Coherent Instability Induced By Electron Cooling

RHIC Electron Cooling and High-Brightness Beams Workshop

Gang Wang

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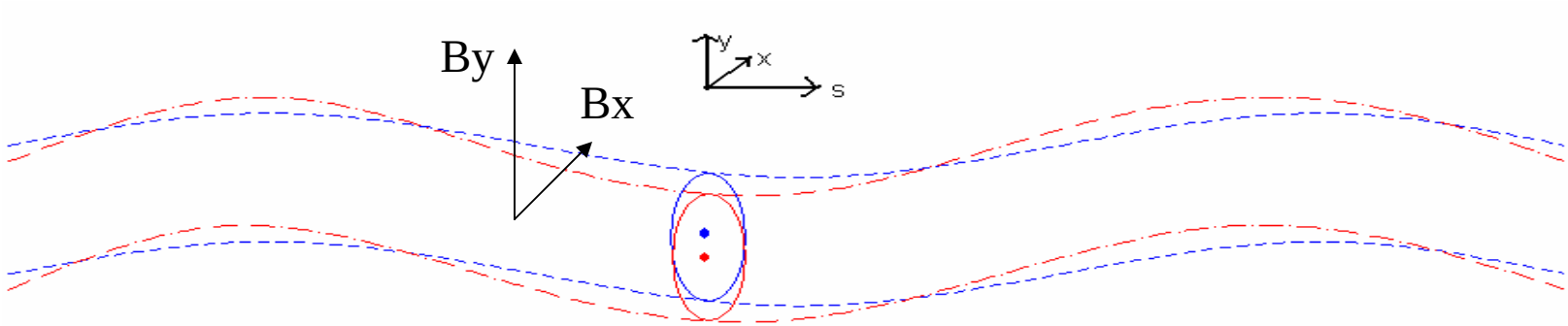
Outline

- Transverse Dipole Instability Induced by non-magnetized electron cooling beam:
  - Ion beam Transversal Instability threshold estimation.
  - Coherent Damping effect due to the designed cooling electron beam.
- Longitudinal Dipole Instability due to electron cooling beam.
- Transverse Quadruple Instability due to envelope oscillation.

Two stream Instability Due to Non-Magnetized Electron Beam



$$B_x = B_\perp \cos(\frac{2\pi s}{\lambda_w}) \qquad B_y = B_\perp \sin(\frac{2\pi s}{\lambda_w}) \qquad B_s = 0$$

$$\frac{d^2}{ds^2} X_i = -\Omega'^2_{ie}(X_i - X_e) - \frac{ZeB_\perp}{M_i \gamma c}\sin(\Omega'^2_w s)$$

$$\frac{d^2}{ds^2} X_e = -\Omega'^2_{ei}(X_e - X_i) + \frac{eB_\perp}{m_e \gamma c}\sin(\Omega'_w s)$$

Designed parameters

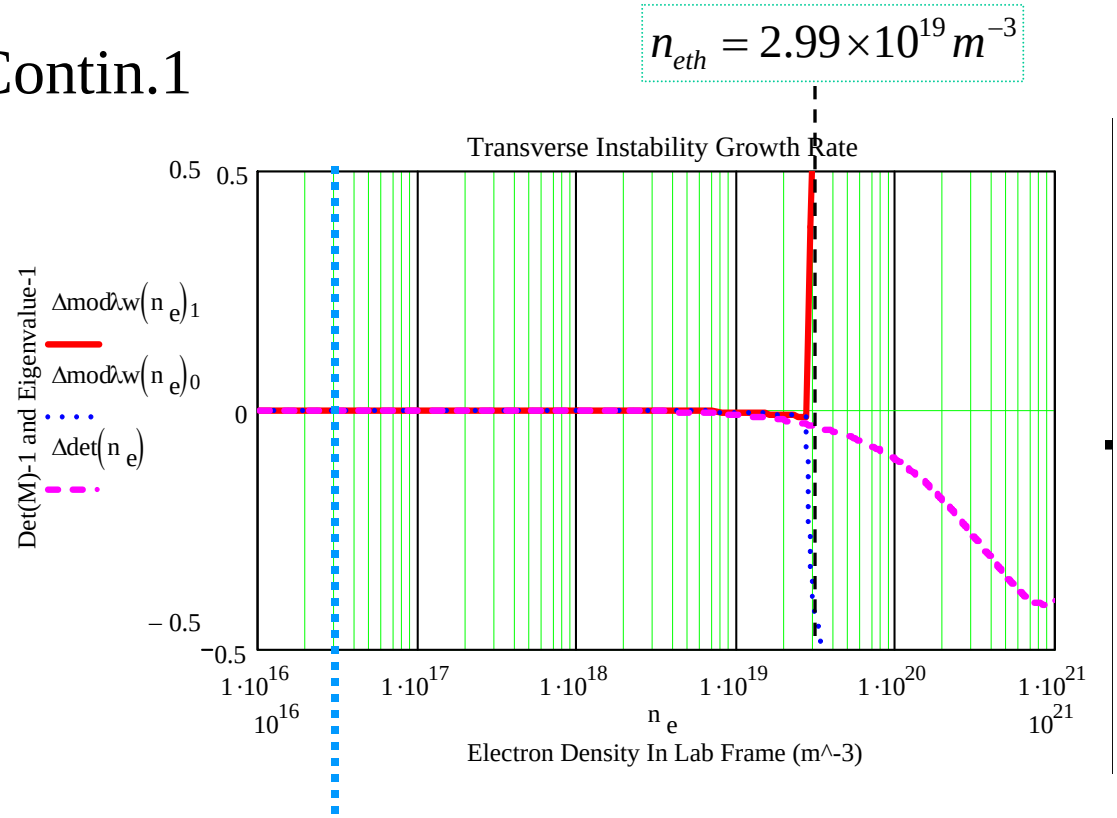
Ion number per bunch	10^9
Ion rms bunch length	0.37 m
Ion rms emittance	2.5 $\pi\mu m$
Wiggler field strength	0.001 T
Wiggler field wavelength	0.15 m
Cooling section ion Beta function	200 m
Cooling section length	60 m
Electron rms beam size	2.36 mm
Electron rms bunch length	0.009 m
Electron beam charge	5 nC
Ion Density (Lab Frame)	$2.31\times10^{13}\ m^{-3}$
Electron Density (Lab Frame)	$2.967\times10^{16}\ m^{-3}$

Non-magnetized Beam Contin.1

$$\begin{pmatrix} X_i \\ X'_i \\ 1 \end{pmatrix}_{n+1} = M_{ring} \begin{pmatrix} X_i \\ X'_i \\ 1 \end{pmatrix}_n$$

$$M_{ring} = R_x L_{drift}(-\frac{l_c}{2}) M_{cool} L_{drift}(-\frac{l_c}{2})$$

The instability threshold is three orders of magnitude bigger than the designed electron density.



Designed Electron Density within 1σ

$$M_{cool} = \begin{pmatrix} 1+\xi'(\cos(\Omega'_0 s)-1) & \frac{1}{\Omega'_0}\left[\Omega'_0 s(1-\xi')+\xi'\sin(\Omega'_0 s)\right] & a(s) \\ -\xi'\Omega'_0 \sin(\Omega'_0 s) & 1+\xi'(\cos(\Omega'_0 s)-1) & a'(s) \\ 0 & 0 & 1 \end{pmatrix}$$

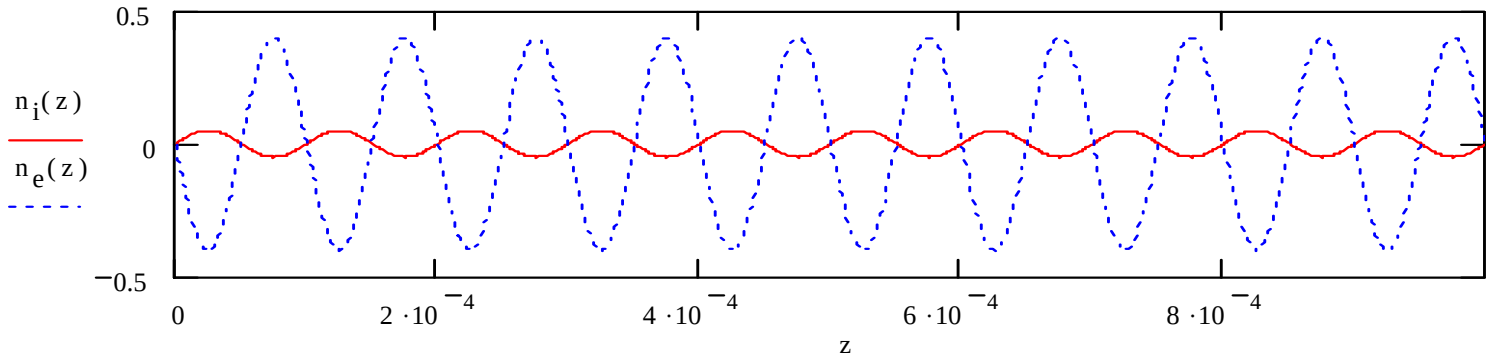
Non-magnetized Beam Contin.2

At the designed electron beam density, the two stream interaction will damp the ion beam coherent oscillation with the damping rate:

$$\left|\det_{transverse}\right| - 1 = -4.01 \times 10^{-6} \quad \text{Per turn}$$

Which corresponds to the damping time of 3.2 seconds.

Longitudinal Instability Induced by electron beam

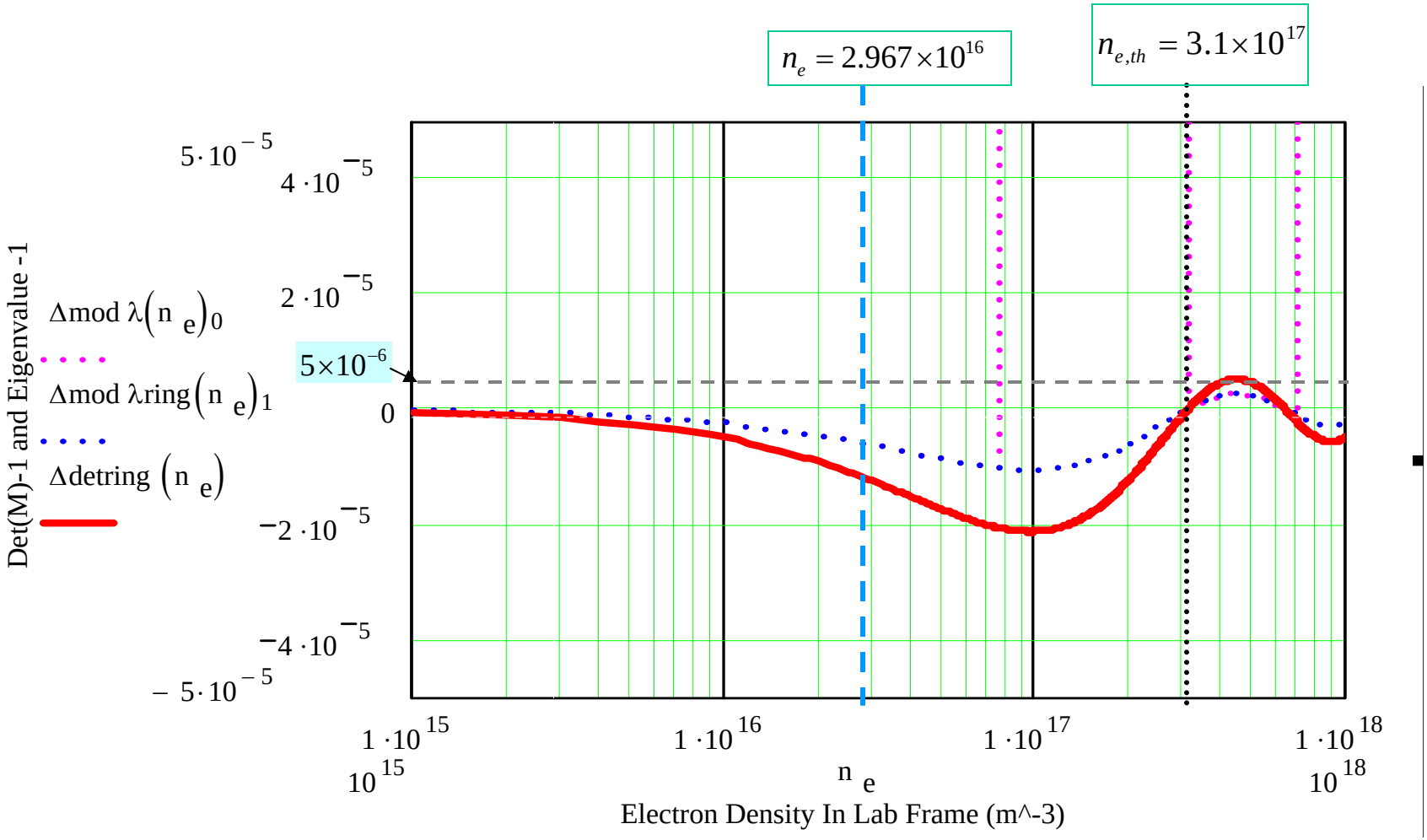


$$\left.\begin{aligned} \frac{d^2 s_i}{ds^2} + \frac{\omega_{pi}^2}{c^2} s_i &= \frac{\omega_{ie}^2}{c^2} s_e \\ \frac{d^2 s_e}{ds^2} + \frac{\omega_{pe}^2}{c^2} s_e &= \frac{\omega_{ei}^2}{c^2} s_i \end{aligned}\right\} \text{Inside Cooling section}$$

$$\frac{d^2 s_i}{ds^2} + \frac{\omega_{pi}^2}{c^2} s_i = 0 \longrightarrow \text{Outside Cooling section}$$

$$\begin{pmatrix} s_i \\ s_i' \end{pmatrix}_{n+1} = M_{ring} \begin{pmatrix} s_i \\ s_i' \end{pmatrix}_n \qquad M_{ring} = M_{cool} M_{out}$$

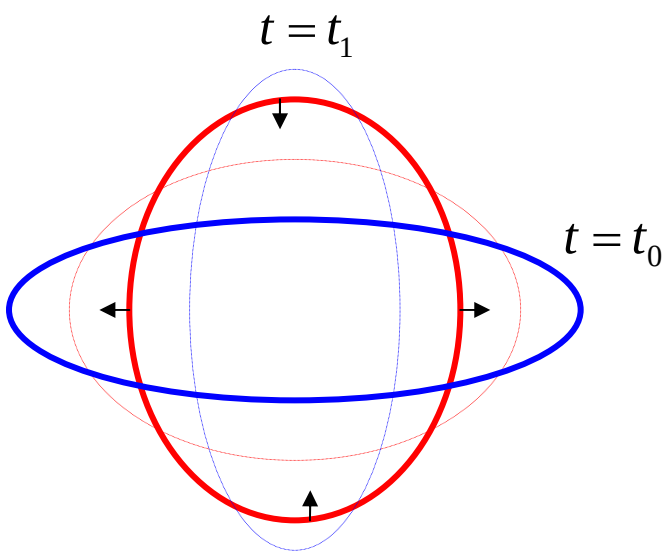
Longitudinal Instability Contin.1(Non-magnetized Beam)



$$M_{cool}=\begin{pmatrix} \xi(\cos(\frac{\omega_0}{c}l_{cool})-1)+1 & \frac{1}{\omega_0}\left[\xi\sin(\frac{\omega_0}{c}l_{cool})+(1-\xi)\frac{\omega_0}{c}l_{cool}\right] \\ -\xi\omega_0\sin(\frac{\omega_0}{c}l_{cool}) & \xi(\cos(\frac{\omega_0}{c}l_{cool})-1)+1 \end{pmatrix}$$

$$M_{out}=\begin{pmatrix} \cos\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] & \frac{\sin\left[\frac{\omega_0}{c}(Cir-l_{cool})\right]}{\omega_0} \\ -\omega_0\sin\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] & \cos\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] \end{pmatrix}$$

Equations of Motion for envelope oscillation



Assumption for space charge:

1. The unperturbed beams envelope are round..
2. The envelope oscillation will not change the area of the beam cross section.

Vlasov's equation:

$$\frac{\partial f}{\partial t} + v_\perp \frac{\partial f}{\partial r_\perp} - K\left[r_\perp \frac{\partial f}{\partial v_\perp}\right] + \frac{e}{\gamma m}\left[E_\perp \frac{\partial f}{\partial v_\perp}\right] + \omega_{He}\left[v_\perp \times \hat{s}\right]\cdot \frac{\partial f}{\partial v_\perp} = 0.$$

Equation of motion continue.1

If no external field, i.e.K=0, one gets

$$\frac{\partial\left\langle \xi_i^2 \right\rangle}{\partial s} - 2\left\langle \xi_i \xi_i' \right\rangle = 0,$$

$$\frac{\partial\left\langle \xi_i \xi_i' \right\rangle}{\partial s} - \left\langle \xi_i'^2 \right\rangle + \left(\frac{2}{\lambda_{ie}^2} - \frac{1}{\lambda_{pi}^2} \right)\left\langle \xi_i^2 \right\rangle + \frac{i}{\lambda_{Hi}}\left\langle \xi_i \xi_i^{'} \right\rangle = \frac{1}{\lambda_{ie}^2}\left\langle \xi_e^2 \right\rangle$$

$$\frac{\partial\left\langle \xi_i'^2 \right\rangle}{\partial s} + \frac{i}{\lambda_{Hi}\lambda_{pi}^2}\left\langle \xi_i^2 \right\rangle + 4\left(\frac{1}{\lambda_{ie}^2} - \frac{1}{\lambda_{pi}^2} \right)\left\langle \xi_i^{'} \xi_i \right\rangle + \frac{2i}{\lambda_{Hi}}\left\langle \xi_i'^2 \right\rangle = \frac{i}{\lambda_{Hi}\lambda_{ie}^2}\left\langle \xi_e^2 \right\rangle$$

Where

$$\left\langle ... \right\rangle = \int ... f(x, y, v_x, v_y) dx dy dv_x dv_y$$

$$\xi = x + iy$$

Result for RHIC e-cooler

Since there is no solenoid in the cooling section, the equation of motion is reduced to the following form,

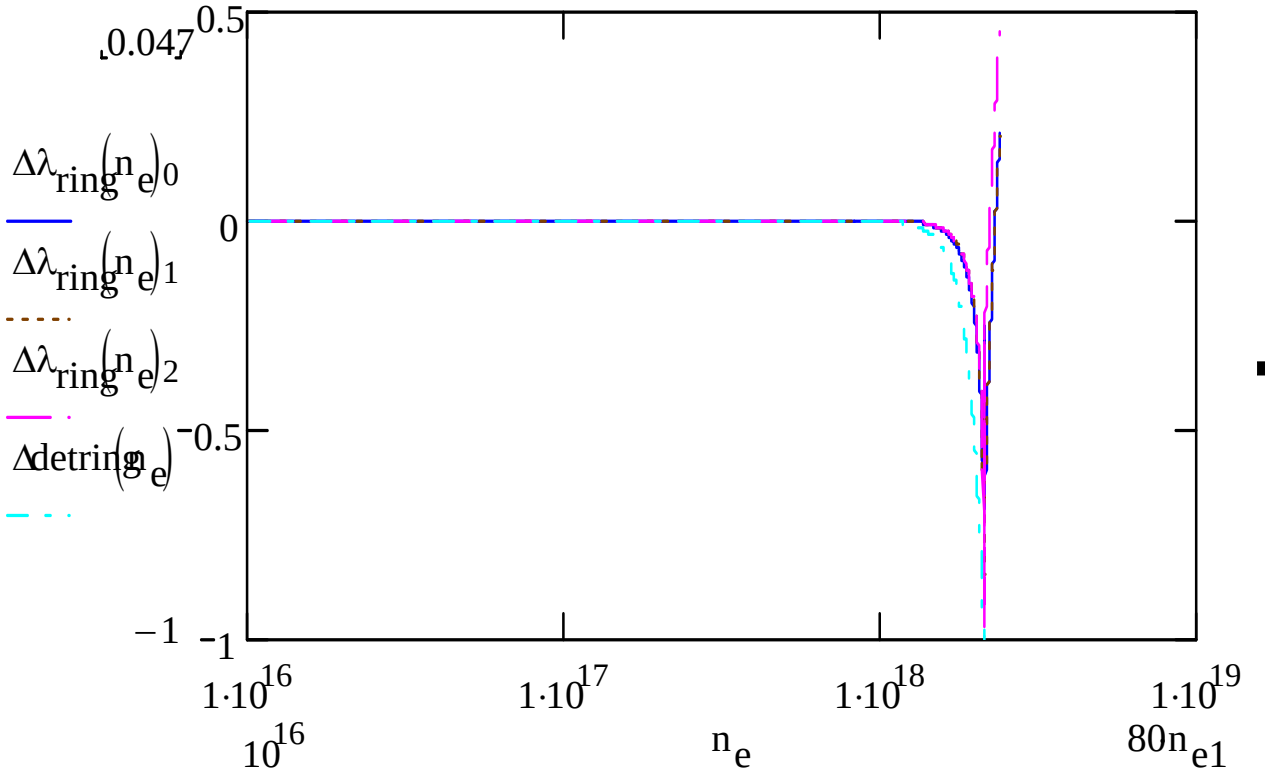
$$\frac{\partial^2}{\partial t^2}\begin{pmatrix}\langle \xi_i^2 \rangle \\ \langle \xi_e^2 \rangle\end{pmatrix} = \begin{pmatrix} 3\omega_{pi}^2 - 4\omega_{ie}^2 & \omega_{ie}^2 \\ \omega_{ei}^2 & 3\omega_{pe}^2 - 4\omega_{ei}^2 \end{pmatrix}\begin{pmatrix}\langle \xi_i^2 \rangle \\ \langle \xi_e^2 \rangle\end{pmatrix} + \begin{pmatrix} 2C_i \\ 2C_e \end{pmatrix}(\beta c)^2$$

where Ci and Ce are determined by the initial conditions. The solution of the equation can also be written into matrix form,

$$\begin{pmatrix}\langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle\end{pmatrix}_t = A_{cool}\begin{pmatrix}\langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle\end{pmatrix}_0$$

The transfer matrix for the ring part has also been included to get the one turn transfer matrix, and thus the eigenvalues for each mode.

Result for RHIC e-cooler continue



The instability threshold is

$$n_{e,th} = 2\times10^{18} m^{-3}$$

which is above the designed electron density $n_e = 2.97\times10^{16} m^{-3}$ by a factor of 60. (All parameters are from MAC'06 design.)

Equation of motion continue.2

$$\frac{\partial}{\partial t}\begin{pmatrix} \langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle \\ \langle \xi_e^2 \rangle \\ \langle \xi_e \dot{\xi}_e \rangle \\ \langle \dot{\xi}_e^2 \rangle \end{pmatrix} = A' \begin{pmatrix} \langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle \\ \langle \xi_e^2 \rangle \\ \langle \xi_e \dot{\xi}_e \rangle \\ \langle \dot{\xi}_e^2 \rangle \end{pmatrix} \qquad A' = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ \frac{\omega_{pi}^2}{2} - \omega_{ie}^2 & -i\omega_{Hi} & 1 & \frac{\omega_{ie}^2}{2} & 0 & 0 \\ -\frac{i\omega_{Hi}\omega_{pi}^2}{2} & 2\omega_{pi}^2 - 2\omega_{ie}^2 & -2i\omega_{Hi} & \frac{i\omega_{Hi}\omega_{ie}^2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ \frac{\omega_{ei}^2}{2} & 0 & 0 & \frac{\omega_{pe}^2}{2} - \omega_{ei}^2 & i\omega_{He} & 1 \\ -\frac{i\omega_{He}\omega_{ei}^2}{2} & 0 & 0 & \frac{i\omega_{He}\omega_{pe}^2}{2} & 2\omega_{pe}^2 - 2\omega_{ei}^2 & 2i\omega_{He} \end{pmatrix}$$

Setting the initial condition of the electron beam to be (0,0,0), above linear differential equation can be solved into the following form

$$\begin{pmatrix} \langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle \end{pmatrix}_t = A_{cool} \begin{pmatrix} \langle \xi_i^2 \rangle \\ \langle \xi_i \dot{\xi}_i \rangle \\ \langle \dot{\xi}_i^2 \rangle \end{pmatrix}_0$$

The instability happens when

$$\left| \det(A_{cool}) \right| > 1$$

Result for FNAL $\bar{p}$ Recycler

Table.1 Parameters used for FNAL Recycler

Solenoid strength	150 gauss
Recycler circumference	3320 meters
Cooling section length	20 meters
Ion rms beam size	1 mm
Ion full bunch length	2000 meter (barrier buchket)
Electron beam size (coasting beam)	4.5 mm
Electron current	500 mA
Beam energy (γ)	9.5
Ion Population per bunch	47.7×10^{10}
Electron Density	$1.65\times10^{14}m^{-3}$
Ion Density	$2.71\times10^{13}m^{-3}$

Result for FNAL$\bar{p}$ Recycler Continue.

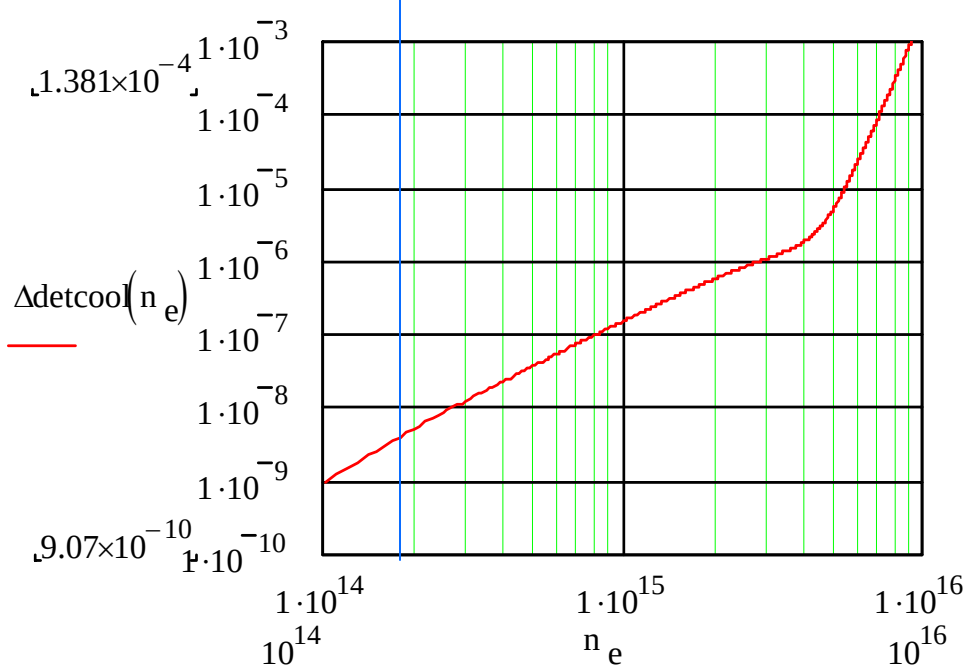
The growth rate for the actual electron density of the $\bar{p}$ Recycler is estimated to be

$$\Gamma = 4.98\times10^{-9} / turn$$

Or

$$\tau_{rise} = 170 \quad \text{seconds}$$

The observed emittance growth is around 1000 seconds, which is slower than the estimated growth rate by a factor of 6.



Summary

1. Under the cosidered V.Parkhomchuk and V.Reva model, the designed electron cooling beam will not cause two stream instability and the threshold for the instability are 3 and 1 orders larger than the currently designed electron beam density for the transversal and longitudinal instability respectively.
2. The calculation under V.Reva model for the envelope oscillation shows the threshold for quadruple oscillation is larger than the designed electron density by a factor of 60. The calculated growth rate for Anti-proton Recycler is 0.0058 which is larger than the observed emittance growth rate by a factor of 6.

Problem to be solved

1. Modify the model for bunched beams in order to take the synchrotron oscillation into account and study the effects of the electron longitudinal motion on the ion beam instability such as electron beam painting.
2. Write computer-simulation codes to track the multi-particle long-range interaction in the cooling section.
3. Experimental measurements of the growth rate and threshold of the electron beam induced instability should be done.