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The Scaling of Lepton Dipole Moments with Lepton Mass

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Abstract

The dipole moments of the leptons and quarks are matrices in flavor space, which can potentially reveal as much about the flavor structure of the theory as do the mass matrices. The off-diagonal elements of the dipole matrices lead to flavor-changing decays such as $\mu \longrightarrow e\gamma$, while the imaginary parts of the diagonal elements give rise to electric dipole moments. We analyze the scaling of the leptonic dipole moments with the lepton masses in theories beyond the standard model. While in many models the dipole moments scale roughly as lepton mass, it is shown that simple models exist in which the dipoles scale as the cube of the mass or in other ways. An explicit example with cubic scaling is presented, which is motivated on independent grounds from large angle neutrino oscillation data. Our results have great significance for the observability of the electric dipole moments d_e, d_μ, d_τ, and the rare decays $\mu \longrightarrow e\gamma$, and $\tau \longrightarrow \mu\gamma$ and will be tested in several forthcoming experiments.

1 Introduction and summary of results

Most of the parameters of the standard model are masses and mixings of the quarks and leptons. These masses and mixings exhibit patterns that suggest to many theorists that there exist underlying "flavor" or "family" symmetries. Over the years many models of fermion masses and flavor symmetry have been proposed, some of them quite elegant. However, it is impossible without further experimental facts to decide among the various theoretical ideas. A determination of neutrino masses and mixings will certainly be enormously important for solving the "flavor problem", but probably even more experimental facts will be required.

Another window on the flavor problem could be provided by the dipole moments of the quarks and leptons, since not only the fermion masses but the dipoles as well should reflect the underlying flavor symmetries of the theory. In fact, potentially there could be as much information about flavor in the dipole moments as in the masses and mixings. Given N fermions f_i having the same Standard Model quantum numbers, their anomalous dipole moments, like their masses, form an $N \times N$ complex matrix, which we shall denote as D_{ij}. This matrix is the coefficient of the effective operator $f^T_{iL} C \sigma_{\mu\nu} F^{\mu\nu} f^c_{jL}$. Whereas in the "physical basis" of the fermions the mass matrix becomes real and diagonal, the anomalous dipole moment matrix would be expected to remain complex and, in general, non-diagonal.

The imaginary part of the diagonal elements of the anomalous dipole moment matrix corresponds to the electric dipole moments of the particles: $Im(D_{ii}) \equiv d_i$. The off-diagonal elements of the anomalous dipole moment matrix are the "transition dipole moments", which can be either magnetic or electric: $Re(D_{ij}) \equiv \mu_{ij}$, and $Im(D_{ij}) \equiv d_{ij}$. The transition dipole moments lead to decays such as $\mu \longrightarrow e\gamma$ [1].

In this paper we shall focus on the dipole moments of the charged leptons. These are quite interesting from the experimental point of view as it is likely that there will soon be great improvements in the limits on several of these quantities. The present limit on the electric dipole moment (edm) of the electron, coming from atomic experiments, is 4.3×10^{-27} ecm [2]. A recently approved experiment on PbO expects to push this down to the level of 10^{-29} ecm in three to five years, and hopes eventually to reach even the level of 10^{-31} ecm [3]. The current limit on the muon edm is 1.1×10^{-18} ecm [4], but there have been recent proposals to push this to 10^{-24} ecm [5]. The present limit on the branching ratio of $\mu \longrightarrow e\gamma$ is 1.2×10^{-11} [6]. There is a proposal to push this to the level of 10^{-14} at PSI [7]. The forthcoming MECO experiment [8] will improve the limit on $\mu - e$ conversion in nuclei to the level of 10^{-16} in branching ratio, which is directly related to the $\mu \to e\gamma$ decay rate. The Brookhaven experiment E821 is currently in the process of improving the precision on muon $g-2$ to the level of (few) $\times 10^{-10}$ [9]. The current limit on $\tau \longrightarrow \mu\gamma$ is 1.1×10^{-6} [10]. This limit will also be substantially improved, at least to the 10^{-7} level, and perhaps to 10^{-8} [11]. The current limit on the edm of the tau lepton is 3.1×10^{-16} ecm [12], which could be improved significantly.

In this paper we shall show that there are reasonable and well-motivated models in which several of these leptonic dipole moments, specifically $D_{\mu e}$, d_{μ}, and $D_{\tau\mu}$, may be within reach of experiment. If this were so, then the pattern of the leptonic edm matrix would be extremely revealing of the underlying flavor structure of the theory. It is also quite possible, of course, that no lepton edm or transition dipole moment is large enough to be seen. Certainly this is the case in the Standard Model, where these quantities are highly suppressed. However, if there is non-trivial flavor structure at scales between a few hundred GeV and a few hundred TeV, then some of these moments may be observable. If there is no new physics all the way to about a thousand TeV, then there is very little chance for seeing these edm's or the rare leptonic decays in the forthcoming experiments. However, new physics such as supersymmetry is anticipated not much above a few TeV, so chances for seeing these effects are great, as we will argue.

In many models there is a close relationship between the anomalous dipole moment matrices and the mass matrices of the fermions. Both kinds of matrix connect the left-handed fermions to the right-handed fermions. Moreover, at the diagrammatic level the Feynman diagrams that contribute to dipole moments and masses are related. In particular, removing the external photon from a dipole moment diagram produces a mass diagram, and adding a photon to a mass diagram of one or more loops produces a dipole diagram. In fact, in many models the anomalous dipole moments of the quarks and leptons are roughly proportional to their masses.

In the minimal supersymmetric Standard Model (MSSM), for instance, there are well-known one-loop contributions to lepton and quark edms coming from gaugino loops [13]. In those diagrams a lepton (or quark) emits a virtual gaugino to become a virtual slepton (or squark) and then reabsorbs the gaugino, coupling in the process to an external photon line. Since a dipole moment operator involves a chirality flip of the fermion, there must be an insertion either of the mass of the external fermion or of the "left-right" mass of the corresponding virtual sfermion. However, the left-right sfermion masses, m^2_{LR}, are proportional to the masses of the corresponding fermions in the MSSM. Thus the edms of the quarks and leptons come out being approximately proportional to their masses.

Proportionality of edms and masses also occurs in multi-Higgs models. In models with more than one Higgs doublet there can be CP violation in the Higgs propagators that can lead to measurable edms [14]. The dominant contribution to the edms of the lighter quarks and leptons come from two-loop diagrams that involve one power of the Higgs Yukawa couplings [15]. These contributions are therefore approximately proportional to the mass of the quark or lepton. (There are also one-loop diagrams, but these are cubic in the Higgs Yukawa couplings and are therefore negligible for the lighter families. These one–loop contributions may however become significant for the muon edm for some range of parameters, especially large $\tan\beta$ [16].)

This feature that the anomalous dipole moments scale roughly as the first power of the fermion mass is characteristic of many models. If it does in fact hold in Nature, then it has several important implications. First, there would be a limit on the muon edm coming from the present limit on the electron edm: $d_\mu \sim (m_\mu/m_e)d_e \leq 8.8\times10^{-25}$ ecm. This is at the margin of what could be seen by the proposed experiments. Second, one would conclude that d_τ and $D_{\tau\mu}$ are unobservably small. For example, one would have $d_\tau \sim (m_\tau/m_e)d_e \leq 1.5 \times 10^{-23}$ ecm, which is 0.5×10^{-7} of the present limit and is well beyond the reach of currently planned experiments.

However, the anomalous dipole moments of the leptons and quarks do not necessarily scale as the first power of the masses. In this paper we shall describe models in which they scale as other powers, in particular as the cube or the square. The idea that electric dipole moments might scale as the cube of the mass is not new. As already noted, the one-loop contributions to the fermion edms in multi-Higgs models do tend to scale with the cube of the mass, and these were for a time naturally thought to be the leading contributions. However, it was then pointed out [15] that two-loop diagrams which scale as the first power of mass dominate for the lighter families.

We present an explicit model in Sec. 2 and 3 where the dipoles obey a cubic scaling. The model is motivated on independent grounds, viz., an explanation of solar and atmospheric neutrino oscillation data in terms of large angle neutrino oscillations. In this model we find that the anomalous dipole moment matrix scales roughly as follows:

$$D_{ij} \sim d_\tau \begin{pmatrix} (m_e/m_\tau)^3 + c\frac{\alpha}{4\pi}(m_e/m_\tau) & - & - \\ (m_e m_\mu^2/m_\tau^3) & (m_\mu/m_\tau)^3 & - \\ (m_e/m_\tau)^2 & (m_\mu m_\tau) & 1 \end{pmatrix}. \qquad (1)$$

Here the dashes indicate elements that are negligibly small. One sees that the diagonal elements — and therefore the edms of the particles — scale as the mass cubed, while those elements below the diagonal are suppressed relative to the diagonal elements by factors that go as the mass ratios (coming from mixing angles).

In Eq. (1), the dipole moment of the electron has a term that is denoted as $c\frac{\alpha}{4\pi}(m_e/m_\tau)d_\tau$. Since the electron mass is so small, one has to take into account two loop diagrams for the dipole moments which may have a linear scaling with the lepton mass [15]. Typically, we find that the coefficient c is of order unity. With $c = \mathcal{O}(1)$,the two loop diagram with linear scaling will win over the one–loop diagram with cubic scaling. But this is significant only for the electron dipole moment. It is also possible that these two–loop diagrams are suppressed in some models, in which case $c \simeq 0$, although in the explicit models that we have constructed, this turns out not to be the case.

Consider the case of $c \simeq 0$ (i.e., the two–loop diagrams being suppressed). The resulting pattern is interesting from the point of view of several kinds of experiments. If we suppose that the transition dipole $D_{\mu e}$ is just at the current limit coming from $B(\mu \longrightarrow e\gamma) \leq 1.2 \times 10^{-11}$, namely $|D_{\mu e}| = 4.5 \times 10^{-27}$ ecm, then the pattern in Eq.

(1) with $c = 0$ would imply the following values of other moments: $d_e \sim 10^{-31}$ ecm, $d_\mu \sim 10^{-24}$ ecm, $d_\tau \sim 5 \times 10^{-21}$ ecm, and $|D_{\tau\mu}| \sim 3 \times 10^{-22}$ ecm (which corresponds to $B(\tau \longrightarrow \mu\gamma) \sim 10^{-4}$). It should be emphasized that these are very rough estimates, which could well be off by an order of magnitude in the dipole moment or two orders of magnitude in a rare decay rate. Making allowances for this, one can say that if the pattern in Eq. (1) is correct then: (a) d_e is probably too small to be seen in the near future, though experiments on molecules like PbO have a chance of seeing it eventually. (And even if not seen, the failure to see it together with the observation of d_μ would confirm a more-than-linear scaling of the edms with mass.) (b) d_μ should be at or near the level which recently proposed experiments can reach. And (c) $\tau \longrightarrow \mu\gamma$ should be very close to the present limit.

In the more realistic case of $c = \mathcal{O}(1)$, the above estimates will still hold, except for the electron edm. d_e should be also observable, at the level of 1×10^{-27} ecm, if the decay $\mu \longrightarrow e\gamma$ is near the present experimental limit.

In section 4, we shall describe, in less detail, a model that illustrates quadratic scaling of the anomalous dipole moments with mass. In that model one finds

$$D_{ij} \sim d_\tau \begin{pmatrix} (m_e/m_\tau)^2 & (m_e m_\mu/m_\tau^2) & (m_e/m_\tau) \\ (m_e m_\mu/m_\tau^2) & (m_\mu/m_\tau)^2 & (m_\mu/m_\tau) \\ (m_e/m_\tau) & (m_\mu/m_\tau) & 1 \end{pmatrix}. \tag{2}$$

In this case the two–loop contribution to d_e which would scale linearly with the electron mass is at best of the same order numerically as the one–loop correction and is not shown in Eq. (2). If $\mu \longrightarrow e\gamma$ is saturated at the present limit, this pattern would lead to the following predictions: (a) $d_e \simeq 2 \times 10^{-29}$ ecm, (b) $d_\mu \simeq 1 \times 10^{-24}$ e-cm, (c) $d_\tau \simeq 3 \times 10^{-22}$ ecm, (d) $|D_{\tau\mu}| \simeq 2 \times 10^{-23}$ ecm (corresponding to $B(\tau \longrightarrow \mu\gamma) \sim 4 \times 10^{-7}$). Again, we see that these predictions can be well tested directly in the near future.

2 Flavor symmetry and the dipole moments

Before describing a specific model that leads to the pattern shown in Eq. (1), we will describe in more general terms how different kinds of scaling of the lepton edms can arise in simple models based on abelian flavor symmetry.

Many models of fermion masses explain the interfamily mass hierarchy by appealing to spontaneously broken flavor symmetry. The idea is that certain elements of the mass matrix would vanish if some abelian symmetry or symmetries were exact, and are therefore suppressed by powers of the expectation values that violate those symmetries. The fields whose expectation values violate flavor symmetry are often called "flavons". If F denotes a flavon field, then different elements of the mass matrix could be suppressed by different powers of $\epsilon \equiv \langle F \rangle / M_F$, where M_F is a scale characterizing the underlying flavor physics. The absolute scale of M_F and $\langle F \rangle$ could be anything from just above the weak scale up to the Planck scale. All that matters for explaining the fermion mass hierarchy is that their ratio ϵ be smaller than, but of order, unity. Some models have several flavon fields, and have the fermion masses depending on several small parameters ϵ_i.

In a typical model of the kind we have been describing the fermion masses arise from tree-level diagrams similar to that shown in Fig. 1(a) [17]. One of the boson lines attached to the fermion line is a Higgs doublet whose vacuum expectation value breaks $SU(2)_L \times U(1)_Y$. The others are flavon fields, whose vacuum expectation values give rise to the requisite powers of the small parameter(s) ϵ_i. It is evident that if two of the boson lines are tied together to make a loop, and an external photon line is attached, the resulting diagram (Fig. 1(b)) gives a non-zero dipole moment to the fermion. If the flavor scale is near the unification scale or Planck scale, the resulting dipole moment is negligible. However, if the flavor scale is near the weak scale, a significant dipole moment can result. One sees, moreover, that the dipole matrix will have a close relationship to the mass matrix.

Consider the following simple toy model as an illustration. This model will be generalized in the next section to a fully realistic model that also explains large neutrino mixing angles as suggested by recent data from solar and atmospheric neutrino experiments. The toy model is over–simplified in that it has vanishing inter–generational mixings. Suppose one has the ordinary three families of leptons (ℓ_i) and three families also of singly-charged vectorlike leptons (X_i):

$$\left(\begin{array}{c} \nu \\ \ell^- \end{array} \right)_{iL} \equiv L_i, \quad \ell^+_{iL}, \quad X^-_{iL}, \quad X^+_{iL}. \tag{3}$$

Suppose the following Yukawa couplings and mass terms:

$$\mathcal{L}_{Yuk} = \sum_i f_i (L_i \ell^+_{iL}) H^\dagger + \sum_{i,K} f^K_i (X^-_{iL} X^+_{iL}) h_K + \sum_i f^S_i (X^-_{iL} \ell^+_{iL}) S_i. \tag{4}$$

This structure can be enforced by a simple flavor symmetry. The field H is the usual Higgs doublet, while S_i and h_K are singlets. Call $f_i \langle H^0 \rangle \equiv \hat{m}_i$, $\Sigma_K f^K_i \langle h_K \rangle \equiv \langle F_i \rangle$, and $f^S_i \langle S_i \rangle \equiv M_i \equiv \epsilon_i^{-1} \langle F_i \rangle$, where it is assumed that $\epsilon_i < 1$ and $m_i \ll M_i$, $\langle F_i \rangle$. Then there is a mass matrix of the form

$$(\ell^-_{iL}, X^-_{iL}) \left(\begin{array}{cc} \hat{m}_i & 0 \\ M_i & \langle F_i \rangle \end{array} \right) \left(\begin{array}{c} \ell^+_{iL} \\ X^+_{iL} \end{array} \right). \tag{5}$$

This matrix has three large eigenvalues that are of order M_i, and three small eigenvalues that are of order $\hat{m}_i$. The three light eigenstates are to be identified as the physical e, μ and τ. Neglecting terms of order $(\hat{m}_i/M_i)$, the large entries in the mass matrix are diagonalized by a change of basis of the positively charged leptons: $\ell^{+\prime}_{iL} = \sin\theta_i \ell^+_{iL} - \cos\theta_i X^+_{iL}$, and $X^{+\prime}_{iL} = \cos\theta_i \ell^+_{iL} + \sin\theta_i X^+_{iL}$, where $\tan\theta_i = \epsilon_i$. (We have implicitly chosen a phase convention wherein the entries in the mass matrices have all been made real. The couplings of the leptons to the scalar fields will however remain complex in this basis.) This would give effective mass terms for the light three fermions of the form $m_i(\ell^-_{iL}\ell^{+\prime}_{iL})$, with

$$m_i = \frac{\langle F_i\rangle}{M_i}\hat{m}_i = \hat{m}_i\epsilon_i, \tag{6}$$

where we have used $\sin\theta_i \cong \tan\theta_i = \epsilon_i$. We suppose that the parameters $\hat{m}_i$ are all of the same order and that the fermion mass hierarchy comes from a hierarchy in the ϵ_i: $\epsilon_1 \ll \epsilon_2 \ll \epsilon_3$.

Diagrammatically, one has at lowest order in the ϵ_i the tree-level diagram in Fig. 2(a). This would give an effective mass term for the light fermions of $(f_i\langle H\rangle)(f_i^S\langle S_i\rangle)^{-1}$ $(\Sigma_K f_i^K\langle h_K\rangle) = \hat{m}_i M_i^{-1}\langle F_i\rangle = \hat{m}_i\epsilon_i$, which of course agrees with the result of diagonalizing the matrix.

If an external photon line is added to the diagram in Fig. 2(a) and the H and h_K boson lines are tied together through the quartic term $\lambda_K H^\dagger H h_K^* h_K$, which must certainly be allowed to exist by symmetry, the diagram shown in Fig. 2(b) results. The contribution to the dipole moment of the i^{th} light lepton ($\ell_1 = e$, $\ell_2 = \mu$, and $\ell_3 = \tau$) will be of the form

$$d_i = \frac{1}{16\pi^2}\sum_K (e\lambda_K f_i f_i^K)\langle H\rangle\langle h_K\rangle M_i I(m_H^2, m_{h_K}^2, M_i^2), \tag{7}$$

where I is the function with dimensions of M^{-4} resulting from doing the momentum integral, given by $I(r_1M, r_2M, M) = \frac{1}{2}M^{-4}(r_1-r_2)^{-1}([1-r_1^2(1+\ln r_1^2)]/(1-r_1)^3 - (r_1 \longrightarrow r_2))$. Let us suppose that the largest mass in the loop is that of the heavy virtual fermion (which Eq. (5) shows to be approximately M_i). Then the integral is given by $I \cong (M_i)^{-4}(\frac{3}{2} + \frac{5}{2}(m_H^2 + m_{h_K}^2)/M_i^2))$, Recalling that $\Sigma_K f_i^K\langle h_K\rangle = \langle F_i\rangle$, define $\Sigma_K\lambda_K f_i^K\langle h_K\rangle \equiv \lambda_i\langle F_i\rangle$. If one neglects the K dependence in the momentum integral I, the dipole moment of the i^{th} light lepton can be written as

$$d_i \cong \frac{3e}{32\pi^2}\lambda_i\frac{\hat{m}_i\langle F_i\rangle}{M_i^3} = \frac{3e}{32\pi^2}\lambda_i\frac{m_i}{M_i^2}\ . \tag{8}$$

It is technically natural to assume that the CP-violating phase of λ_i is of order one. It should be noted that there will be such a phase in the physical basis of the leptons, in general, if there is more than one type of boson h_K. The point is that the mass of the i^{th} light lepton, which is a real number in the physical basis, is proportional to $\Sigma_K f_i^K\langle h_K\rangle$, whereas the dipole of that lepton depends on $\Sigma\lambda_K f_i^K\langle h_K\rangle$, which has no reason to be real in the same basis. Thus the electric dipole moment of the i^{th} lepton is of the same order as the result in Eq. (8)

One can see that this edm can be quite large for reasonable values of the parameters. For example, if $M_2 = 3$ TeV and $\lambda_2 = 1$, then the edm of the muon would be of order 3×10^{-24} ecm. The scaling of the edms with mass depends upon how the family mass hierarchy is assumed to arise. This hierarchy is given by $m_i/m_j \sim \epsilon_i/\epsilon_j = (\langle F_i \rangle / \langle F_j \rangle)(M_i/M_j)^{-1}$. Thus, it can come from (a) a hierarchy in the $\langle F_i \rangle$, with all the M_i being comparable, (b) a hierarchy in the M_i, with all the $\langle F_i \rangle$ being comparable, or (c) a combination of these. If (a) is the case, then Eq. (8) would imply that the edms of the leptons scale approximately *linearly* in lepton mass. However, if (b) is the case, then the edms scale with the cube of the masses, since $m_i/m_j \sim M_j/M_i$ and thus $d_i/d_j \sim (m_i/m_j)(M_j/M_i)^2 \sim (m_i/m_j)^3$.

Since the one–loop dipoles scale as the cube of the lepton masses if the hierarchy is in M_i, one should examine whther two–loop diagrams induce larger contributions, especially for d_e. We find that this is indeed the case, as in multi–Higgs models. Diagonalizing the mass matrix of Eq. (5) to lowest order in ϵ_i and $\hat{m}_i/M_i$ will lead to the following Yukawa couplings of the light fermions to the scalars:

$$\mathcal{L}_{\rm Yuk} = \ell_{iL}^{-} \ell_{iL}^{+\prime} \epsilon_i f_i H + \ell_{iL}^{-} \ell_{iL}^{+\prime} \frac{\hat{m}_i}{M_i} f_i^K h_k + H.C. \tag{9}$$

In the physical basis, $\epsilon_i \hat{m}_i$ are real, but f_i^K are complex in general. Thus, an edm of the electron can be induced through a two–loop diagram with h_K propagating. The dominant contribution to d_e will arise when h_K decays into a real and a virtual photon. This happens, for example, by a diagram in which h_K emits an $X_{iL}^+ X_{iL}^-$ pair which closes by emitting two photons, or one where h_K becomes an H (through quartic scalar couplings), which then decays to two photons through a top quark (and other) loop. All these diagrams, while they are suppressed by an additional loop factor, scale linearly with the lepton mass and can be dominant over the one–loop diagram. For $m_{h_K} \simeq 300$ GeV, d_e turns out to be of order 4×10^{-27} ecm [15], if the relevant phases are all of order one.

3 A realistic model with mixing of charged lepton families

The simple example just discussed is perhaps too simple in that it does not have any mixing among the families. It is true that the leptonic mixing seen in neutrino oscillations might arise from the neutrino sector rather than the charged lepton sector. However, the CKM mixing of the quarks and the likelihood that quarks and charged leptons are unified in some way suggest otherwise. In this section we present a unifiable model of lepton mass in which the charged lepton mass matrix is off-diagonal in the flavor basis. This model is actually a realization of an idea proposed several years ago to explain both the fermion mass hierarchy and the largeness of neutrino mixing angles [18]. The model is thus well-motivated on grounds having nothing to do with dipole moments. But it naturally leads to the pattern given in Eq. (1).

The model has the following leptons and scalar fields:

$$\begin{array}{lccccccc} \text{field :} & \ell_i^- & \ell_i^+ & X_i^- & X_i^+ & H & S_i & h_K \\ F \text{ charge :} & 0 & 0 & q_i & -q_i & 0 & -q_i & 0 \end{array} \tag{10}$$

where F is an abelian flavor symmetry. This symmetry allows the following Yukawa terms for the leptons:

$$\mathcal{L}_{Yuk} = \sum_{i,j} f_{ij}(\ell_i^- \ell_j^+)H^0 + \sum_i f_i^K (X_i^- X_i^+) h_K + \sum_{i,j} f_{ij}^S (X_i^- \ell_j^+) S_i. \tag{11}$$

The crucial difference between this model and the toy model discussed in the last section is that here the Yukawa coupling f_{ij} is a matrix. The coupling f_{ij}^S is also a matrix, but this is of less significance, as we shall see. It will simplify the discussion without affecting the qualitative conclusions if we assume that f_{ij}^S is diagonal: $f_{ij}^S = f_i^S \delta_{ij}$. This assumption will be relaxed later.

Let us define the matrix $\hat{m}_{ij} = f_{ij} \langle H^0 \rangle$. There will be a 6×6 mass matrix of the charged leptons, which has the same form as in the toy model (cf. Eq. (5)) except that $\hat{m}_{ij}$ is here non-diagonal:

$$(\ell_{iL}^-, X_{iL}^-) \left(\begin{array}{cc} \hat{m}_{ij} & 0 \\ M_i \delta_{ij} & \langle F_i \rangle \delta_{ij} \end{array} \right) \left(\begin{array}{c} \ell_{jL}^+ \\ X_{jL}^+ \end{array} \right). \tag{12}$$

The quantities M_i and $\langle F_i \rangle$ appearing here are defined in the same way as in the toy model, and $\epsilon_i = \langle F_i \rangle / M_i$. After diagonalizing the large, i.e. $O(M_i)$, elements in the mass matrix, the light leptons (namely ℓ_i^- and $\ell_j^{+\prime} \cong X_j^+ - \epsilon_j \ell_j^+$) have an effective mass matrix given by

$$m_{ij} = \hat{m}_{ij} \frac{\langle F_j \rangle}{M_j} = \hat{m}_{ij} \epsilon_j. \tag{13}$$

As in the toy model of the last section, we assume that all the f_{ij} and hence all the $\hat{m}_{ij}$ are of the same order, and that the fermion mass hierarchy comes from the ϵ_i: $\epsilon_1 \ll \epsilon_2 \ll \epsilon_3$. From Eq. (13) it is obvious that the 3×3 light lepton mass matrix has a column form. That is, the first, second and third columns of the matrix have elements that are, respectively, small, medium, and large. That means that in diagonalizing it, the rotations that are done from the left (i.e. of the ℓ_{iL}^-) are of order one, whereas those done from the right (i.e. of the $\ell_{iL}^{+\prime}$) are suppressed by the hierarchy factors $\epsilon_j / \epsilon_k \sim m_j / m_k$, $j < k$. This means that there would be large leptonic mixing angles, as is observed in atmospheric neutrino oscillations, and perhaps also in solar neutrino oscillations. If one were to embed this model in $SU(5)$ then the mass matrix of the down quarks d, s, and b would have a similar structure, except transposed (since $SU(5)$ relates M_L to M_D^T, where M_L and M_D are the mass matrices of the charged leptons and down quarks). As the down quark mass matrix would thus have a *row* structure instead of a column structure, it would be the mixing angles of the d_{iL} that would be of order m_i/m_j, while those of the d^c_{iL} would be of order one. Thus, such a model would explain the fact that at least certain of the leptonic (MNS) mixings are large while the quark (CKM) mixings are small [19].

Returning to the leptons, it is easy to see that their dipole matrix also has a column form. In fact, the same calculation that led to Eq. (8) gives

$$D_{ij} \cong \frac{3e}{32\pi^2}\hat{m}_{ij}\frac{\lambda_j\langle F_j\rangle}{M_j^3} = \frac{3e}{32\pi^2}m_{ij}\frac{\lambda_j}{M_j^2}. \tag{14}$$

Both the dipole matrix and the mass matrix of the light leptons have a column form, and the corresponding columns of the two matrices are in fact proportional to each other. However, the hierarchies among the columns are different in the two matrices. (Compare Eqs. (13) and (14).) The case we are most interested in is where the hierarchy in the ϵ_i parameters comes from a hierarchy in the M_i, with the $\langle F_i\rangle$ all being of the same order. Then the columns of m_{ij} are in the ratio $\epsilon_1 : \epsilon_2 : \epsilon_3$, while the columns of D_{ij} are roughly in the ratios $\epsilon_1^3 : \epsilon_2^3 : \epsilon_3^3$. This shows that $d_e : d_\mu : d_\tau = m_e^3 : m_\mu^3 : m_\tau^3$.

To find the magnitudes of the off-diagonal elements of D_{ij} is simple. The first thing to be noticed is that because of the fact that the columns of m_{ij} and D_{ij} are proportional to each other, the same unitary transformation acting from the left will make zero all the elements above the diagonal in both matrices. This is a crucial fact, since otherwise there would be large off-diagonal elements in D_{ij}. For instance, in the matrix in Eq. (14) the (12) element is of the same order as the (22) element, which leads to the danger that $d_\mu \sim |D_{e\mu}|$. That would mean that the limit on $\mu \longrightarrow e\gamma$ would put a severe constraint on the muon edm. As it is, the elements above the diagonal can be made zero simultaneously in both matrices,

To complete the diagonalization of m_{ij} requires a unitary transformation from the right. Because there is a strong hierarchy among the columns of m_{ij} this unitary transformation involves rotations U_{ij}, $i<j$, that are of order $\epsilon_i/\epsilon_j \ll 1$. This will not eliminate the elements of D_{ij} that are below the diagonal, since the hierarchy among the columns of D_{ij} is different. Rather, the result of this unitary transformation will be to make D_{ji}, $i<j$, be of order $(\epsilon_i/\epsilon_j)D_{jj}$. This gives the form shown in Eq. (1).

An interesting prediction that arises from the nearly triangular form in Eq. (1), aside from the ones already discussed in the Introduction, is that in the decay $\mu^- \longrightarrow e^-\gamma$ the electron will be almost purely right-handed. A similar remark applies for the decay $\tau^- \longrightarrow \mu^-\gamma$.

Up to this point we have oversimplified the analysis of this model by assuming that the Yukawa matrix f^S_{ij} appearing in Eq. (11) is diagonal. The symmetries do not require it to be diagonal, so this is an unnatural assumption. However, eliminating this assumption makes no qualitative difference to the form of the dipole matrix. Suppose that f^S_{ij} is allowed to take a general form. Then the 6×6 mass matrix of the leptons is

$$(\ell^-_{iL}, X^-_{iL}) \left(\begin{array}{cc} \hat{m}_{ij} & 0 \\ M_{ij} & \langle F_i \rangle \delta_{ij} \end{array} \right) \left(\begin{array}{c} \ell^+_{jL} \\ X^+_{jL} \end{array} \right), \tag{15}$$

where $M_{ij} \equiv f^S_{ij} \langle S_i \rangle$. As before, we will assume that the hierarchy comes from $M_{ij} = f^S_{ij} \langle S_i \rangle$ rather than from $\langle F_i \rangle$. Since f^S_{ij} is assumed to be an arbitrary matrix, this means that the hierarchy comes from $\langle S_i \rangle$. In other words, $\langle S_1 \rangle \gg \langle S_2 \rangle \gg \langle S_3 \rangle$. This means that M_{ij} has a row structure, with the first row being much larger than the second, which in turn is much larger than the third. By rotating among the ℓ^+_{jL} one can go to a basis where M_{ij} is triangular, with zeros above the diagonal. This will also change the form of the Yukawa matrix f_{ij}, but since that matrix is not assumed to have any special form anyway, this makes no difference. Thus without loss of generality, one can take M_{ij} to have this triangular form. To completely diagonalize it then requires a rotation among the X^-_i by angles that go as $\langle S_j \rangle / \langle S_i \rangle \sim \epsilon_i/\epsilon_j$, $i < j$. This rotation will have the effect of making the coupling of X^-_i to X^+_j non-diagonal. This is the only change from our previous analysis. However, the off-diagonal elements thus introduced are suppressed by the small hierarchy factors ϵ_i/ϵ_j, $i < j$. One can easily trace through the effect of these small off-diagonal elements, and one finds that they give rise to a mixing of the i^{th} and j^{th} columns of the dipole matrix that is of order ϵ_i/ϵ_j, $i < j$. This does not change the form of the dipole matrix given in Eq. (1).

As in the previous section, there are two–loop diagrams that are suppressed by additional factors of $(\alpha/4\pi) \sim 10^{-3}$, but these are still more significant to d_e. The dominant diagrams involve the exchange of h_K fields. The coupling matrix of h_K to the light fermions are neither diagonal nor real in the physical basis for the leptons. As in the toy model, we would expect $d_e \sim 4 \times 10^{-27}$ ecm if the masses of h_K are of order 300 GeV. These two–loop diagrams have negligible effects for the muon and the tau lepton.

We have a model of mixing that is satisfactory for the leptons. As already mentioned, it is possible to embed this structure in an $SU(5)$ model. The simplest way to do this would be to add to the usual three families of $\mathbf{10}_i + \overline{\mathbf{5}}_i$ three vectorlike pairs $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$. The Higgs S_i and h_K would be $SU(5)$ singlets, while H would be a $\mathbf{5}$. The charge assignments in Eq. (10) and the couplings in Eq. (11) would be generalized in the obvious way. This would lead to a mass matrix for the down quarks that was simply the transpose of that for the charged leptons, just as in the minimal $SU(5)$ model. A realistic $SU(5)$ version of this model would have to introduce non-minimal features, such as larger Higgs multiplets or higher-dimensional effective operators contributing to light fermion masses. That would allow Clebsch coefficients to appear in the mass matrices which differed for the down quarks and the charged leptons. Nonetheless, one would expect that the down quark mass matrix, M_D, would be closely related to the transpose of the charged lepton mass matrix, M_L^T, even if not exactly equal to it. As noted above, this would elegantly explain the curious fact that all the CKM quark mixing angles have been observed to be small, while at least some of the MNS leptonic angles are large. For M_L has a column structure, while M_D has a row structure.

A question arises, however, whether the model can be extended to include quarks without running into a problem with excessive flavor changing in the quark sector. In particular, will one-loop diagrams involving the scalar fields H, S_i, and h_K lead to excessive $K^0 - \overline{K}^0$ mixing? The answer turns out to be no. Consider, for example, an $SU(5)$ version of this model where one takes the parameters to have the following orders of magnitude: $f_{ij} \sim 10^{-2}$, $f_{ij}^S \sim 1$, $f_{ij}^K \sim 1$, $\langle H \rangle \sim 300$ GeV, $\langle h_K \rangle \sim m_{h_K} \sim 300$ GeV — 1 TeV, $\langle S_3 \rangle \sim 300$ GeV — 1 TeV, $\langle S_2 \rangle \sim (\epsilon_3/\epsilon_2)\langle S_3 \rangle$, and $\langle S_1 \rangle \sim (\epsilon_3/\epsilon_1)\langle S_3 \rangle$, where $\epsilon_1 : \epsilon_2 : \epsilon_3 \sim m_e : m_\mu : m_\tau \sim m_d : m_s : m_b$. It can be shown that none of the box diagrams involving the scalar fields H, h_K, and S_i, and the new heavy vectorlike fermions contribute excessively to either the real or imaginary parts of M_{12} in the neutral kaon system. Some of the diagrams are suppressed by small mixings of order m_d/m_b and m_s/m_b or powers thereof, or by the large masses of the vectorlike fermions, especially X_1^- and X_2^-, or by the small values of the f_{ij}, or by combinations of all these factors. This fact, namely that one can have in this model one-loop dipole moments that are of interesting magnitude — including flavor-changing ones — without having excessive one-loop flavor changing in the kaon system, is non-trivial.

It might be suspected that the mixing of the light fermions with the heavy vector fermions could lead to large flavor–changing Z couplings that can arise at the tree level. However, this turns out to be not the case in this model. For example, in the lepton sector, there is no flavor changing Z coupling at the tree level in the right–handed sector. There are such couplings in the left–handed sector, but they are suppressed by $(\hat{m}/M)^2$ in the amplitude, which is at most of order 10^{-6} for $f_{ij} \sim 10^{-2}$ and M of order TeV. The resulting branching ratio for $Z \longrightarrow \tau\mu$ decay for example is at the level of 10^{-12}, which is too small to be observable, but is not much below the current limit for the decay $\mu \longrightarrow 3e$.

It should be observed that the $SU(5)$ version of this model described above could not be supersymmetrized if we wish to maintain perturbative unification at a scale of 10^{16} GeV. Having three vectorlike $\mathbf{10} + \overline{\mathbf{10}}$ pairs along with supersymmetry would destroy the perturbativity of the gauge couplings around 10^9 GeV.

4 Other scalings of the lepton dipole matrix

We have seen examples of linear and cubic scaling of the electric dipole moments with fermion mass. It is not difficult to construct models in which the scaling is quadratic. For example, consider a model in which there are the following leptons and Higgs fields. Leptons: $L_i \equiv (\nu_{iL}, \ell_{iL}^-)^T$, ℓ_{iL}^+, n_{aL}, n_{aL}^c. Higgs: $H \equiv (H^0, H^-)^T$, $h_a \equiv (h_a^0, h_a^-)^T$, s_a^-. Here n_a are neutral leptons which are taken to be Dirac particles. It is assumed that the fields h_a and s_a have vanishing VEVs. Let there be the following couplings: $\Sigma_a m_a (n_{aL}^c n_{aL}) + \Sigma_{a,i} f_{ia} h_a^\dagger (L_i n_a) + \Sigma_{a,i} f'_{ia} s_a^- (\ell^+ n_a^c) + \Sigma_a \lambda_a h_a H (s_a^-)^*$. This form can be enforced easily with abelian flavor symmetries that distinguish fields with different values of the index 'a'.

Suppose that the masses of the known leptons $\ell^{\pm}$ arise predominantly from the loops shown in Fig. 3. Let the heaviest particles in the loops be the bosons h_a and s_a^-, which are assumed to have masses $\sim M_a$ that are in a hierarchy: $M_3 \ll M_2 \ll M_1$. The largest contribution to the lepton mass matrix m_{ij} therefore comes from the diagram in Fig. 3 having $a = 3$. This gives a contribution to m_{ij} that is of rank 1. This would generate the mass of the τ lepton. The second largest contribution comes from the diagram with $a = 2$ and also is rank 1. This would give mass to the muon. The diagram with $a = 1$, finally, gives the electron mass [20]. If we assume that the only hierarchy in the underlying parameters is in the M_a, then it is apparent from simple power counting that the lepton masses have a hierarchy that goes roughly as $m_e : m_\mu : m_\tau \sim M_1^{-2} : M_2^{-2} : M_3^{-2}$, since the loop diagram of Fig. 3 is quadratically convergent.

If one adds an external photon line to the loop, it must couple to the boson line, since the virtual lepton n_a is neutral. The resulting diagram for the anomalous dipole moment is quartically convergent. Thus one obtains $d_e : d_\mu : d_\tau \sim M_1^{-4} : M_2^{-4} : M_3^{-4} \sim m_e^2 : m_\mu^2 : m_\tau^2$. In fact, it is not difficult to show that the whole anomalous dipole matrix has the scaling given in Eq. (2).

One difference from the model of Sec. 2 and 3 is that numerically the magnitudes of the dipole moments are enhanced here by a loop factor, which is roughly $(1/16\pi^2)^{-1} \sim 100$. The observability of the edm is thus greater in this case.

It is also possible to generate the dipole matrix of Eq. (2) by introducing singlet charged leptons and neutral scalars instead of neutral leptons and charged scalars.

There is an obvious generalization of this model to the quark sector. We can introduce singlet quarks with electric charges $+2/3$ and $-1/3$. The ordinary quarks can couple to these vector quarks through the higgs doublets $h_a = (h_a^0, h_a^-)$ and neutral higgs singlets s_a which do not acquire VEVs. One–loop diagrams analogous to Fig. 3 will induce quark masses as well as dipole moments. If the masses of the h_a fields (and/or the s_a fields) are hierarchical, the observed fermion mass hierarchy can be reproduced, with all Yukawa couplings being of the same order. This might be an interesting radiative mass hierarchy model for all the fermions, except for the top quark.

In the supersymmetric standard model, if there is non–trivial flavor structure in the trilinear A terms, analogous mass hierarchy can be induced, with interesting dipole moments [21]. In this case, the diagrams will involve exchange of the gaugino and squarks.

5 Conclusions

The magnetic and electric dipole moment matrices of quarks and leptons can potentially carry a wealth of information about the underlying flavor structure. In a large class of models we have shown that these are experimentally accessible, through the electric dipole moments of the electron, muon and the tau lepton, and through rare decays such as $\mu \longrightarrow e\gamma$ and $\tau \longrightarrow \mu\gamma$. We have addressed the scaling of the leptonic dipole moments with the lepton mass. While linear scaling with the mass is quite generic, in a class of models motivated on independent grounds by large angle neutrino oscillation solution to the solar and atmospheric neutrino data, we have found interesting scalings, where the dipoles go as the cube or the square of the lepton mass.

Dipole moments of quarks and leptons have been analyzed in the literature in various other contexts. Their scaling with mass in grand unified supersymmetric models is typically linear [22, 23], in contrast to the cubic scaling that we have found. It is also possible that significant contributions to leptonic dipole moments arise through SUSY exchange from interactions that are responsible for heavy right–handed neutrino masses [24]. Such interactions do not lead to any simple power law scaling. Experiments in the near future will be capable of telling these theories apart. Indeed, very important information about the flavor structure will be gained by the proposed improvements in the dipole moment measurements as well as in the radiative decays of μ and τ.

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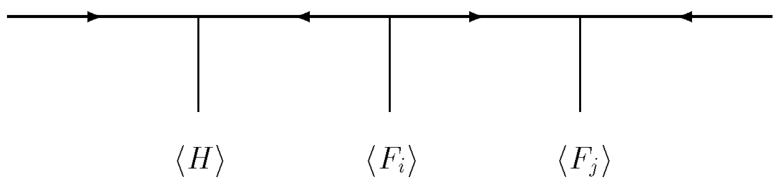

Fig. 1(a)

A typical tree diagram for light fermion mass through flavon fields.

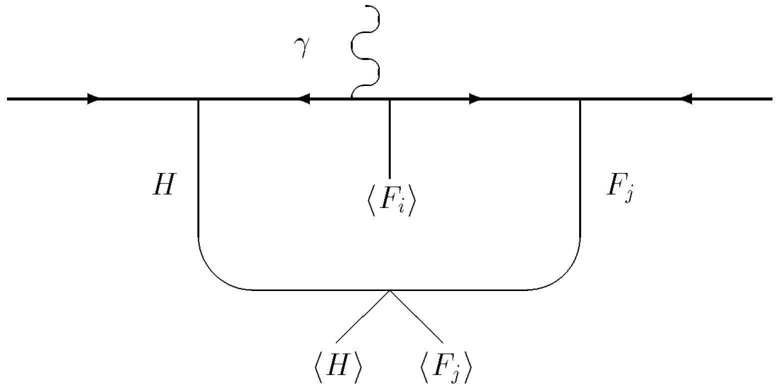

Fig. 1(b)

Dipole moment of the fermion induced by folding the legs of Fig. 1(a).

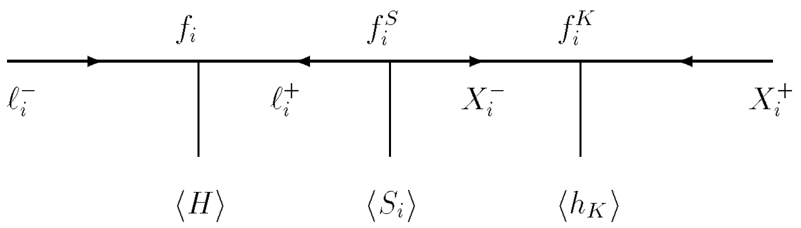

Fig. 2(a)

Diagram that induces lepton masses in the model of Sec. 2.

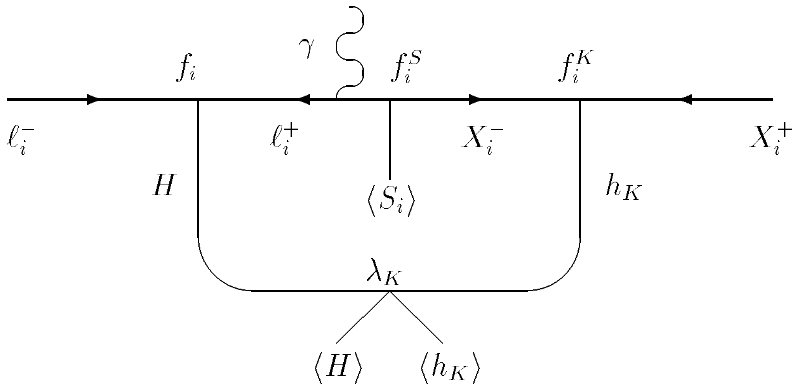

Fig. 2(b)

Lepton dipole moment contribution in the model of Sec. 2.

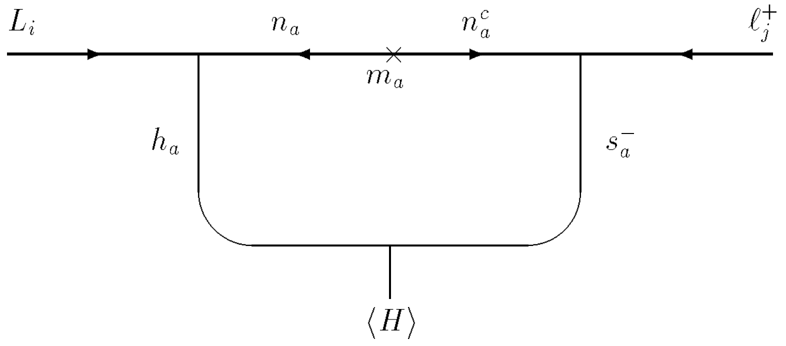

Fig. 3

Diagram that induces radiative masses for the leptons in the model of Sec. 4.