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Development of Simulation Environment UAL for Spin Studies in EDM

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Outline

- Brief History
- Equations of Particle and Spin Motion
- Computational Approaches in UAL
- Applications for Testing UAL
- Goals

History of Developing the Simulation Tools

- **First order transport maps (6 x 6 matrix) from MAD => SPINK**
  - Advantage: fast, symplectic, good enough for large machine (AGS, RHIC)
  - Limitation : linear approximation in some element (like sextupoles) disable the effect on spin motion due to particle nonlinear motion

- **High order transport maps from UAL => SPINK**
  - Advantage: particle motion more precise
  - Limitation : store the lattice information (like length, field gradient, beta function, dispersion, etc) and transport maps for each element, need consider the balance between the accuracy of cut slices of each element from the simulation point of view and the performance from the time consuming point of view

- **Simultaneous orbit and spin tracking in UAL (TEAPOT+SPINK)**
  - Advantage: thin length approximation, no storage of lattice information and maps needed, faster performance

Equation of Spin and Particle Motion

- **Thomas- BMT Equation** (due to magnetic dipole moment):

$$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{F}_\mu \qquad \vec{F}_\mu = \frac{ev_0}{p_0 c}\left[(a\gamma+1)\vec{B} - \frac{a\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]$$

$a = (g-2)/2$: Anomalous magnetic moment

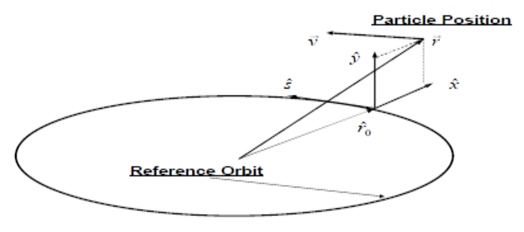
$a\gamma \frac{ev_0}{p_0 c} B_\perp$: (g-2) frequency

- **Lorentz Equation** :

$$\frac{d\vec{p}}{dt} = e(\vec{E} + \vec{\beta}\times\vec{B}) \qquad \frac{d\varepsilon}{dt} = e\vec{v}\cdot\vec{E}$$

- ✓ Frenet-Serret coordinate system in accelerator physics:

$$\vec{r}(s) = \vec{r}_0(s) + x\hat{e}_x(s) + y\hat{e}_y(s)$$



$$\frac{dt}{ds} = \frac{1+hx}{\frac{p_s}{p_0}}\frac{1}{v_0}$$

Computational Approaches in UAL

- **Spin motion (**Thomas BMT equation**) -- SPINK module**

Courtesy of A.U.Luccio

$$S'''_{(x,y,s)} + \omega^2 S'_{(x,y,s)} = 0 \qquad \omega^2 = f_x^2 + \left(f_y - \frac{1}{\rho}\right)^2 + f_s^2 \qquad \vec{f} = \vec{F}_\mu \cdot \frac{dt}{ds}$$

$$\Downarrow$$

$$S = C_1 + C_2 \cos(\delta\mu) + C_3 \sin(\delta\mu) \qquad \delta\mu = \omega\delta s$$

Spin rotation angle
(spin kick)

$$\Downarrow$$

$$M = \begin{pmatrix} 1-(B^2+C^2)c & ABc+Cs & ACc-Bs \\ ABc-Cs & 1-(A^2+C^2)c & BCs+As \\ ACc+Bs & BCc-As & 1-(A^2+B^2)c \end{pmatrix}$$

$c = (1-\cos(\delta\mu))$

$s = \sin(\delta\mu)$

$A = f_x / \omega$

$B = \left(f_y - \frac{1}{\rho}\right) / \omega$

$C = f_s / \omega$

Computational Approaches in UAL

- **Particle motion (**Lorentz equation**) --TEAPOT module**

Courtesy of R.Talman

$$\frac{d}{ds}\left(\frac{p_x}{p_0}\right) = \frac{e}{p_0 c}\left[\frac{\vec{p}}{p_0}\times\vec{B}\right]_x\left(\frac{dt}{ds}\cdot v_0\right) + h\frac{p_z}{p_0}$$

$$\frac{d}{ds}\left(\frac{p_y}{p_0}\right) = \frac{e}{p_0 c}\left[\frac{\vec{p}}{p_0}\times\vec{B}\right]_y\left(\frac{dt}{ds}\cdot v_0\right)$$

$$\frac{d}{ds}\left(\frac{\varepsilon}{p_0 c}\right) = 0$$

$$\frac{d}{ds}\left(\frac{p_x}{p_0}\right) = \frac{e}{p_0 c}\left[\frac{\gamma}{\gamma_0}.\frac{\vec{E}}{\beta_0} + \frac{\vec{p}}{p_0}\times\vec{B}\right]_x\left(\frac{dt}{ds}\cdot v_0\right) + h\frac{p_z}{p_0}$$

$$\frac{d}{ds}\left(\frac{p_y}{p_0}\right) = \frac{e}{p_0 c}\left[\frac{\gamma}{\gamma_0}.\frac{\vec{E}}{\beta_0} + \frac{\vec{p}}{p_0}\times\vec{B}\right]_y\left(\frac{dt}{ds}\cdot v_0\right)$$

$$\frac{d}{ds}\left(\frac{\varepsilon}{p_0 c}\right) = \left(\frac{e\vec{E}}{p_0 c}\right).\frac{\vec{p}}{p_0}.\left(\frac{dt}{ds}\cdot v_0\right)$$

References:

- M.Berz, NIM in Phys. Research A298 (1990)
- F.Lin etc. ICAP'09 (2009)

Applications

- **Orbit Motion Test (Compare with analytical estimate):**

$$\nu_x = \sqrt{1-0.37} = 0.793725$$

$$\nu_y = \sqrt{0.37} = 0.608276$$

$$T = 2\pi R / \upsilon = 0.876\,\mu s$$

$$x\_oscillation = \nu_x \times \frac{10^{-5}}{T} = 9.0646$$

$$y\_oscillation = \nu_y \times \frac{10^{-5}}{T} = 6.9467$$

$$\frac{<\Delta R>}{R} \approx -\frac{1}{1-n} . \frac{\Psi_0^2}{2}$$

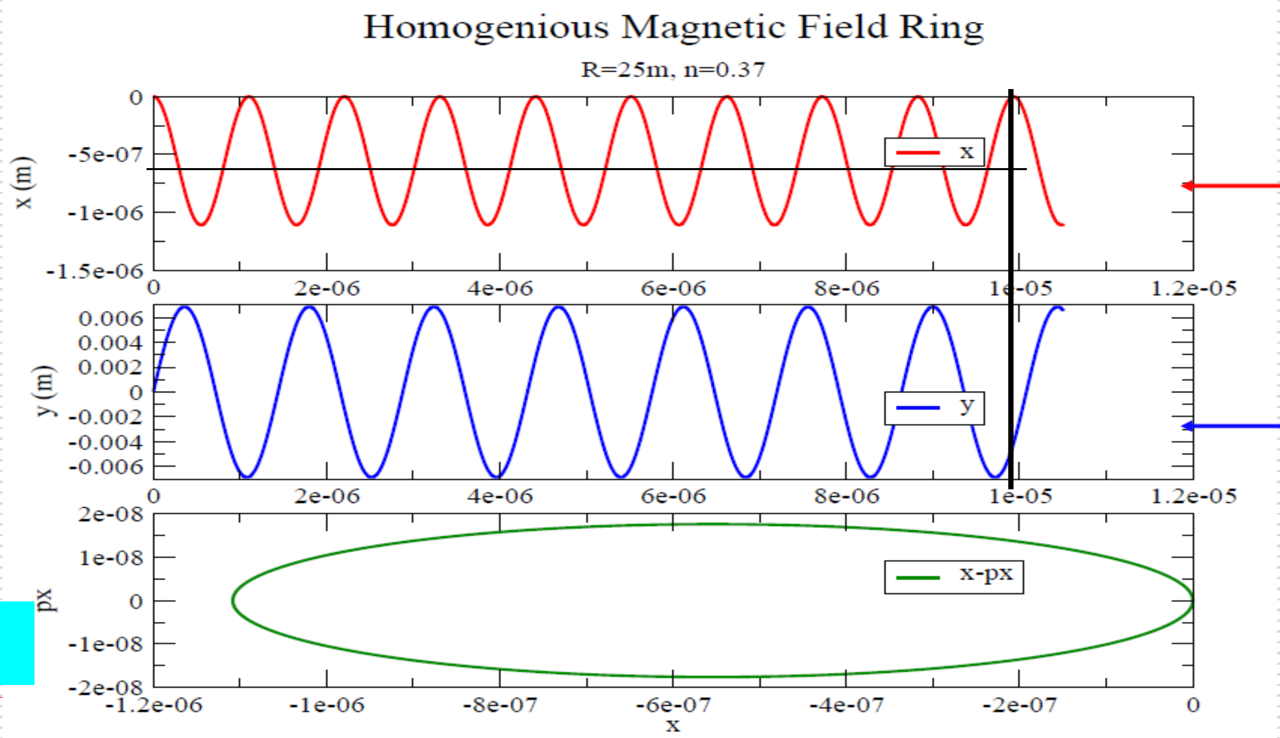
$$\Downarrow$$

$$<\Delta R> \approx -\frac{1}{1-n} . \frac{\Psi_0^2}{2} . R$$

$$\Downarrow$$

$$<\Delta R> \sim -5.3 \times 10^{-7} m$$

Agree Well !



Applications

- **Orbit Motion Test (Compare with COSY-INFINITY):**

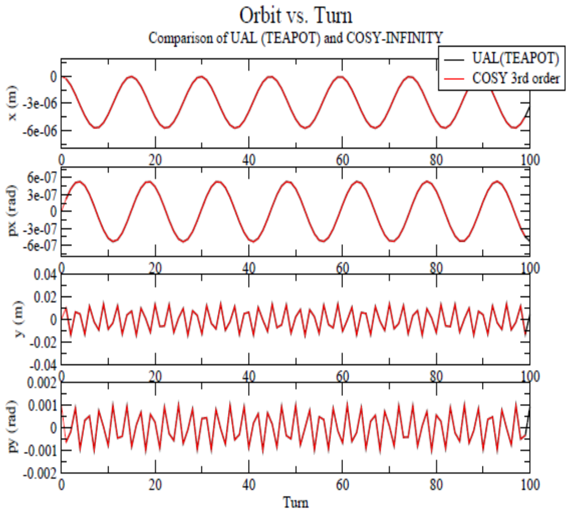


Figure 1: Particle horizontal and vertical motion in the first 1-100 t
UAL and COSY-INFINITY. The averaged radius shifts are both
2.85×10^{-6}.

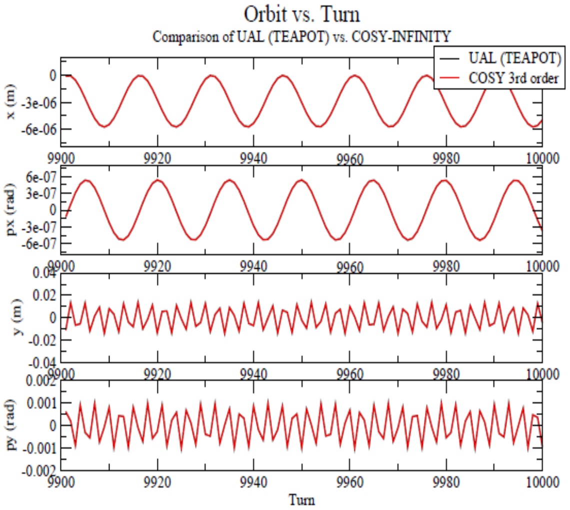


Figure 2: Particle horizontal and vertical motion in the last 9900-10000 turn from UAL and COSY-INFINITY. The averaged radius shifts are both around 2.85×10^{-6}.

Agree Well !

Applications

- **Spin Motion Test in the Presence of Pitch Effect:**

  The correction on the (g-2) spin precession frequency of the spin motion of a circulating particle in a continuous ring with weak magnetic focusing, when the particle moves with a vertical oscillation. The pitch corrected frequency should be related to the uncorrected frequency.

$$B_x(x,y) = B_0(-\frac{n}{\rho_0}y)$$

$$B_y(x,y) = B_0(1-\frac{n}{\rho_0}x+\frac{n}{2\rho_0^2}y^2)$$

$$B_s(x,y) = 0$$

When ω_y increases and arrives to the limit of fast pitch changes ($\omega_y \gg \omega_0$), it has

$$\omega_a = \omega_0\{1-\frac{1+2a}{4}\psi_0^2+\frac{n}{2(1-n)}\psi_0^2+\frac{1}{4}\psi_0^2\}.$$

(g-2) frequency

Initial vertical angle

Extended (g-2) frequency due to pitch correction

Applications

- **Testing lattice**

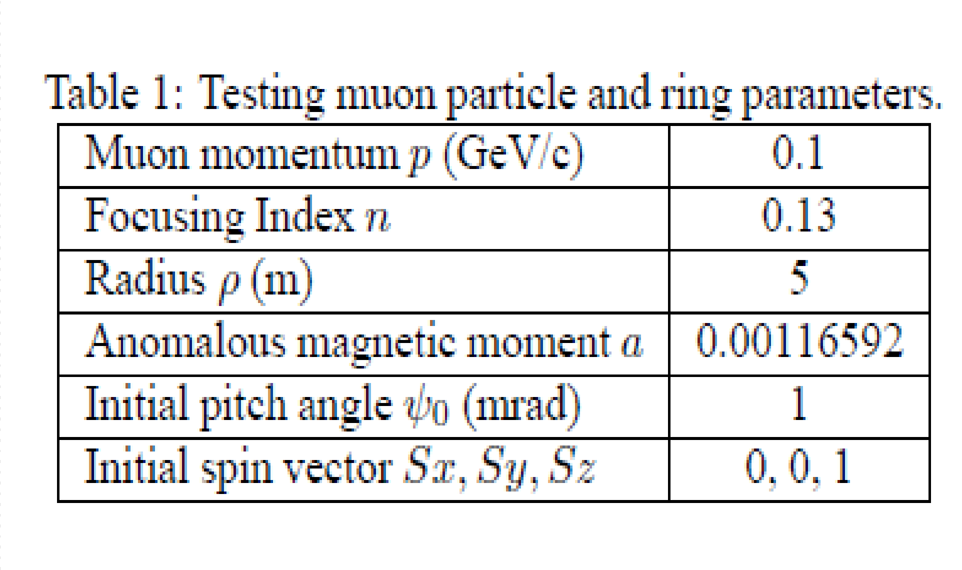


Table 1: Testing muon particle and ring parameters.

Muon momentum p (GeV/c)	0.1
Focusing Index n	0.13
Radius ρ (m)	5
Anomalous magnetic moment a	0.00116592
Initial pitch angle ψ_0 (mrad)	1
Initial spin vector Sx, Sy, Sz	0, 0, 1

$$\nu_y = 0.3606$$

$$\nu_s = 0.0016$$

$$\nu_y / \nu_s \approx 225$$

Given the initial vertical angle $\psi_0 = 1$ mrad

Analytical calculation : $(\omega_a - \omega_0) / \omega_0 \approx 7.42 \times 10^{-8}$

Applications

- **Simulation Results:**

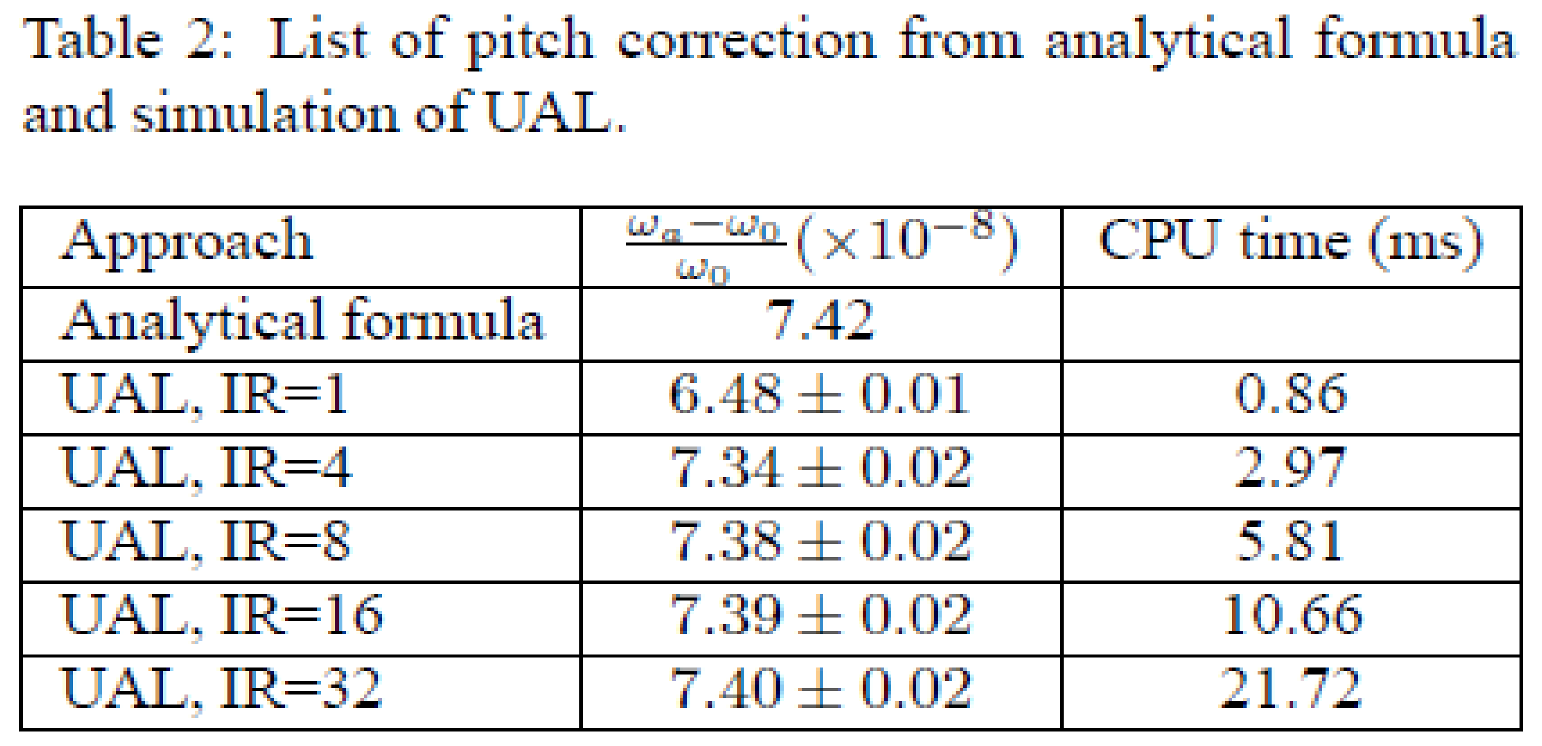


Table 2: List of pitch correction from analytical formula and simulation of UAL.

Approach	$\frac{\omega_a-\omega_0}{\omega_0}$ $(\times 10^{-8})$	CPU time (ms)
Analytical formula	7.42	
UAL, IR=1	6.48 ± 0.01	0.86
UAL, IR=4	7.34 ± 0.02	2.97
UAL, IR=8	7.38 ± 0.02	5.81
UAL, IR=16	7.39 ± 0.02	10.66
UAL, IR=32	7.40 ± 0.02	21.72

Agree Well!

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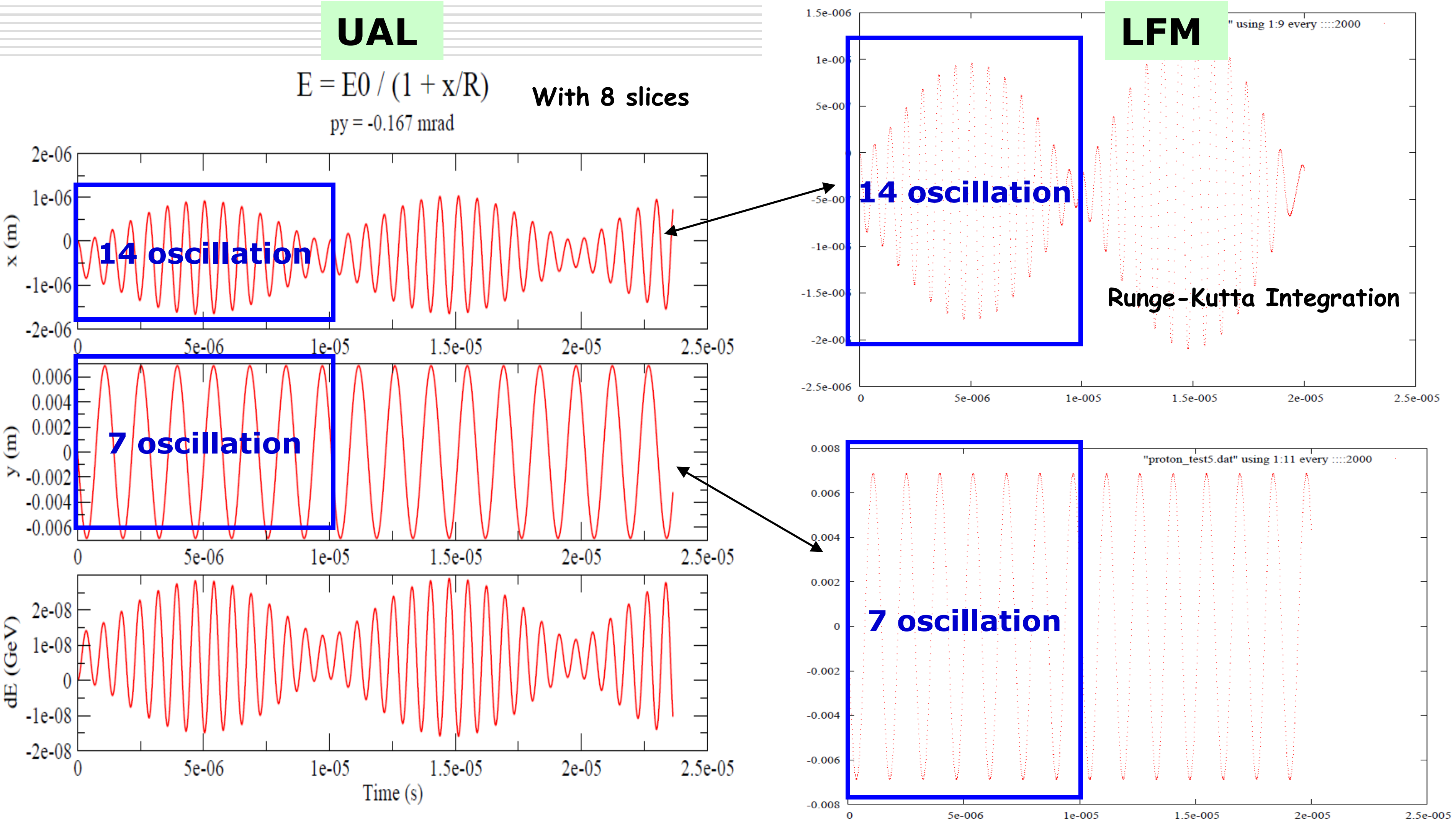
Applications

- **Lattice:**
  - Proton with a=1.7928456 and pc=0.700741Gev
  - A ring lattice R = 25m composed of homogeneously distributed magic electric fields Er0=16.77MV with n=0.37

$$\vec{E} = \frac{\vec{E}r_0}{1 + \frac{x}{R}}$$

  - Compare the tracking results with those from the Local Field Method (LFM: integrate the equations of motion):
    - Initial condition : px = 0 mrad, py = 0.167 mrad

Comparison (I)



UAL

$E = E0 / (1 + x/R)$

$py = -0.167$ mrad

With 8 slices

LFM

14 oscillation

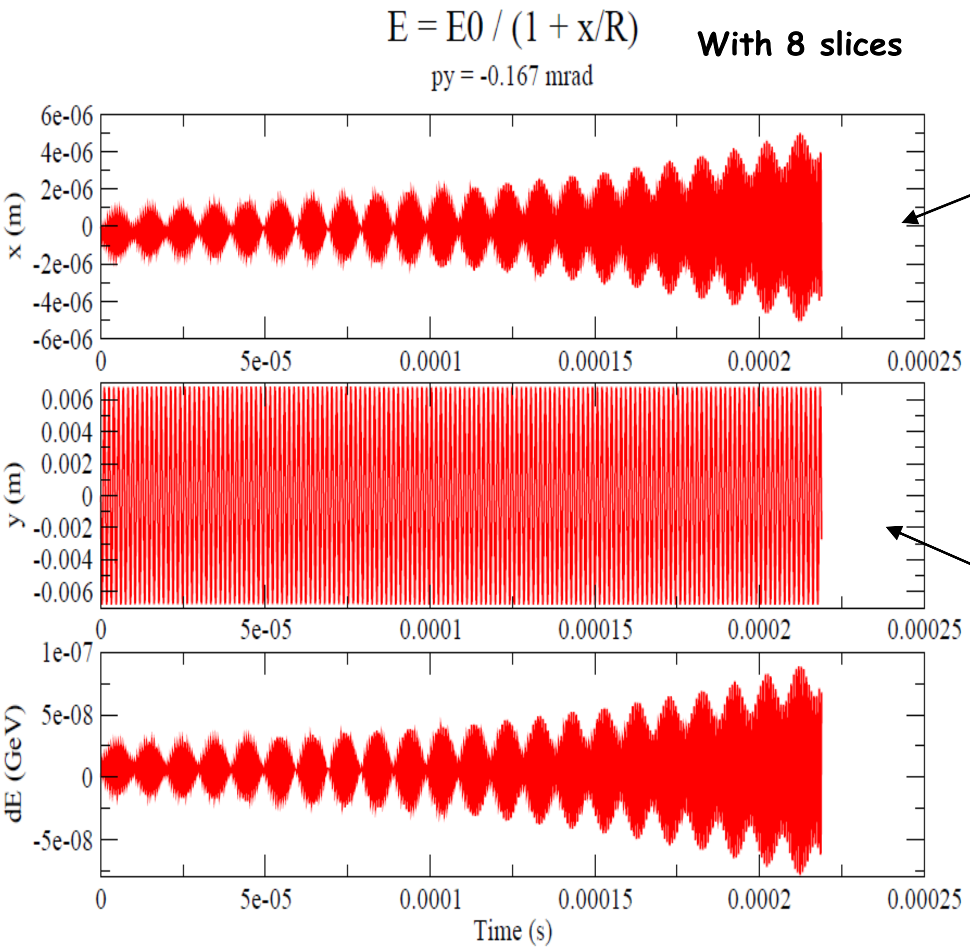
7 oscillation

Runge-Kutta Integration

Courtesy of Y.K.Semertzidis

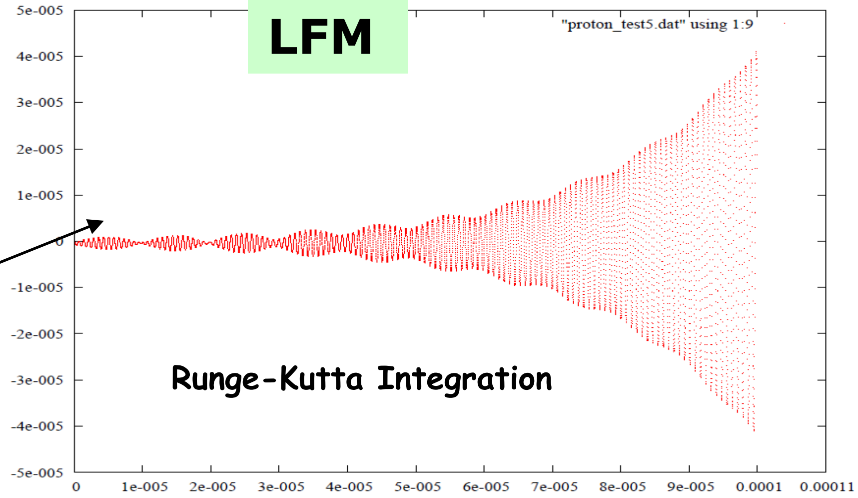
Comparison (II)

UAL

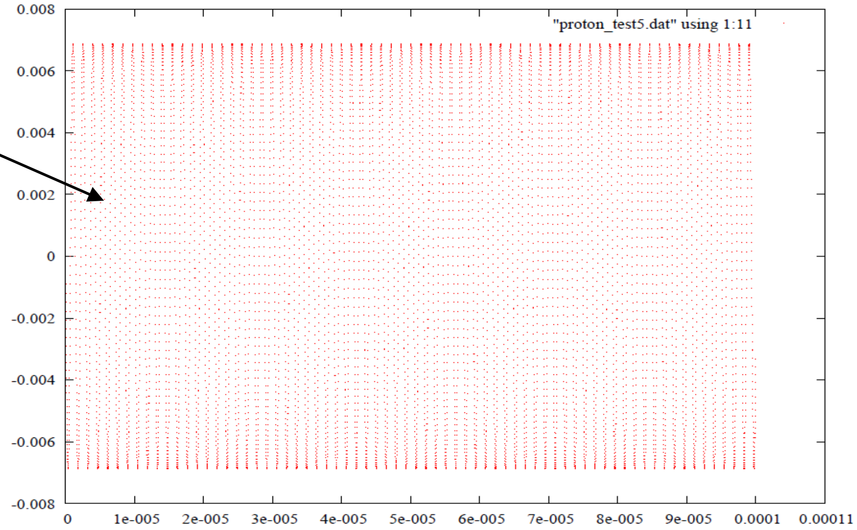


With 8 slices

LFM

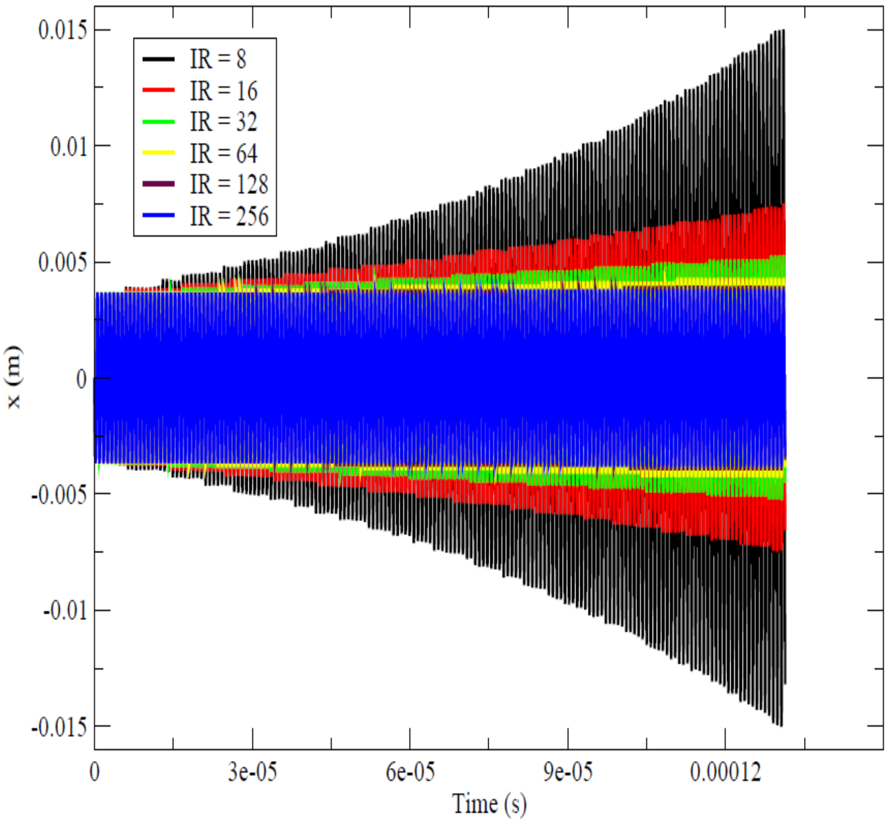


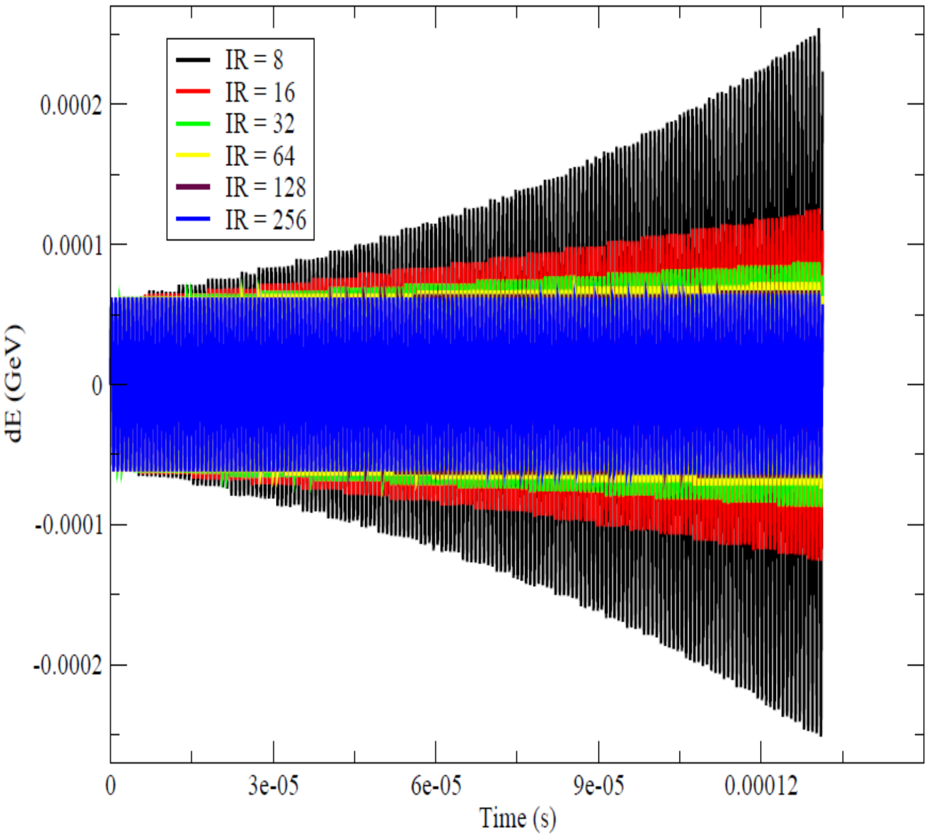
Runge-Kutta Integration



Courtesy of Y.K.Semertzidis

UAL Tracking With More Slices





More slices, less amplitude growth !

Goals – One Year

- Develop and test the computer model UAL-SPINK to track particle orbit and spin motion in the proposed EDM storage ring, based on electric field bends, for protons and deuterons
- The ring will contain, other than the electric bends, magnetic quadrupoles, sextupoles and RF cavities
- The orbit tracking will be in the full 6 dimensional phase spaces
- Specific items to be addressed are
  - The tracking should provide a stable orbit, with all invariants of the motion conserved, for a number of turns of the order of 10^9 in the ring
  - Spin coherence for a time of the order of 1000 should be reached mainly with the use of sextupoles and manipulation of beam bunching
  - Effects of alignment and field errors in the magnet and the effects of fringe fields should be carefully addressed

Longer Term Commitments

Year	Tasks	Approach
Year 1	• Import RHIC experience to EDM project, benchmarking, etc.	• SPINK essential elements such as snakes, electric elements, etc. and post-processing tools
	• Symplectic lab frame tracking	• THINSPIN
	• Beam simulation, longitudinal studies with electric field, etc. Spin coherence time and related studies	• BNL cluster
Year 2	• Preliminary EDM simulation studies with errors, misalignments, etc.	• Blue Gene/L
	• Long term spin evolution studies	• High order orbit-spin Taylor maps
Year 3	• Full scale EDM modeling, EDM beam studies, commissioning modeling	• TBD

Thank you for your attention !

Notes on Goals (One Year)

- The thin element tracker to be used in this study: TEAPOT (via UAL) has been in use for several years, proving itself very stable and capable of orbit tracking to high order. Originally, TEAPOT did not deal with electric fields, that have recently been added by F.Lin and N.Malitsky. Some issues of propagation in electric fields, that act on the energy of the particle, are still been worked out, with very encouraging preliminary results.

- The algorithm for spin coherence calculation of election, based on the analysis of the width of the spin tune line obtained by parallel tracking of many particles representing the beam, proved an effective way to address the problem. Tests of correction of spin decoherence correction using sextupolar lenses, similarly to what has been done to correct the chromaticity in an accelerator have been carried on (on the EDM and COSY), with good results (F.Lin, A.U.Luccio).

- Effects of stable orbit distortion due to misalignment and field errors have been satisfactorily treated in the past in the orbital part of the spin tracker SPINK, using MAD matrices. We expect the same will happen using TEAPOT instead of MAD.

Time and Cost (One Year)

The cost of this work will be mainly for manpower

- Time estimated to reach the exposed goals: 12 months, including 100% contingency
- Manpower:
  - One full time experienced physicist. Cost including overhead: $350,000.
  - One part time computer expert as in-house consultant: $50,000.
  - One graduate student: $50,000.
  - We assume that computing, including parallel computing, will be provided at no cost by the laboratory.