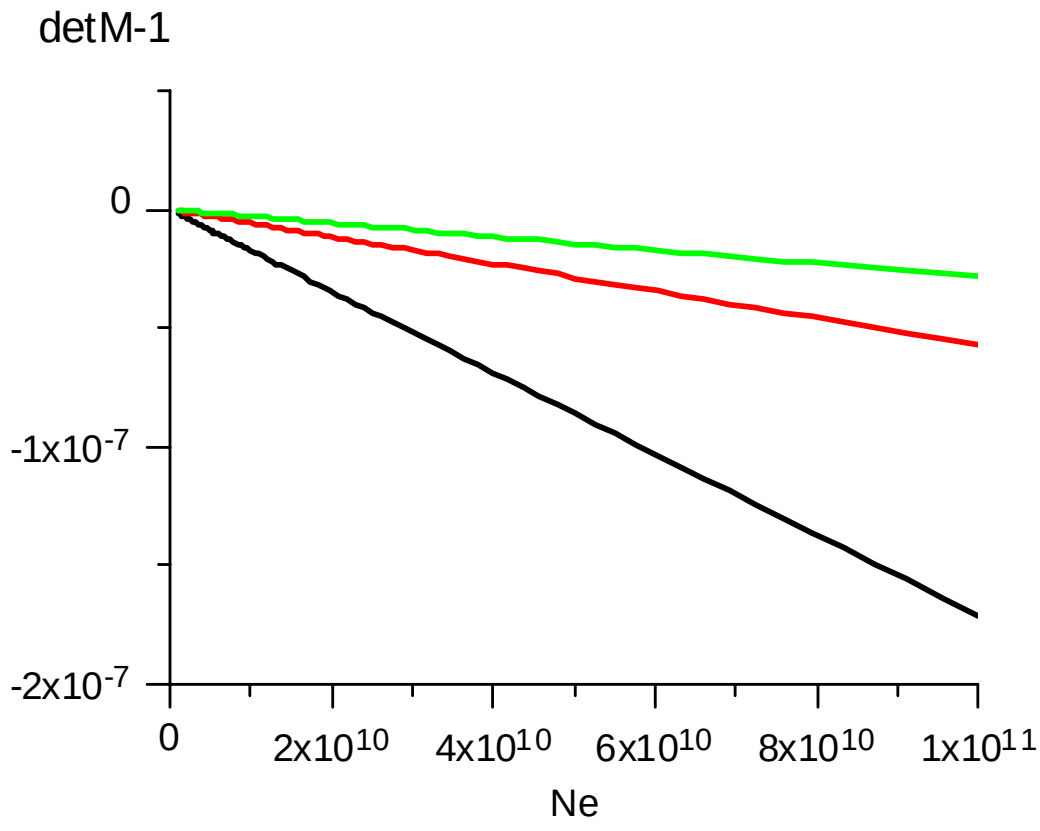
$$\omega_{ie}{}^2 = \frac{2\pi n_e Z_i e^2}{m_i}$$

k – is typical wave length

a – is beam sizes

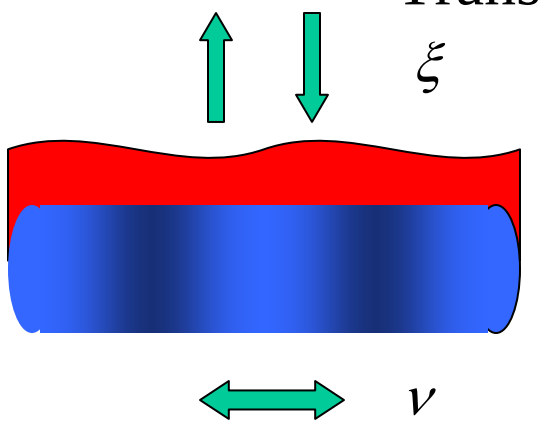
b – is sizes of vacuum chamber

Only decrement, no increment



Matrix determinant versus number of the electron per bunch

Transverse – longitudinal coherent interaction



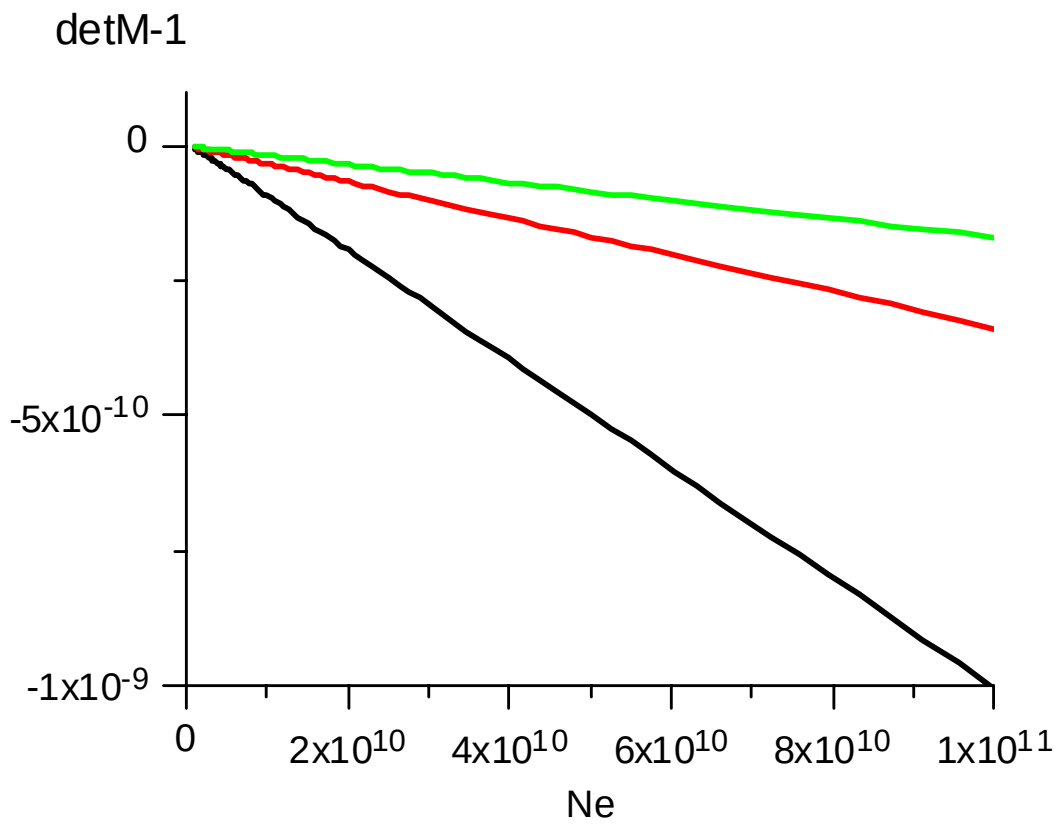
Ions – small perturbation like snake

Electrons – longitudinal type of motion (at presence of the longitudinal magnetic field is very natural hypothesis)

$$\frac{\partial^2 \xi}{\partial^2 t} + \omega_{ie}{}^2 \xi = -(ika)\,\omega_{ie}{}^2\, Z_k^2 \nu$$

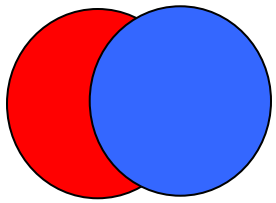
$$\frac{\partial^2 \nu}{\partial^2 t} + \omega_{pe}{}^2 Z_k^2 \nu = (ika)\omega_{ei}{}^2\ \xi$$

Only decrement, no increment



Matrix determinant versus number of the electron per bunch

Beam space-charge interaction into the longitudinal magnetic field



$\oplus$ Electron

B – magnetic field

$$\frac{\partial^2 \langle z_i \rangle}{\partial s^2} + \frac{i}{\lambda_{Hi}} \frac{\partial \langle z_i \rangle}{\partial s} + \frac{1}{\lambda_{ie}^2} \langle z_i \rangle = \frac{1}{\lambda_{ie}^2} \langle z_e \rangle$$

$$\frac{\partial \langle z_e \rangle}{\partial s} + i \frac{\lambda_{He}}{\lambda_{ei}^2} \langle z_e \rangle = i \frac{\lambda_{He}}{\lambda_{ei}^2} \langle z_i \rangle$$

RIHC BINP Project

$N_e = 3 \cdot 10^{10}$

$\sigma_x = \sigma_y = 0.43\,\text{cm}$

$\sigma_s = 1.0\,\text{cm}$

$N_{Ai} = 10^9$

$\sigma_x = \sigma_y = 0.3\,\text{cm}$

$\sigma_s = 20\,\text{cm}$

$B = 10\,\text{T}$

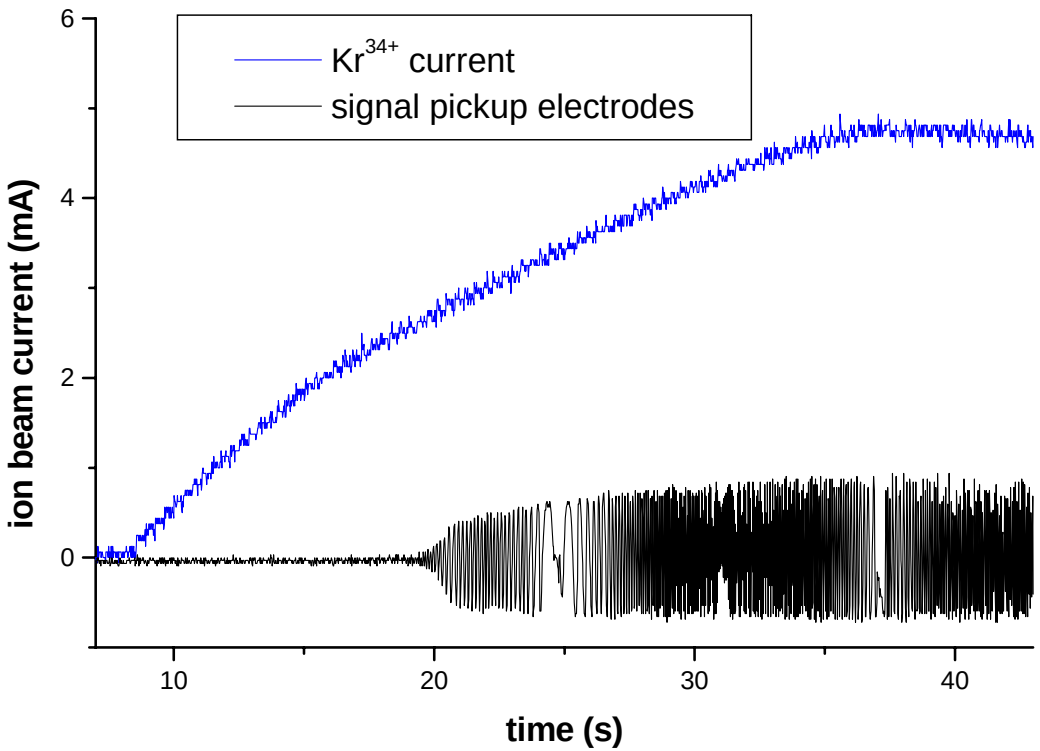
$\lambda = 1$

$$\lambda_{ie} = \frac{m_i \gamma^3 (\beta c)^2}{2\pi e^2 Z_i n_e}, \quad \lambda_{ei} = \frac{m_e \gamma^3 (\beta c)^2}{2\pi e^2 Z_i n_i}, \quad \lambda_{Hi} = \frac{\gamma \beta m_i c^2}{Z_i e B}, \quad \lambda_{He} = \frac{\gamma \beta m_e c^2}{eB}$$

$$\lambda^2 = 1 \pm \frac{1}{2} \beta_\perp L_{cool}^2 \frac{\lambda_{He}}{\lambda_{ie}^2 \lambda_{ei}^2}$$ instability increment

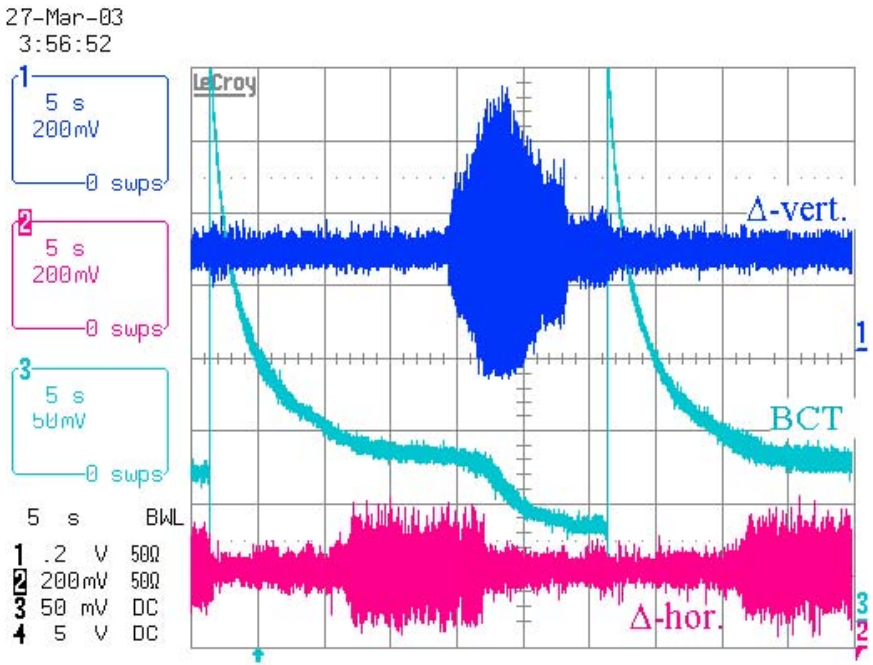
Experimental Observation

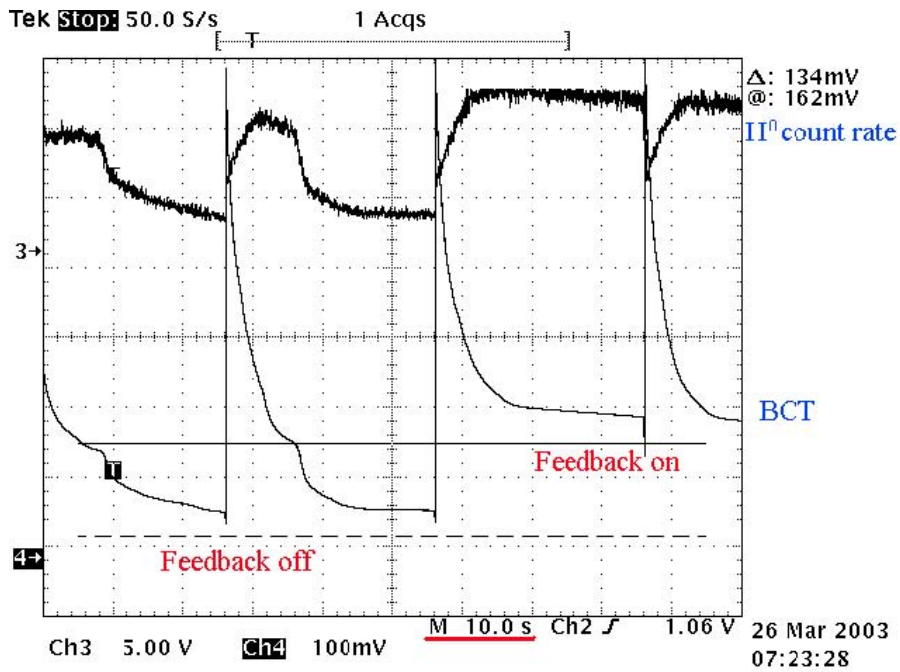
Coherent instability:
COSY, CELSIUS, NAP-M, SIS …



Signal of the ion beam pickup electrode during accumulation at SIS. The intensity limitation for a Kr^{+34} beam is observed.

Feedback stabilization – COSY experience





$J_e = 200\,\mathrm{mA}$ $\quad E_p = 45\,\mathrm{MeV}$

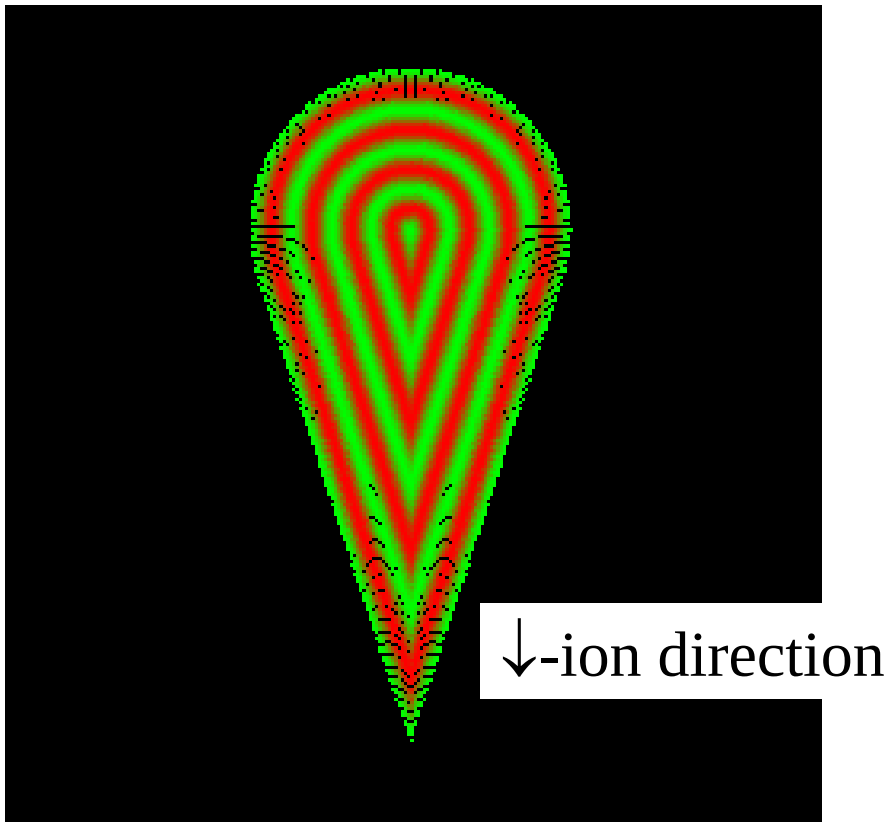
$a_e = 1.27\,\mathrm{cm}$ $\quad J_p = 1\,\mathrm{mA}$

$a_p = 0.1\,\mathrm{cm}$

True coherent instability

Parkhomchuk wake model of electron heating (fluctuation point of view):

after come in electron beam around fluctuation of space charge at ion beam exited plasma oscillations producing enhanced diffusion.



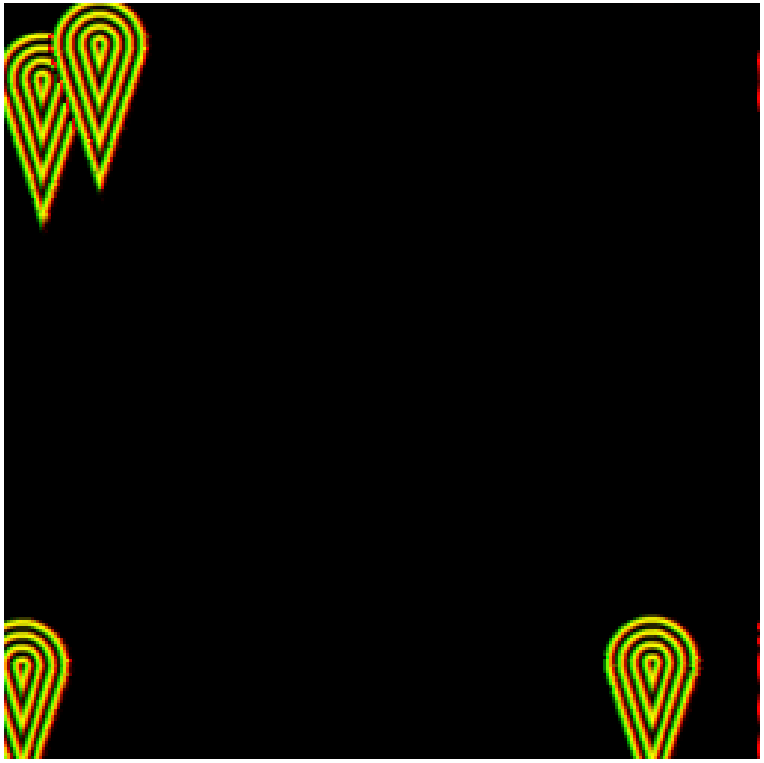
Ion passed down same distance and at tail exited plasma oscillations similar of wake waves after passing a ship at sea.

$$\Delta p^2 = -2\,p\cdot\delta p + \frac{4\pi}{3}\,n_i\,\rho_{\max}^3\,\delta p^2$$

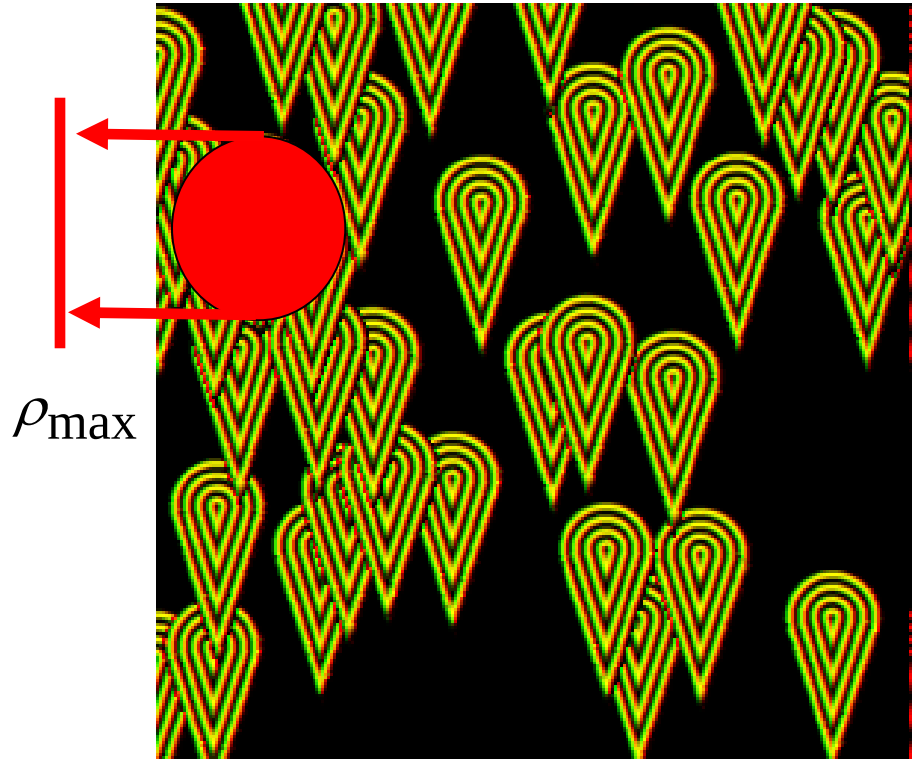
$$\rho_{\max} = \tau\sqrt{V^2 + V_{eff}^2}$$

$$\left(\frac{\Delta p}{p}\right)^2 = -2\,\frac{F\tau}{p}\left(1 - g\,\omega_{ee}{}^2\,\omega_{ii}{}^2\,\tau^4\right)$$

Wake field at small current



Wake field at large current



no interaction and cooling
each protons individual

many overlapping wake fields
zone and some specific heating

Heating criterion $G = \omega_{ee}^2 \omega_{ii}^2 \tau^4 \propto 1$

ω_{ee} electron beam plasma frequency

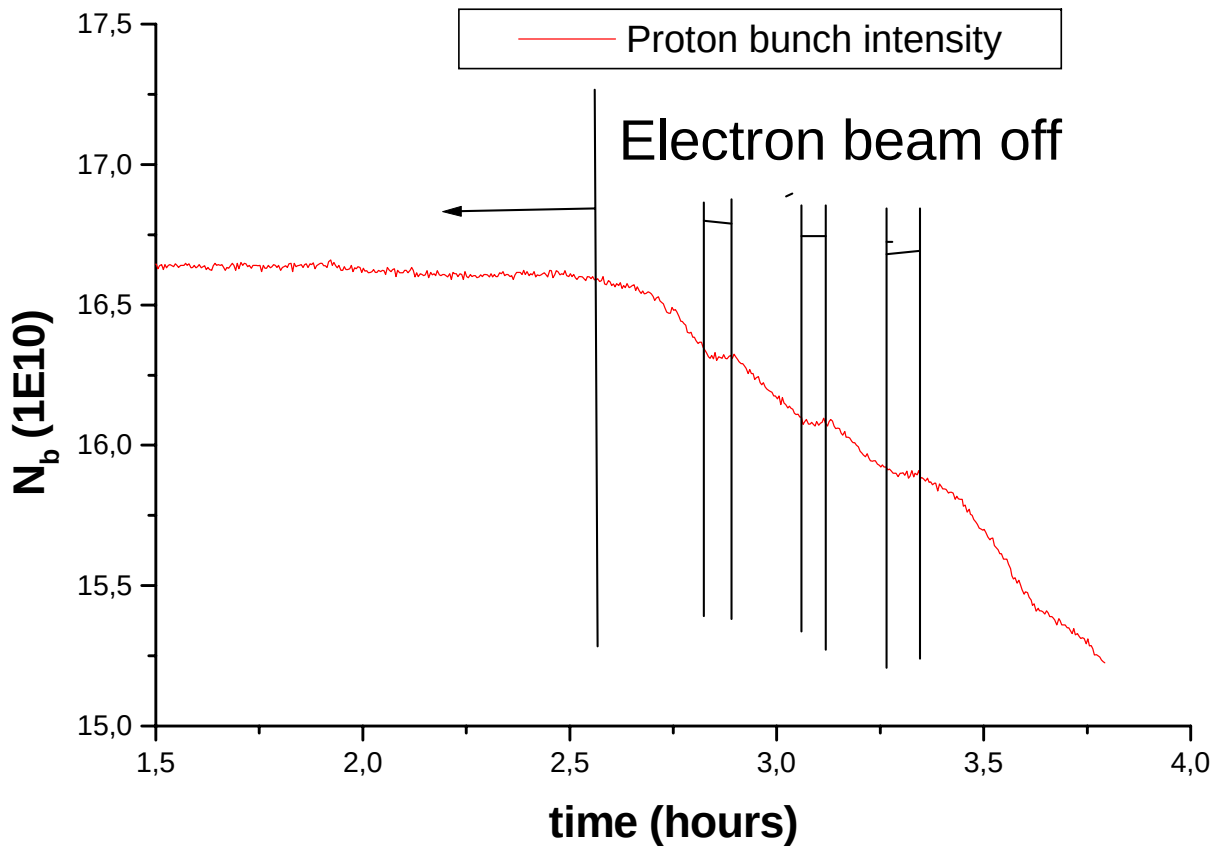
ω_{ii} ion beam plasma frequency

τ time of flight cooling section

$G = 5 \cdot 10^{-5}$ RIHC

TeV electron lens experiment

Degradation of life time at presence of electron beam - without e life time 300 hours - with electrons on only 8-10 hours



There is no coherent oscillation. The slow growth of emittances and fatal degradation of the beam life-time

FNAL experience

coherent instability of the “over” cooling beam with large antiprotons number ?

Direct scale

the coherent interaction of the proton and electrons should be proportional their densities in the co-moving system and the duration of the interaction.

COSY

$J_e = 200\ \mathrm{mA}$ $\quad E_p = 45\ \mathrm{MeV}$

$a_e = 1.27\ \mathrm{cm}$ $\quad J_p = 1\ \mathrm{mA}$

$\tau = 2 \cdot 10^{-8}\ \mathrm{s}$ $\quad a_p = 0.1\ \mathrm{cm}$

$n_e = 2 \cdot 10^{7}\ \mathrm{cm}^{-3}$ $\quad n_i = 2 \cdot 10^{7}\ \mathrm{cm}^{-3}$

CELSIUS

$J_e = 200\ \mathrm{mA}$ $\quad E_p = 48\ \mathrm{MeV}$

$a_e = 1.00\ \mathrm{cm}$ $\quad J_p = 25\ \mathrm{mA}$

$\tau = 2.6 \cdot 10^{-8}\ \mathrm{s}$ $\quad a_p = 1\ \mathrm{cm}$

$n_e = 2.1 \cdot 10^{7}\ \mathrm{cm}^{-3}$ $\quad n_i = 1.5 \cdot 10^{7}\ \mathrm{cm}^{-3}$

SIS

$J_e = 600\ \mathrm{mA}$ $\quad E_{Kr^{34+}} = 11.4\ \mathrm{MeV/u}$

$a_e = 2.2\ \mathrm{cm}$ $\quad J_{Kr^{34+}} = 2\ \mathrm{mA}$

$\tau = 7 \cdot 10^{-8}\ \mathrm{s}$ $\quad a_{Kr^{34+}} = 0.4\ \mathrm{cm}$

$n_e = 5.2 \cdot 10^{7}\ \mathrm{cm}^{-3}$ $\quad n_i = 2 \cdot 10^{8}\ \mathrm{cm}^{-3}$

RHIC

$n_e = 2.1 \cdot 10^{8}\ \mathrm{cm}^{-3}$

$n_i = 2.6 \cdot 10^{8}\ \mathrm{charge/cm}^{3}$

$\tau = 2.5 \cdot 10^{-9}\ \mathrm{s}$

Point for discussion

1. Pure electron – ion interaction look not dangerous. The electron and ion densities is not very large in the co-moving reference system but the interaction time is very small.
2. This report doesn’t deal with the passing of the intensive electron beam in the cooler structure. The total charge of the electron bunch is not very small and some “impedances” problem may be investigated.