

## Enhanced Electric Dipole Moment of the Muon in the Presence of Large Neutrino Mixing

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The electric dipole moment (edm) of the muon ( $d_\mu^e$ ) is evaluated in supersymmetric models with nonzero neutrino masses and large neutrino mixing arising from the seesaw mechanism. It is found that if the seesaw mechanism is embedded in the framework of a left-right symmetric gauge structure, the interactions responsible for the right-handed neutrino Majorana masses lead to an enhancement in  $d_\mu^e$  to values as large as  $5 \times 10^{-23} e$  cm, with a correlated value of  $(g-2)_\mu \approx 13 \times 10^{-10}$ . This should provide a strong motivation for improving the edm of the muon to the level of  $10^{-24} e$  cm as has recently been proposed.

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It has long been recognized that electric dipole moments (edm) of fermions can provide a unique window to probe into the nature of the forces that are responsible for  $CP$  violation [1]. Experimental limits on the edm of neutron have reached the impressive level of  $6 \times 10^{-26} e$  cm [2] and have already helped constrain and sometimes exclude theoretical models of  $CP$  violation. Electric dipole moment of the electron has severely been constrained by atomic measurements in Cs ( $d_e^e \leq 10^{-26}$ ) and Tl ( $d_e^e \leq 4.3 \times 10^{-27} e$  cm) [3]. The limits on the muon edm, on the other hand, are much weaker, the present limit derived from the CERN  $(g-2)$  experiment [4] is  $d_\mu^e \leq 1.1 \times 10^{-18} e$  cm. There has been a recent proposal to improve this limit on  $d_\mu^e$  to the level of  $10^{-24} e$  cm [5]. In this paper we will argue that there is a strong motivation for this proposed improvement, related to the observation of neutrino masses and oscillations.

In a large class of models, a generic scaling law holds for leptonic edm given by  $d_\mu^e/d_e^e \approx m_\mu/m_e$ . The present limit on  $d_e^e$  would then imply that  $d_\mu^e \leq 10^{-24} e$  cm. Examples where such a scaling law holds are (i) multi-Higgs models where the dominant contribution to the leptonic edm arises from a two-loop diagram involving  $\gamma$ - $V$ -Higgs vertex, with  $V = Z, W$  [6], and (ii) the minimal supersymmetric standard model (MSSM) with the usual assumption of universality of scalar masses and proportionality of trilinear  $A$  terms [7]. In both cases,  $e - \mu$  universality in edm is broken only by the lepton masses, and hence the scaling law. (Recently an extended Higgs model [8] has been analyzed, where it has been shown that for large values of the parameter  $\tan\beta$ , the one-loop Higgs exchange diagram can compete with the two-loop diagram [6], leading to order one violation of the scaling law.)

In the light of Super-Kamiokande [9], MSSM must be extended to incorporate small neutrino masses. A natural place is left-right (LR) symmetric gauge theories [10] with the seesaw mechanism. We have recently advocated a simple supersymmetric realization of left-right symmetry (SUSYLR) [11], where we simply embed the MSSM into a

LR gauge structure at a high scale  $\nu_R \sim 10^{11} - 10^{15}$  GeV. The effective theory that emerges from this model at scales below  $\nu_R$  is a constrained MSSM with far fewer number of phases. In particular, it has a built-in solution to the SUSY  $CP$  problem [11,12]. In this paper we study  $d_\mu^e$  in this class of models and show that the interactions responsible for the Majorana masses of the  $\nu_R$  will lead to an enhancement of  $d_\mu^e$ . Our main effect arises through the renormalization group extrapolation from the Planck scale to  $\nu_R$  [13]. In this interval the Majorana Yukawa couplings of the  $\nu_R$  fields, as well as the associated trilinear  $A$  terms, will affect the soft supersymmetry breaking parameters of the effective MSSM, leading to the enhancement of  $d_\mu^e$ . Since the Majorana Yukawa couplings do not obey  $e - \mu$  universality, the scaling law  $d_\mu^e/d_e^e = m_\mu/m_e$  is not obeyed by these new diagrams.

*The model.*—The electroweak gauge group of the model is  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  with the standard assignment of quarks and leptons—left-handed quarks and leptons ( $Q, L$ ) transform as doublets of  $SU(2)_L$ , while the right-handed ones ( $Q^c, L^c$ ) are doublets of  $SU(2)_R$ . The Dirac masses of fermions arise through their Yukawa couplings to a Higgs bidoublet  $\Phi(2, 2, 0)$ . We will confine to the minimal version with only one such  $\Phi(2, 2, 0)$  field. The  $SU(2)_R \times U(1)_{B-L}$  symmetry is broken to  $U(1)_Y$  by  $B - L = 2$  triplet scalar fields, the left triplet  $\Delta$  and right triplet  $\Delta^c$  (accompanied by  $\bar{\Delta}$  and  $\bar{\Delta}^c$  fields, their conjugates to cancel anomalies). These fields also couple to the leptons and are responsible for inducing large Majorana masses for the  $\nu_R$ . The gauge invariant matter part of the superpotential involving these fields is

$$W = \mathbf{Y}_q Q^T \tau_2 \Phi \tau_2 Q^c + \mathbf{Y}_l L^T \tau_2 \Phi \tau_2 L^c + (\mathbf{f} L^T i \tau_2 \Delta L + \mathbf{f}_c L^c T i \tau_2 \Delta^c L^c). \quad (1)$$

Under left-right parity,  $Q \leftrightarrow Q^{c*}, L \leftrightarrow L^{c*}, \Phi \leftrightarrow \Phi^\dagger, \Delta \leftrightarrow \Delta^{c*}$ , along with  $W_{SU(2)_L} \leftrightarrow W_{SU(2)_R}^*, W_{B-L} \leftrightarrow W_{B-L}^*$ , and  $\theta \leftrightarrow \bar{\theta}$ . Here the transformations apply to the respective superfields. As a consequence,  $\mathbf{Y}_q = \mathbf{Y}_q^\dagger$ ,

$\mathbf{Y}_l = \mathbf{Y}_l^\dagger$ , and  $\mathbf{f} = \mathbf{f}_c^*$  in Eq. (1). Furthermore, the trilinear  $A_q$  and  $A_l$  terms will be Hermitian, the gluino mass term will be real, and the supersymmetric mass term for  $\Phi$  (the  $\mu$  term) as well as the supersymmetry breaking  $B\mu$  term will be real. Departures from these boundary conditions below  $v_R$  due to the renormalization group extrapolation is small. The model thus provides a natural resolution to the supersymmetric  $CP$  problem.

Below  $v_R$ , the effective theory is the MSSM with the  $H_u$  and  $H_d$  Higgs multiplets. The single coupling matrix  $\mathbf{Y}_q$  of Eq. (1) describes the flavor mixing in the MSSM in both the up and the down sectors leading to the relations

$$\mathbf{Y}_u = \gamma \mathbf{Y}_d, \quad \mathbf{Y}_{\nu^p} = \gamma \mathbf{Y}_\ell, \quad (2)$$

which we call up-down unification [11]. Here  $\gamma$  is a mixing parameter characterizing how much of  $H_u$  and  $H_d$  of MSSM are in the bidoublet  $\Phi$ —in general,  $\Phi$  will mix with other bidoublets/doublets of the high scale theory. The case of  $H_{u,d}$  entirely in  $\Phi$  will correspond to  $\gamma = 1$  and  $\tan\beta = m_t/m_b$ . The first of Eq. (2) will lead to vanishing quark mixings and unacceptable quark mass ratios. We showed in Ref. [11] that including the one-loop diagrams involving the gluino and the chargino and allowing for a flavor structure for the  $A$  terms, there exists a large range of parameters (though not the entire range possible in the usual MSSM) where correct quark mixings as well as masses can be obtained consistent with flavor changing constraints. We found a class of solutions for large  $\tan\beta \sim 35\text{--}40$  ( $\gamma = 1$ ), and for small  $\tan\beta \sim 4$  where all quark masses mixings and  $CP$  violating phenomena could be explained. The smaller value  $\tan\beta$  requires larger values of  $\gamma$ , since  $\gamma \tan\beta = m_t/m_b$  is fixed. In this paper, we use small  $\tan\beta$  scenarios which is less constrained.

A concrete example of the high scale theory which results in  $\gamma \neq 1$  and also maintains automatic  $R$ -parity of the left-right model involves the addition of the fields  $\rho(2, 2, 2) + \bar{\rho}(2, 2, -2)$  and  $\Omega_L(3, 1, 0) + \Omega_R(1, 3, 0)$ . It can be shown (see, e.g., Ref. [14]) that this model has a ground state where  $\langle \Omega_R \rangle \sim \langle \Delta^c \rangle = \langle \bar{\Delta}^c \rangle \sim v_R$  and  $\langle \Omega_L \rangle = \langle \Delta \rangle = \langle \bar{\Delta} \rangle = 0$ . All the coupling and the mass parameters in the superpotential are guaranteed to be real by parity symmetry,  $P$ , defined earlier, in combination with the charge conjugation symmetry  $C$  under which all superfields (except  $\rho$  and  $\Phi$ ) transform as  $\Psi \rightarrow \Psi^c$ , where  $\Psi$  stands for a relevant superfield in the theory;  $W_L \rightarrow W_R$  and  $B \rightarrow -B$ . The fields  $\rho$  and  $\Phi$  transform as follows:  $\rho \rightarrow \tau_2 \bar{\rho}^T \tau_2$  and  $\Phi \rightarrow \tau_2 \Phi^T \tau_2$ . We will assume that the supersymmetry breaking terms respect only  $P$  and not  $C$ . The superpotential terms  $W \supset \lambda_1 \text{Tr}[(\rho \Delta^c \Phi) + (\bar{\rho} \Delta \Phi)] + \lambda_2 \text{Tr}[(\bar{\rho} \bar{\Delta}^c \Phi) + (\rho \bar{\Delta} \Phi)]$  mix the doublets in  $\Phi$  and  $\rho$ , resulting in MSSM fields given by  $H_u = \cos\theta_1 \Phi_u + \sin\theta_1 \rho_u$  and  $H_d = \cos\theta_2 \Phi_d + \sin\theta_2 \rho_d$ . Here  $\theta_1$  is the  $\rho_u - \Phi_u$  mixing angle, which is unrelated (since  $\lambda_1 \neq \lambda_2$ ) to  $\theta_2$ , the  $\rho_d - \Phi_d$  mixing angle. This gives  $\gamma = \frac{\cos\theta_1}{\cos\theta_2}$ , which can take any arbitrary value.

We note that due to the combination of  $P$  and softly broken  $C$  symmetry, all dimension four couplings are real.

The VEV's (vacuum expectation values) of order  $v_R$  are also real, which will render all entries of the Higgsino mass matrix to be real. The effective  $\mu$  and  $B\mu$  terms are then real, leading to a solution to the SUSY  $CP$  problem. Such a scheme is completely stable under renormalization, since only the dimension 3 and 2 terms of the SUSY breaking Lagrangian are assumed to break  $C$  (but not  $P$ ).

Unlike the large  $\tan\beta$  case (corresponding to  $\gamma = 1$ ) [15], we are finding that  $CP$  violation in the quark sector has to arise from soft terms (due to  $C$  invariance of  $d = 4$  terms). We have analyzed this possibility in Ref. [11] and shown its consistency.

In the absence of the  $\Omega_R$  field, the doubly charged field  $\Delta^{c++}$  in  $\Delta^c$  (as well as its conjugate) will remain massless—it will pick up mass of order  $v_R^2/M_{\text{string}}$  if nonrenormalizable operators are included. Inclusion of  $\Omega_R$  lifts the mass of  $\Delta^{c++}$  to the scale  $v_R$  [14]. We will analyze two cases, one with the inclusion of  $\Omega_{L,R}$  fields, and one without.

*Leptonic CP violation and muon edm.*—To discuss  $CP$  violation in the lepton sector, we need to specify, in addition to Eq. (1), the most general soft breaking Lagrangian in the lepton superpartners:

$$-\mathcal{L}_{\text{soft}}^\ell = \mathbf{m}_{LL}^2 \tilde{L}^\dagger \tilde{L} + \mathbf{m}_{RR}^2 \tilde{L}^c \tilde{L}^c + [A_l \tilde{L} \Phi \tilde{L}^c + A_f (\tilde{L} \tilde{L} \Delta + \tilde{L}^c \tilde{L}^c \Delta^c) + \text{H.c.}] \quad (3)$$

To generate a nonvanishing muon edm, one needs a complex valued  $(A_l)_{22}$  and/or complex soft mass-squared terms. But above the scale where the parity symmetry is valid,  $A_l$  is Hermitian, and therefore its diagonal elements are all real. This element can, however, be complex due to radiative corrections below the parity breaking scale. There are two ways this can happen: (i) if only parity symmetry is broken but gauge symmetry  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  is unbroken at the string scale by introduction of parity odd singlets [16]; (ii) if both parity and the left-right gauge symmetry are broken, but some remnant of the  $\mathbf{f}$  and  $A_f$  couplings remain below the  $v_R$  scale. This happens in SUSYLR model without the  $\Omega_{L,R}$  fields [14,17] where the  $\Delta^{c++}$  field from  $\Delta^c$  will have a mass of order  $v_R^2/M_{\text{string}} \sim 10^{12}$  GeV. So between  $M_{\text{string}}$  and  $M_{\Delta^{c++}}$ , the effects of  $\mathbf{f}$  and  $A_f$  couplings will be felt, and  $(A_l)_{22}$  can become complex.

In case (i),  $(A_l)_{ij}$  become complex due to the RGE (renormalization group equation) effect,  $16\pi^2 \frac{dA_l}{dt} = (4A_f \mathbf{f}^\dagger \mathbf{Y}_l + 2\mathbf{f} \mathbf{f}^\dagger A_l + \dots)$ . The first term will introduce phase in  $A_l$ . Note that  $A_f$  is not constrained to be Hermitian at the string scale by parity symmetry.

Below the  $D$ -parity breaking scale, the soft mass parameters  $\mathbf{m}_{LL}^2$  and  $\mathbf{m}_{RR}^2$  will evolve differently. In particular,  $\mathbf{m}_{RR}^2$  will feel the effects of  $\mathbf{f}$  and  $A_f$  couplings. In order to explain the large oscillation angle needed for the atmospheric neutrino data, we will find that  $f_{23}$  is not much smaller than  $f_{33}$ .  $(\mathbf{m}_{RR}^2)_{23}$  will then become large and complex through the RGE  $16\pi^2 \frac{d\mathbf{m}_{RR}^2}{dt} = (2A_f A_f^\dagger + \dots)$ .

The dominant contribution to the edm of muon arises from a diagram which has right- and left-handed muon in the external legs and a lighter stau inside the loop. It utilizes the above-mentioned 2–3 mixing which is large and complex. For example, the diagram can have  $\mu_L - \tilde{\tau}_R$  and  $\tilde{\tau}_L - \mu_R$  vertices along with the stau mass flip inside the loop or it can involve just the  $\mu_L - \tilde{\tau}_R$  and  $\tilde{\tau}_R - \mu_R$  vertices. It might be suspected that similar diagrams will also induce large edm for the electron. However, in this model, since  $f_{13}$  and  $f_{12}$  are much smaller, such contributions are negligible. Essentially, we have a scenario where  $e - \mu$  flavor symmetry is broken by a large amount by the  $\mathbf{f}$  and  $A_f$  terms. As a result the scaling law alluded to in the introduction does not hold. Because of the new  $\mathbf{f}$  couplings above the  $\nu_R$  scale, the  $\tilde{\tau}_1$  mass is lower than the usual SUGRA (supergravity) model for the same values of the parameters. This is why the diagram involving the  $\tilde{\tau}$  tends to dominate in  $d_\mu^e$ .

$$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = (6.27 \times 10^{-6}, 2.5 \times 10^{-3}, 5.2 \times 10^{-2}) \text{ eV} \quad (4)$$

$$(|U_{e2}|, |U_{e3}|, |U_{\mu 3}|) = (3.1 \times 10^{-2}, 2.9 \times 10^{-2}, 0.71).$$

Here  $|U_{e2}|$  is the mixing relevant for solar neutrinos, and  $|U_{\mu 3}|$  is for atmospheric neutrinos. Note that we have taken all Yukawa couplings to be real, consistent with our assumption that  $C$  and  $P$  symmetry are respected by  $d = 4$  terms.

It is possible to obtain the large angle MSW solution by choosing at  $\nu_R \sim 10^{15.6}$  GeV,  $\hat{\mathbf{f}} = (-1.77 \times 10^{-7}, -1.42 \times 10^{-6}, 0, -3.9 \times 10^{-3}, -6.4 \times 10^{-2}, -1.28)$ . The neutrino masses are then  $(1.7 \times 10^{-3}, 2.0 \times 10^{-3}, 3.4 \times 10^{-2})$  eV with  $(|U_{e2}|, |U_{e3}|, |U_{\mu 3}|) = (0.45, 2.8 \times 10^{-2}, 0.72)$ .

We use the one-loop Yukawa and two-loop gauge RGE to extrapolate all parameters between the string scale and  $\nu_R$ . For simplicity we choose a universal scalar mass  $m_0$  at the string scale. We also assume a common trilinear mass  $A_0(\times \mathbf{Y}_l)$  for all generations. For  $A_f$  we use a structure similar to  $\mathbf{f}$ . But we do not impose  $A_f \propto \mathbf{f}$ . We demand electroweak symmetry to be broken radiatively.

In this model, the neutralino-slepton loop contribution dominates the edm of muon. Sizable  $d_\mu^e$  results if  $A_f$  and  $\mathbf{f}$  are not proportional. We give an explicit example for this case below. It should be mentioned that large values of  $A_f$  reduces stau mass while it increases  $d_\mu^e$ . So in exploring regions of large  $d_\mu^e$ , we need to consider the experimental limits on stau. In our calculation we take the lightest stau mass ( $\tilde{\tau}_1$ ) to be  $\geq 80$  GeV (which is above the current experimental limit of 70 GeV [18] at  $\sqrt{s} = 202$  GeV). In Fig. 1 (solid lines) we plot the muon edm parameter  $k_\mu \equiv \log_{10}[\frac{d_\mu^e}{1 \times 10^{-23} \text{ e cm}}]$  for case (i) for  $\tan\beta = 3$  and for the small angle MSW fit given above ( $d_\mu^e$  is not much different for the large angle MSW solution which is not shown). This case corresponds to  $D$  parity broken at the string scale, but left-right gauge symmetry broken at  $\nu_R \approx 10^{15.3}$  GeV. At the string scale (taken to be  $10^{17}$  GeV),

*Results.*—First we discuss the neutrino mass fits. We start with a basis where the charged lepton masses are diagonal and Dirac neutrino masses are given by  $M_{\nu^D} = \gamma \tan\beta M_l$ , where  $M_l = \text{Diag}(m_e, m_\mu, m_\tau)$ . The light Majorana neutrino mass matrix is then given by  $M_\nu = \frac{\gamma^2 \tan^2\beta}{\nu_R} M_l \mathbf{f}^{-1} M_l$ , where  $\mathbf{f}$  is the right-handed Majorana Yukawa coupling matrix.

In our fit, we first use the small angle MSW (Mikheyev-Smirnov-Wolfenstein) oscillations for the solar neutrinos with  $\Delta m_{e\mu}^2 \approx (0.3-1) \times 10^{-5}$  eV<sup>2</sup> and  $2 \times 10^{-3} \leq \sin^2 2\theta_{e\mu} \leq 2 \times 10^{-3}$ . We also use the  $\nu_\mu \rightarrow \nu_\tau$  oscillation scenario for atmospheric neutrinos with  $\Delta m_{\mu\tau}^2 \approx (0.1-1) \times 10^{-2}$  eV<sup>2</sup> and  $\sin^2 2\theta_{\mu\tau} \approx 0.8-1$  [9]. For  $\tan\beta = 3$ , we find a good fit to the solar and atmospheric neutrino data by choosing the matrix elements of  $\mathbf{f}$  as  $\hat{\mathbf{f}} \equiv (f_{11}, f_{12}, f_{13}, f_{22}, f_{23}, f_{33}) = (-1.0 \times 10^{-4}, 8.8 \times 10^{-4}, -2.2 \times 10^{-5}, -1.3 \times 10^{-2}, 1.03 \times 10^{-1}, -1.59)$ . At  $\nu_R = 10^{15.3}$  GeV. The resulting neutrino masses and mixing angles are

we have assumed (in GeV units throughout) the elements of  $A_f$  (in a notation analogous to  $\hat{\mathbf{f}}$ ) to be  $\hat{A}_f = (-2 \times 10^{-3}, 1 \times 10^{-2}, 0, -1 \times 10^2 e^{i\pi/2}, 4.7 \times 10^2 e^{i\pi/2}, 3.3 \times 10^2 e^{-i\pi/2})$ . We put  $A_0 = -120$  GeV (where  $A_l = A_0 \mathbf{Y}_l$ ). The upper solid line in Fig. 1 is drawn for  $m_0 = 160$  GeV. The extreme left corner of the curve corresponds to lighter stau mass ( $\tilde{\tau}_1$ ) = 82 GeV. At the same spot in the parameter space, the lightest chargino ( $\chi_1^\pm$ ) and the lightest neutralino masses ( $\chi_1^0$ ) are 106 and 52 GeV, respectively. We see that the muon edm can be as large as  $\sim 3 \times 10^{-23} e$  cm in this case. The lower solid line is drawn for  $m_0 = 170$  GeV for the same set of input values.

In Fig. 1 the dashed line is the muon edm parameter  $k_\mu$ , for case (ii) with  $\tan\beta = 3$  and  $m_0 = 160$  GeV. This case

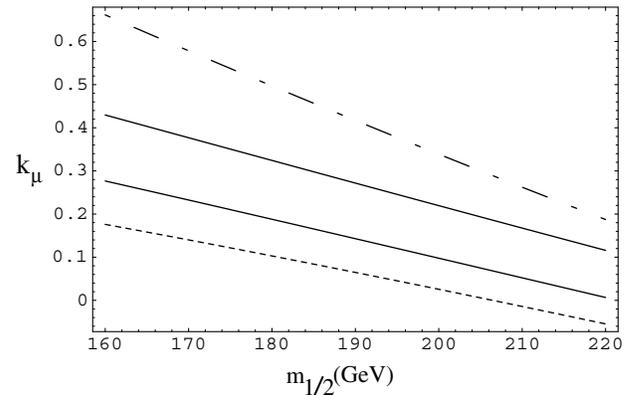


FIG. 1.  $k_\mu (\equiv \log_{10}[\frac{d_\mu^e}{1 \times 10^{-23} \text{ e cm}}])$  is plotted against  $m_{1/2}$  for  $\tan\beta = 3$ . The solid lines are for case (i). The upper (lower) solid line is for  $m_0 = 160$  (170) GeV. The dashed line ( $m_0 = 160$  GeV) is for case (ii). The dot-dashed line ( $m_0 = 85$  GeV) is for the model with horizontal symmetry. The other inputs are described in the text.

corresponds to  $\Delta^{c++}$  surviving below  $\nu_R$ , with its mass taken to be  $10^{12}$  GeV. We have used the universal scenario for the slepton masses and the same  $\mathbf{f}$  matrix as before. At the string scale, we take  $\hat{A}_f = (-2 \times 10^{-3}, 1 \times 10^{-2}, 0, -1 \times 10^1 e^{i\pi/2}, 3.0 \times 10^2 e^{i\pi/2}, 1.1 \times 10^2 e^{-i\pi/2})$ . We take  $A_0 = 0$ . The extreme left corner of the dashed curve corresponds to lighter stau mass ( $\tilde{\tau}_1$ ) mass of 80 GeV. As can be seen from the figure, large values of  $d_\mu^e$  are possible, as large as  $2 \times 10^{-23} e$  cm.

We have assumed nonproportionality of  $A_f$  and  $\mathbf{f}$  in the preceding two examples. We will argue that this is not unnatural. First of all, there are no strong experimental hints that suggest proportionality of the two (unlike the case of  $A_l$  and  $\mathbf{Y}_l$ ). Second, we have proposed recently a model based on horizontal gauge symmetry  $H$  which allows for all parameters of the soft breaking sector to be arbitrary, subject only to the constraints of  $H$  [19]. The symmetry  $H$  was taken to be  $SU(2)_H \times U(1)_H$ , with the first two generations of fermions falling into  $SU(2)_H$  doublets and the third generation into singlets.  $H$  is spontaneously broken by a pair of doublet  $[\phi(+1), \bar{\phi}(-1)]$  and singlet  $[\chi(+1), \bar{\chi}(-1)]$  scalar fields whose VEV's are below the string scale. We denote  $\epsilon_\phi \equiv \langle \phi \rangle / M_{\text{string}}$ ,  $\epsilon_\chi \equiv \langle \chi \rangle / M_{\text{string}}$  with  $\epsilon_\phi \sim 1/7$ ,  $\epsilon_\chi \sim 1/25$ . The effective Yukawa couplings involving the light fermions will be proportional to powers of  $\epsilon_\phi$  and  $\epsilon_\chi$ .

The dot-dashed line in Fig. 1 shows the enhanced muon edm within this horizontal symmetric framework. We have embedded the model of Ref. [19] into left-right symmetry at a high scale. Unlike in Ref. [19], all the CKM (Cabibbo-Kobayashi-Maskawa) mixing will vanish at tree level now. In a basis where the Dirac Yukawa couplings are diagonalized, the elements of the Majorana neutrino coupling are taken to be  $\hat{\mathbf{f}} = (0.81\epsilon_\chi^4/\epsilon_\phi^2, 0.56\epsilon_\chi^2, 0.125\epsilon_\chi^2/\epsilon_\phi, 0.59\epsilon_\phi^2, 0.73\epsilon_\phi, -1.59)$ . The light neutrino masses are then  $(6.27 \times 10^{-6}, 2.9 \times 10^{-3}, 4.4 \times 10^{-2})$  eV, and  $(|U_{e2}|, |U_{e3}|, |U_{\mu 3}|) = (3.2 \times 10^{-2}, 2.9 \times 10^{-2}, 0.74)$ . The elements of the soft mass-squared matrix are taken as  $85^2(1, \frac{1}{2}\epsilon_\chi^2, \frac{1}{7}\epsilon_\chi^2/\epsilon_\phi, 1, \epsilon_\phi, 1.8)$  with  $m_{LL}^2 = m_{RR}^2$ . The elements of  $A_l$  are taken as  $30(\epsilon_\chi^4/\epsilon_\phi^2, \frac{1}{2}\epsilon_\chi^2, \frac{1}{3}\epsilon_\chi^2/\epsilon_\phi, \frac{1}{3}\epsilon_\phi^2, \frac{1}{3}e^{i\pi/3}\epsilon_\phi, -4)$ , while  $\hat{A}_f = 500(\epsilon_\chi^4/\epsilon_\phi^2, \frac{1}{3}\epsilon_\chi^2, \frac{1}{7}\epsilon_\chi^2/\epsilon_\phi, -3\epsilon_\phi^2 e^{i\pi/2}, 4e^{i\pi/2}\epsilon_\phi, 0.6e^{-i\pi/2})$ . Note that  $A_l$  is Hermitian, while  $A_f$  is not. In this new scenario, for case (i), the muon edm can be enhanced to  $5 \times 10^{-23} e$  cm.

Now we turn to the evaluation of  $(g-2)$  of the muon. In MSSM, the  $(g-2)_\mu$  gets contributions from the chargino and neutralino diagrams. The relevant expressions can be found in Ref. [20]. We find the chargino contribution to be somewhat bigger than the neutralino loop. The magnitude of  $(g-2)_\mu$  is  $(6-13) \times 10^{-10}$  for the curves in Fig. 1.

As for other rare processes, the branching ratio of  $\tau \rightarrow \mu\gamma$  is 1 to 2 orders of magnitude below the present experimental limit. Since this process cannot be made much smaller, it will be of great interest to improve the present

limit by 2 orders of magnitude. In all cases that we studied, the edm for electron is of order  $10^{-28} e$  cm. As for  $\mu \rightarrow e\gamma$ , it is 3 to 4 orders of magnitude smaller than current limits for cases (i) and (ii), and 1 order of magnitude smaller than current limits in the case of horizontal symmetry.

In conclusion, we have found that probing  $d_\mu^e$  at the level of  $10^{-23} e$  cm could reveal the underlying structure responsible for  $CP$  violation as well as for the generation of neutrino masses.

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