Calculation of Field Components Outside the Pole Tip Radius in the 17D120 Dipoles

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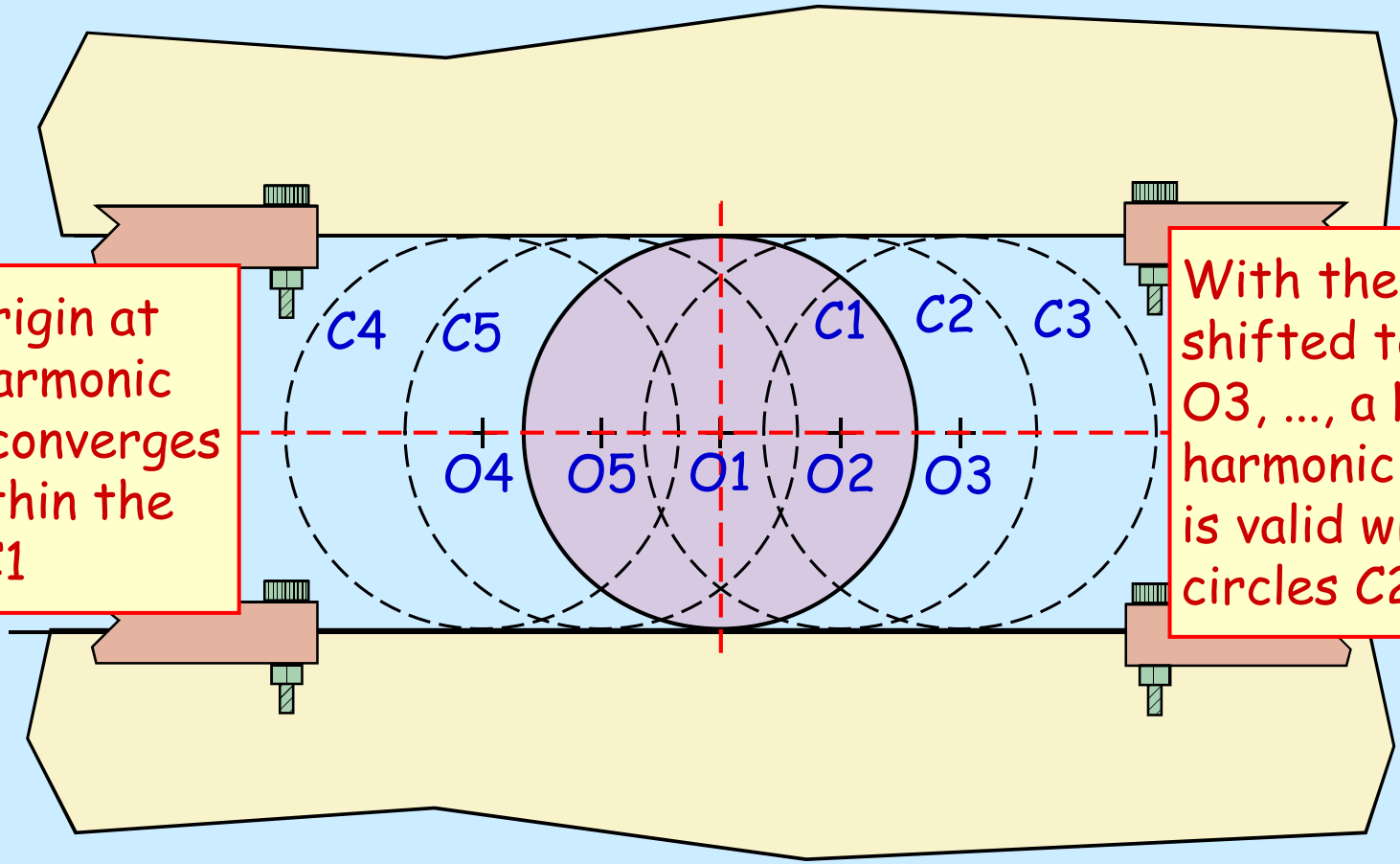
Introduction

- The maximum horizontal and vertical beam excursions in the 17D120 dipoles for the SNS are 89.2 mm and 64.4 mm respectively.
- The horizontal excursion is larger than the 85 mm vertical pole tip radius (half of the pole gap), within which a single harmonic series expansion is convergent in general.
- Harmonics were measured at five horizontal positions, allowing calculations of field at any point in the rectangular aperture. However, beam optics programs can not handle multiple sets of harmonics for a single magnet.
- **Question:** Can we use the harmonics measured in the central position alone to calculate fields outside the radius of convergence? What are the errors if we do so?

Field in a Non-circular Aperture



With origin at O1, a harmonic series converges only within the circle C1

With the origin shifted to O2, O3, ..., a NEW harmonic series is valid within circles C2, C3, ...

By having a significant overlap between the various circles of convergence, one can verify the integrity and accuracy of data by comparing results in the overlap regions.

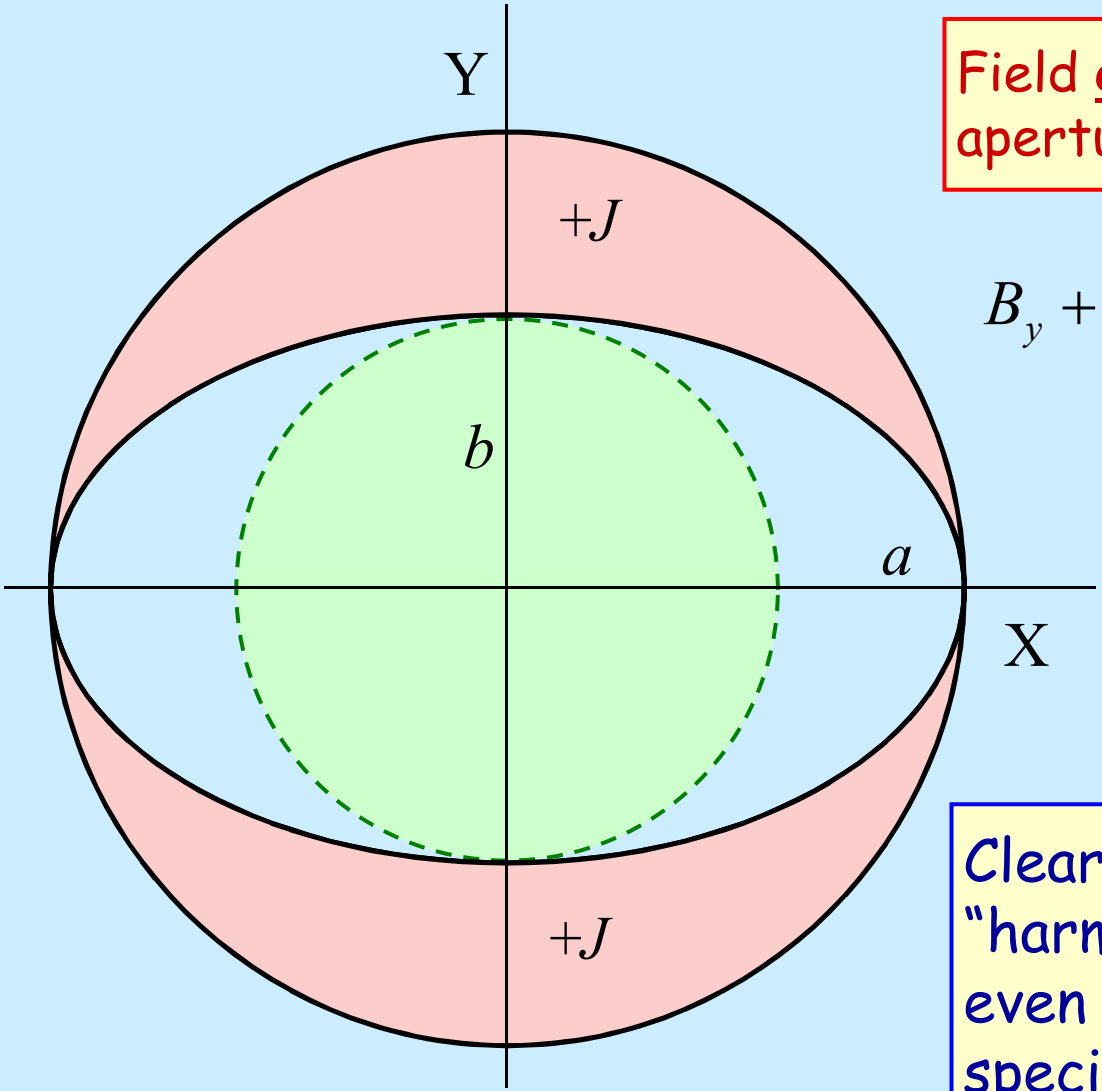
Calculation of Field

- The field components at any point (x, y) can be computed from the normal and skew harmonics measured at (x_0, y_0) using:

$$B_y(x,y) + iB_x(x,y) = \sum_{n=0}^{\infty} (B_n + iA_n) \left(\frac{(x - x_0) + i(y - y_0)}{R_{ref}} \right)^n$$

- For accurate calculations of field, the origin (x_0, y_0) should be chosen such that the field point (x, y) is as close as possible, and is well within the circle of convergence defined by the pole tips.
- In special situations, such as when the higher order terms are identically zero, convergence of the series may not be a problem, and it ***may*** be possible to calculate the field outside the radius of convergence using the same (B_n, A_n).

An Example of Elliptical Aperture



Field **everywhere** in the elliptical aperture is a pure quadrupole field:

$$B_y + iB_x = \left(\frac{\mu_0 J}{2}\right)\left(\frac{a-b}{a+b}\right)(x+iy)$$

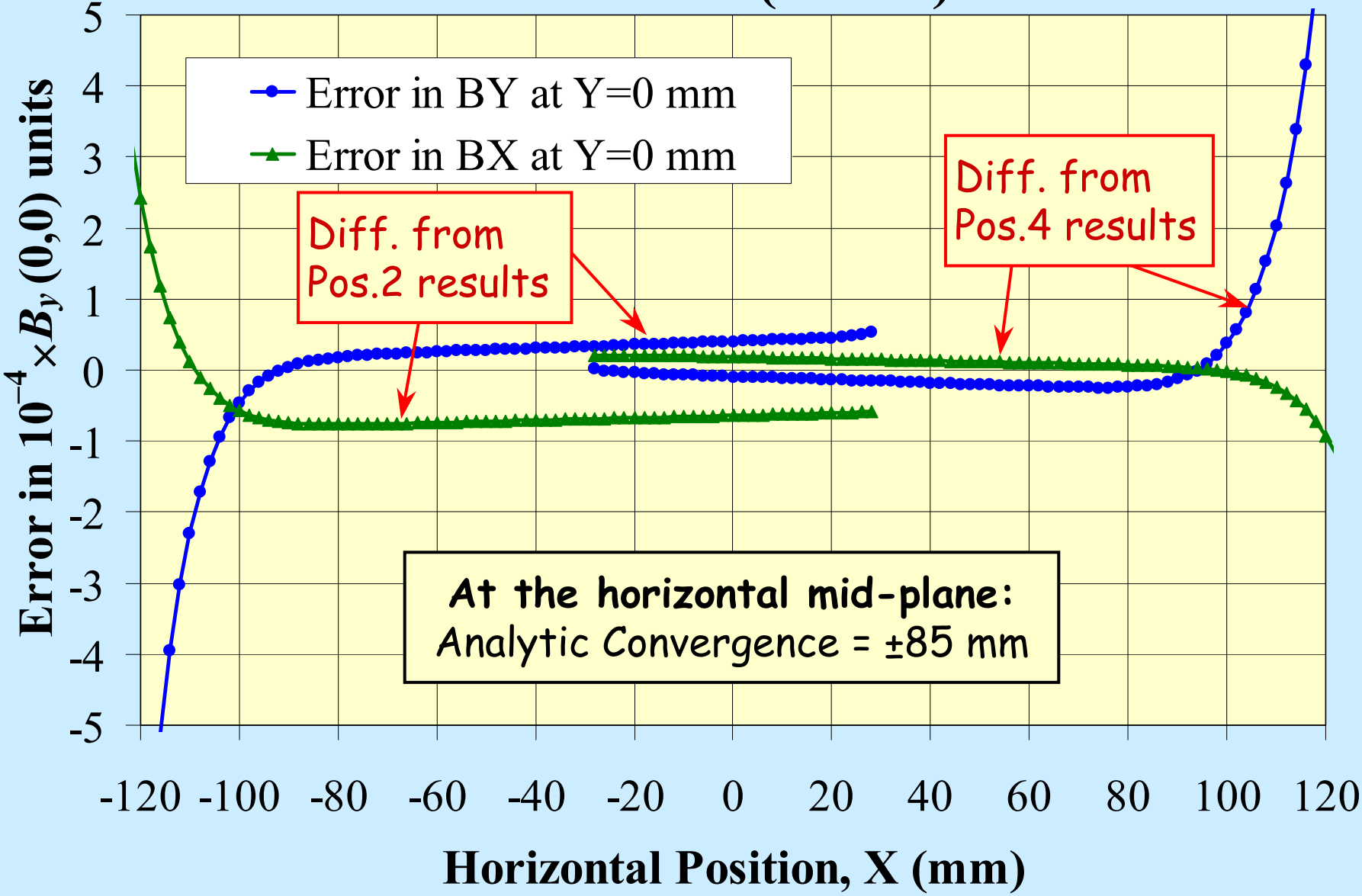
Convergence of the harmonic series, in general, is assured only inside the circle of radius *b*.

Clearly, the single term "harmonic expansion" is valid even outside the circle in this special case.

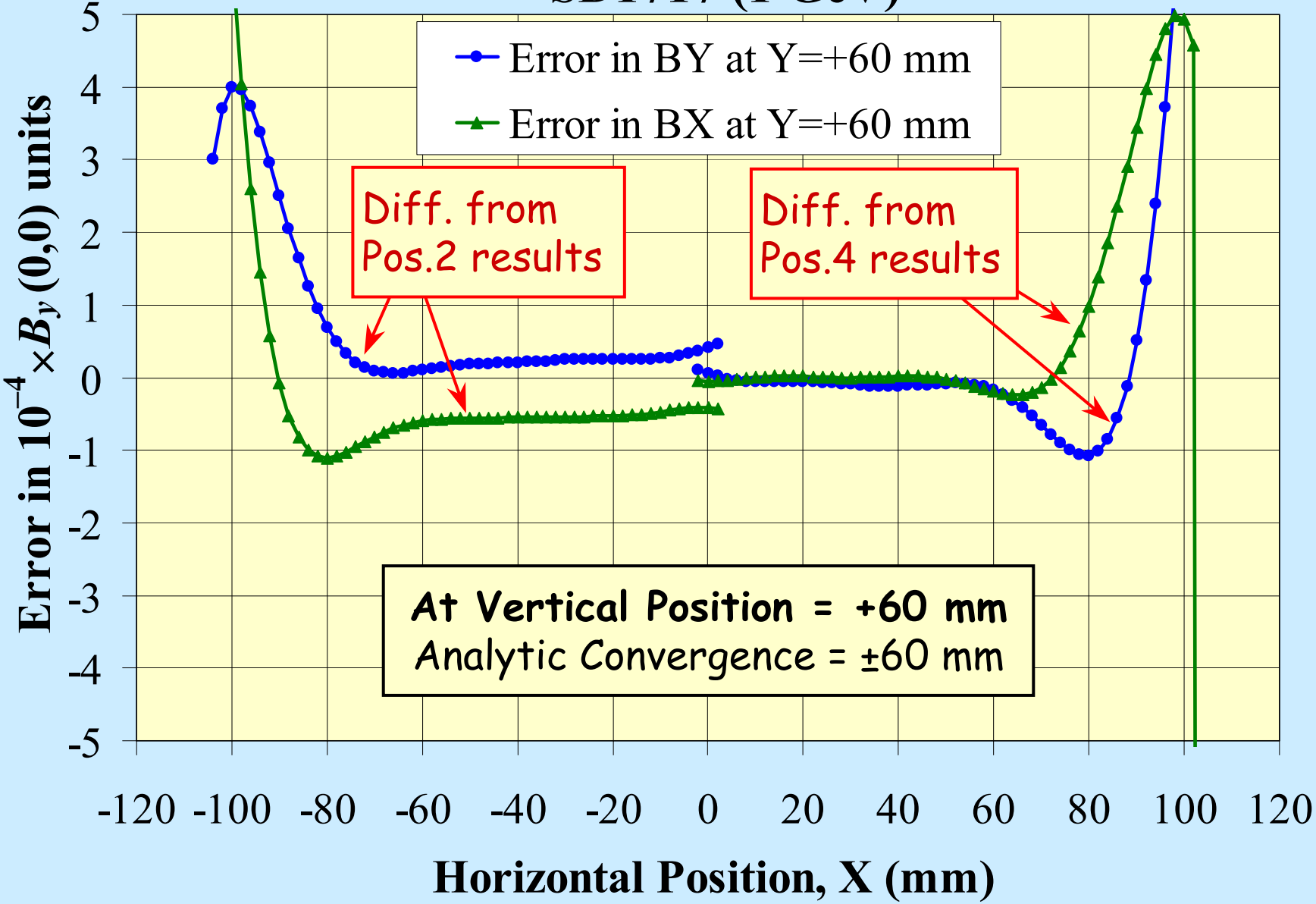
Case of 17D120 Dipoles

- We have measurements of harmonics at five positions.
- Accurate values of field components can be calculated at any point in the aperture using harmonics measured at a suitable position. In most cases, multiple measurements can be used to calculate field at the same point.
- We can ***blindly*** compute field components, using the central position harmonics alone, in an extended zone of radius 130 mm, say (but only at points inside the aperture).
- The blindly computed field components can be compared with the more accurate results obtained by using harmonics measured at different locations.
- Such a comparison will tell how far away from the center, in practice, we may go using only one set of harmonics.

Error Due to Extending Radius of Pos.3 SD1717 (1 GeV)



Error Due to Extending Radius of Pos.3
SD1717 (1 GeV)



Conclusions

- Within a region of "analytic convergence", the results from various measurements match very well. There is a nearly constant offset in this region, which may be a result of small errors (~10^{-5}) in the measurement of transfer function and/or field angle.
- The errors tend to increase outside the 85 mm radius zone. However, it is seen that there is ***no catastrophic failure*** of the harmonic expansion outside this zone.
- If one accepts errors at the level of 10^{-4} of the central field, the field components in 17D120 can be obtained within a circle of ~100 mm radius using a single set of harmonics measured at the central position #3.