BETACOOL program
Accuracy of numerical models

Anatoly Sidorin
Electron cooling group
JINR

Contents

- BETACOOL algorithms
- Electron cooling simulation
- IBS simulation
- Luminosity evaluation

Collaboration with Science Organizations

- BNL (USA)
- Fermilab (USA)
- RIKEN (Japan)
- NIRS (Japan)
- Kyoto Univ. (Japan)
- CERN
- GSI (Germany)
- FZJ (Germany)
- ITEP (Russia)
- BINP (Russia)
- Erlangen Univ. (Germany)
- Uppsala Univ. (Sweden)
- Kyiv Univ. (Ukraine)

http://lepta.jinr.ru/betacool.htm

General goal :

- To simulate long-term processes (in comparison with the ion revolution period) leading to variation of the ion distribution function in 6 dimensional phase space.
- The ion beam motion inside a storage ring is supposed to be stable and is treated in linear approximation.

Advantages:

- Many different effects (ECOOL, IBS, Target, etc.) can be simulated simultaneously at the same parameters using different algorithms
- Fast estimations on PC, accurate calculations on supercomputer

RMS dynamics

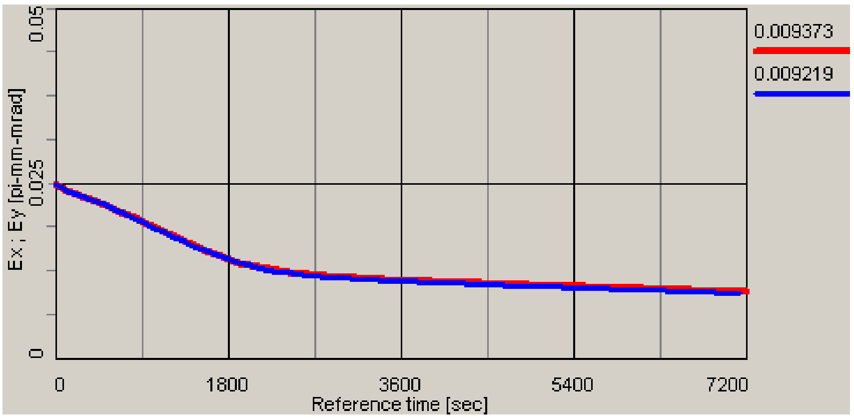
- evolution of RMS parameters (emittances, momentum spread, particle number)

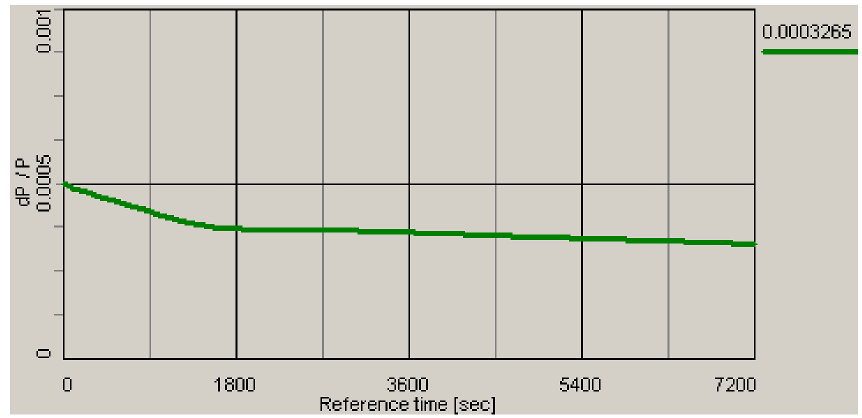
  of ion beam (Gaussian distribution)

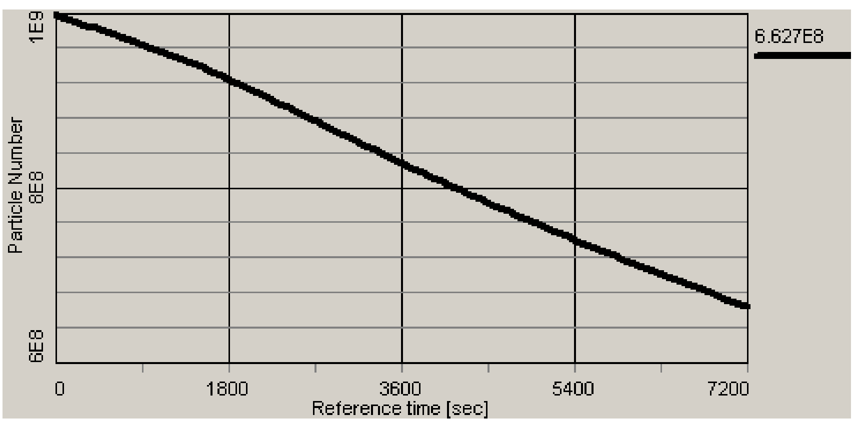
Analytical models for all general effects, fast calculations

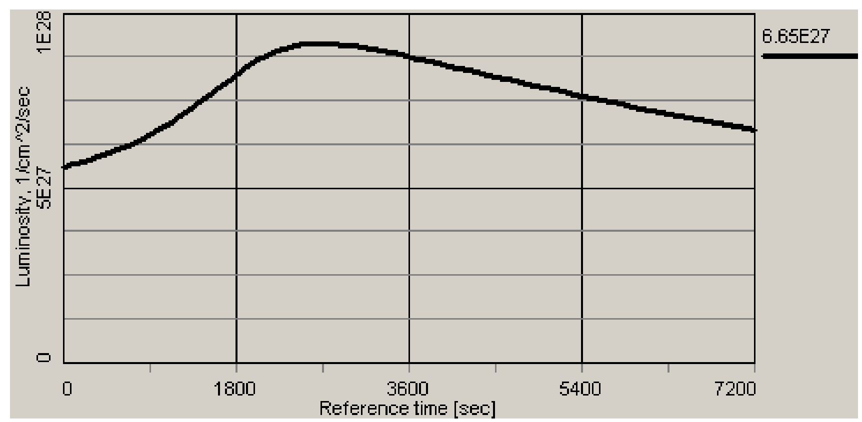
Disadvantage:
Friction force strongly depends on ion velocity
that leads to formation of dense core of the beam,
which determines the luminosity value

Electron cooling with undulator, IBS, particle loss in IP









$$\frac{1}{L}\frac{dL}{dt} = \frac{1}{N}\frac{dN}{dt} - \frac{1}{\varepsilon}\frac{d\varepsilon}{dt} \qquad \varepsilon \sim N^{0.3} \qquad \frac{1}{L}\frac{dL}{dt} \approx 0.7\frac{1}{N}\frac{dN}{dt}$$

Model Beam algorithm

Array of particles $\vec{X} = \left(x, \frac{p_x}{p}, y, \frac{p_y}{p}, s - s_0, \frac{\Delta p}{p} \right)$

Each effect (ECOOL, IBS, particle loss in IP) calculates:

- Momentum variation in accordance with Langevin equation

$$\left(p_{x,y,s} / p\right)_{fin} = \left(p_{x,y,s} / p\right)_{in} + \Lambda_{x,y,s} \Delta T + \sqrt{D_{x,y,s} \Delta T} \xi_{x,y,s}$$

$$\left(p_{x,y,s} / p\right)_{fin} = \left(p_{x,y,s} / p\right)_{in} + \Lambda_{x,y,s} \Delta T + \sqrt{\Delta T} \sum_{i=1}^{3} C_{j,i} \xi_i$$

- Particle loss probability

Electron cooling simulation

Peculiarities of RHIC cooling system

- Bunched electron beam
- Absence of guiding magnetic field
- Undulator for recombination suppression

Friction force

Gaussian bunch

$$f(v_e) = \left(\frac{1}{2\pi}\right)^{3/2} \frac{1}{\Delta_x \Delta_y \Delta_\parallel} \exp\left(-\frac{v_x^2}{2\Delta_x^2} - \frac{v_y^2}{2\Delta_y^2} - \frac{v_\parallel^2}{2\Delta_\parallel^2}\right)$$

Three components of the force

$$\vec{F} = -\sqrt{\frac{2}{\pi}} \frac{Z^2 e^4 n_e}{m\Delta_x \Delta_y \Delta_\parallel} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{(\vec{V} - \vec{v}) \exp\left(-\frac{v_x^2}{2\Delta_x^2} - \frac{v_y^2}{2\Delta_y^2} - \frac{v_\parallel^2}{2\Delta_\parallel^2}\right)}{\left((V_\parallel - v_\parallel)^2 + (V_x - v_x)^2 + (V_y - v_y)^2\right)^{3/2}} dv_x dv_y dv_\parallel$$

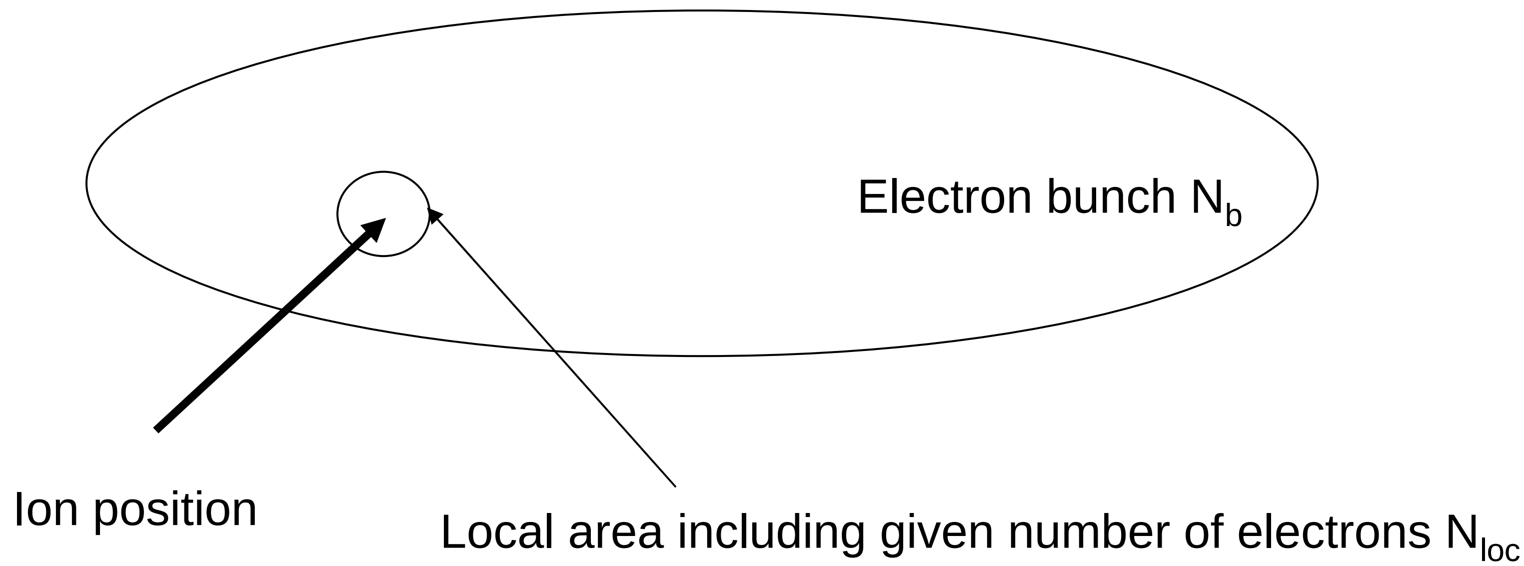
Electron density is uniform inside ρ_{max}

$$n_e = \frac{N_e \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{s^2}{2\sigma_s^2}\right)}{2\pi\sqrt{2\pi}\sigma_x \sigma_y \sigma_s}$$

$$\rho_{\min} = \frac{Ze^2}{m} \frac{1}{\left|\vec{V} - \vec{v}\right|^2} \xrightarrow{\text{Undulator}} \rho_{\min} = \max\left(\rho_{\min}, \frac{eB\lambda^2}{4\pi^2 pc}\right)$$

General case

The electron bunch parameters are calculated with external program (PARMELA) as an array of particles



All the parameters for friction force calculation are functions of ion co-ordinates:
-Local density
-Local mean velocity
-Local velocity spread

Array calculated with PARMELA

N_b = 20000 particles $\longrightarrow$ 5 nQ

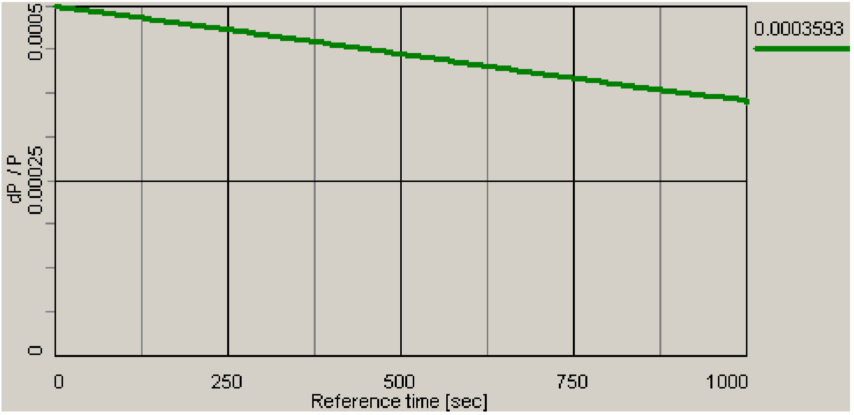
	Co-ordinate	Angular spread
horizontal	3.4463 mm	5.5997 E-6
Vertical	3.4187 mm	5.5688 E-6
Longitudina	1.0794 cm	3.2317 E-4

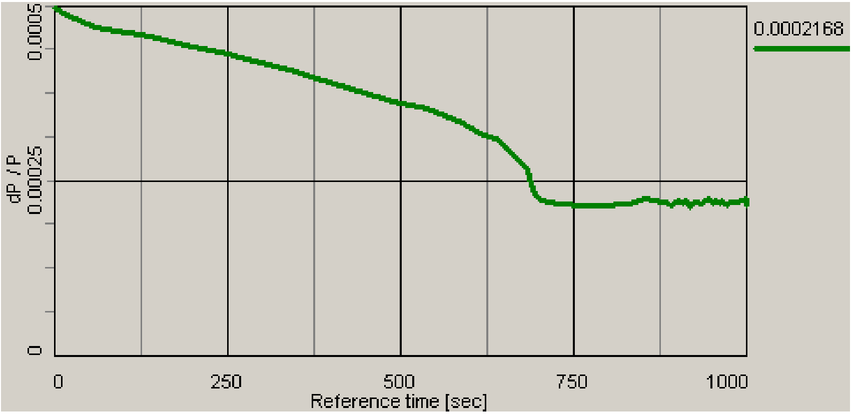
	PARMELA array (300 local)	Gaussian array (20000 particle 300 local)
$1/\tau_\perp$	-8.63E-4 $\pm$ 3.7%	-5.4E-4 $\pm$ 5.2%
$1/\tau_{\|\|}$	-3.61E-3 $\pm$ 8.5%	-2.54E-3 $\pm$ 2.9%

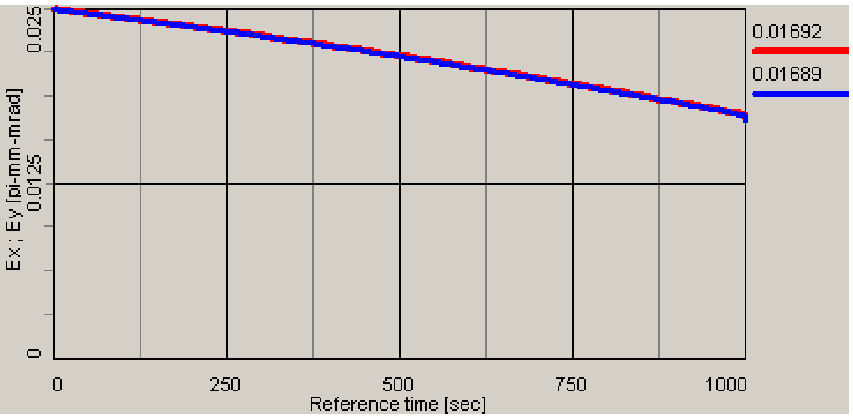
RMS dynamics, ECOOL + IBS

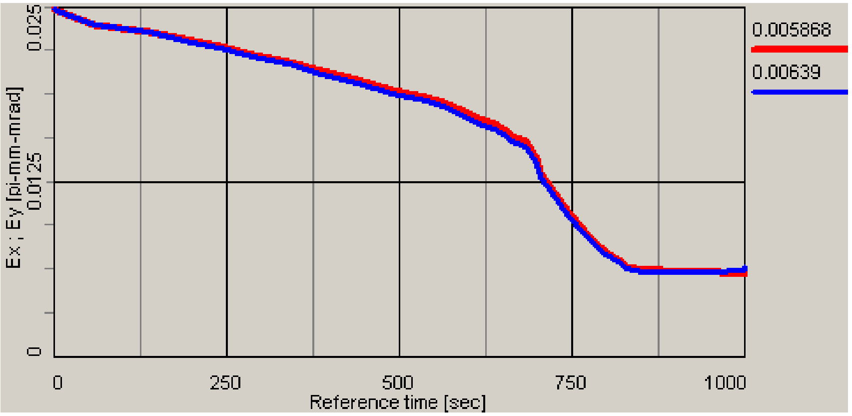
Gaussian bunch

Array of electrons



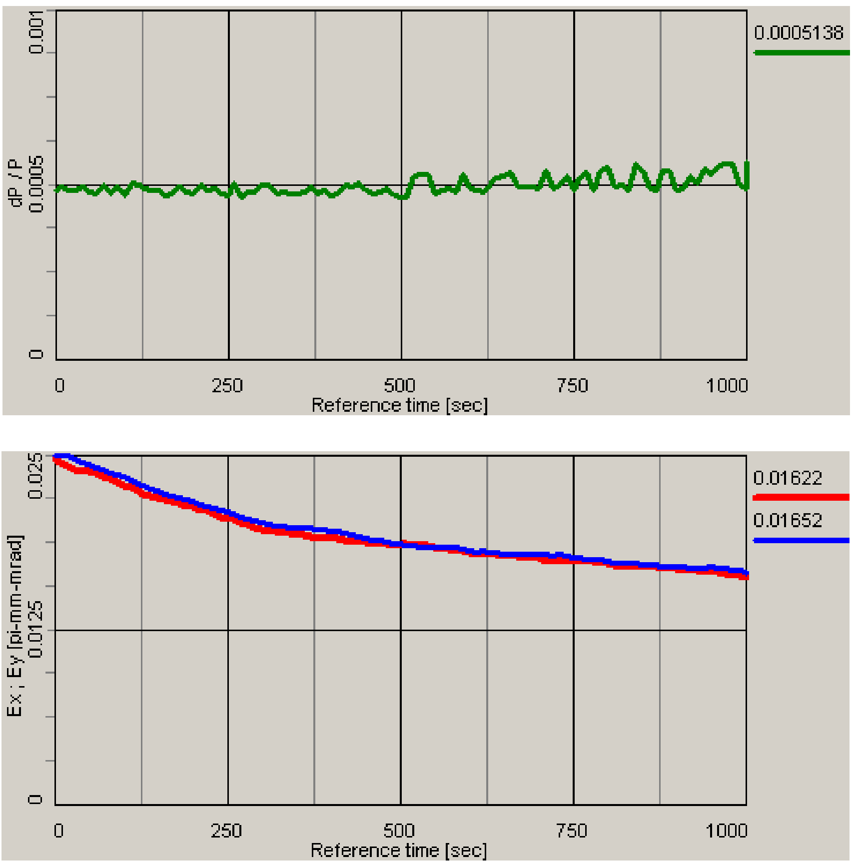




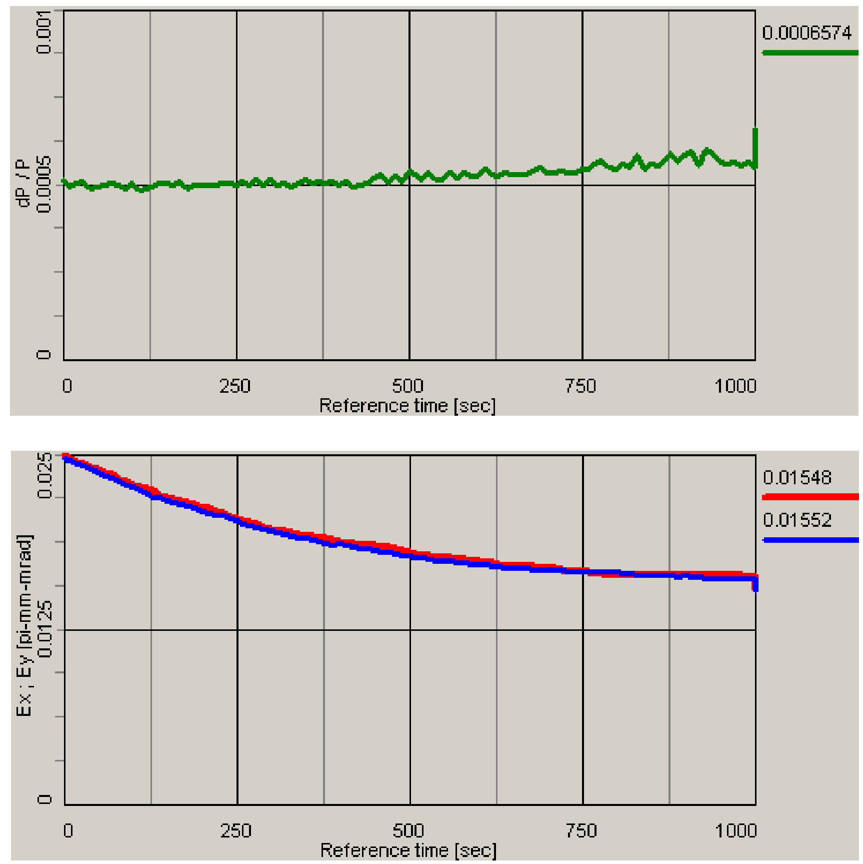


Model beam, 2000 particles, ECOOL + IBS

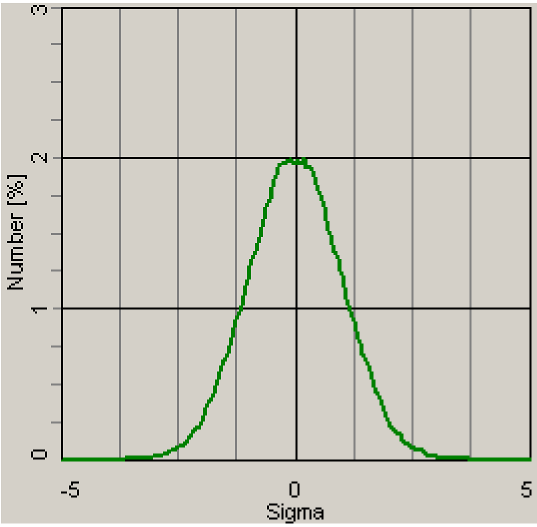
Gaussian bunch



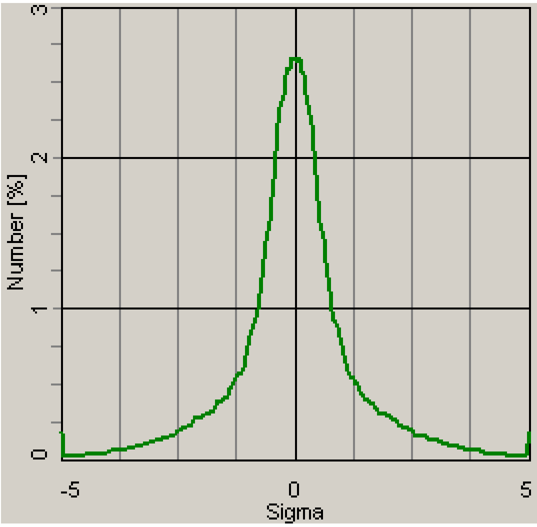
Array of electrons



Initial momentum spread



After 1000 sec of cooling



Loss probability

$$P_{loss} = \int_0^{l_{cool}} \frac{\alpha_r n_e}{\gamma^2 \beta c} ds$$

$$\alpha_r = \int (V_i - v_e)\sigma(V_i - v_e) f(v_e) d^3 v_e$$

$$\sigma = A\left(\frac{h\nu_0}{E}\right)\left(\ln\sqrt{\frac{h\nu_0}{E}} + 0.1402 + 0.525\left(\frac{E}{h\nu_0}\right)^{1/3}\right)$$

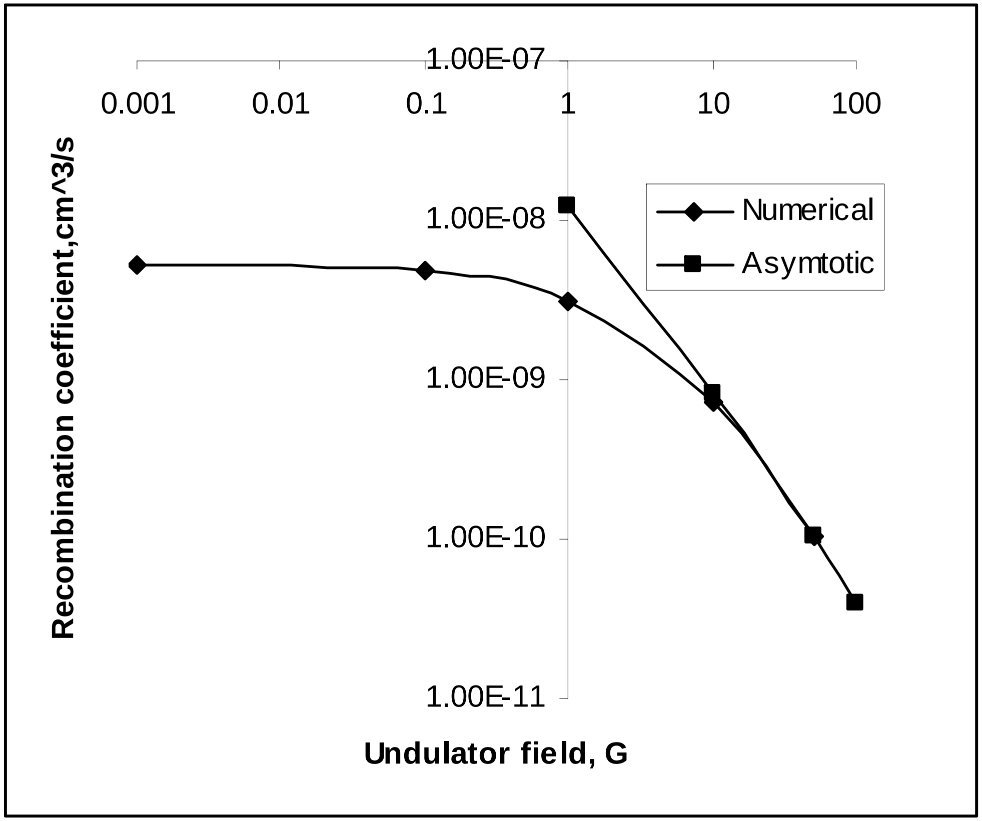
At flattened velocity distribution $\alpha_r = B \cdot Z^2 \sqrt{\frac{1}{T_\perp}}\left[\ln\left(\frac{11.32Z}{\sqrt{T_\perp}}\right) + 0.14\left(\frac{T_\perp}{Z^2}\right)^{1/3}\right]$

$$f(v)d^3v = \left(\frac{m}{2\pi}\right)^{3/2} \frac{1}{T_\perp \sqrt{T_{II}}} e^{-m(v_\perp + v_{und})^2/2T_\perp - m v_{II}^2/2T_{II}} \, 2\pi v_\perp dv_\perp dv_{II}$$

$$v_{und} = c\beta\gamma\theta \qquad \theta = \frac{eB\lambda}{2\pi pc}$$

At $v_{und} >> v_e$ $\qquad \alpha_r = v_{und}\,\sigma(v_{und})$

λ = 12.6 cm, $T_{\perp}$ = 5 eV, $T_{||}$ = 0.005 eV



Now the ion velocity assumed to be sufficiently less then electron one.
It leads to overestimation of recombination coefficient.

Models of IBS in BETACOOL

1. Gaussian beam – analytical models for characteristic times (Piwinski, Martini, Bjorken-Mtingwa,
Jie Wei, gas relaxation)

2. Arbitrary distribution
- kinetic model (Zenkevich, Bolshakov);
– Burov's model,
- model by G. Parzen,
- core-tail model (Bi-Gaussian distribution),
- local model.

Kinetic model:

Constant diffusion and linear friction

The components of the friction force and diffusion tensor
are calculated in accordance with Bjorken-Mtingwa formulae

RHIC:
at time step of a few sec more important term is longitudinal diffusion

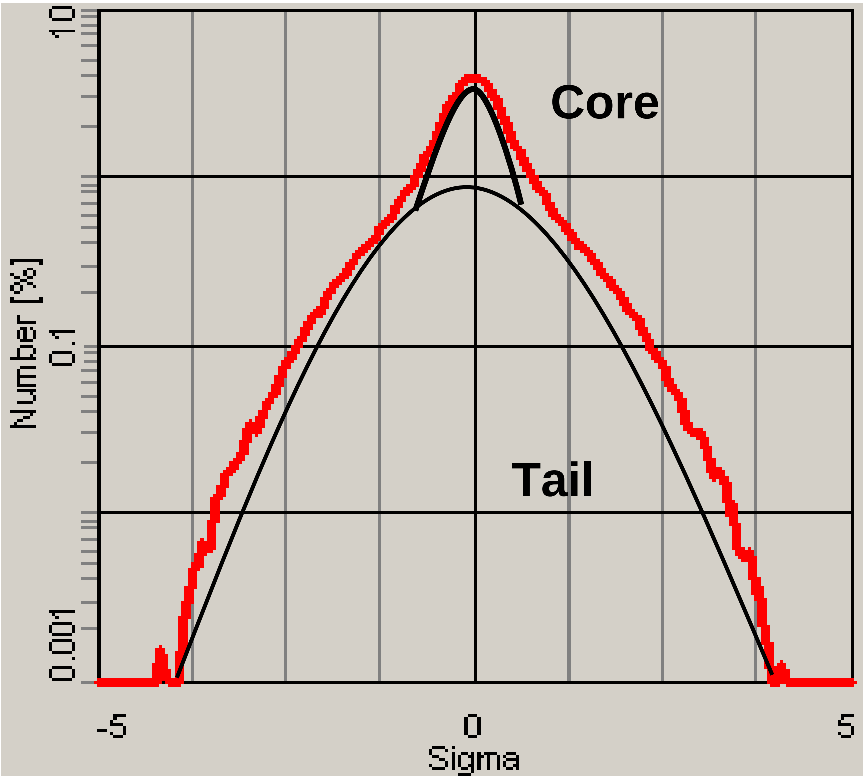
Burov's model:

Longitudinal diffusion is calculated as a function
of particle motion invariants
under assumption of flattened Gaussian distribution

Bi-Gaussian model by G. Parzen:

Real distribution is approximated as a sum of two Gaussian distributions
The heating rate has the same value for all particles

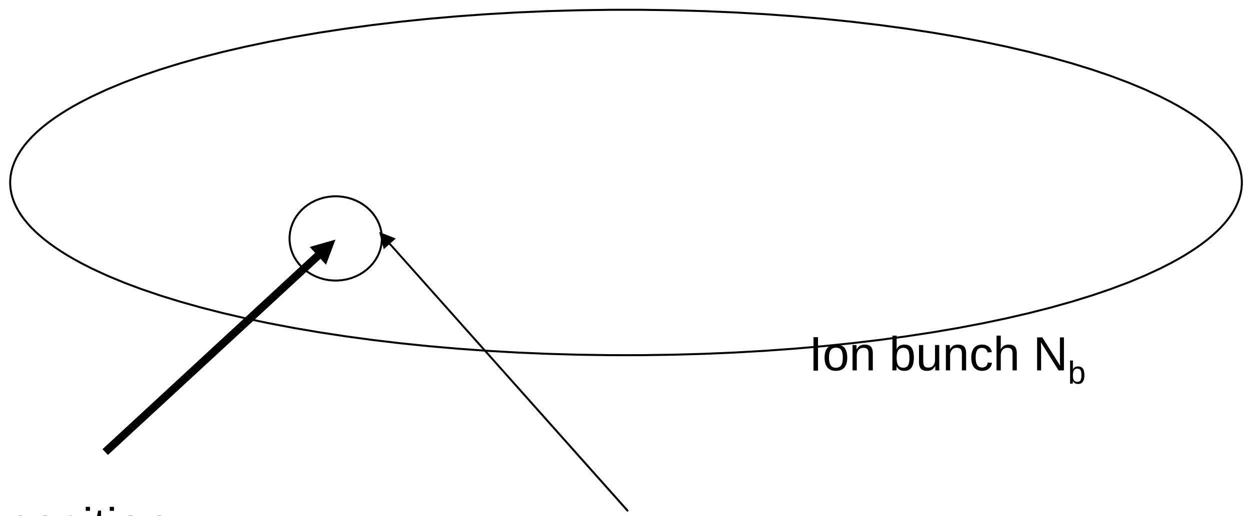
Core-Tail model



Horizontal profile
Bi-Gaussian fit

Diffusion is different for "core" and "tail" particles

Local IBS simulation



Program calculates:
-Local density $\longrightarrow$ Formula for Diffusion
-Local velocity spread

real RHIC lattice, 1000 particles, 50 local particles

T ~ 1 h

$T \sim N_b^2 N_{loc}$

Luminosity calculation

The loss probability depends on the ion co-ordinates inside the bunch

$$P_i = \frac{N_{IP}}{N_b} L_i \sigma_{loss}$$

L_i – luminosity of individual particle

$$L_i = \frac{N_b \rho_i}{T_{rev}}$$

ρ_I – density in ions/cm^2 along the ion trajectory

$$L = \sum_i^N L_i$$

Gaussian bunch

$$\rho(x, y, s, t) = \frac{N_1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_s}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{z^2}{2\sigma_y^2} - \frac{(s - vt)^2}{2\sigma_s^2}\right)$$

$$\sigma_{x,y}^2 = \sigma_{x,y}^{*2}\left(1 + \frac{s^2}{\beta_{x,y}^{*2}}\right)$$

Bi-Gaussian distribution

Bunch is a superposition of two Gaussian bunches

Local density calculation from the ion co-ordinates

Analogous to ECOOL and IBS local models

Conclusions

RMS dynamics model gives fast estimates by the order of magnitude.
At current parameters of RHIC cooling system it predicts correctly general behavior of the cooling process. It can be used for preliminary optimization.

Model beam

1. Model for electron cooling simulation reflects all general peculiarity of RHIC cooling system. Fast estimates can be provided within an accuracy of 10%. Recombination?

2. Core-tail model for IBS simulation includes a few free parameters. Depending on them a spread of equilibrium emittances is about 30-50%. Accuracy of the model can be estimated only with local model using supercomputer

3. Local model for the luminosity calculation provides high level of noise at small number of the model particles.
Simulations with supercomputer.