

**Measurement of the  
polarization vector  
of the  $e^+$   
from the decay of polarized  $\mu^+$   
as a test  
of time reversal invariance**

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# **1. Collaborators**

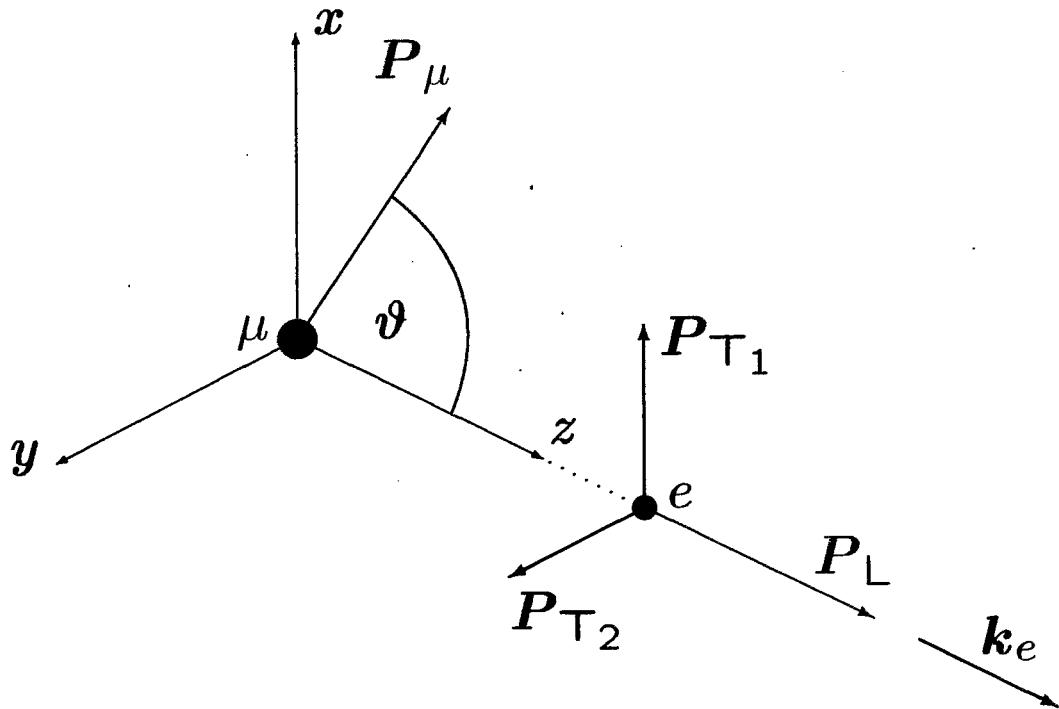
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$$P_{T_1} = f_1(E, \vartheta, \eta, \eta'')$$

$$P_{T_2} = f_2(E, \vartheta, \frac{\alpha}{A}, \frac{\beta}{A})$$

The standard model predicts:

$$\langle P_{T_1} \rangle_E = 0.003$$

$$P_{T_2} \equiv 0$$

A nonzero  $P_{T_2}$  would signal violation of time reversal invariance. This is the only purely leptonic reaction for which TRI has been tested up to now.

### 3. Matrix Element

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^{\gamma} \langle \bar{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle \bar{\nu}_m | \Gamma_{\gamma} | (\mu)_{\mu} \rangle$$

The index  $\gamma$  labels the type of interaction:

$\Gamma^S$	=	4-scalar
$\Gamma^V$	=	4-vector
$\Gamma^T$	=	4-tensor

The indices  $\varepsilon, \mu$  indicate the chiralities of the spinors of the observed (charged) leptons. The chiralities  $n, m$  of the neutrinos are uniquely determined for given  $\gamma, \varepsilon$  and  $\mu$ .

The transverse polarization component  $P_{T_1}$  yields the low energy parameter  $\eta$  *without* the suppression factor  $m_e/m_{\mu}$  of  $\eta$  in the energy spectrum of the decay positron. These interference terms allow for sizeable effects.

$$\begin{aligned} \eta = & \frac{1}{2} \text{Re} \{ g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} \\ & + g_{LR}^V (g_{RL}^{S*} + g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + g_{LR}^{T*}) \} \end{aligned}$$

In the standard model

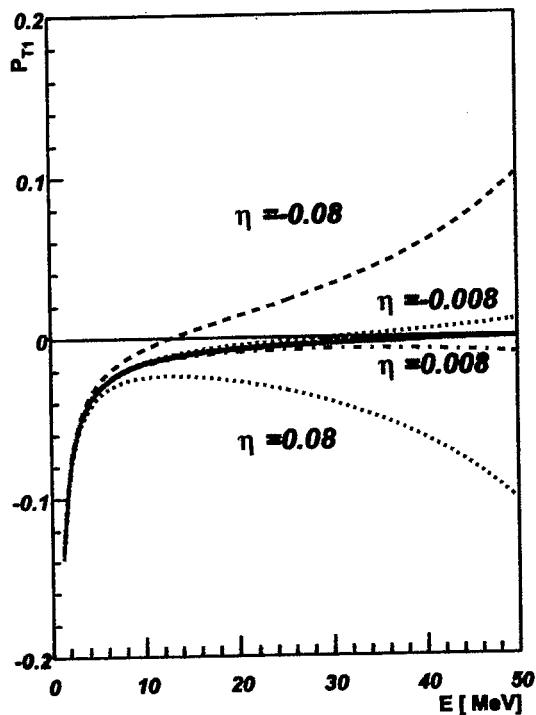
$$g_{LL}^V = 1$$

$$g_{\varepsilon\mu}^{\gamma} = 0 \quad (\text{all other interactions})$$

Assuming 1 additional interaction and knowing that

$$g_{LL}^V \approx 1,$$

one deduces:



$$P_{T_1}(E_e) \rightarrow \eta \approx \frac{1}{2} \operatorname{Re}\{g_{RR}^S\}$$

$$P_{T_2}(E_e) \rightarrow \frac{\beta'}{A} \approx \frac{1}{4} \operatorname{Im}\{g_{RR}^S\}$$

### Main scientific interests:

$P_{T_1}$ : Precise determination of Fermi coupling constant  $G_F$

$P_{T_2}$ : Test of time reversal invariance

## 4. Fermi coupling constant

Should be independent of masses and radiative corrections:

Universal coupling constant

$$G_F^2 = 192\pi^3 \cdot \frac{\hbar}{\tau_\mu} \cdot \frac{1}{m_\mu^5} \cdot$$

$$\left\{ 1 + \frac{\alpha}{2\pi} \left( \pi^2 - \frac{25}{4} \right) \right\} \cdot \left\{ 1 - \frac{3}{5} \left( \frac{m_\mu}{m_W} \right)^2 \right\}.$$

$$\left\{ 1 - 4\eta \cdot \frac{m_e}{m_\mu} - 4\lambda \cdot \frac{m_{\nu_\mu}}{m_\mu} + 8 \left( \frac{m_e}{m_\mu} \right)^2 + 8 \left( \frac{m_{\nu_\mu}}{m_\mu} \right)^2 \right\}$$

New:

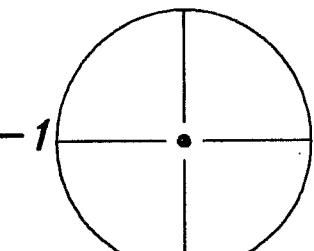
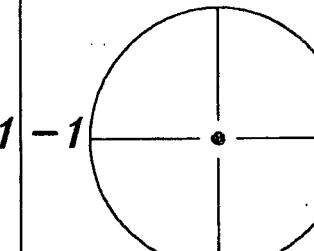
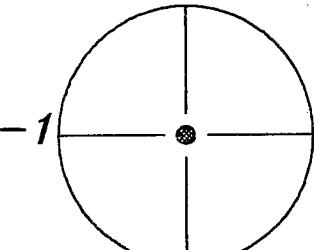
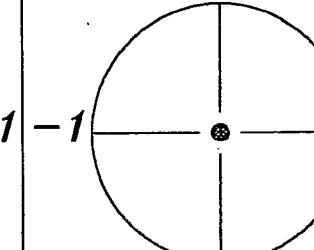
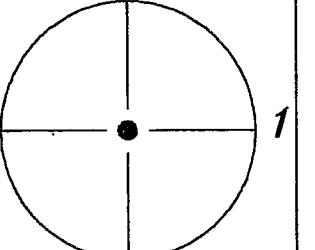
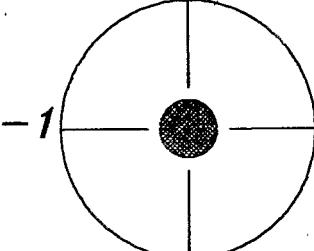
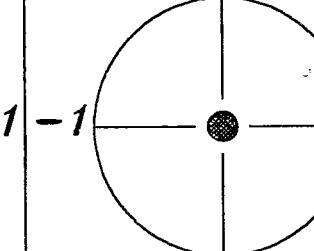
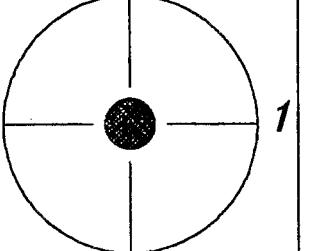
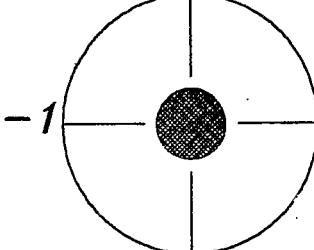
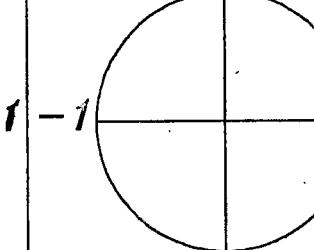
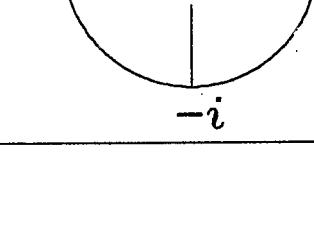
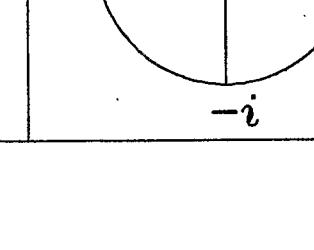
$$\lambda \approx \frac{1}{2} \operatorname{Re} \{ g_{LL}^V \cdot g_{LR}^{V*} \}$$

In left-right symmetric models with mixing angle  $\zeta$ :

$$\lambda \approx \frac{1}{2} \zeta$$

Contribution from	$\frac{\Delta G_F}{G_F} [10^{-6}]$	
	$\mu^+ \rightarrow \bar{\nu}_\mu e^+ \nu_e$	$\tau^+ \rightarrow \bar{\nu}_\tau \mu^+ \nu_\mu$
$\Delta m_W$	0.0	1
$\Delta m_{\mu,\tau}$	0.2	421
$\Delta \tau$	9.1	2 070
$\Delta(\lambda m_{\bar{\nu}})$	70.0	22 500
$\Delta \eta$	193.0	6 500
$\Delta \Gamma_{\tau \rightarrow \mu}$	-	100 000

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

$g_{\varepsilon\mu}^{\gamma_1}$	$S$	$V$	$T$
	$i$ 	$i$ $V+A$ 	
$e_R \mu_R$	$i$ 	$i$ 	$i$ 
$e_L \mu_R$	$i$ 	$i$ 	$i$ 
$e_R \mu_L$	$i$ 	$i$ $V-A$ 	
$e_L \mu_L$	$i$ 	$i$ 	

## 5. Experimental method

Measure the complete polarization vector  
of the decay positrons:

$$\mathbf{P}_{e^+} = \begin{pmatrix} P_{T_1} \\ P_{T_2} \\ P_L \end{pmatrix} \equiv \begin{pmatrix} P_T \cdot \cos \varphi \\ P_T \cdot \sin \varphi \\ P_L \end{pmatrix}$$

with 3 simultaneous and independent measurements:

Observable	Method
$P_T$	Time dependence of annihilation
$\varphi$	Remnant $\mu$ SR effect
$P_L$	Spatial dependence of annihilation

## 6. Experimental setup

6.1 Highly polarized  $\mu^+$  beam at  $\mu$ E1 area  
of PSI: (91%)

6.2 Muon stop rate in Be target:  
 $(20 - 80) \times 10^6 \text{ s}^{-1}$

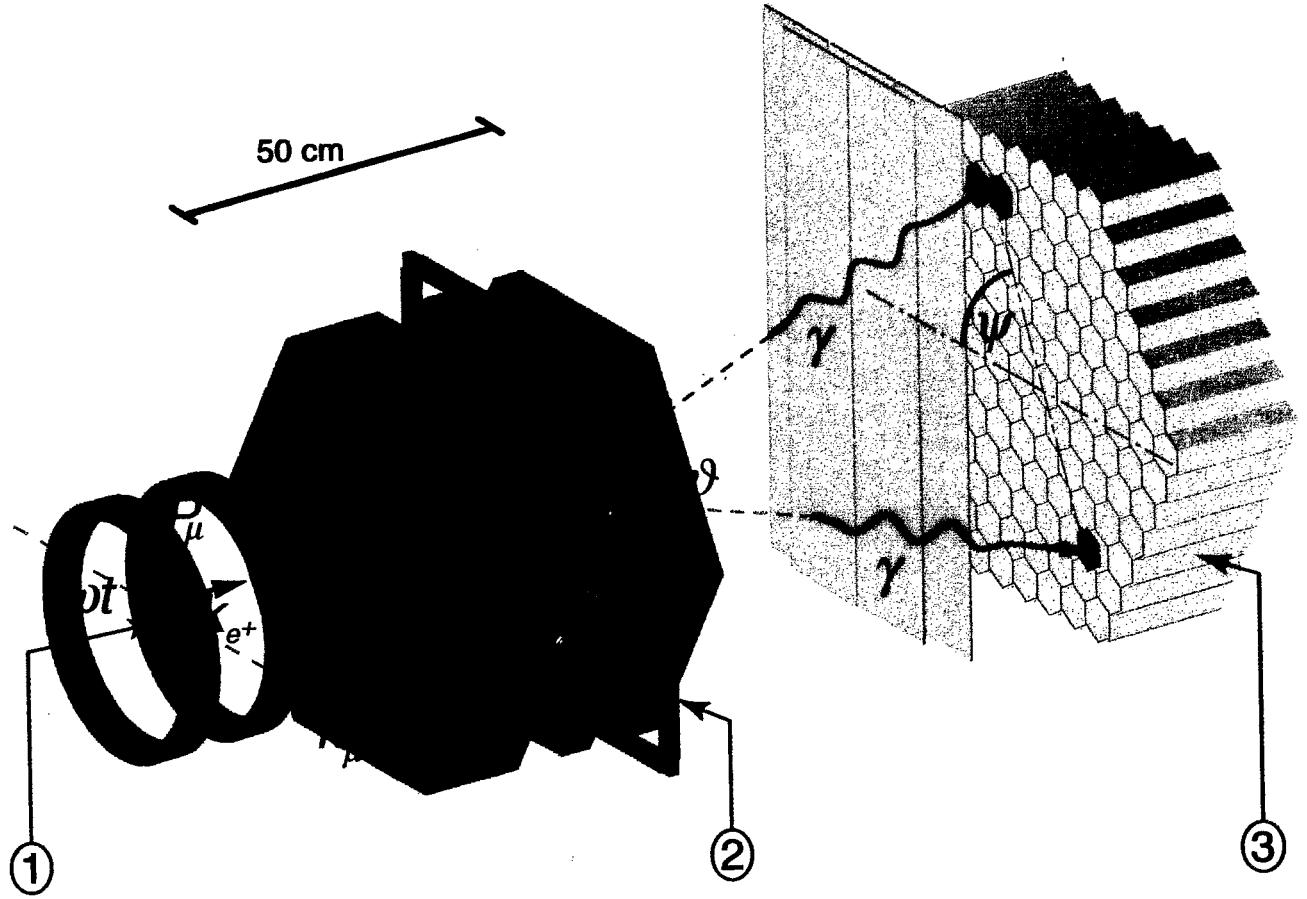
6.3 Precession in homogeneous  $B$  field;  
precession frequency = cyclotron frequency  
(50.8 MHz)

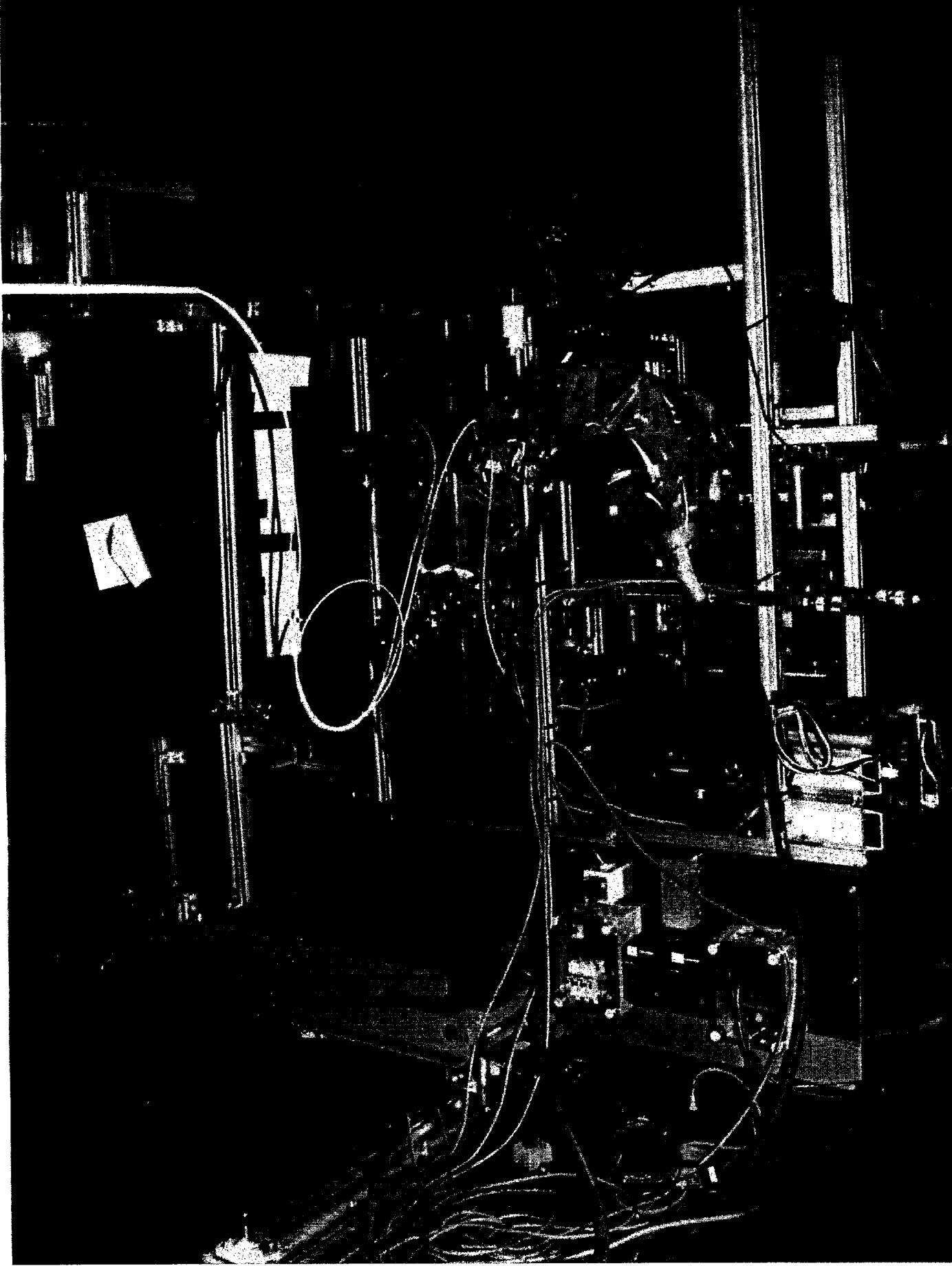
6.4 Burst width 3.9 ns (FWHM)  
 $\implies$  80% muon polarization in Be stop target

6.5 Positron tracking with drift chambers

6.6 Annihilation with polarized  $e^-$

6.7 Detection of annihilation quanta with 127 BGO  
crystals





# **7. Event reconstruction and data analysis**

**7.1  $e^+$  annihilation-in-flight as analysing reaction for transverse polarization**

**7.2 Muon decay asymmetry ( $\mu$ SR yields time zero)**

**7.3 Energy calibration with cosmic rays**

**7.4 Cluster recognition**

**7.5 Background suppression**

**7.6 Analysing power of annihilation events**

**7.7 Energy distribution of transverse polarization**

**7.8 Longitudinal polarization**

$P_\mu$

$P_{e^-}$

Selected Positrons •

$\mu$ SR Effect

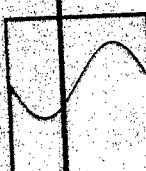
$P_T$

Selected Positrons •

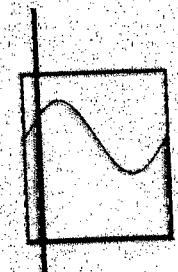
$P_T$  Effect



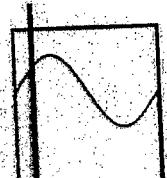
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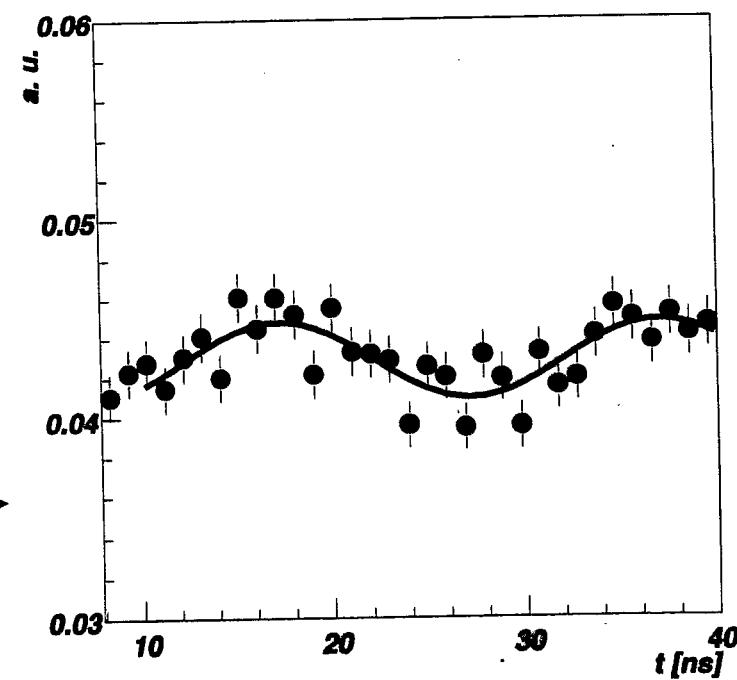
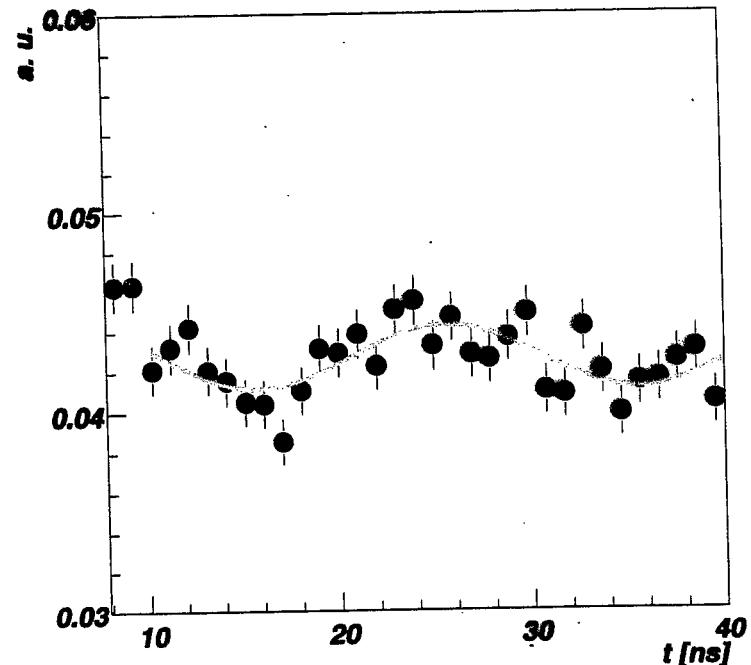
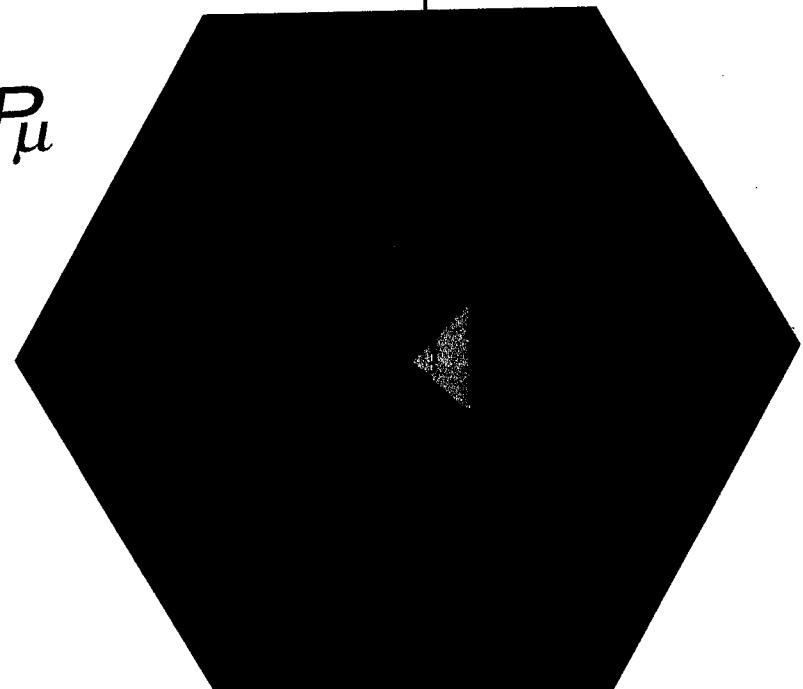


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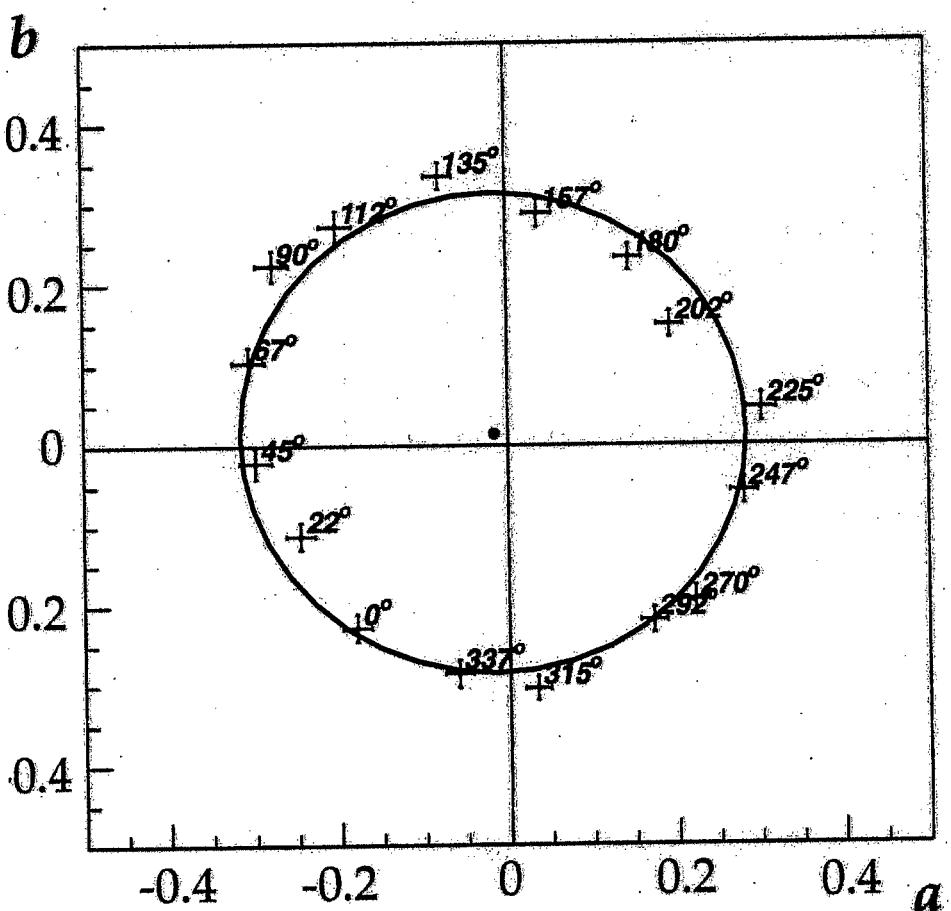
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$P_\mu$



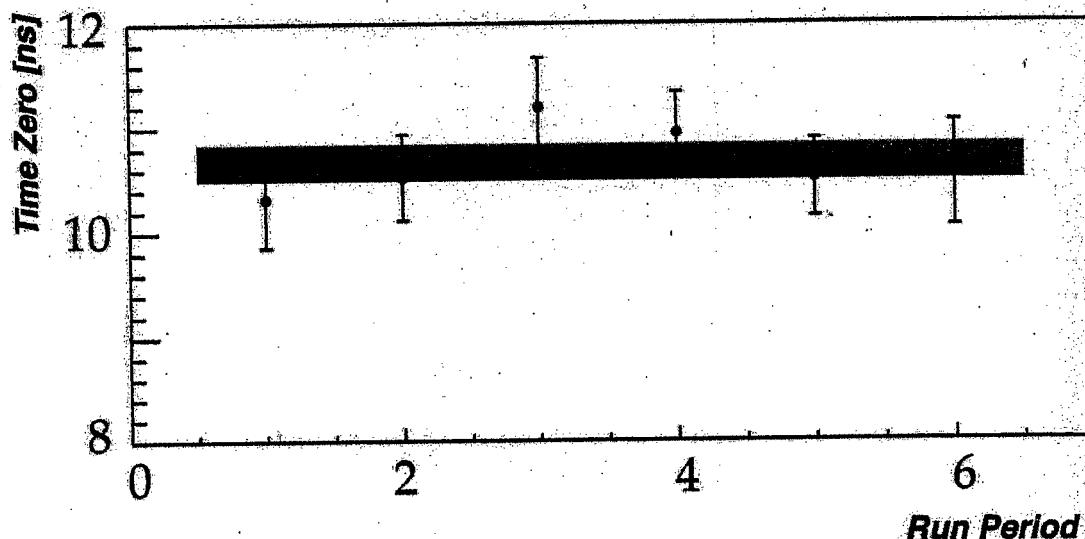
# Phases and Amplitudes for all azimuthal bins ( $\phi$ )

$$N \sim (1 + a \cos \omega t + b \sin \omega t)$$



Average  $\mu$ SR amplitude:  $0.297 \pm 0.004$ ,  
consistent with theory.

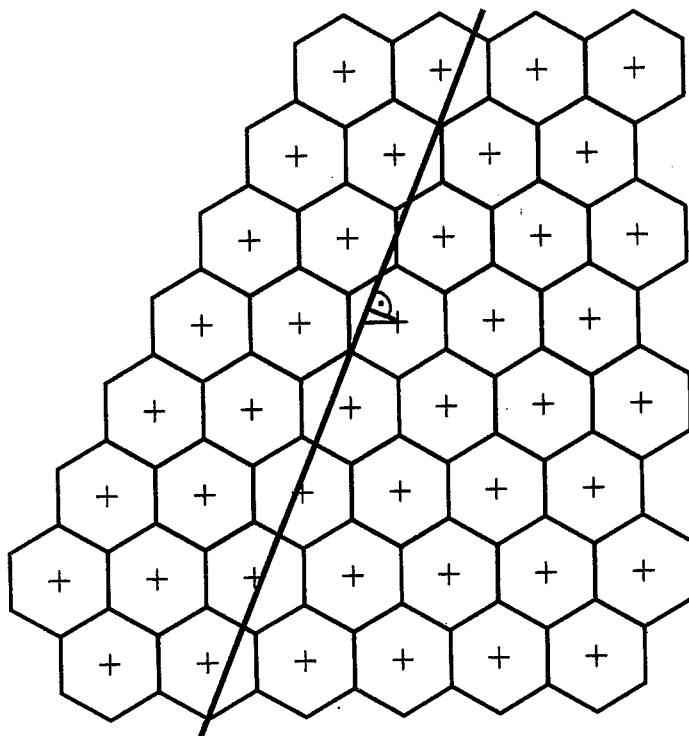
Stability of the Time-Zero during the whole run



Time-Zero :  $10.67 \pm 0.187$  ns gives the time,  
when the muon spin shows upward.

# **Data is Calibrated with Cosmic Muons**

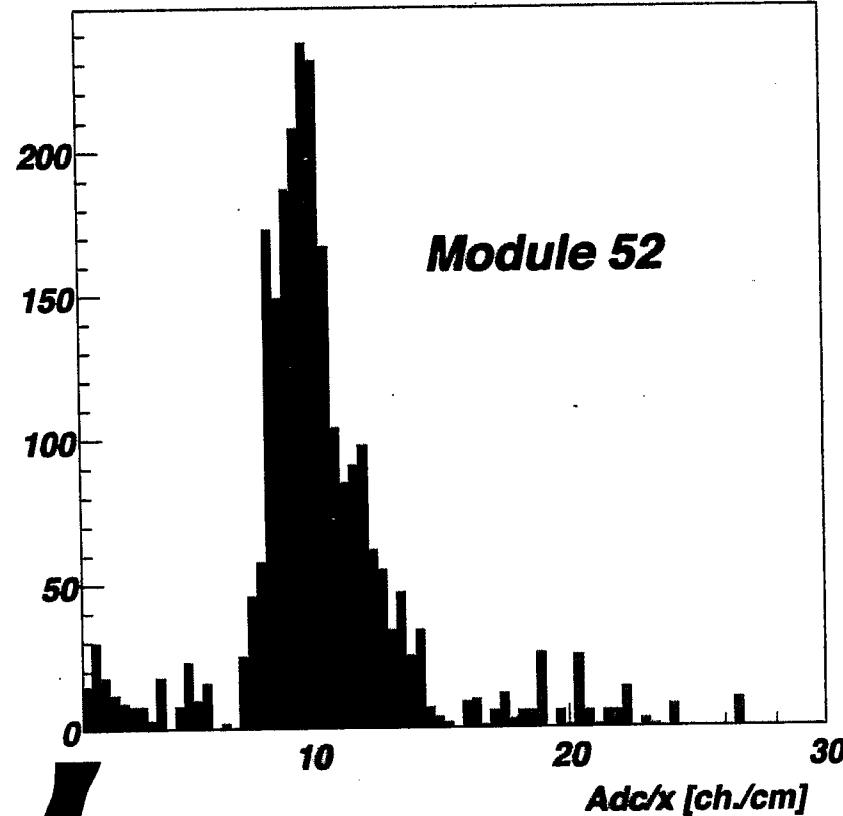
**Reconstructing the cosmic tracks  
gives the tracklength in BGOs**



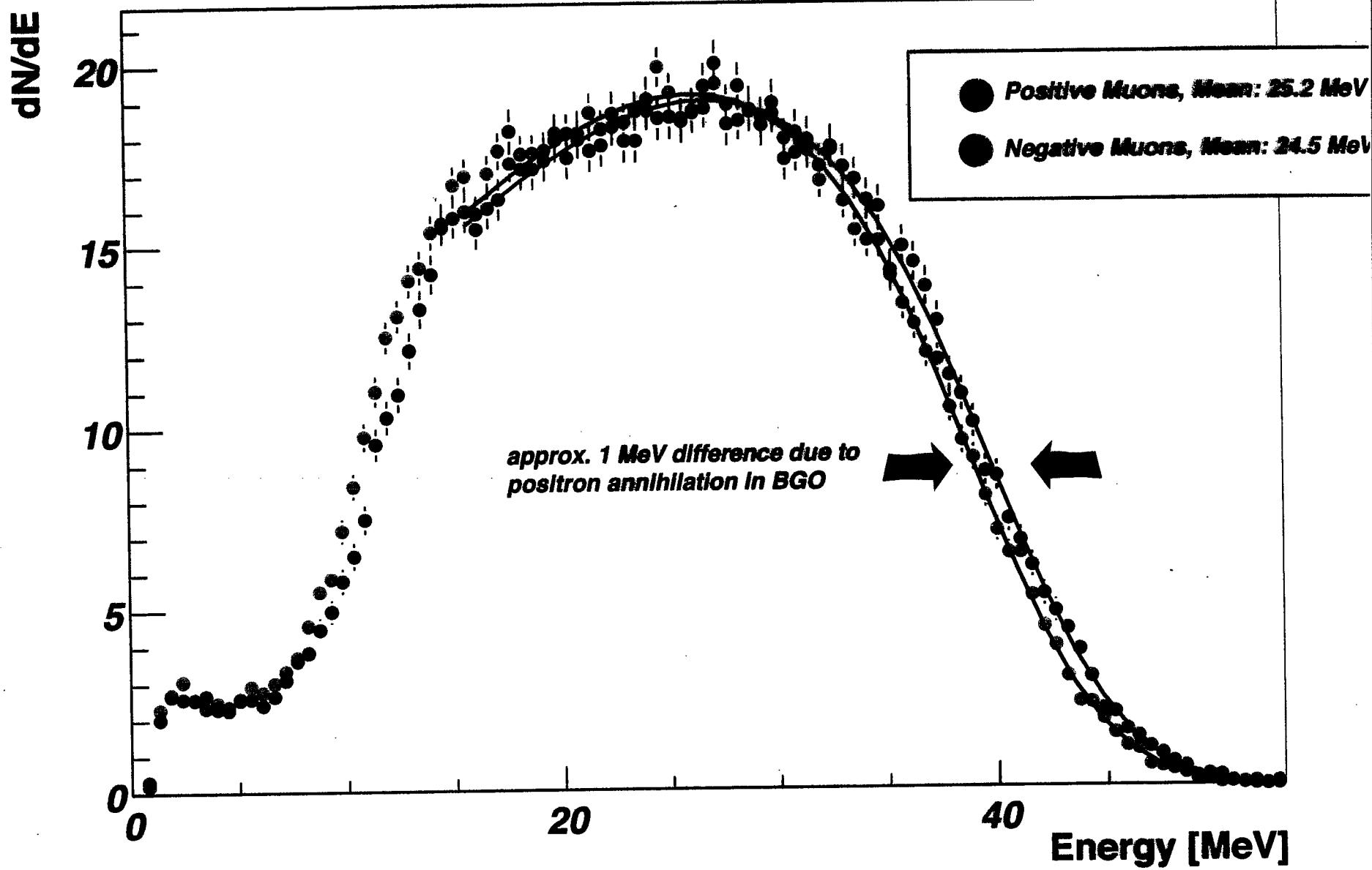
**Monte Carlo**

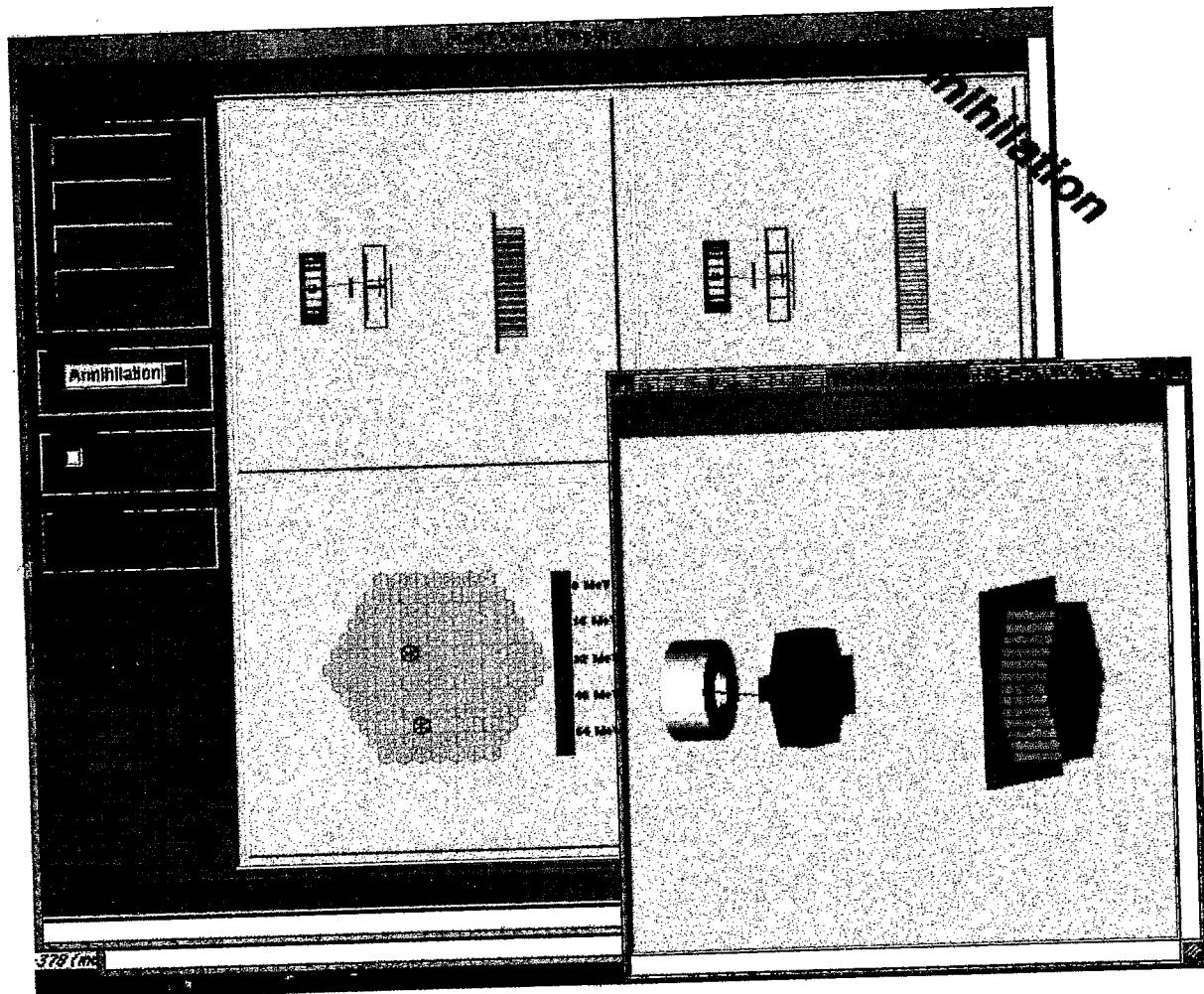
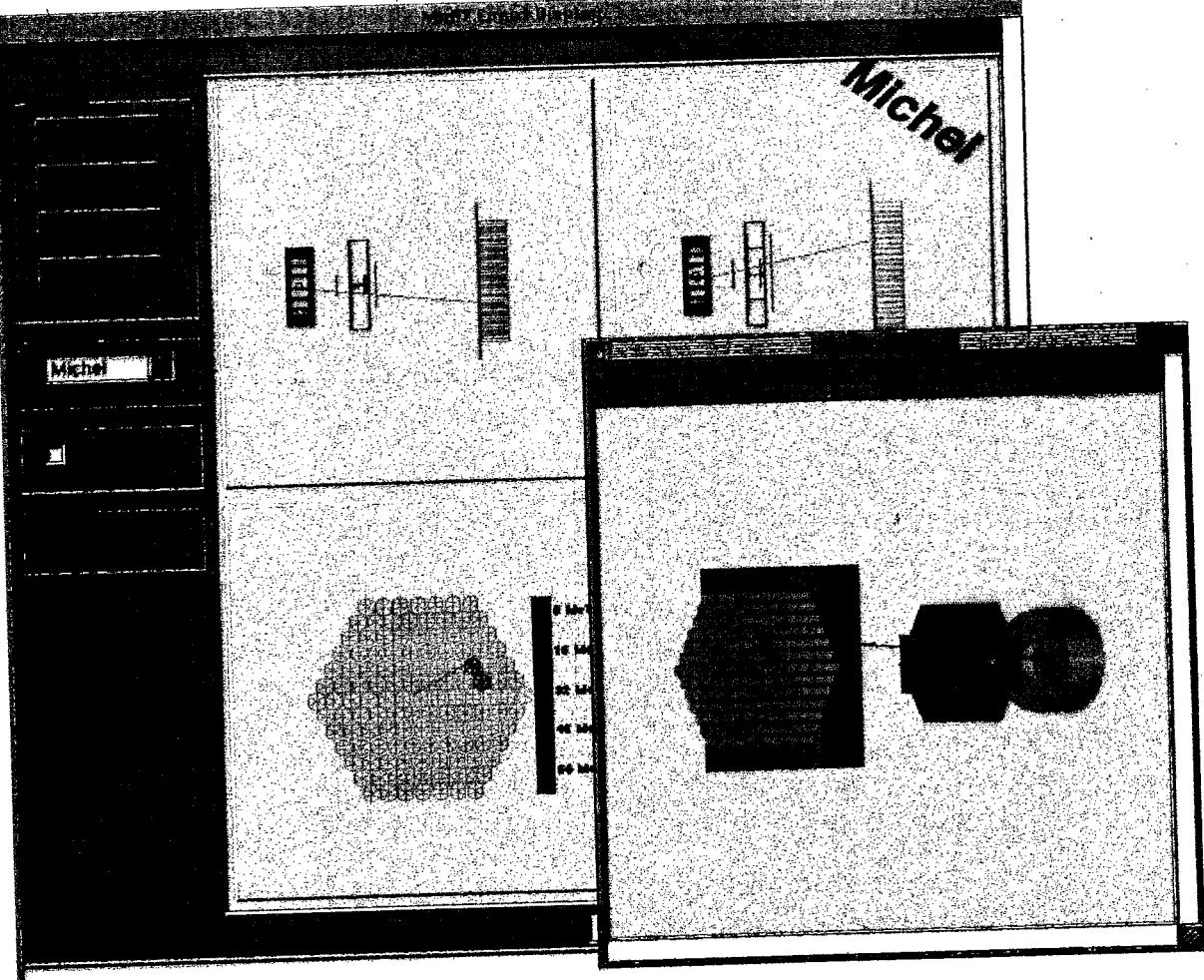
$$\frac{1}{E_i/x_i} \times \frac{ADC_i}{x_i} = \frac{ADC_i}{E_i} = c_i$$

**Histogram Adc/x for cosmics**



**Calibration constant  
for BGO number  $i$**





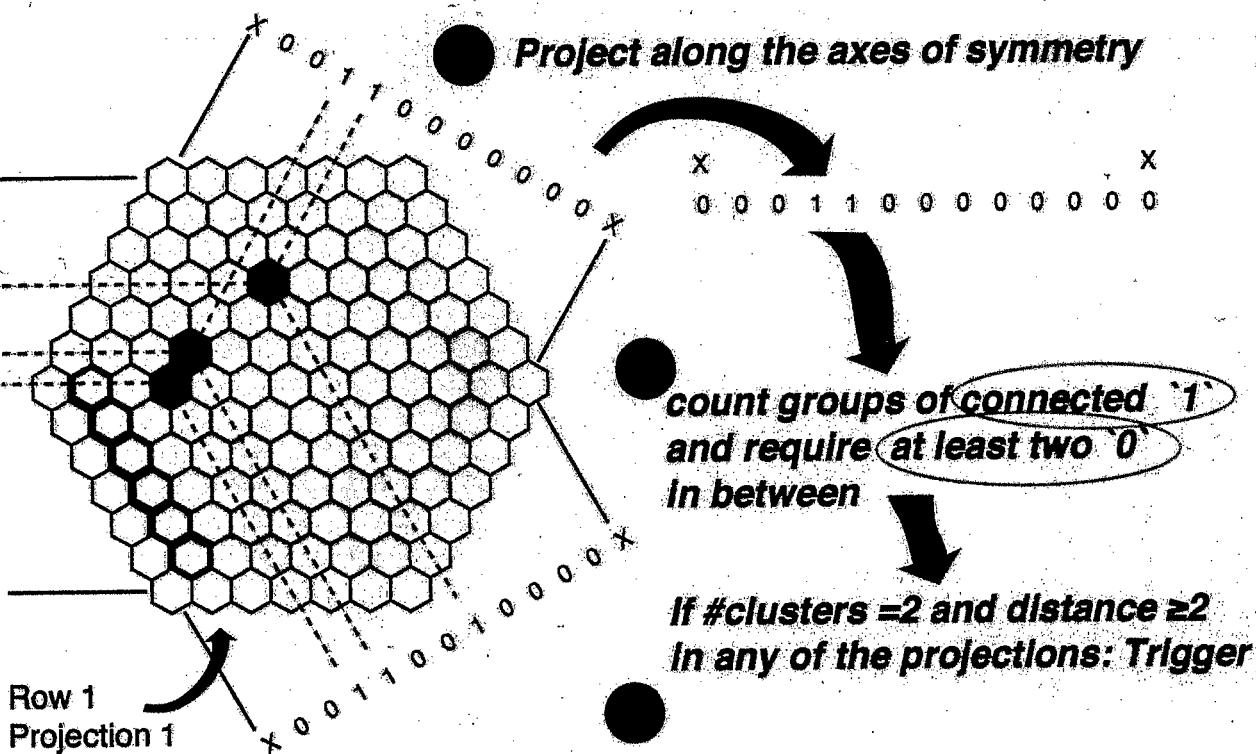
# Triggering on Two Photons at a minimal Distance: The Cluster Recognition Unit

How does it work?

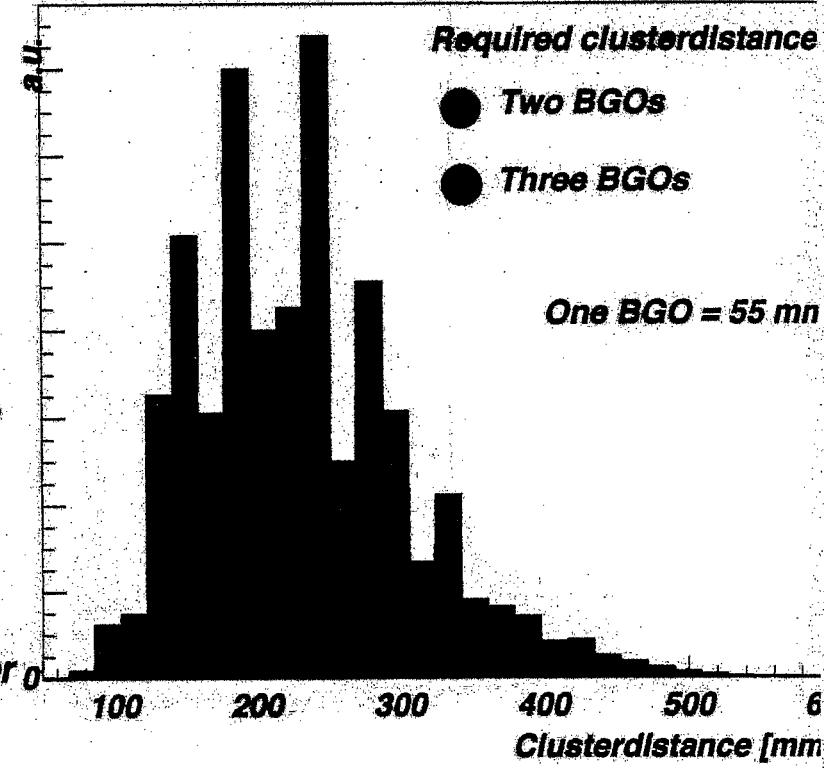
Kinematics require a  
minimal cluster distance

$$\cos \theta = 1 - m_e \frac{4}{E_{e^+}} \quad d = 2z \tan \frac{\theta}{2}$$
$$d_{min} = d(E_{e^+} = 50 \text{ MeV}) \approx 16 \text{ cm}$$

FPGA approach allows redefining  
trigger conditions 'on the fly'

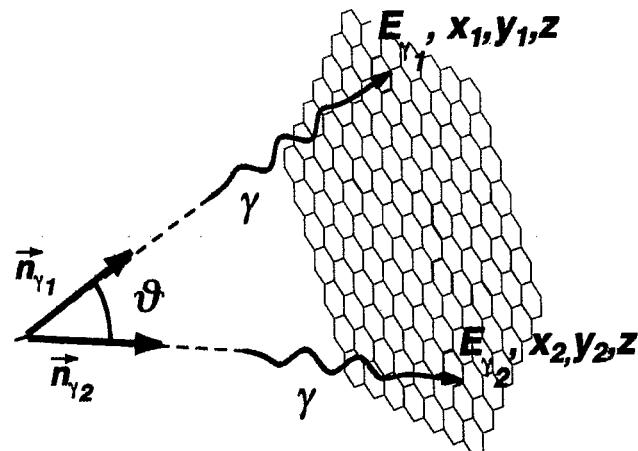


Distance between two clusters



# *Cut on the Kinematics to Extract the 'Good' Events*

**Calculate  $\vartheta$  in two different ways:**



**Energy:**

$$\cos \vartheta = 1 - m_e \frac{E_{\gamma_1} + E_{\gamma_2}}{E_{\gamma_1} \cdot E_{\gamma_2}}$$



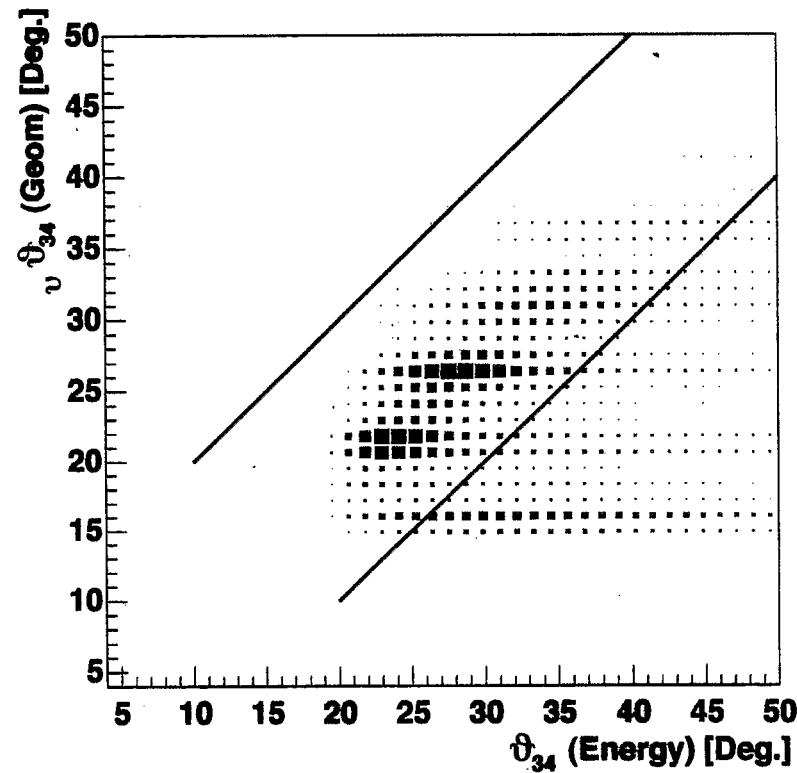
$$\vartheta^{\text{Geom}} = \vartheta^{\text{Energy}}$$

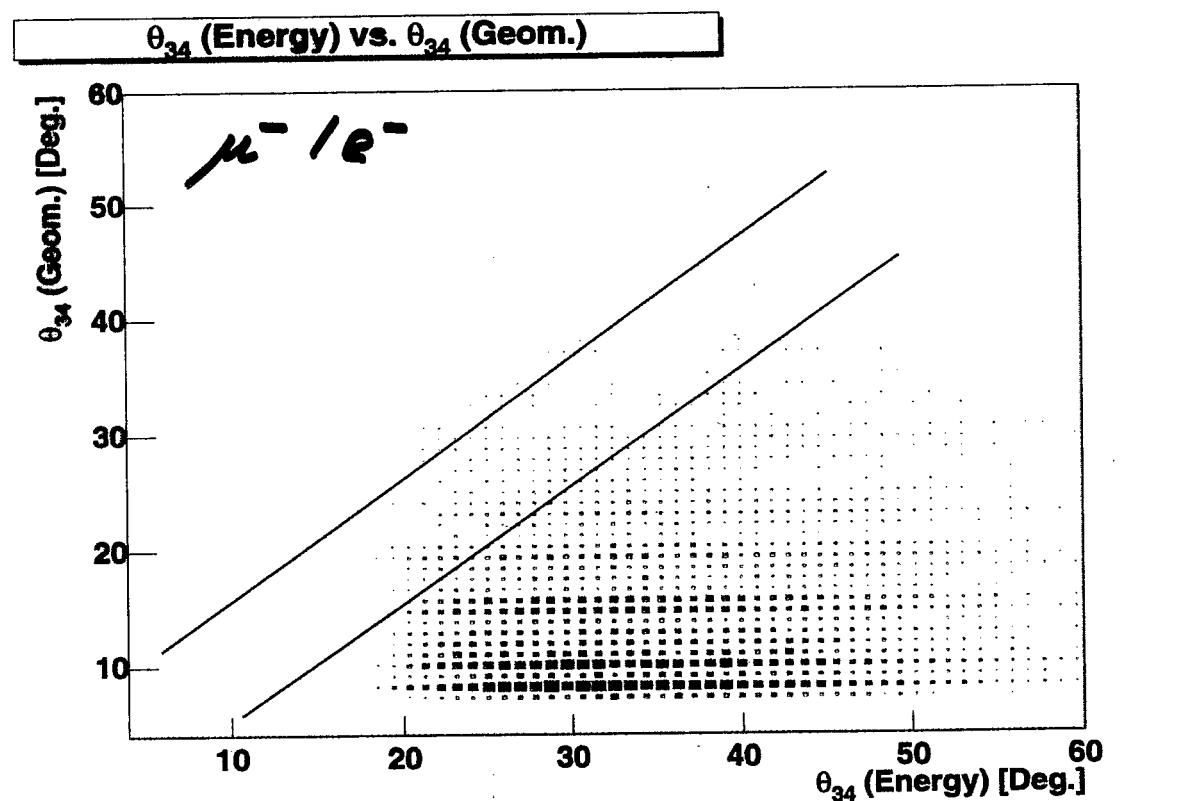
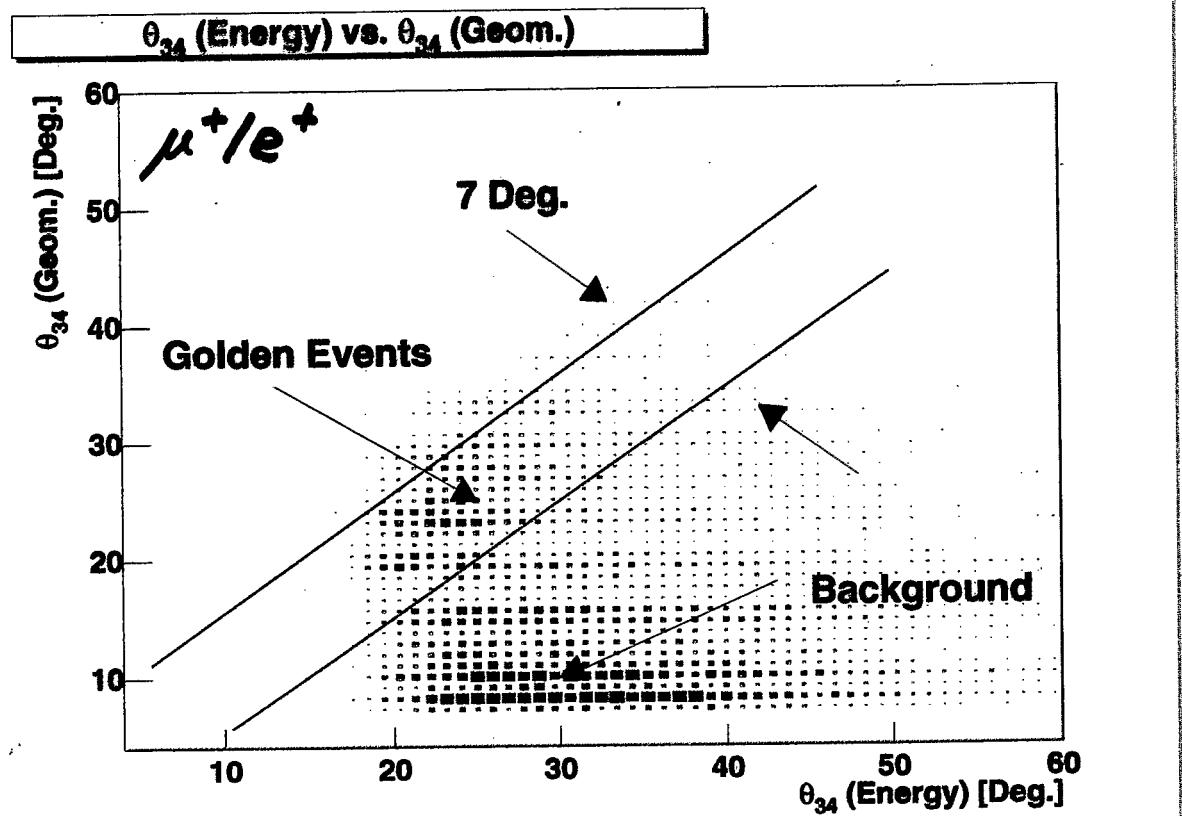
**Geometry**

$$\cos \vartheta = \vec{n}_{\gamma_1} \cdot \vec{n}_{\gamma_2}$$

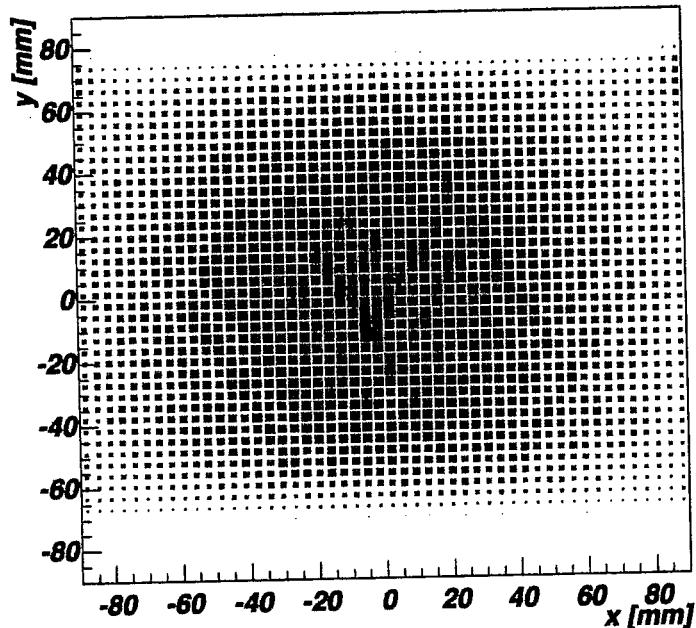
**for 'good' annihilations**

**Sample from the last run**

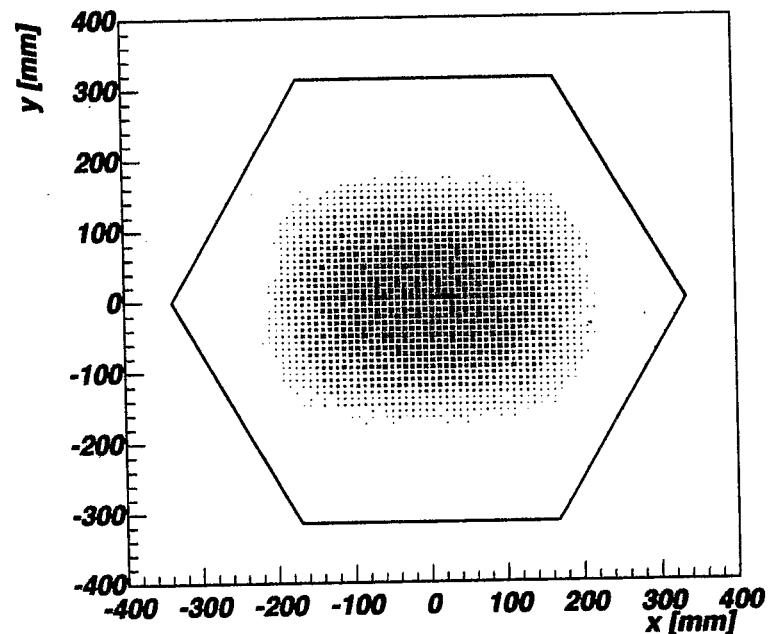




***Where do 'good' annihilations  
come from,  
and where do they go?***



***reconstructed position  
of annihilations on the foil***

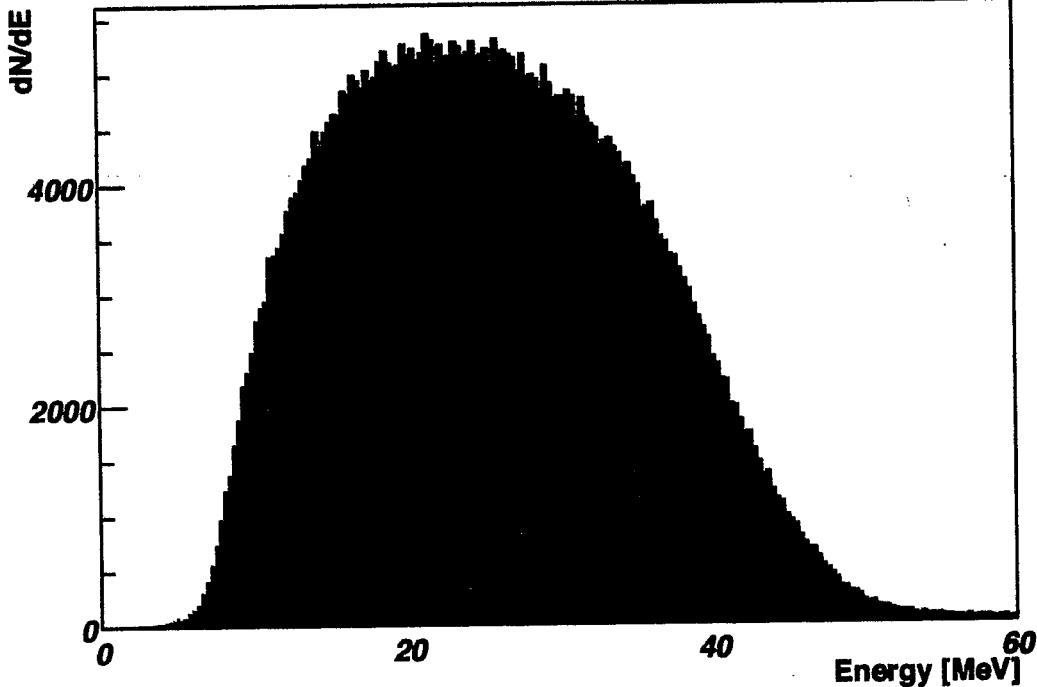


***center of energy on the BGO-wall***

## **Analysis, Step Two: From Raw Data to 'Good' Events**

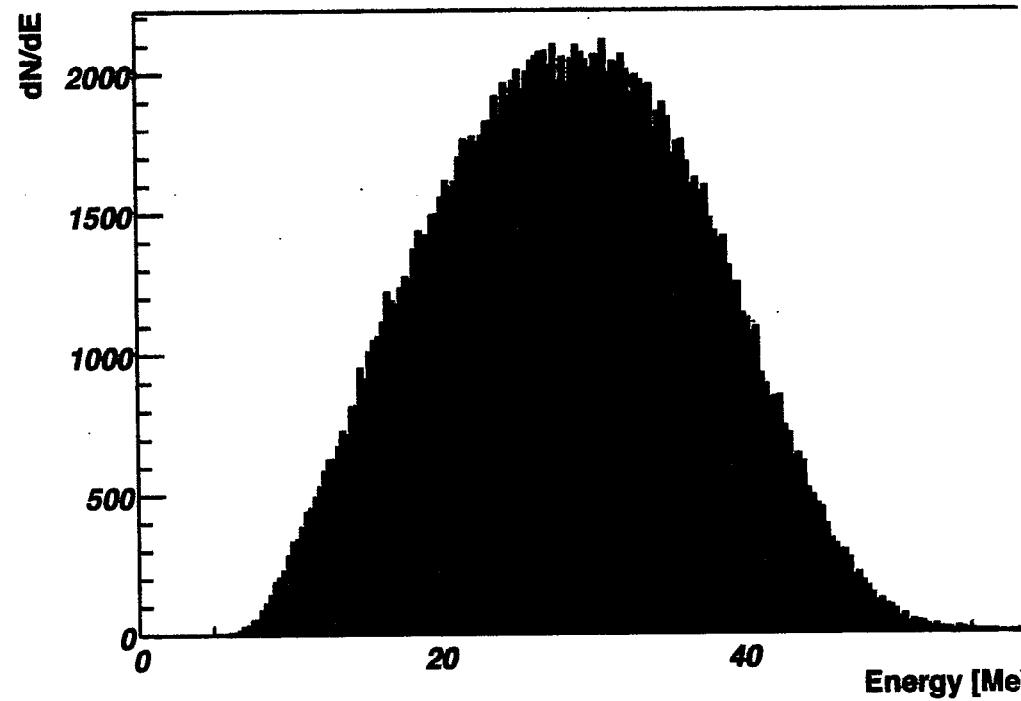
**Energy ( $E_{\text{tot}} = E_{\gamma_1} + E_{\gamma_2}$ ) spectra of annihilation events**

*After reconstruction ( $\approx 50\% \text{ Eff.}$ )*



*charged track reconstructed,  
exactly two clusters in the BGO.*

*After selection ( $\approx 34\% \text{ Eff.}$ )*



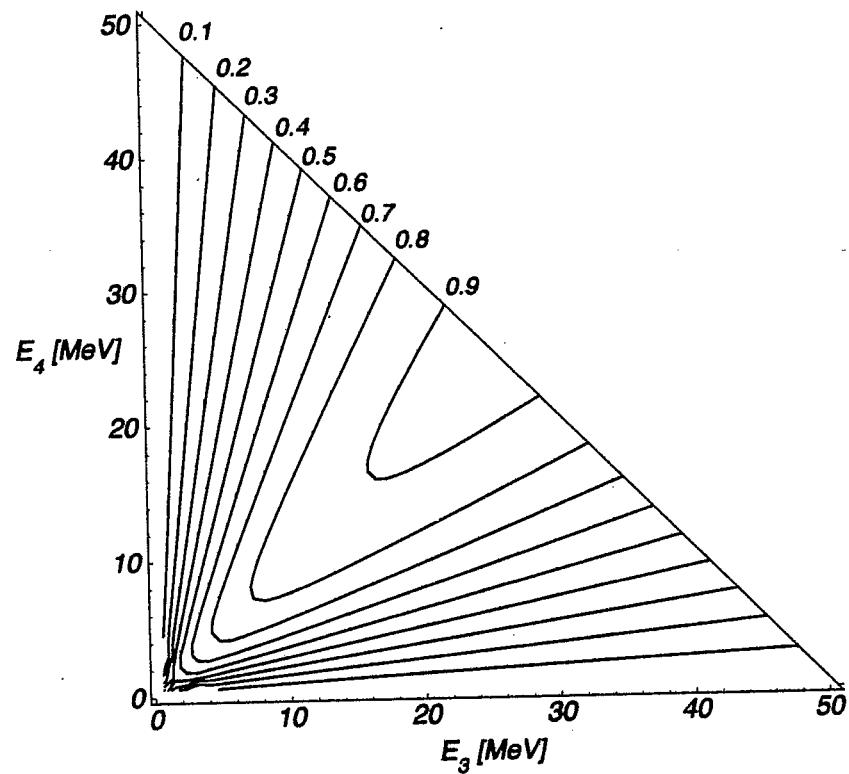
*event kinematics must be  
consistent with annihilation  
hypothesis.*

*After all cuts  $\approx 17\%$  'good' annihilation events remain.*

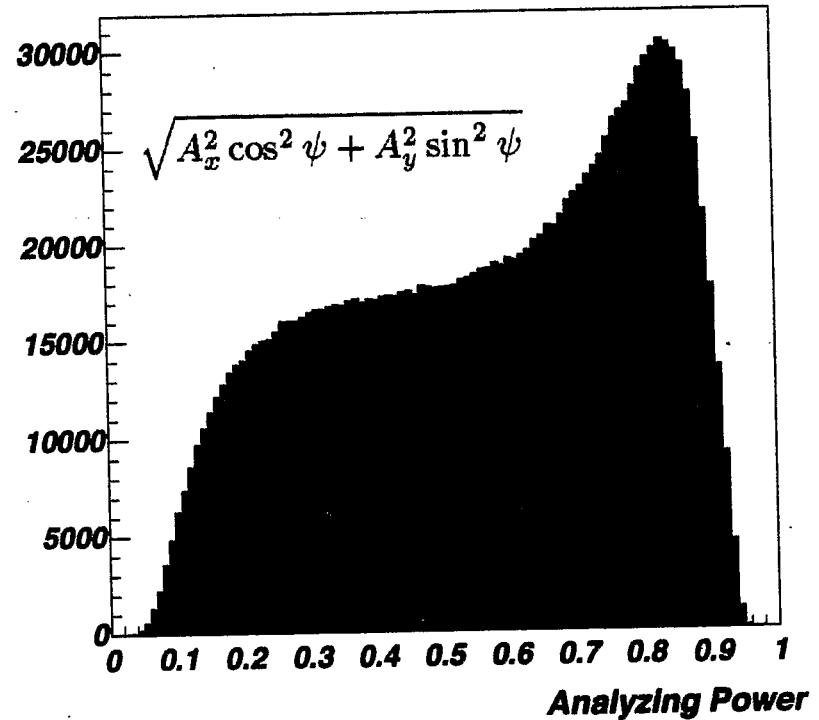
# Theoretical and Experimental Analyzing Power

**The analyzing power is the amplitude of the expected oscillation**

$$A = S \cdot \sqrt{P_1^2 + P_2^2} \cdot \sqrt{A_x^2 \cos^2 \psi + A_y^2 \sin^2 \psi}$$



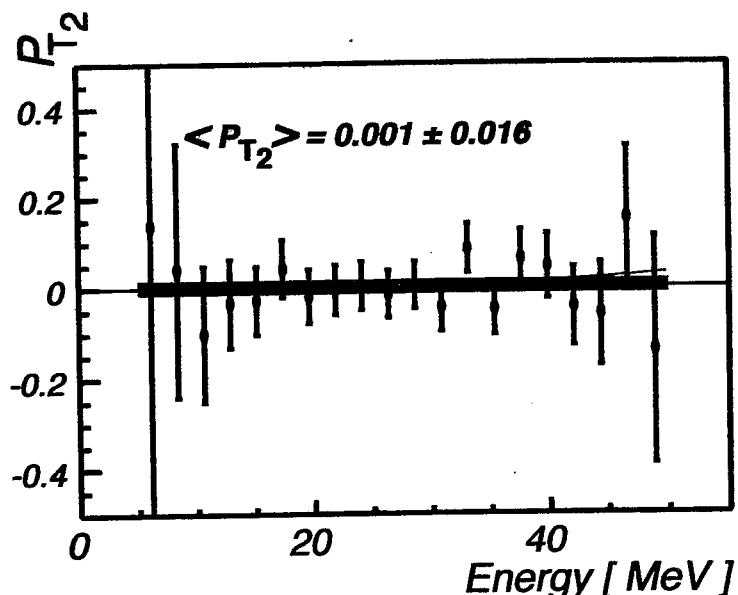
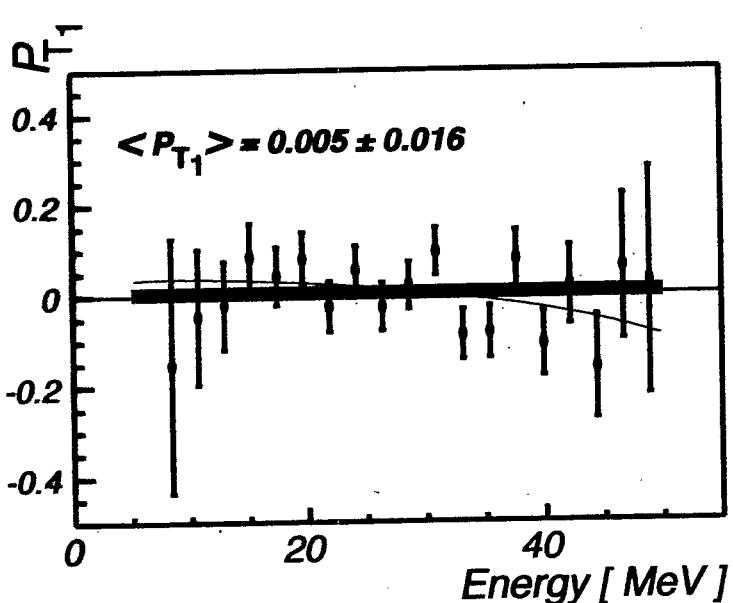
Theoretical analyzing power,  
 $\Psi = 90^\circ$ ,  $P_{e^-} = 100\%$ ,  $|IP_T| = 1$



Analyzing Power of 'good' annihilation events.  
 $A_x$  and  $A_y$  are functions of the photon energies

## Transverse Polarization Components $P_{T_1}$ and $P_{T_2}$

- time zero from  $\mu$ SR Effect (→ orientation of muon spin relative to  $P_1$  and  $P_2$ )
- rotation of transverse polarization components in the field of the spin precessing magnet ( MC )
- convolution with energy loss of the positron in the apparatus ( MC )
- sums of  $P_{T_1}$  and  $P_{T_2}$  for negative and positive foil polarization

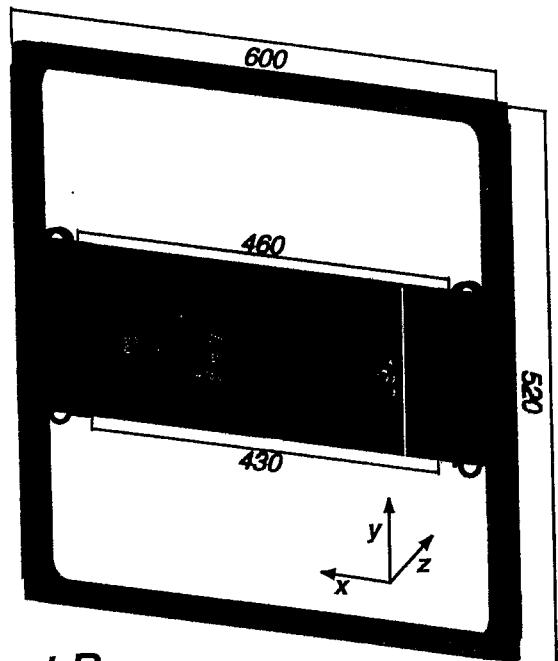


# Measurement of the Longitudinal Polarization

using information about position  
on magnetized Vacoflux foil  
(determined by tracks reconstructed  
from drift-chamber data)  
where annihilations take place

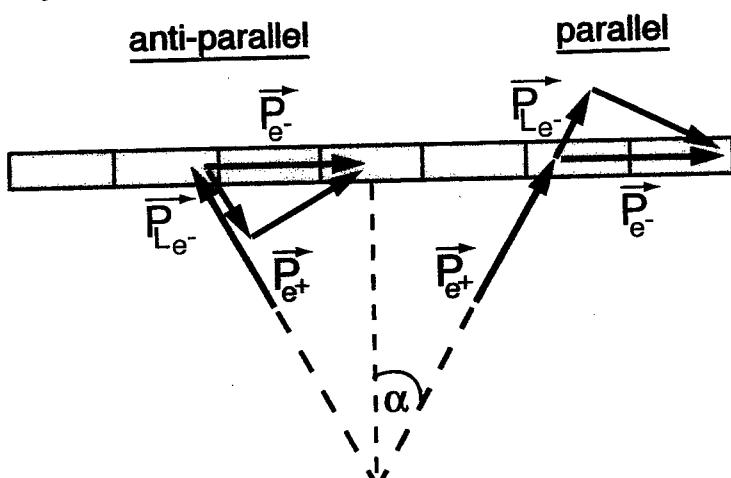
area on foil taken into account:  $140^2 \text{ mm}^2$

area divided into rectangular bins (ij),  
17 bins in x- and y-direction, respectively



*Tracks that do not hit the center  
of the foil 'see' a longitudinal component  $P_{L e^-}$   
of the polarization of the electrons in the foil.*

*This  $P_{L e^-}$ - can either be parallel or anti-parallel  
to the positron polarization :*



# Longitudinal Polarization $P_L$ of the Positrons

annihilation cross section depends  
on relative orientations of spins;  
it is larger if both spins  
are anti-parallel

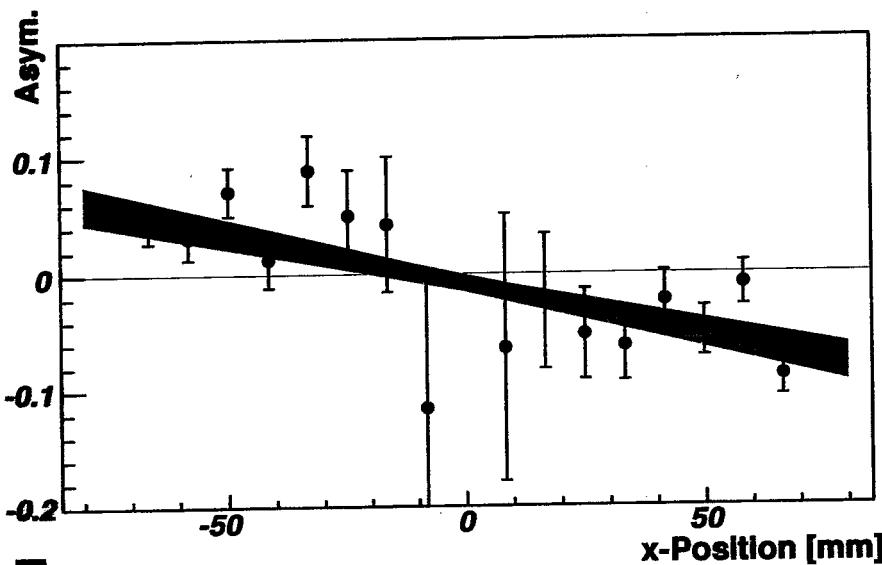


$$\text{Asymmetry: } A_{ij} = \frac{n_{ij}^- - n_{ij}^+}{n_{ij}^- + n_{ij}^+}$$

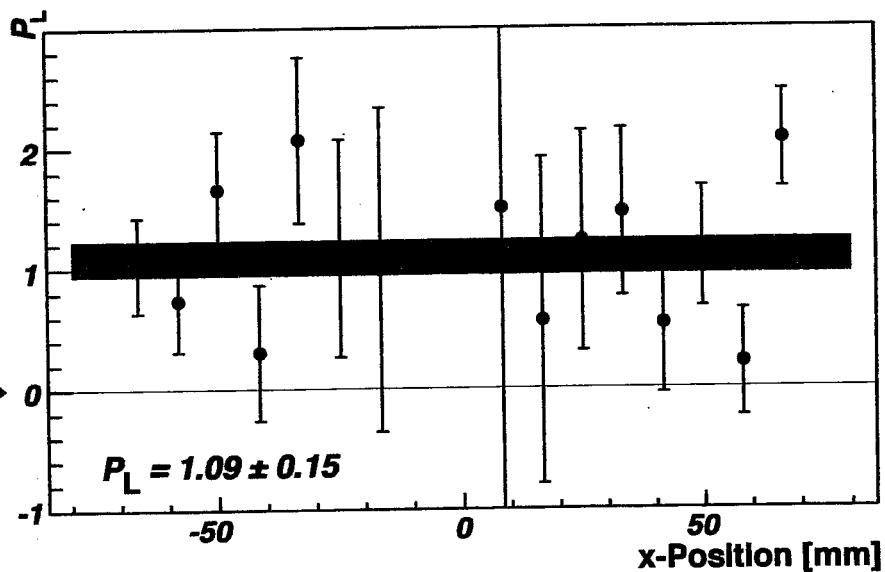
where  $n_{ij}^+$  : number of annihilations in bin  $ij$   
for positive foil polarization

$N^+$  : total number of annihilations  
for positive polarization

$n_{ij}^-, N^-$ : same for negative polarization



- angle  $\alpha$
- elektron polarization in foil ( $P_{e^-} = 7.2\%$ )
- analysing power of 0.79
- background factor of 0.75 (backgr. ratio 25 %, mainly due to bremsstrahlung)



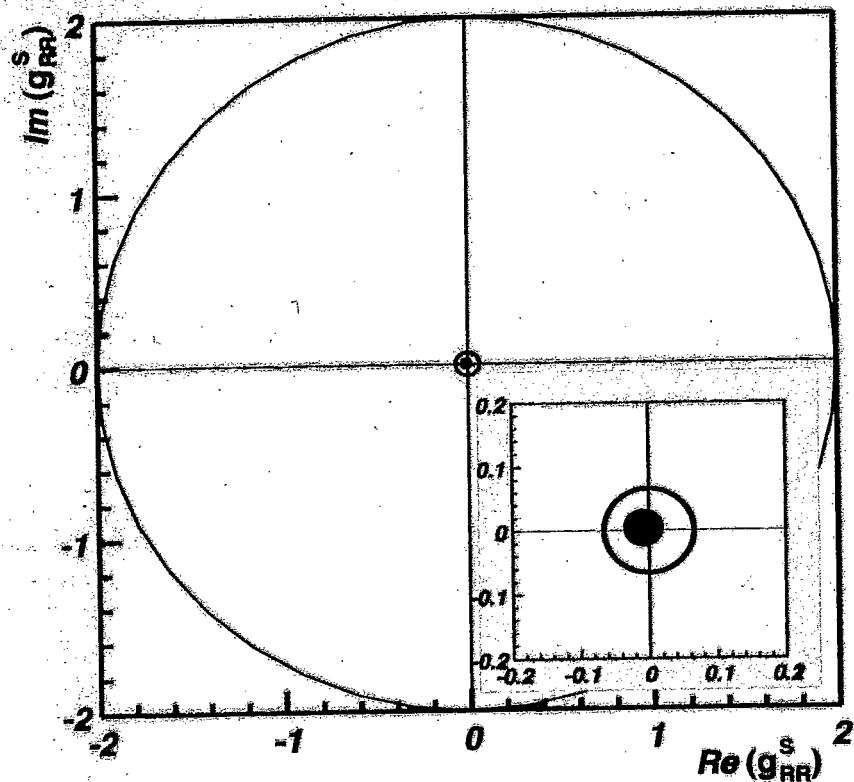
## 8. Preliminary Results

	General analysis	$V - A$ + $g_{RR}^S$
$10^3 \times \langle P_{T_1} \rangle_E$	5 ± 16	5 ± 16
$10^3 \times \langle P_{T_2} \rangle_E$	1 ± 16	1 ± 16
$10^3 \times \eta$	95 ± 60	-4 ± 14
$10^3 \times \eta''$	98 ± 57	$-10^3 \times \eta$
$10^3 \times \alpha'/A$	-13 ± 29	0
$10^3 \times \beta'/A$	8 ± 16	1 ± 7
$10^3 \times \text{Re } g_{RR}^S$	—	-8 ± 28
$10^3 \times \text{Im } g_{RR}^S$	—	4 ± 28
$10^3 \times  g_{RR}^S $	—	9 ± 28



## Implications from the Results

### 1. No Evidence for Additional Scalar Couplings in Muon Decay



Green circle: Result of a general analysis including all possible left- and righthanded scalar, vector and tensor couplings.

Red circle: Only one additional righthanded scalar coupling interferes with the lefthanded vector coupling in the SM.

$$\eta = -0.004 \pm 0.014$$

$$\frac{\beta'}{A} = 0.001 \pm 0.007$$

$$Im(g_{RR}^S) = 0.004 \pm 0.028$$

$$Re(g_{RR}^S) = -0.008 \pm 0.028$$

$$|g_{RR}^S| = 0.009 \pm 0.028$$

## 8. Outlook

Improve precision of previous experiment [1] by almost one order in magnitude to:

$$\Delta \langle P_{T_1} \rangle = 0.004$$

$$\Delta \langle P_{T_2} \rangle = 0.004$$

Assuming  $V - A$  and one additional coupling , this will reduce the limits for  $\eta$  and  $g_{RR}^S$  to

$$\Delta\eta = 0.004$$

$$\Delta Re \{ g_{RR}^S \} = 0.009$$

$$\Delta Im \{ g_{RR}^S \} = 0.009$$

[1] H. Burkard, F. Corriveau, J. Egger, W. Fettscher, H.-J. Gerber, K.F. Johnson, H. Kaspar, H-J. Mahler, M. Salzmann, F. Scheck

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