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Theoretical Expectations for the Muon's Electric Dipole Moment

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Abstract

We examine the muon's electric dipole moment d_μ from a variety of theoretical perspectives. We point out that the reported deviation in the muon's $g-2$ can be due partially or even entirely to a new physics contribution to the muon's *electric* dipole moment. In fact, the recent $g-2$ measurement provides the most stringent bound on d_μ to date. This ambiguity could be definitively resolved by the dedicated search for d_μ recently proposed. We then consider both model-independent and supersymmetric frameworks. Under the assumptions of scalar degeneracy, proportionality, and flavor conservation, the theoretical expectations for d_μ in supersymmetry fall just below the proposed sensitivity. However, non-degeneracy can give an order of magnitude enhancement, and lepton flavor violation can lead to $d_\mu \sim 10^{-22}$ e cm, two orders of magnitude above the sensitivity of the d_μ experiment. We present compact expressions for leptonic dipole moments and lepton flavor violating amplitudes. We also derive new limits on the amount of flavor violation allowed and demonstrate that approximations previously used to obtain such limits are highly inaccurate in much of parameter space.

I. INTRODUCTION

Electric dipole moments (EDMs) of elementary particles are predicted to be far below foreseeable experimental sensitivity in the standard model. In extensions of the standard model, however, much larger EDMs are possible. Current EDM bounds are already some of the most stringent constraints on new physics, and they are highly complementary to many other low energy constraints, since they require CP violation, but not flavor violation.

The field of precision muon physics will be transformed in the next few years [1]. The EDM of the muon is therefore of special interest. A new experiment [2] has been proposed to measure the muon's EDM at the level of

$$d_\mu \sim 10^{-24}\ e\ \text{cm}\ , \tag{1}$$

more than five orders of magnitude below the current bound [3]

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19}\ e\ \text{cm}\ . \tag{2}$$

The interest in the muon's EDM is further heightened by the recent measurement of the muon's anomalous magnetic dipole moment (MDM) $a_\mu = (g_\mu - 2)/2$, where g_μ is the muon's gyromagnetic ratio. The current measurement $a_\mu^{\text{exp}} = 11\ 659\ 202\,(14)\,(6) \times 10^{-10}$ [4] from the Muon $(g-2)$ Experiment at Brookhaven differs from the standard model prediction a_μ^{SM} [5,6] by 2.6σ:

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (43 \pm 16) \times 10^{-10}\ . \tag{3}$$

The muon's EDM and a_μ arise from similar operators, and this tentative evidence for a non-standard model contribution to a_μ also motivates the search for the muon's EDM.

In this study, we examine the prospects for detecting a non-vanishing muon EDM from a variety of theoretical perspectives. We first note that the reported deviation in the muon's $g-2$ can be due partially or even entirely to a new physics contribution to the muon's *electric* dipole moment. In fact, at present the result from the Muon $(g-2)$ Experiment provides the most stringent bound on d_μ. We derive this bound and comment on the conclusions that may be drawn about d_μ from the a_μ measurement alone.

We then move to more concrete frameworks, where additional correlations constrain our expectations. In particular, we consider supersymmetry and examine quantitatively the implications of the electron EDM and lepton flavor violating processes [7,8]. Our aim is to impose as few theoretical prejudices as possible and draw correspondingly general conclusions. For studies of the muon EDM in specific supersymmetric models, see, *e.g.*, Refs. [9,10].

Finally, although we use exact expressions for all flavor-conserving amplitudes in this study, we also provide compact expressions in the mass insertion approximation for branching ratios of radiative lepton decays and for leptonic EDMs and MDMs both with and without lepton flavor violation. These include all leading supersymmetric effects and are well-suited to numerical calculations.

II. MODEL-INDEPENDENT BOUNDS FROM THE MUON $(g-2)$ EXPERIMENT

Modern measurements of the muon's MDM exploit the equivalence of cyclotron and spin precession frequencies for $g = 2$ fermions circulating in a perpendicular and uniform magnetic field. Measurements of the anomalous spin precession frequency are therefore interpreted as measurements of a_μ.

The spin precession frequency also receives contributions from the muon's EDM, however. For a muon traveling with velocity $\boldsymbol{\beta}$ perpendicular to both a magnetic field $\boldsymbol{B}$ and an electric field $\boldsymbol{E}$, the anomalous spin precession vector is

$$\boldsymbol{\omega}_a = -a_\mu \frac{e}{m_\mu}\boldsymbol{B} - d_\mu \frac{2c}{\hbar}\boldsymbol{\beta}\times\boldsymbol{B} - \frac{e}{m_\mu c}\left(\frac{1}{\gamma^2-1} - a_\mu\right)\boldsymbol{\beta}\times\boldsymbol{E} - d_\mu \frac{2}{\hbar}\boldsymbol{E} \; . \tag{4}$$

In recent experiments, the third term of Eq. (4) is removed by running at the 'magic' $\gamma \approx 29.3$, and the last term is negligible. For highly relativistic muons with $|\boldsymbol{\beta}| \approx 1$, then, the anomalous precession frequency is

$$|\boldsymbol{\omega}_a| \approx |\boldsymbol{B}| \left[\left(\frac{e}{m_\mu}\right)^2 \left(a_\mu^{\mathrm{SM}\,2} + 2a_\mu^{\mathrm{SM}} a_\mu^{\mathrm{NP}}\right) + \left(\frac{2c}{\hbar}\right)^2 d_\mu^{\mathrm{NP}\,2}\right]^{1/2} \; , \tag{5}$$

where NP denotes new physics contributions, and we have assumed $a_\mu^{\mathrm{NP}} \ll a_\mu^{\mathrm{SM}}$ and $d_\mu^{\mathrm{NP}} \gg d_\mu^{\mathrm{SM}}$.

The observed deviation from the standard model prediction for $|\boldsymbol{\omega}_a|$ has been assumed to arise entirely from a MDM and has been attributed to a new physics contribution of size Δa_μ. However, from Eq. (5), we see that, more generally, it may be due to some combination of magnetic and electric dipole moments from new physics. More quantitatively, the effect can also be due to an EDM contribution

$$\left|d_\mu^{\mathrm{NP}}\right| \approx \frac{\hbar e}{m_\mu c}\sqrt{\frac{1}{2}\, a_\mu^{\mathrm{SM}}\left(\Delta a_\mu - a_\mu^{\mathrm{NP}}\right)} \approx 3.0\times 10^{-19}\ e\ \mathrm{cm}\,\sqrt{1 - \frac{a_\mu^{\mathrm{NP}}}{43\times 10^{-10}}} \; , \tag{6}$$

where a_μ^{NP} has been normalized to the current central value given in Eq. (3). In Fig. 1 we show the regions in the $(a_\mu^{\mathrm{NP}}, d_\mu^{\mathrm{NP}})$ plane that are consistent with the observed deviation in $|\boldsymbol{\omega}_a|$.

In fact, the observed anomaly may, in principle, be due entirely to the muon's EDM! This is evident from Eqs. (2) and (6), or from Fig. 1, where the current 1σ and 2σ upper bounds on d_μ^{NP} [3] are also shown. Alternatively, in the absence of fine-tuned cancellations between a_μ^{NP} and d_μ^{NP}, *the results of the Muon $(g-2)$ Experiment also provide the most stringent bound on d_μ to date*, with 1σ and 2σ upper limits

$$\Delta a_\mu < 59\ (75)\times 10^{-10} \Longrightarrow \left|d_\mu^{\mathrm{NP}}\right| < 3.5\ (3.9)\times 10^{-19}\ e\ \mathrm{cm} \; . \tag{7}$$

This discussion is completely model-independent. In specific models, however, it may be difficult to achieve values of d_μ large enough to saturate the bound of Eq. (7). For example, in supersymmetry, assuming flavor conservation and taking extreme values of superparticle

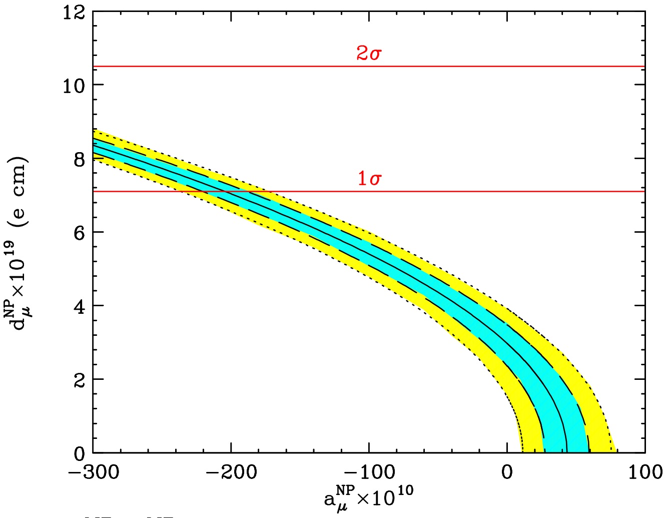


FIG. 1. Regions in the $(a_\mu^{\rm NP}, d_\mu^{\rm NP})$ plane that are consistent with the observed $|\boldsymbol{\omega}_a|$ at the 1σ and 2σ levels. The current 1σ and 2σ bounds on $d_\mu^{\rm NP}$ [3] are also shown.

masses ($\sim$ 100 GeV) and $\tan\beta$ ($\tan\beta\sim 60$) to maximize the effect, the largest possible value of a_μ is $a_\mu^{\rm max}\sim 10^{-7}$ [11]. Very roughly, one therefore expects a maximal muon EDM of order $(e\hbar/2m_\mu c)a_\mu^{\rm max}\sim 10^{-20}$ e cm in supersymmetry.

Of course, the effects of d_μ and a_μ are physically distinguishable: while a_μ causes precession around the magnetic field's axis, d_μ leads to oscillation of the muon's spin above and below the plane of motion. This oscillation is detectable in the distribution of positrons from muon decay, and further analysis of the recent a_μ data should tighten the current bounds on d_μ. Such analysis is currently in progress [12]. The proposed dedicated d_μ search will provide a definitive answer, however, by either measuring a non-zero d_μ or constraining the contribution of d_μ to $|\boldsymbol{\omega}_a|$ to be insignificant.

III. THEORETICAL EXPECTATIONS FROM THE MUON'S MDM

The muon's EDM and anomalous MDM are defined through[1]

$$\mathcal{L}_{\rm EDM} = -\frac{i}{2} d_\mu^{\rm NP}\, \bar\mu \sigma^{mn}\gamma_5 \mu\, F_{mn} \tag{8}$$

$$\mathcal{L}_{\rm MDM} = a_\mu^{\rm NP} \frac{e}{4m_\mu}\, \bar\mu\sigma^{mn}\mu\, F_{mn}\ , \tag{9}$$

where $\sigma^{mn} = \frac{i}{2}[\gamma^m,\gamma^n]$ and F is the electromagnetic field strength. These operators are closely related. In the absence of all other considerations, one might expect their coefficients to be of the same order. Parameterizing them as $d_\mu^{\rm NP}/2 = \mathcal{I}m A$ and $a_\mu^{\rm NP} e/(4m_\mu) = \mathcal{R}e A$ with $A\equiv |A|e^{i\phi_{\rm CP}}$, one finds

$$d_\mu^{\rm NP} = 4.0\times 10^{-22}\ e\ {\rm cm}\ \frac{a_\mu^{\rm NP}}{43\times 10^{-10}}\ \tan\phi_{\rm CP}\ . \tag{10}$$

[1]Here and below we set $\hbar = c = 1$.

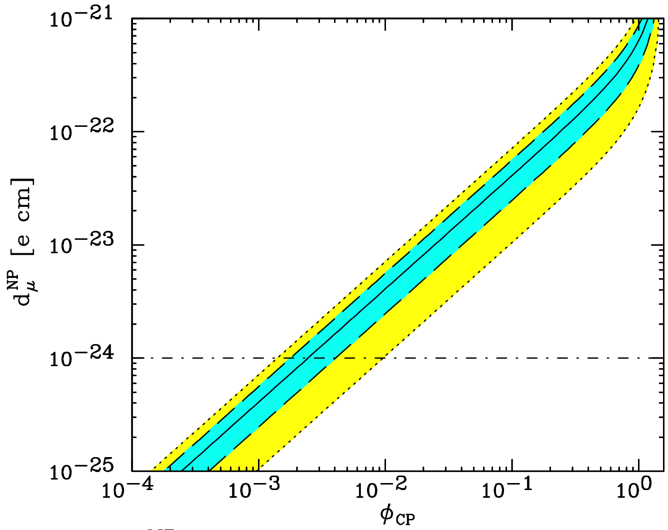


FIG. 2. Regions of the $(\phi_{\rm CP}, d_\mu^{\rm NP})$ plane allowed by the measured central value of $|\boldsymbol{\omega}_a|$ (solid) and its 1σ and 2σ preferred values (shaded). The horizontal line marks the proposed experimental sensitivity to $d_\mu^{\rm NP}$.

The measured discrepancy in $|\boldsymbol{\omega}_a|$ then constrains $\phi_{\rm CP}$ and $d_\mu^{\rm NP}$. The preferred regions of the $(\phi_{\rm CP}, d_\mu^{\rm NP})$ plane are shown in Fig. 2. For 'natural' values of $\phi_{\rm CP} \sim 1$, $d_\mu^{\rm NP}$ is of order 10^{-22} e cm. With the proposed $d_\mu^{\rm NP}$ sensitivity of Eq. (1), all of the 2σ allowed region with $\phi_{\rm CP} > 10^{-2}$ yields an observable signal.

At the same time, while this model-independent analysis indicates that natural values of $\phi_{\rm CP}$ prefer $d_\mu^{\rm NP}$ well within reach of the proposed muon EDM experiment, very large values of $d_\mu^{\rm NP}$ also require highly fine-tuned $\phi_{\rm CP}$. For example, the contributions of $d_\mu^{\rm NP}$ and $a_\mu^{\rm NP}$ to the observed discrepancy in a_μ are roughly equal only if $|\pi/2 - \phi_{\rm CP}| \sim 10^{-3}$. This is a consequence of the fact that EDMs are CP-odd and $d_\mu^{\rm SM} \approx 0$, and so $d_\mu^{\rm NP}$ appears only quadratically in $|\boldsymbol{\omega}_a|$. Without a strong motivation for $\phi_{\rm CP} \approx \pi/2$, it is therefore natural to expect the EDM contribution to $|\boldsymbol{\omega}_a|$ to be negligible, and we assume in the following that the $|\boldsymbol{\omega}_a|$ measurement is indeed a measurement of a_μ.

IV. THE ELECTRON EDM AND NAIVE SCALING

The EDM operator of Eq. (8) couples left- and right-handed muons, and so requires a mass insertion to flip the chirality. The natural choice for this mass is the lepton mass. On dimensional grounds, one therefore expects

$$d_\mu^{\rm NP} \propto \frac{m_\mu}{\tilde{m}^2} \ , \tag{11}$$

where $\tilde{m}$ is the mass scale of the new physics. If the new physics is flavor blind, $d_f \propto m_f$ for all fermions f, which we refer to as 'naive scaling.' In particular,

$$d_\mu \approx \frac{m_\mu}{m_e} d_e \ . \tag{12}$$

The current bound on the electron EDM is $d_e = 1.8\,(1.2)\,(1.0) \times 10^{-27}$ e cm [13]. Combining the statistical and systematic errors in quadrature, this bound and Eq. (12) imply

$$d_\mu \lesssim 9.1 \times 10^{-25}\ e\ \mathrm{cm}\ , \tag{13}$$

at the 90% CL, which is barely below the sensitivity of Eq. (1). Naive scaling must be violated if a non-vanishing d_μ is to be observable at the proposed experiment. On the other hand, the proximity of the limit of Eq. (13) to the projected experimental sensitivity of Eq. (1) implies that even relatively small departures from naive scaling may yield an observable signal.

V. VIOLATIONS OF NAIVE SCALING IN SUPERSYMMETRY

Is naive scaling violation well-motivated, and can the violation be large enough to produce an observable EDM for the muon? To investigate these questions quantitatively, we consider supersymmetry. (For violations of naive scaling in other models, see, for example, Ref. [14].) Many additional mass parameters are introduced in supersymmetric extensions of the standard model. These are in general complex and so are new sources of CP violation. These parameters may be correlated by a fundamental theory of supersymmetry breaking that includes a specific mechanism for mediating the breaking. In fact, all viable mechanisms of mediating supersymmetry breaking are designed to suppress flavor violation, and so CP-violating observables that also involve flavor violation, such as ϵ_K, are also suppressed. However, EDMs do not require flavor violation, and constraints on quark and electron EDMs are some of the main challenges for supersymmetric models. For a recent discussion of the supersymmetric CP problem in various supersymmetry breaking schemes, see Ref. [15].

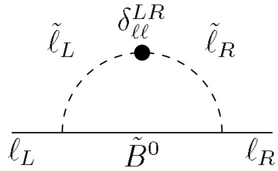
In full generality, the relevant dimensionful supersymmetry parameters for leptonic EDMs are the slepton mass matrices $\boldsymbol{m}^2$, trilinear scalar couplings $\boldsymbol{A}$, gaugino masses M_1 and M_2, the Higgsino mass μ, and the dimension two Higgs scalar coupling B. Schematically, these enter the Lagrangian through the terms

$$\boldsymbol{\mathcal{L}} \supset \boldsymbol{m}^2_{LL\,ij} \tilde{L}_i^* \tilde{L}_j,\ \boldsymbol{m}^2_{RR\,ij} \tilde{E}_i^* \tilde{E}_j,\ \boldsymbol{A}_{ij} H_d \tilde{L}_i \tilde{E}_j,\ M_1 \tilde{B}\tilde{B},\ M_2 \tilde{W}\tilde{W},\ \mu \tilde{H}_u \tilde{H}_d,\ B H_u H_d\ , \tag{14}$$

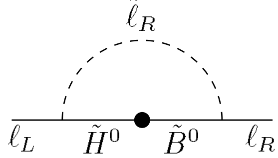
where i, j are generational indices, L and E denote SU(2) doublet and singlet leptons, respectively, H_u and H_d are the up- and down-type Higgs multiplets, and $\tilde{B}$ and $\tilde{W}$ are gauginos, the U(1) Bino and SU(2) Winos. The parameter $\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$, the ratio of Higgs boson vacuum expectation values, will also enter below.

The $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_{PQ}$ symmetries allow us to remove two phases — we choose M_2 and B real. Throughout this study, we assume that the gaugino masses have a common phase, as is true in many well-motivated theories where the gaugino masses are either unified at some scale or otherwise have a common origin. We also begin by assuming supersymmetric flavor conservation, that is, that the sfermion masses and trilinear couplings are diagonal in the fermion mass basis. (We will consider flavor violation in Sec. V C.) With these assumptions, only the μ and A_ℓ parameters are complex. We define $\phi_\mu \equiv \mathrm{Arg}(\mu)$ and $\phi_{A_\ell} \equiv \mathrm{Arg}(A_\ell)$.

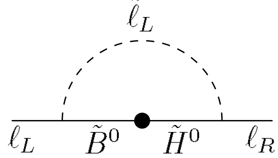
Leptonic electromagnetic dipole moments arise at one-loop from chargino-sneutrino and neutralino-charged slepton diagrams. In all figures and results presented here, we evaluate flavor-conserving amplitudes exactly. However, for purposes of exposition, it is convenient to consider the fermion mass basis and to adopt the mass insertion approximation for sleptons, neutralinos, and charginos. (We have checked that the mass insertion approximation is accurate to about 5% in almost all of parameter space, justifying the intuition derived from this simplification.) In the mass insertion approximation, for large and moderate $\tan\beta$ and neglecting subdominant terms, there are five contributions with the following Feynman diagrams and amplitudes [16]:



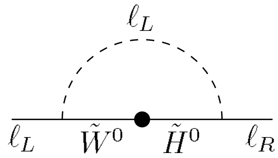
$$\mathcal{A}^a_\ell = -g'^2 M_1 \left[A_\ell \langle H^0_d\rangle - m_\ell\,\mu\tan\beta\right] K_N(M_1^2, m^2_{\tilde{\ell}_L}, m^2_{\tilde{\ell}_R})$$



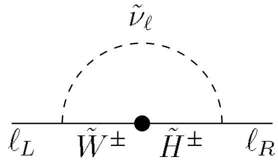
$$\mathcal{A}^b_\ell = -g'^2 M_1 m_\ell\,\mu\tan\beta\, K_N(m^2_{\tilde{\ell}_R}, |\mu|^2, M_1^2)$$



$$\mathcal{A}^c_\ell = \frac{1}{2} g'^2 M_1 m_\ell\,\mu\tan\beta\, K_N(m^2_{\tilde{\ell}_L}, |\mu|^2, M_1^2)$$



$$\mathcal{A}^d_\ell = -\frac{1}{2} g_2^2 M_2 m_\ell\,\mu\tan\beta\, K_N(m^2_{\tilde{\ell}_L}, |\mu|^2, M_2^2)$$



$$\mathcal{A}^e_\ell = g_2^2 M_2 m_\ell\,\mu\tan\beta\, K_C(m^2_{\tilde{\nu}_\ell}, |\mu|^2, M_2^2)\ . \tag{15}$$

In these diagrams, an external photon connected to any charged internal line is implicit, and $\delta^{LR}_{\ell\ell} \equiv (A_\ell\langle H^0_d\rangle - m_\ell\,\mu\tan\beta)/m^2_{\tilde{\ell}}$. The functions K_N and K_C are mass dimension -4 functions entering the neutralino and chargino diagrams, respectively, and are given in the Appendix.

Defining

$$\mathcal{A}^{\rm tot}_\ell \equiv \mathcal{A}^a_\ell + \mathcal{A}^b_\ell + \mathcal{A}^c_\ell + \mathcal{A}^d_\ell + \mathcal{A}^e_\ell\ , \tag{16}$$

The EDM and anomalous MDM of a lepton ℓ are simply

$$d_\ell = \frac{1}{2} e\,\mathcal{I}m\mathcal{A}^{\rm tot}_\ell\ , \quad a_\ell = m_\ell\,\mathcal{R}e\mathcal{A}^{\rm tot}_\ell\ . \tag{17}$$

From the amplitudes of Eq. (15), naive scaling is seen to require

- Degeneracy: Generation-independent $m_{\tilde{\ell}_R}$, $m_{\tilde{\ell}_L}$, and $m_{\tilde{\nu}_\ell}$.
- Proportionality: The A terms must satisfy $\mathcal{I}m(A_\ell) \propto m_\ell$.
- Flavor conservation: Vanishing off-diagonal elements of $\boldsymbol{m}^2_{LL}$,$\boldsymbol{m}^2_{RR}$, and $\boldsymbol{A}$.

The last requirement, flavor conservation, has been assumed in all of our discussion so far. As we will see, relaxing this assumption also leads to naive scaling violation. We now consider violations of each of these properties in turn.

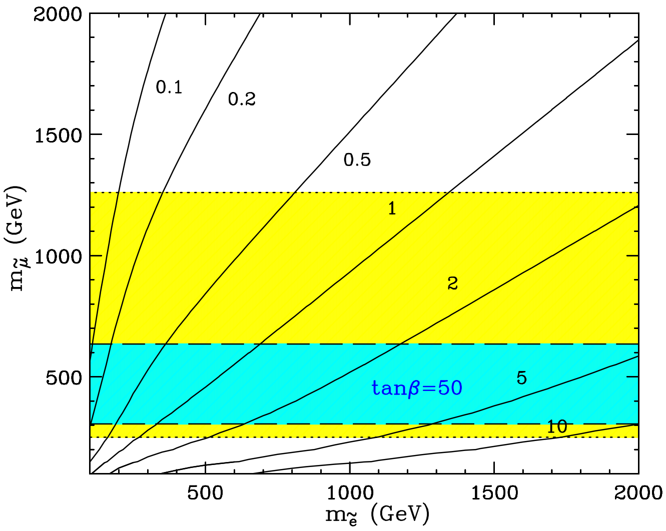


FIG. 3. Contours of d_μ in units of 10^{-24} e cm for varying $m_{\tilde{e}_R} = m_{\tilde{e}_L} = m_{\tilde{e}}$ and $m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = m_{\tilde{\mu}}$ for vanishing A terms, fixed $|\mu| = 500$ GeV and $M_2 = 300$ GeV, and $M_1 = (g_1^2/g_2^2)M_2$ determined from gaugino mass unification. The parameter combination $\sin\phi_\mu \tan\beta$ is assumed to saturate the bound $d_e < 4.4 \times 10^{-27}$ e cm. The shaded regions are preferred by a_μ at 1σ and 2σ for $\tan\beta = 50$.

A. Non-degeneracy

Scalar degeneracy is the most obvious way to reduce flavor changing effects to allowable levels. Therefore many schemes for mediating supersymmetry breaking try to achieve degeneracy. However, in many of these, with the exception of simple gauge mediation models, there may be non-negligible contributions to scalar masses that are generation-dependent. Furthermore, there are classes of models that do not require scalar degeneracy at all. For example, scalar non-degeneracy is typical in alignment models [17], where flavor-changing effects are suppressed by the alignment of scalar and fermion mass matrices rather than by scalar degeneracy. Scalar non-degeneracy is also typical in models with anomalous U(1) contributions to the sfermion masses. In fact, in models where the anomalous U(1) symmetry determines both sfermion and fermion masses, the sfermion hierarchy is often inverted relative to the fermion mass hierarchy [18–20], and so smuons are lighter than selectrons, as required for an observable d_μ. In summary, there is a wide variety of models in which deviations from scalar degeneracy exist.

We now consider a simple model-independent parameterization to explore the impact of non-degenerate selectron and smuon masses. We set $m_{\tilde{e}_R} = m_{\tilde{e}_L} = m_{\tilde{e}}$ and $m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = m_{\tilde{\mu}}$ and assume vanishing A parameters. For fixed values of M_1, M_2, $|\mu|$, and large $\tan\beta$, then, to a good approximation both d_e and d_μ are proportional to $\sin\phi_\mu \tan\beta$, and we assume that $\sin\phi_\mu \tan\beta$ saturates the d_e bound.

Contours of d_μ are given in Fig. 3. The contributions to d_μ have been evaluated exactly (without the mass insertion approximation). Observable values of d_μ are possible even for small violations of non-degeneracy; for example, for $m_{\tilde{\mu}}/m_{\tilde{e}} \lesssim 0.9$, muon EDMs greater than 10^{-24} e cm are possible. The current value of a_μ also favors light smuons and large EDMs. The smuon mass regions preferred by the current a_μ anomaly are given in Fig. 3 for

$\tan\beta = 50$. Within the 1σ preferred region, d_μ may be as large as 4 (10) $\times 10^{-24}$ e cm for $m_{\tilde{e}} < 1$ (2) TeV. Our assumed value of $\tan\beta$ is conservative; for smaller $\tan\beta$, the preferred smuon masses are lower and the possible d_μ values larger.

B. Non-proportionality

Naive scaling is also broken if the $\mathcal{I}m A_\ell$ are not proportional to Yukawa couplings y_ℓ. Just as in the case of non-degeneracy, deviations from proportionality are found in many models. Even in models constructed to give proportionality, there are often corrections to the A terms, so that

$$\boldsymbol{A}_{ij} = \boldsymbol{y}_{ij} A_0 + \boldsymbol{a}_{ij} \ , \tag{18}$$

where the second term is smaller than the first term, but violates proportionality. In flavor models, the A terms do not obey proportionality at all. Rather they are of the form

$$\boldsymbol{A}_{ij} = c_{ij}\, \boldsymbol{y}_{ij} A_0 \ , \tag{19}$$

where c_{ij} are order one coefficients. Clearly, violations of proportionality may affect d_μ not only by changing the magnitude of the A terms, but also through the possible appearance of new phases, either in $\boldsymbol{a}_{ij}$ or in c_{ij}. Note that these possibilities also lead to flavor violation, the subject of the following section.

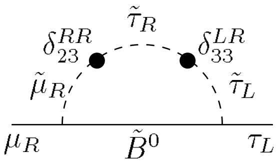
We will not study the possibility of non-proportionality in detail. For large $\tan\beta$, the A term contribution to the EDM is suppressed relative to the typically dominant chargino contribution by roughly a factor of $(g_2^2 M_2/g'^2 M_1)(y_\mu \mathcal{I}m\mu \tan\beta/\mathcal{I}m A_\mu)$, where we have used the amplitudes of Eq. (15). However, there are many possibilities that may yield large effects. In Ref. [8], for example, it was noted that A_e may be such that the chargino and neutralino contributions to d_e cancel, while, since $A_e \neq A_\mu$, there is no cancellation in d_μ, and observable values are possible.

C. Flavor Violation

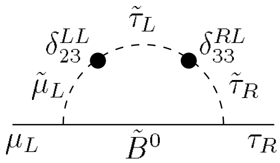
In all of the discussion so far, we have neglected the possibility of supersymmetric lepton flavor violation. However, such flavor violation is present, at least at sub-leading order, in most models of high-scale supersymmetry breaking [15]. Moreover, large smuon-stau mixing, of particular importance here, is well-motivated by the evidence for large $\nu_\mu - \nu_\tau$ mixing observed in atmospheric neutrinos [21]. Explicit examples of this connection in models with left-right gauge symmetry are given in Refs. [9,10]. The relation between neutrino and slepton mixing is also a general feature of Abelian flavor models [22]. In the simplest of these models, highly mixed states have similar masses, contradicting the most straightforward interpretation of the neutrino data. This difficulty may be circumvented in less minimal models by generating hierarchical neutrino masses from the neutrino mass matrix and large mixing by arranging for the gauge and mass eigenstates of the SU(2) lepton doublets to be related by large rotations. However, because lepton doublets contain charged leptons in addition to neutrinos, these rotations also generate large misalignments between the charged leptons and sleptons, producing large slepton flavor mixing.

Smuon-stau mixing leads to a potentially significant enhancement in d_μ, because it breaks naive scaling by introducing contributions with a tau mass insertion so that $d_\mu \propto m_\tau/\tilde{m}^2$. However, to evaluate the significance of this enhancement, we must first determine how large the flavor violation may be. This effect may be isolated by assuming that all charged sleptons are roughly degenerate with characteristic mass $m_{\tilde{\ell}}$. In the basis with lepton mass eigenstates and flavor-diagonal gauge interactions, slepton flavor violation enters through off-diagonal masses in the slepton mass matrix. As usual, we parameterize the chirality-preserving off-diagonal masses by $\delta_{23}^{LL} \equiv \boldsymbol{m}_{L\,23}^2/m_{\tilde{\ell}}^2$ and $\delta_{23}^{RR} \equiv \boldsymbol{m}_{E\,23}^2/m_{\tilde{\ell}}^2$. There may also be flavor violation in the left-right couplings; we parameterize these by δ_{23}^{LR} and δ_{23}^{RL}. We begin by assuming real δs; however, very large effects are possible for imaginary δs, and we consider this possibility at the end of this section.

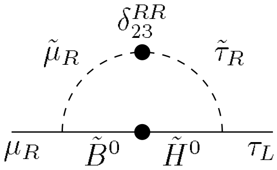
The off-diagonal masses induce $\tau \to \mu\gamma$ transitions. Eight contributions are parametrically enhanced by m_τ/m_μ (retaining the possibility that $\delta_{23}^{LR,RL} \propto m_\tau$). Their Feynman diagrams and amplitudes are



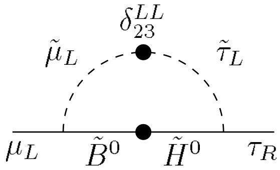
$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_R}^2}\delta_{23}^{RR}$$



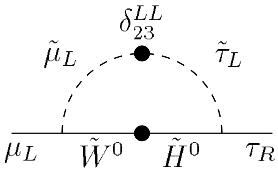
$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2}\delta_{23}^{LL}$$



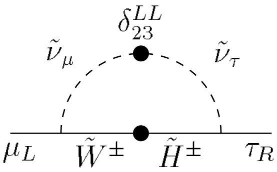
$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^b}{\partial \ln m_{\tilde{\mu}_R}^2}\delta_{23}^{RR}$$



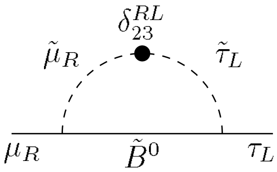
$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^c}{\partial \ln m_{\tilde{\mu}_L}^2}\delta_{23}^{LL}$$



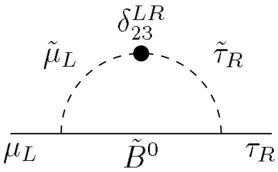
$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^d}{\partial \ln m_{\tilde{\mu}_L}^2}\delta_{23}^{LL}$$



$$\frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^e}{\partial \ln m_{\tilde{\nu}_\mu}^2}\delta_{23}^{LL}$$



$$\mathcal{A}_\mu^a \frac{1}{\delta_{22}^{LR}}\delta_{23}^{RL}$$



$$\mathcal{A}_\mu^a \frac{1}{\delta_{22}^{LR}}\delta_{23}^{LR} \ , \tag{20}$$

where the amplitudes $\mathcal{A}_\ell^i$ are given in Eq. (15). The branching ratio may then be written as

$$B(\tau \to \mu\gamma) \ = \ \frac{12\pi^3\alpha}{G_F^2 m_\tau^2}\left[|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2\right] B(\tau \to e\bar{\nu}_e\nu_\tau) \ , \tag{21}$$

where

$$\begin{aligned}
\mathcal{M}_L &= \frac{m_\tau}{m_\mu}\left[\frac{\partial \mathcal{A}_\mu^{\text{tot}}}{\partial \ln m_{\tilde{\mu}_L}^2} + \frac{\partial \mathcal{A}_\mu^{\text{tot}}}{\partial \ln m_{\tilde{\nu}_\mu}^2}\right]\delta_{23}^{LL} + \mathcal{A}_\mu^a \frac{1}{\delta_{22}^{LR}}\delta_{23}^{LR} \\
\mathcal{M}_R &= \frac{m_\tau}{m_\mu}\frac{\partial \mathcal{A}_\mu^{\text{tot}}}{\partial \ln m_{\tilde{\mu}_R}^2}\delta_{23}^{RR} + \mathcal{A}_\mu^a \frac{1}{\delta_{22}^{LR}}\delta_{23}^{RL} \ .
\end{aligned} \tag{22}$$

Equations (15), (16), (21) and (22) provide a compact form for the branching ratio for radiative lepton decays in the mass insertion approximation and are well-suited to numerical

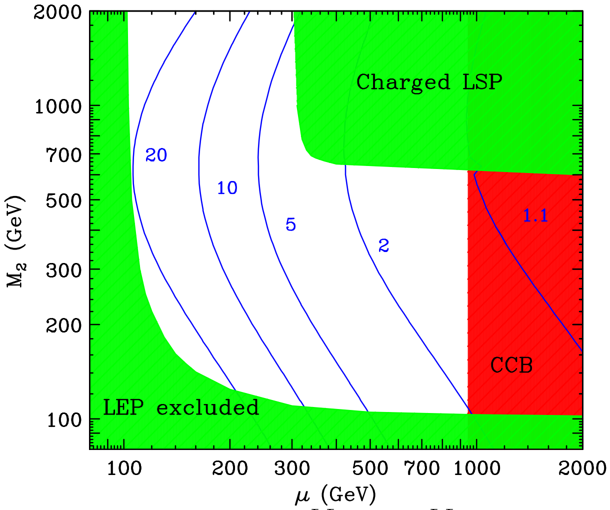
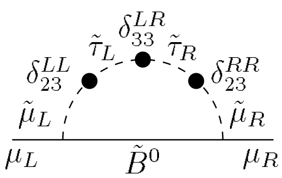


FIG. 4. The ratios $\delta_{23\,\tilde{B}\,\mathrm{only}}^{LL\,\mathrm{max}}/\delta_{23}^{LL\,\mathrm{max}}$ (left) and $\delta_{23\,\tilde{B}\,\mathrm{only}}^{RR\,\mathrm{max}}/\delta_{23}^{RR\,\mathrm{max}}$ (right). The values $\delta_{23}^{\mathrm{max}}$ and $\delta_{23\,\tilde{B}\,\mathrm{only}}^{\mathrm{max}}$ are the upper bounds allowed by $\tau \to \mu\gamma$ determined with all leading contributions and with only the Bino-mediated contribution, respectively. We fix $m_{\tilde{\ell}} \approx 300$ GeV and $\tan\beta = 50$. The shaded regions are excluded by LEP, the requirement of a neutral LSP, and the condition that the vacuum not be charge-breaking.

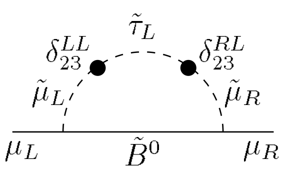
evaluation. (Note that terms subleading in m_τ/m_μ and linear in the δs are also easily computed as $\mathcal{M}_L = (\partial \mathcal{A}_\mu^{\mathrm{tot}}/\partial \ln m_{\tilde{\mu}_R}^2)\delta_{23}^{RR}$ and $\mathcal{M}_R = (\partial \mathcal{A}_\mu^{\mathrm{tot}}/\partial \ln m_{\tilde{\mu}_L}^2)\delta_{23}^{LL}$, but we neglect them in the analysis to follow.)

The flavor-violating mass insertions are bounded by the the current constraint $B(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ [23]. It is important to note that the Higgsino-mediated decays give the dominant contribution unless $\mu \gg M_2, M_1$. The often-used bounds of Gabbiani, Gabrielli, Masiero and Silvestrini [24] assume a photino neutralino, and so effectively include only the Bino-mediated contribution. In Fig. 4, we show contours of the ratio of the upper bound on $\delta_{23}^{LL,RR}$ determined with only the Bino-mediated contribution included to the upper bound determined with all of the leading diagrams included. We see that the Bino-only bounds are reasonably accurate only for $|\mu| \gtrsim 1$ TeV, a region that is forbidden by the requirement that electromagnetic charge be an unbroken symmetry. (We have assumed that the diagonal entries of the stau and smuon mass matrices are equal; in the forbidden region, $m_{\tilde{\tau}_1}^2 < 0$.) Analyses based solely on the Bino contribution are highly misleading in most regions of parameter space, especially for moderate and large $\tan\beta$. In particular, for δ_{23}^{LL}, the constraint from $\tau \to \mu\gamma$ is always far more stringent than one would conclude from a Bino-only analysis. Similar conclusions apply to constraints from $\tau \to e\gamma$, $\mu \to e\gamma$, and $\mu - e$ conversion and will be presented elsewhere [25].

We now determine what effect flavor violation may have on d_μ. Flavor violation induces four m_τ/m_μ enhanced contributions to the muon's EDM:



$$\frac{m_\tau}{m_\mu}\frac{\partial^2 \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2\,\partial \ln m_{\tilde{\mu}_R}^2}\delta_{23}^{LL}\delta_{23}^{RR}$$



$$\frac{\delta_{23}^{RL}}{\delta_{22}^{LR}}\frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2}\delta_{23}^{LL}$$

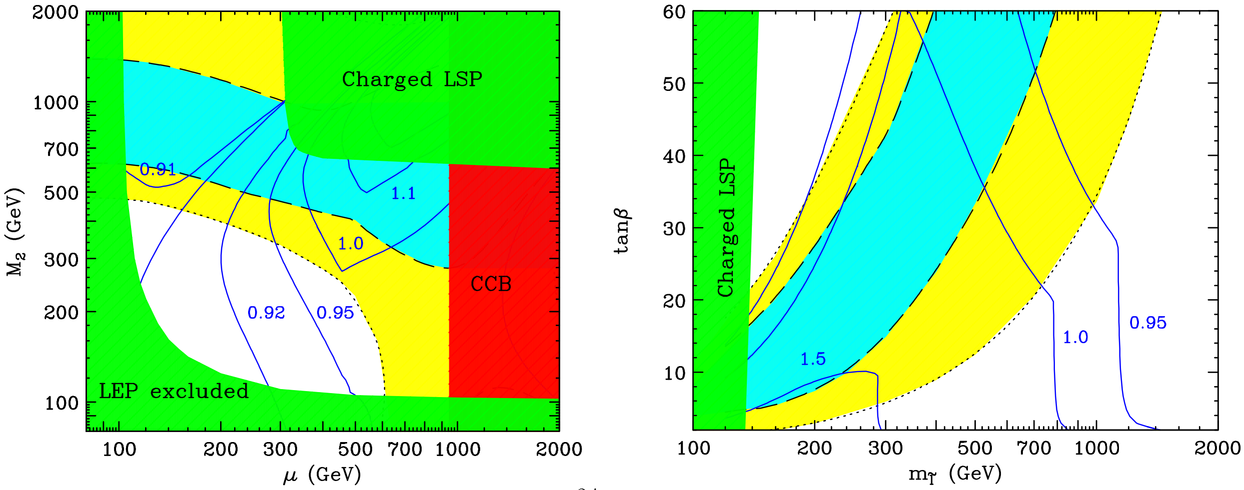
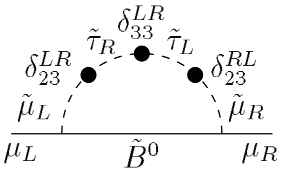
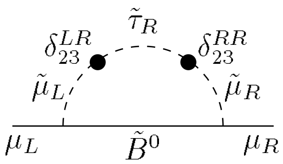


FIG. 5. Contours of $d_\mu^{\max}$ in units of 10^{-24} e cm in the presence of flavor violation. Regions consistent with the observed Δa_μ at the 1σ and 2σ levels are also shown. On the left, $m_{\tilde{\ell}} \approx 300$ GeV and $\tan\beta = 50$, and on the right, $M_2 = 300$ GeV and $|\mu| = 500$ GeV. M_1 is fixed by gaugino mass unification. The excluded regions are as in Fig. 4.



$$\frac{m_\tau}{m_\mu} \frac{\partial^2 \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2 \, \partial \ln m_{\tilde{\mu}_R}^2} \delta_{23}^{LR} \delta_{23}^{RL}$$



$$\frac{\delta_{23}^{LR}}{\delta_{22}^{LR}} \frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_R}^2} \delta_{23}^{RR} \; .$$

The additional flavor-violating contribution to the muon's EDM is then simply $d_\mu^{FV} = \frac{1}{2} e \mathcal{I}m \mathcal{A}_\mu^{FV}$, where

$$\begin{aligned} \mathcal{A}_\mu^{FV} = & \frac{m_\tau}{m_\mu} \frac{\partial^2 \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2 \, \partial \ln m_{\tilde{\mu}_R}^2} \left[\delta_{23}^{LL} \delta_{23}^{RR} + \delta_{23}^{LR} \delta_{23}^{RL} \right] \\ & + \frac{\delta_{23}^{RL}}{\delta_{22}^{LR}} \frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_L}^2} \delta_{23}^{LL} + \frac{\delta_{23}^{LR}}{\delta_{22}^{LR}} \frac{\partial \mathcal{A}_\mu^a}{\partial \ln m_{\tilde{\mu}_R}^2} \delta_{23}^{RR} \; . \end{aligned} \tag{23}$$

Contours of the maximal possible EDM $d_\mu^{\max}$ in the presence of LL and RR flavor violation are given in Fig. 5. To obtain $d_\mu^{\max}$, ϕ_μ is taken to saturate the constraint from d_e, and the sign of the flavor-violating contribution is chosen to add constructively to the flavor-conserving piece. The parameters $\delta_{23}^{LL,RR}$ are also taken to maximize d_μ given the constraint from $\tau \to \mu\gamma$; we make use of the fact that, subject to the constraint $ax^2 + by^2 \le c$ with $a, b, c > 0$, the product xy is maximized for $ax^2 = by^2 = c/2$. We also require $\delta_{23}^{LL,RR} \le 1/2$ so that the mass insertion approximation is valid. We find that flavor violation may enhance d_μ. While the enhancement is not enormous, it does bring the maximal possible value of d_μ into the range of the proposed experimental sensitivity in parts of the parameter space.

We have performed a similar analysis for chirality-violating flavor violation. In the case of non-vanishing $\delta_{23}^{LR,RL}$, enhancements above the proposed sensitivity are not found.

Note, however, that, to investigate various effects independently, we have assumed real off-diagonal masses. In fact, however, this division of new physics effects is rather artificial, as off-diagonal masses need not be real and generically have $\mathcal{O}(1)$ phases.

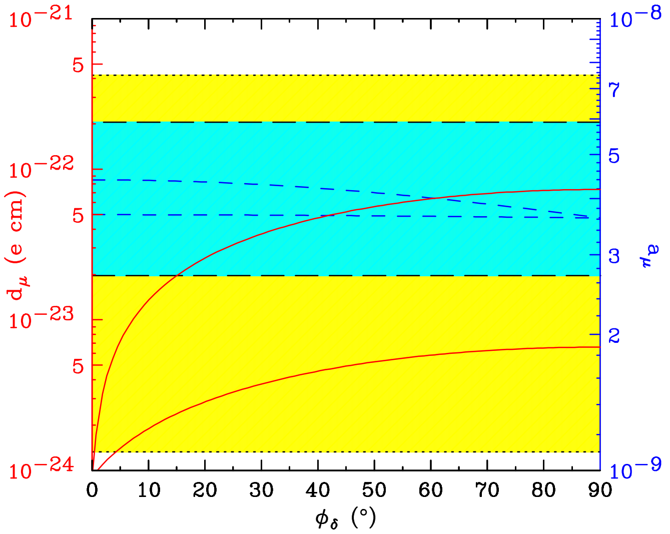


FIG. 6. Contours of d_μ (solid) and a_μ (dashed) as functions of ϕ_δ, the CP-violating phase present in flavor-violating slepton mass terms. In each pair of contours, the upper is for $\mathrm{Arg}(\delta_{23}^{LL}\delta_{23}^{RR}) = \phi_\delta$ and $\delta_{23}^{LR} = \delta_{23}^{RL} = 0$, while the lower is for $\delta_{23}^{LL} = \delta_{23}^{RR} = 0$ and $\mathrm{Arg}(\delta_{23}^{LR}\delta_{23}^{RL}) = \phi_\delta$. We fix $m_{\tilde{\ell}} = 300$ GeV, $\tan\beta = 30$, $|\mu| = 500$ GeV, and $M_2 = 300$ GeV, and M_1 is fixed by gaugino mass unification.

For concreteness, we consider two cases: in the first, we take $\mathrm{Arg}(\delta_{23}^{LL}\delta_{23}^{RR}) = \phi_\delta$ and $\delta_{23}^{LR} = \delta_{23}^{RL} = 0$, while in the second, we let $\delta_{23}^{LL} = \delta_{23}^{RR} = 0$ and $\mathrm{Arg}(\delta_{23}^{LR}\delta_{23}^{RL}) = \phi_\delta$. In both cases, the phase ϕ_δ is irrelevant for $B(\tau \to \mu\gamma)$, as only the magnitudes of the δs enter in Eq. (21). This phase is also not constrained by d_e, as it has no direct couplings to the first generation. However, it contributes directly to d_μ, as is clear in Eq. (23).

In Fig. 6, we show the dependence of d_μ on the phase ϕ_δ for both cases. We see that in the LL/RR case, it is easy to achieve values of $d_\mu \sim 10^{-22}$ e cm, two orders of magnitude above the proposed sensitivity. For the LR/RL case, d_μ well above the proposed sensitivity is also possible. Such values are consistent with all present constraints. In particular, note that we have also given values of a_μ including both the flavor-conserving contribution and the flavor-violating amplitude of Eq. (23). As may be seen in Fig. 6, the values of ϕ_δ are also perfectly consistent with the currently preferred a_μ.

VI. CONCLUSIONS

The proposal to measure the muon EDM at the level of 10^{-24} e cm potentially improves existing sensitivities by five orders of magnitude. Such a leap in sensitivity is rare in studies of basic properties of fundamental particles and merits attention.

In this study we have considered the muon EDM from a number of theoretical perspectives. We noted that the recent results from the Muon $(g-2)$ Experiment, although widely interpreted as evidence for a non-standard model contribution to a_μ, may alternatively be ascribed entirely to a non-standard model contribution to d_μ. In fact, these results provide the most stringent constraints on the muon EDM at present. Theoretical prejudices aside, this ambiguity will be definitively resolved only by improved bounds on (or measurements of) d_μ, such as will be possible in the proposed d_μ experiment.

Considering only the indications from a non-standard model contribution to a_μ, 'naturalness' implies muon EDMs far above the proposed sensitivity. In more concrete scenarios, however, additional constraints, notably from the electron's EDM and lepton flavor violation, impose important restrictions. Nevertheless, we have noted a number of well-motivated possibilities in supersymmetry: non-degeneracy, non-proportionality, and slepton flavor violation. Each of these may produce a muon EDM above the proposed sensitivity.

For simplicity, we have focused for the most part on one effect at a time. From a model-building point of view, however, this is rather unnatural. For example, non-degeneracy of the diagonal elements of the scalar mass matrices is typically accompanied by flavor violation. At the very least, if the soft masses are diagonal but non-degenerate in a particular interaction basis, off-diagonal elements will be generated upon rotating to the fermion mass basis. As noted above, non-proportionality will also typically be accompanied by flavor violation. In both of these cases, then, the lepton flavor violation leads to new contributions to d_μ, as well as to new constraints from lepton flavor violating observables. The amount of flavor violation present is highly model-dependent, and so we have not considered this in detail. In Sec. V C, however, we have noted that d_μ may be greatly enhanced by two or more simultaneous effects, leading to values of $d_\mu \sim 10^{-22}\ e$ cm, far above the proposed sensitivity.

If a non-vanishing d_μ is discovered, it will be unambiguous evidence for physics beyond the standard model. At the currently envisioned sensitivity, it will also imply naive scaling violation, with important implications for many new physics models. In addition, the measurement of non-standard model contributions to both d_μ and a_μ will provide a measurement of new CP-violating phases, with little dependence on the overall scale of the new physics. Such information is difficult to obtain otherwise. Low energy precision experiments may not only uncover evidence for new physics before high energy collider experiments, but may also provide information about the new physics that will be highly complementary to the information ultimately provided by colliders.

ACKNOWLEDGMENTS

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APPENDIX

The functions K_N and K_C of Eq. (15) are

$$K_N(x^2, y^2, z^2) = J_5(x^2, x^2, y^2, z^2, z^2) + J_5(x^2, x^2, y^2, y^2, z^2)\ , \tag{24}$$

$$K_C(x^2, y^2, z^2) = 2I_4(x^2, y^2, z^2, z^2) + 2I_4(x^2, y^2, y^2, z^2) - K_N(x^2, y^2, z^2)\ , \tag{25}$$

where the loop functions J_n and I_n are defined iteratively [16] through

$$I_n(x_1^2,\ldots,x_n^2) = \frac{1}{x_1^2 - x_n^2}\ \left[I_{n-1}(x_1^2,\ldots,x_{n-1}^2) - I_{n-1}(x_2^2,\ldots,x_n^2)\right]\ , \tag{26}$$

$$J_n(x_1^2,\ldots,x_n^2) = I_{n-1}(x_1^2,\ldots,x_{n-1}^2)\ +\ x_n^2\ I_n(x_1^2,\ldots,x_n^2)\ , \tag{27}$$

with

$$I_2(x_1^2,x_2^2)\ =\ -\frac{1}{16\pi^2}\left\{\frac{x_1^2}{x_1^2-x_2^2}\ln\frac{x_1^2}{\Lambda^2} + \frac{x_2^2}{x_2^2-x_1^2}\ln\frac{x_2^2}{\Lambda^2}\right\}\ . \tag{28}$$

Their arguments are the gaugino masses M_1 and M_2, the Higgsino mass parameter μ, and the scalar soft supersymmetry breaking masses

$$\begin{aligned}
m_{\tilde{\ell}_L}^2 &= (\boldsymbol{m}_{LL}^2)_{\ell\ell} + \left(-\frac{1}{2} + \sin^2\theta_W\right) m_Z^2\cos 2\beta \\
m_{\tilde{\ell}_R}^2 &= (\boldsymbol{m}_{RR}^2)_{\ell\ell} - \sin^2\theta_W m_Z^2 \cos 2\beta \\
m_{\tilde{\nu}_\ell}^2 &= (\boldsymbol{m}_{LL}^2)_{\ell\ell} + \frac{1}{2} m_Z^2\cos 2\beta\ .
\end{aligned} \tag{29}$$

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