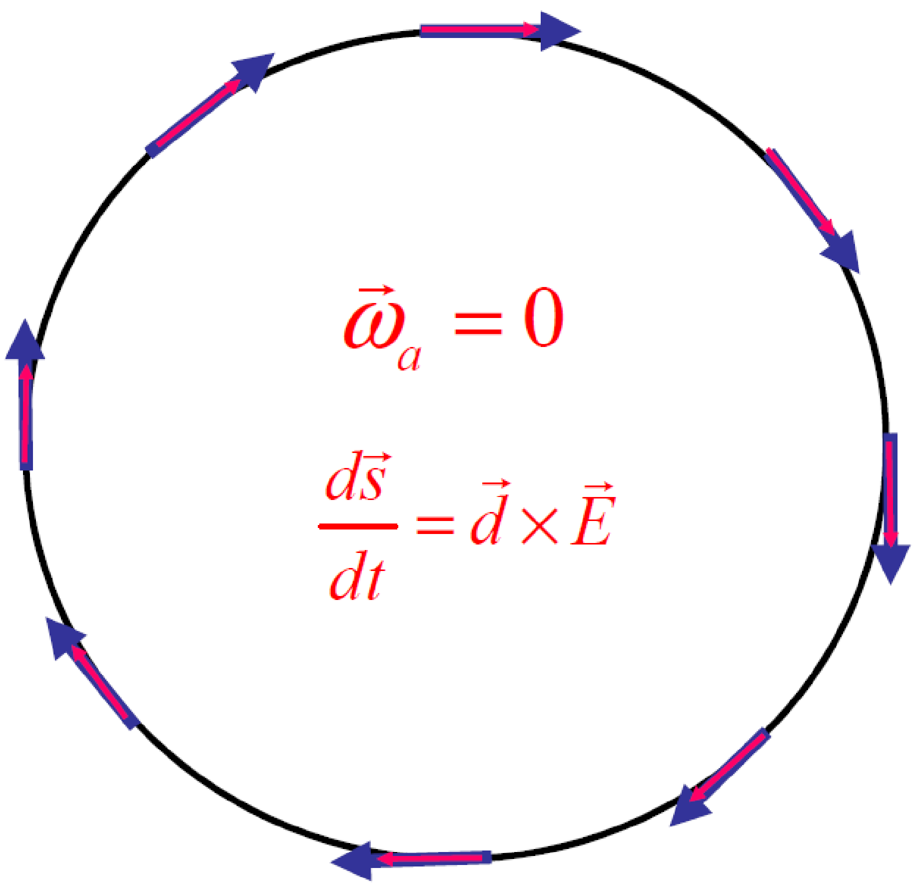
Storage Ring EDM Collaboration

R&D development plan for a Proton EDM Experiment with sensitivity of 10^{-29}e·cm

Part I, Physics Motivation and Experimental method



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Proton R&D Plan

Storage Ring EDM Collaboration, 25 November 2009

1. Introduction

Electric dipole moments (EDM) violate both T-time and P-parity symmetries and conserve C-charge symmetry. Assuming conservation of the combined CPT symmetry, T-violation also means CP-violation. The weak interaction CP-violation contributes a very small EDM, orders of magnitude below current experimental limits. However, most models beyond the SM predict EDM values near the current experimental limits. Hence, the study of EDMs is a search for CP-violation beyond the standard model (SM). If a non-zero EDM value is found it will point to a new, strong CP-violation source needed to solve the mystery of baryon-antibaryon asymmetry of our universe (BAU).

We are planning to search for EDM of the proton in a storage ring with a statistical sensitivity of $\sim 2.5\times 10^{-29}$e·cm/year. At this level it will be an order of magnitude more sensitive than the currently planned neutron EDM experiments at SNS (Oak Ridge), and ILL (Grenoble-France). The ring, after a major upgrade, can accommodate a deuteron EDM experiment with similar sensitivity.

The method employs a radial E-field to steer the proton beam in the ring, magnetic or electric quadrupoles to form a strong focusing lattice (e.g. FODO) and internal polarimeters to probe the proton spin state as a function of storage time. An RF-cavity and sextupole magnets will be used to prolong the spin coherence time (SCT) of the proton beam. It requires building a storage ring with a highly uniform radial E-field with strength of ~17 MV/m between stainless steel plates 2 cm apart. The bending radius will be ~25m and including the straight sections it will have a physical radius of ~30 m.

The g-2 precession frequency of protons at the so called magic momentum (0.7 GeV/c) is zero. The spins of vertically polarized protons injected into the EDM ring can be rotated into the horizontal plane by turning on a solenoidal magnetic field in a straight section of the EDM ring and turning it off at the appropriate time. The Booster facility at BNL can provide the beams as required by the storage ring EDM experiment.

The parameters of the proton beam from the Booster into the EDM ring are:
Polarization state: vertical, polarization value as high as possible (we will assume ≥80%)
Intensity: 2×10^{10} in two bunches every $\sim 10^{3}$ s to be split into $\sim 10^{2}$ bunches after injection in the EDM ring.
Emittance (95%, un-normalized): 3-mm-mrad horizontally and 10-mm-mrad vertically. These emittance values are to be obtained by scraping in the Booster ring a beam with intensity 2×10^{11} in one bunch.
Beam momentum: 0.7GeV/c (kinetic energy: 232 MeV)
Full momentum spread: $(dp/p)_{max}=10^{-3}$, or $(dp/p)_{rms}=2.5\times 10^{-4}$
Running time per year: 10^{7} s.
The total energy of the beam is 1J, and the instantaneous current is 3mA. The required beam vacuum in the EDM storage ring needs to be $<10^{-9}$ Torr.

2. Motivation for Proton and Deuteron EDM Measurements

Modern interest in elementary particle and bound-state electric dipole moments (EDMs) stems from the pioneering work of Normal Ramsey and his collaborators [1]. Their more than 50-year quest to find a neutron EDM anticipated parity (P) and time-reversal (T or CP) violation, necessary ingredients for the existence of a non-zero EDM. Over the years, improvements in the bound on d_n have been used to rule out or severely constrain many models of CP violation, a strong testament to the power of sensitive null results.

As a result of those efforts, the neutron EDM bound currently stands at

$$\left|d_n\right| < 3\times10^{-26}\,\mathrm{e}\cdot\mathrm{cm} \quad (1)$$

Complementary to the bound, elegant (neutral) atomic physics experiments have obtained improved atomic edm constraints. Examples are

$$\left|d_{Tl}\right| < 9\times10^{-25}\,\mathrm{e}\cdot\mathrm{cm} \quad (2)$$

$$\left|d_{Xe}\right| < 6\times10^{-28}\,\mathrm{e}\cdot\mathrm{cm} \quad (3)$$

$$\left|d_{Hg}\right| < 3.1\times10^{-29}\,\mathrm{e}\cdot\mathrm{cm} \quad (4)$$

Those bounds have been used to constrain "new physics" scenarios and provide the indirect charged particle bounds (from *Tl* and *Hg* respectively)

$$\left|d_e\right| < 1.6\times10^{-27}\,\mathrm{e}\cdot\mathrm{cm} \quad (5)$$

$$\left|d_p\right| < 7.9\times10^{-25}\,\mathrm{e}\cdot\mathrm{cm} \quad (6)$$

Although the indirect $|d_p|$ bound from atomic experiments has improved considerably over recent years, it is still a factor of 26 worse than $|d_n|$ and not really competitive. Here, we discuss an experimental opportunity, provided by storage ring technology, to push the direct measurement of d_p and d_D (deuteron) to 10^{-29}e·cm sensitivity, an improvement by nearly 5 orders of magnitude. Such dramatic improvement is made possible by new ideas and techniques described in this document.

What would we learn from the measurement of a non-zero EDM? The standard $SU(3)_C\times SU(2)_L\times U(1)_Y$ model predicts non-vanishing EDMs; however, their magnitudes are expected to be unobservably small $\left|d_e^{SM}\right| < 10^{-38}\,\mathrm{e}\cdot\mathrm{cm}$ and $\left|d_N^{SM}\right| < 10^{-32}\,\mathrm{e}\cdot\mathrm{cm}$, $N=n,p$. Hence, discovery of a non-zero EDM between the current bounds and standard model predictions would signal "new physics" CP violation. Uncovering such a phenomenon could prove crucial in understanding the matter-antimatter asymmetry of our universe which seems to require (suggest) new large sources of CP violation beyond standard model expectations. That fundamental connection with the origin of our very existence coupled with the popularity of well motivated "new physics" scenarios such as supersymmetry (SUSY) with potentially large new sources of CP violation make searches for EDMs exciting and at the forefront of high energy and nuclear physics. Indeed, it is anticipated that the next generation of EDM experiments with several orders of magnitude improved sensitivity may be on the verge of a major discovery with far reaching implications.

Of course, several new neutron EDM experiments are already mounted worldwide. They aim to eventually approach $|d_n| \sim 10^{-28}\,\mathrm{e\cdot cm}$ sensitivity. At that level, the $\bar{\theta}$ parameter of QCD, SUSY phases, Left-Right symmetric models, multi-Higgs scenarios etc. are being probed. With that backdrop, what is the added value of proton and deuteron edm experiments with goals exceeding the d_n searches?

The obvious answer is that storage ring studies aim for $|d_p|$ and $|d_D|$ sensitivities of 10^{-29} e·cm, more than an order of magnitude beyond $|d_n|$ expectations. Hence, they represent the possibility of significant improvement beyond already forefront efforts. However, even at lower 10^{-28}e·cm level, roughly comparable to d_n, they are extremely complementary to d_n and will be of crucial follow-up importance should a non-zero value of d_n or any other EDM be measured.

To put d_n, d_p and d_D into perspective, we note that a priori, all are independent and could have significantly different values. Only when interpreted within the context of a specific theoretical framework, do their values become related and comparison is meaningful. If d_n is found to differ from zero, d_p and d_D will prove crucial in unfolding the new source of CP violation responsible for it. To sort out its structure, the I=1 and 0 isospin combinations

$$d_N^{I=1} = \left(d_p - d_n\right)/2 \tag{7}$$

$$d_N^{I=0} = \left(d_p + d_n\right)/2 \tag{8}$$

along with d_D (which samples various isospin effects) will be complementary.

To illustrate the combined utility, we consider several examples.

1) The QCD CP Violating $\bar{\theta}$ Parameter

The $\bar{\theta}$ CP-violating parameter of QCD can be set to zero in lowest order, but will reemerge from high scale physics via loop level contributions to the quark mass matrix. For nucleons, one expects from leading chiral logs (ln m_p/m_π terms) the isovector relation

$$d_n \simeq -d_p \simeq 3\times 10^{-16}\bar{\theta}\ \mathrm{e\cdot cm} \tag{9}$$

From the bound on equation (1), the restrictive constraint $\bar{\theta} < 10^{-10}$ already follows. The sensitivity will improve to better than 10^{-13} if the storage ring goal of $d_p \sim 10^{-29}$e·cm is achieved. More interesting, should a non-vanishing d_n be measured, it will be necessary to determine d_p to see if the isospin relation of equation (9) is respected. That will, of course, require a measurement of d_p with sensitivity comparable to d_n. Also, even a primarily isovector $\bar{\theta}$ effect, $|d_D|$ is expected to be smaller than $|d_N|$ due to leading log cancellations between d_n and d_p but not zero. Indeed, from non-logarithmic contributions, one roughly anticipates

$$d_D\left(\bar{\theta}\right)/d_N\left(\bar{\theta}\right) \approx 1/3 \tag{10}$$

Confirming or negating $\bar{\theta}$ effects will certainly require measurements of d_n, d_p and d_D.

2) Supersymmetry

Supersymmetry (SUSY) and the new particles associated with it (sparticles) represent a popular, well motivated extension of the standard model. If real, it suggests that a plethora of new particles will be revealed at the LHC. New CP phases associated with SUSY interactions could lead to electromagnetic quark EDMs, d_q with $q=u$ or d, as well as quark color edms, d_q^c, all of which are rather independent. One expects [2]

$$d_n \simeq 1.4\left(d_d - 0.25 d_u\right) + 0.83 e\left(d_u^c + d_d^c\right) - 0.27 e\left(d_u^c - d_d^c\right) \tag{11}$$

$$d_p \simeq 1.4\left(d_d - 0.25 d_u\right) + 0.83 e\left(d_u^c + d_d^c\right) + 0.27 e\left(d_u^c - d_d^c\right) \tag{12}$$

$$d_D \simeq \left(d_u + d_d\right) - 0.2 e\left(d_u^c + d_d^c\right) - 6 e\left(d_u^c - d_d^c\right) \tag{13}$$

or in terms of I=1 and 0 components

$$d_N^{I=1} \simeq 0.87\left(d_u - d_d\right) + 0.27 e\left(d_u^c - d_d^c\right) \tag{14}$$

$$d_N^{I=0} \simeq 0.5\left(d_u + d_d\right) + 0.83 e\left(d_u^c + d_d^c\right) \tag{15}$$

Notice that d_D is very sensitive to the isovector combination $d_u^c - d_d^c$ due to the 2-body pion exchange and represents our most sensitive probe of that quantity by more than an order of magnitude. On the other hand $d_N^{I=1}$ is more sensitive to the electromagnetic d_u-d_d while $d_N^{I=0}$ would determine the isoscalar electromagnetic and color combination in equation (15). Although measurements of d_n and d_p and d_D might not uniquely determine the underlying "new physics" source of CP violation; they will take us quite far in unfolding its structure.

An alternative to the above light quark scenario might be one dominated by heavy quark edm effects. In that case, one would expect isoscalar dominance and

$$d_n \simeq d_p \tag{16}$$

$$d_D \simeq d_p + d_n \tag{17}$$

To test those relations, requires measurements of d_n and d_p and d_D with similar sensitivity.
Based on the above examples, one can very roughly approximate sensitivity relationships among potential future EDM experiments. In table 1, we give current and anticipated EDM bounds and sensitivities for nucleons, atoms and the deuteron. The last column provides a rough measure of their probing power relative to d_n.

Table 1. Current EDM limits in units of [e·cm], and long-term goals for the neutron, ^{199}Hg, ^{129}Xe, proton, and deuteron are given here. The neutron equivalent indicates the EDM value for the neutron to have the same physics reach as the indicated system.

Particle/Atom	Current EDM limit	Future Goal	~d_n equivalent
Neutron	$<1.6\times10^{-26}$	$\sim10^{-28}$	10^{-28}
^{199}Hg	$<3.1\times10^{-29}$	$\sim10^{-29}$	10^{-26}
^{129}Xe	$<6\times10^{-27}$	$\sim10^{-30}$-10^{-33}	10^{-26}-10^{-29}
Proton	$<7.9\times10^{-25}$	$\sim10^{-29}$	10^{-29}
Deuteron		$\sim10^{-29}$	3×10^{-29}-5×10^{-31}

3) Dimensional Analysis

To roughly estimate the scale of "new physics" probed by EDM experiments, we often assume on dimensional grounds

$$d_i \approx \frac{m_i}{\Lambda^2} e \sin\phi, \tag{18}$$

where m_i is the quark or lepton mass, sinϕ is the result of CP-violating phases, and Λ is the "new physics" scale. For m_q~10 MeV and sinϕ of order ½, one finds

$$\left|d_p\right| \sim \left|d_D\right| \sim 10^{-22} \left(\frac{1\text{TeV}}{\Lambda}\right)^2 \text{e}\cdot\text{cm}. \tag{19}$$

So d_p and d_D ~10^{-29}e·cm sensitivity probe Λ~3000 TeV. More realistically, the d_i generally results from a quantum loop effect and there is a further $g^2/16\pi^2$~1/100 suppression. So, for example, in supersymmetry one might expect

$$\left|d_p\right| \sim \left|d_D\right| \sim 10^{-24} \left(\frac{1\text{TeV}}{M_{\text{SUSY}}}\right)^2 \sin\phi \;\; \text{e}\cdot\text{cm}. \tag{20}$$

In such a theory, with $M_{\text{SUSY}} \leq$1 TeV, $\sin\phi$ would have to be very small, $\leq 10^{-5}$ if a d_p or $d_D \geq 10^{-29}$e·cm were not observed. Of course, one hopes that the LHC may actually observe squarks in the TeV or lower range and that $\sin\phi \geq 10^{-5}$. If that is the case d_p and d_D will provide precise EDM measurements that will unveil their CP-violating nature and perhaps help to explain the matter-antimatter asymmetry of our universe.

Other new models of CP-violation from Left-Right symmetric gauge theories, additional Higgs scalars, etc. can also be studied using EDM experiments. In such cases d_p and d_D at 10^{-28}e·cm is competitive with or better than other EDM measurements, while at 10^{-29} e·cm they become our best hope for finding new sources of CP-violation. Couple that sensitivity with the relative theoretical simplicity of the proton and deuteron and it becomes clear that they hold great discovery potential. The storage ring method should therefore be vigorously pursued.

References:

1. J.H. Smith, E.M. Purcel, and N.F. Ramsey, Phys. Rev. **108**, 120 (1957).
2. I.B. Khriplovich, R.A. Korkin, Nucl. Phys. A **665**, 365 (2000), nucl-th/9904081; C.P. Liu and R.G.E. Timmermans, Phys. Rev. C **70**, 055501 (2004); M. Pospelov and A. Ritz, Ann. Phys. **318**, 119 (2005), hep-ph/0504231; O. Lebedev *et al.*, Phys. Rev. D **70**, 016003 (2004).

3. Experimental method

Two of the storage ring EDM experimental methods are described in refs [1] and [2]. Here we describe the magic-momentum proton EDM case, which we chose to pursue since it is the simplest of all experiments with great sensitivity. EDMs (d) couple to electric fields and MDMs (μ) couple to magnetic fields and the spin precession in the presence of both electric and magnetic fields is given by

$$\frac{d\vec{s}}{dt} = \vec{d}\times\vec{E} + \vec{\mu}\times\vec{B} \tag{1}$$

where (assuming the EDM vector is orthogonal to E and B-fields)

$$\frac{d\vec{s}}{dt} = \vec{\omega}\times\vec{s}, \text{ with } \frac{ds}{dt} = \frac{1}{2}\hbar\omega, \text{ and } \frac{ds}{dt} = \hbar\omega \tag{2}$$

for spin ½ (e.g. proton) and 1 (e.g. deuteron) respectively.

Even though in studying the MDM of fundamental particles it is possible to place them in a magnetic field for a considerable amount of time, it is not always possible to do the same for the EDM studies. Placing a charged particle in an electric field region is more challenging since a Coulomb force will act on it. That needs to be compensated without canceling the EDM effect. One way to do this is to place charged particles in a storage ring where the steering field is a radial electric field. The method will be most sensitive when the spin vector is kept along the momentum vector for the duration of the storage, as shown in Figure 1. The spin is frozen in the horizontal plane along the momentum direction whereas, if there is an EDM, it will precess vertically, out of plane.

It turns out the required condition can always be met at one specific momentum for particles with a positive anomalous magnetic moment (defined as $a = (g\text{-}2)/2$). The g-2 precession in the presence of electric fields only is given by (in S.I. units, for $\vec{\beta}\cdot\vec{E} = 0$)

$$\vec{\omega}_a = -\frac{q}{m}\left[a - \left(\frac{m}{p}\right)^2\right]\frac{\vec{\beta}\times\vec{E}}{c} \tag{3}$$

with $q=\pm e$ the charge of the particle with e the absolute value of the electron charge, m the mass of the particle, p its momentum, β its velocity in units of the speed of light c, and E the electric field. For the proton (a = 1.8) there is one momentum, the so-called "magic" momentum, at which $\omega_a = 0$, which can be estimated from eq. (3) to be

$$p = \frac{m}{\sqrt{a}} = 0.7 \text{ GeV/c} \tag{4}$$

for the proton. The magic momentum for muons is 3.1 GeV/c, the momentum at which the muon g-2 experiment ran at BNL[1].

[1] The muon g-2 experiment was performed at the magic momentum where the radial electric field from the electrostatic quadrupoles used for the beam focusing does not contribute to the g-2 frequency.

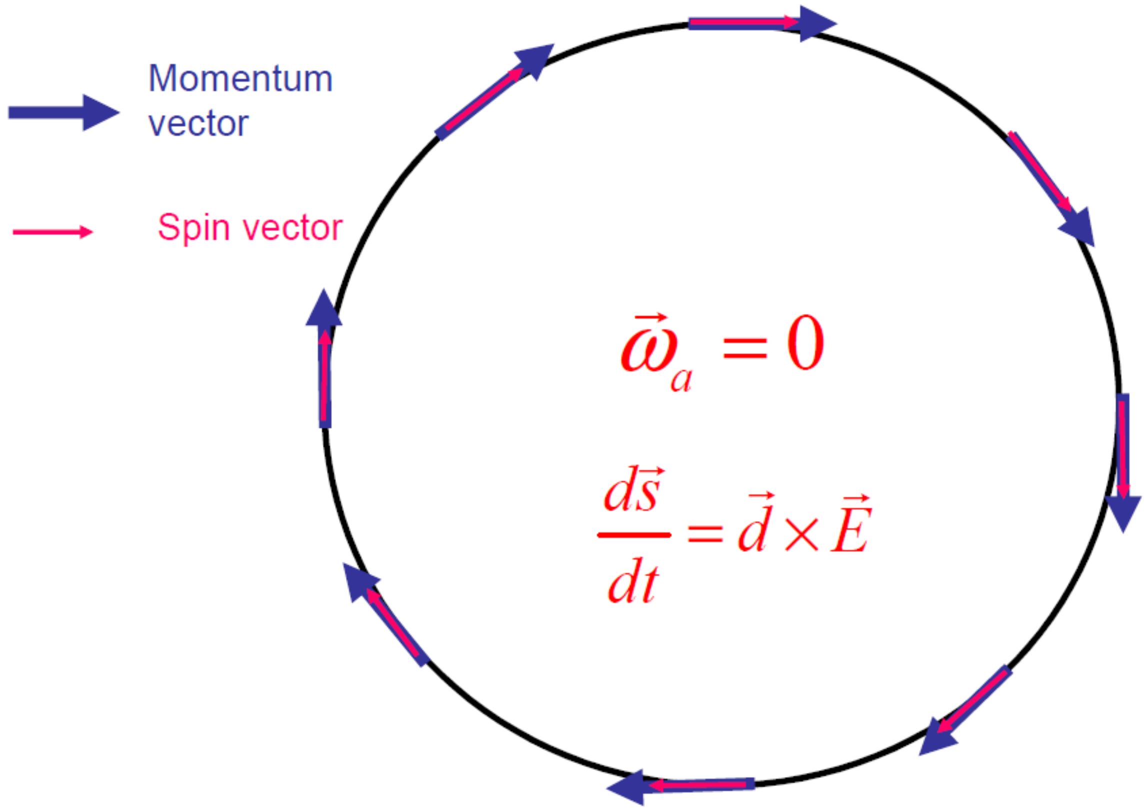

Figure 1. A top view of an ideal storage ring EDM experiment. The spin and momentum vectors are kept aligned for the duration of the storage, i.e. the in-plane g-2 precession $\omega_a = 0$. If the EDM vector (*d*) is not zero the particle spin will precess out of plane as a function of storage time due to the radial E-field.

For particles with negative anomalous magnetic moments, like the deuteron ($a = -0.14$), there is no "magic" momentum and a combination of B&E-fields is needed to achieve the same result. The g-2 precession in the presence of both B&E-fields (for $\vec{\beta} \cdot \vec{B} = \vec{\beta} \cdot \vec{E} = 0$) is

$$\vec{\omega}_a = -\frac{q}{m}\left\{ a\vec{B} + \left[a - \left(\frac{m}{p} \right)^2 \right] \frac{\vec{\beta} \times \vec{E}}{c} \right\} \tag{5}$$

and the radial E-field to cancel the g-2 precession is given by

$$E = \frac{aBc\beta\gamma^2}{1 - a\beta^2\gamma^2} \approx aBc\beta\gamma^2 \tag{6}$$

with the approximation holding when the denominator in equation (6) is approximately equal to one.

The combined E&B-fields method to freeze the spin can be applied to all particles with both positive and negative anomalous magnetic moment values. In this document we will focus on the proton EDM using the magic momentum and utilizing only radial E-fields.

3.1 Overview of the proton EDM experiment

A schematic of the proton EDM ring is shown in Figure 2.1. The straight sections are shown in Figures 2.2, the beta-functions and dispersions are shown in Figure 2.3 and the ring parameters are given in table 2.

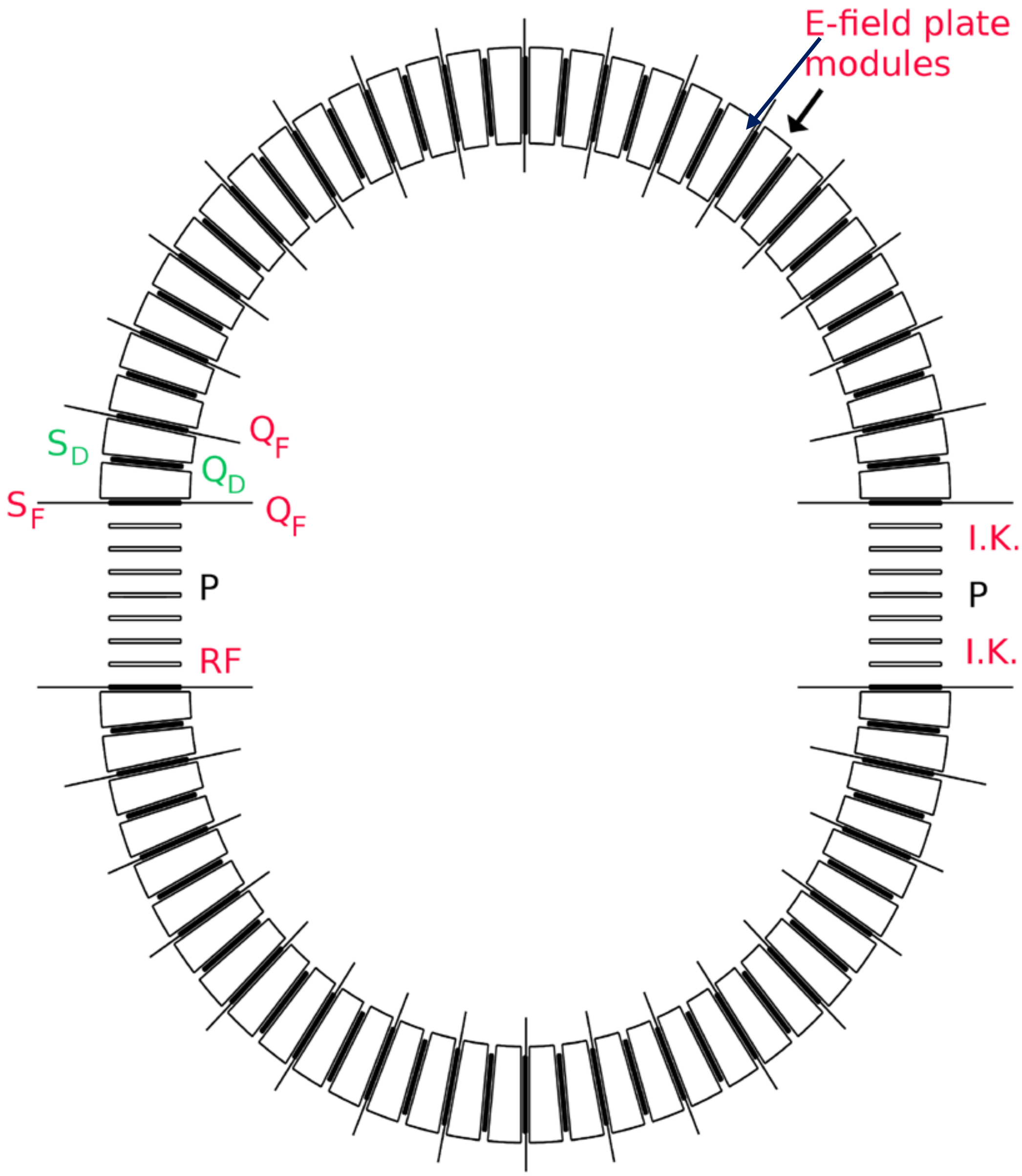

Figure 2.1 The working model of the proton EDM ring is shown here. The bend sections (64 sections) are radial E-field plates (about 3 m long each) interleaved with small straight sections (about 1 m long each) for the quadrupoles (denoted as Q_F & Q_D) and sextupoles (S_F & S_D) as well as the pick-up electrodes and resonant cavity BPMs. The two long straight sections (about 10 m long each) are to be used for injection kickers (I.K.), the placement of the RF-cavity (RF), polarimeters (P), etc. The bending radius for a radial E-field of 17 MV/m is ~25 m. The ring circumference is about 240 m and the E-field covers more ~3/4 of it.

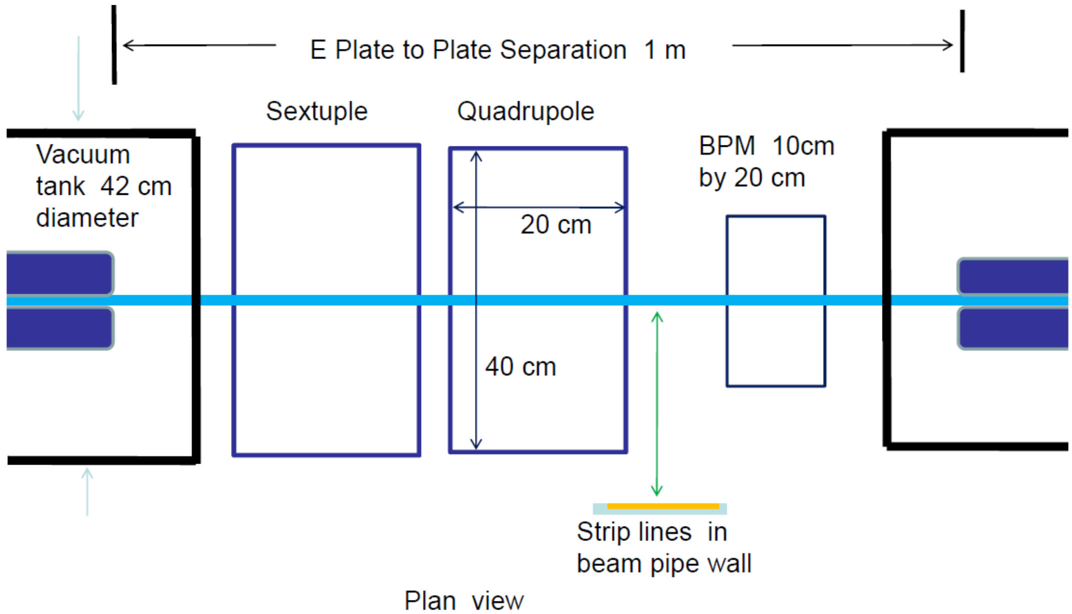

Figure 2.2a The interface between two electric field regions is shown here with the magnetic quadrupoles, the beam position monitors (BPM), and sextupoles.

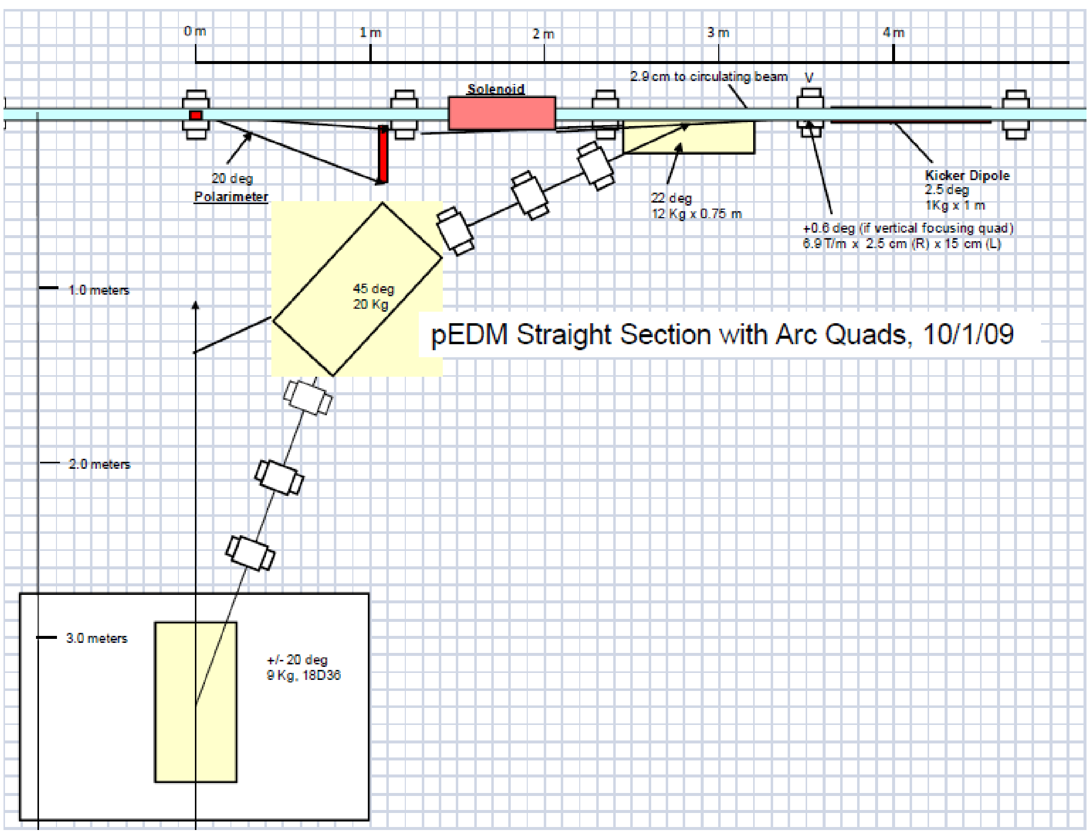

Figure 2.2b An enlarged drawing of part of one of the straight sections showing some of the details of the various elements that are planned to occupy them: kickers for injection, solenoid for spin rotation, quads, and polarimeter (see polarimeter Appendix for further details).

The proton EDM experiment requires:

1. A polarized proton source, a LINAC, an accumulator (BOOSTER) and a transfer-line capable of delivering 2×10^{10} polarized protons into the EDM ring. The beam characteristics are given in the Introduction section.
2. An injection system capable of injecting the beam clockwise (CW) and counter-clockwise (CCW) into the EDM ring.
3. State of the art radial electric field plates capable of delivering ~17 MV/m between two parallel stainless steel plates 2 cm apart and about 20 cm high.
4. An RF-system that will provide a synchrotron tune of ≥0.01 eliminating the first order spin de-coherence effects due to momentum spread of the beam.
5. A focusing system (Fig. 2.1 shows a FODO system). This system needs to be either magnetic or electric. The two cases have very different systematic errors.
6. Sextupole magnets installed at strategic locations around the EDM ring to prolong the beam spin coherence time (SCT). **We are pushing this system beyond the state of the art.**
7. State of the art beam position monitors (BPMs) at most straight sections to locate the beam with high resolution. **For simultaneous CW & CCW storage in the same place we need to develop BPMs similar to the current state of the art.**
8. State of the art internal polarimeters located in the straight sections that can monitor the proton spin components as a function of time with low systematic errors. Our polarimeter work at COSY shows that our anticipated systematic errors are smaller than the expected EDM signal.
9. Extraction striplines that will be used to slowly extract the beam onto the polarimeter target. The striplines will randomly kick the beam horizontally, vertically or both ways using “white noise”. This “white noise” will induce random walk on the beam betatron oscillation amplitudes enlarging the beam phase space in a controlled way. We have already used a similar extraction system at our runs at COSY.
10. A vacuum system capable of delivering $<10^{-9}$ Torr.

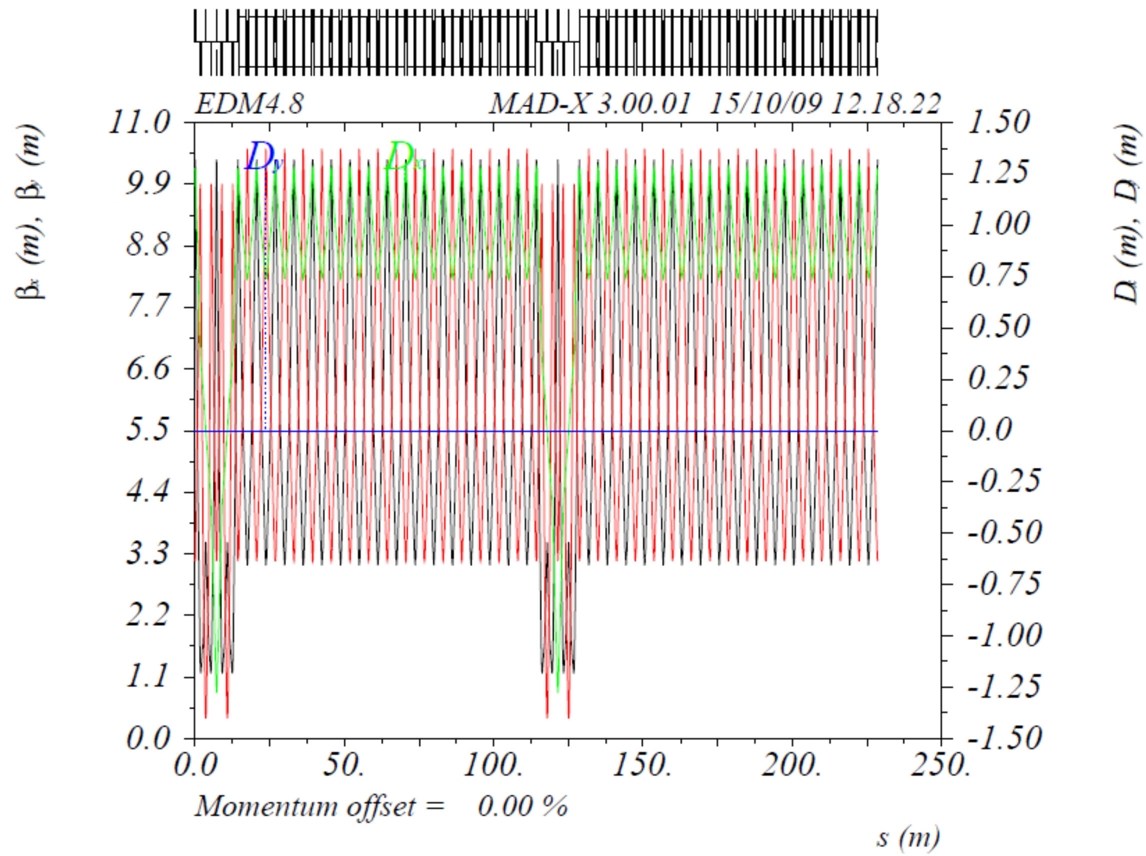

Figure 2.3 The horizontal and vertical beta functions (left vertical scale) and D-functions (right vertical scale) are shown here for the proton working lattice. The total length of this lattice is about 250 m.

Table 2. The table of parameters for the proton EDM ring is shown here. The lattice has been estimated using an effective dipole magnetic field (for estimating the lattice parameters only-for the actual tracking E-field is used). This case is not exactly equivalent to having a radial E-field and the horizontal tune will be different from the stated one. The particle spin tracking software uses the radial E-field and not an equivalent B-field.

Proton Momentum (GeV/c)	0.700741
Rigidity E_R/β ($B\rho$) (Tm)	2.3374
Radial Electric field E_R (MV/m)	17.0
Effective magnetic field B_v (T)	0.0948
Length of E field section (m)	2.422
arc section radius R (m)	24.666
Distance between two E field sec. (m)	0.7
Drift between E field and quads (m)	0.2
Length of orbit L (m)	**222.58**
Horizontal tune	7.939
Vertical tune	7.816
$\beta_{x,max}$ (m)	**10.341**
$\beta_{y,max}$ (m)	**10.391**
Dispersion maximum $D_{x,max}$ (m)	**1.278**
GammaTr γ_T	6.11
Momentum compaction factor α	0.0268
Length of Sextupoles in in E field sectionm (m)	0.15
Strength of Sextupoles in in E field sectionm (T/m^{-2})	0.0
Length of Focusing & Defocusing Quads in E field sectionm (m)	**0.15**
Strength of Focusing & Defocusing Quads in E field section (T/m)	$\pm$ **5.485**
Length of quads in straight section (m)	**0.30**
Strength of quads in straight section (T/m)	$\pm$ **6.495**
Drift between quads in s.s. (m)	**1.5**
Length of straight section (m)	**14.4**

$***$ **The phase advance at the straight section is π.**

3.2 Statistical sensitivity of the proton EDM experiment

The expected EDM signal, assuming the spin is along the momentum, is estimated by

$$\frac{d\vec{s}}{dt} = \vec{d} \times \vec{E} \Rightarrow \frac{1}{2}\hbar\omega = dE \Rightarrow \frac{d\theta}{dt} = \frac{2dE}{\hbar} \Rightarrow$$
$$\theta(t) = \theta_0 + \frac{2dE}{\hbar}t \qquad (7)$$

(assuming *d*, *E* are orthogonal to each other) which for E=17 MV/m, and an EDM of $d=10^{-29}$ e·cm we have for the rate of change in the vertical spin component to be

$$\theta(t) = \theta_0 + \frac{2dEc}{\hbar c}t = \theta_0 + \frac{2\times10^{-31}\mathrm{e\cdot m}\times17\mathrm{MV/m}\times3\times10^{8}\mathrm{m/s}}{197\ \mathrm{MeV\ fm}}t \Rightarrow$$
$$\theta(t) = \theta_0 + 5\frac{\mathrm{nrad}}{\mathrm{s}}t \qquad (8)$$

We can make the following observations from equations (7) and (8):

1) The vertical component of the proton spin grows linearly with time. This growth, however, will be in practice limited by the spin coherence time (SCT) of the stored proton beam.
2) Equation (7) implies that the EDM effect is proportional to the E-field applied on the proton EDM.
3) Analytical estimations show that 10^3 s of SCT is possible yielding about 5 μrad of early to late change in the vertical component of the proton spin.
4) Lastly, a high efficiency polarimeter with high analyzing power that can detect the beam polarization as a function of time is essential to the success of the experiment. Using existing experimental data we have estimated an average efficiency of 1.1% summing over the 2π azimuthal angle and an average analyzing power of 0.6 (i.e. 60%) for 0.7 GeV/c protons scattered off solid carbon target.

Therefore the maximum expected early to late normalized change in counting asymmetry related to EDM is of order of 3 ppm.

3.2.1 Electric field strength

In the last twenty years there has been tremendous progress towards developing the highest possible E-field gradients on metallic surfaces. It was mainly driven by the linear collider R&D as well as work on energy recovery LINACs. One safe conclusion is that the metal surface preparation is very important factor in achieving the highest possible E-field gradients.

For our experiment, since we are planning to have large area surfaces (of order of 10^2 m^2) with high electric field gradient, three parameters are important:

1) The electric field gradient that can be applied safely between the plates
2) The dark current induced by the E-field gradient and

3) The so-called patch effect, affecting the field homogeneity as a function of position.

The first, i.e. the electric field gradient, depends on the distance between the plates [3,4] and more specifically the maximum E-field strength goes as $1/\sqrt{d}$, with d the distance between the plates. The prevailing model explaining this functional form is given in [3] and reproduced here. Sparks are generated when material dislodged from one electrode lands on the surface of the other electrode and acquires enough energy to melt and evaporate. When the material evaporates it raises the pressure locally above a certain, critical level creating a local glow discharge and a spark. We can assume the charge on the material to be proportional to the E-field gradient $q=kE$. The breakdown energy will be $W_{\mathrm{br}} = qV$ and

$$W_{\mathrm{br}} = qV = kE^2 d \Rightarrow E = const. \times \frac{1}{\sqrt{d}} \tag{9}$$

On the other hand the R&D on developing better surface treatments that can yield higher electric field gradients has been focused on monitoring the induced dark current as a function of the E-field gradient. "Sub-micron-scale surface roughness and contamination cause field emission that can lead to high-voltage breakdown of electrodes, …" [5] clearly states that a smaller dark current correlates with a more reliable high voltage operation.

The recommended surface treatment sequence includes:

1) producing a low roughness surfaces by machining
2) electro-polish to achieve optical quality surface
3) apply high pressure water rinsing (HPR) [6] to manually dislodge loose material from the metallic surface or
4) coating the metallic surface with 0.5-1 μm silicon oxy-nitride coating on stainless steel [7] to significantly reduce the effectiveness of the local emitters or
5) apply gas cluster ion beam (GCIB) surface treatment [5] where asperities and scratch marks from polishing are removed

and so on. The achievement of the last three methods above is similar in getting a dark current density of 1 pA/cm^2 for 30 MV/m and ~5 mm separation. The goal of the photocathode R&D work at Cornell University is to achieve 15 MV/m for 5 cm electrode separation [8]. Since the dark current scales only with the electric field gradient and not the distance, it is reasonable to assume we will be able to achieve a dark current density of less than 1 pA/cm^2 for an electric field gradient below 20 MV/m. This will result to a total dark current load of <1 μA for the sum of all the E-field plates in the ring.

E-field strength R&D goals:

1) Our minimum goal will be to achieve 15 MV/m.
2) The R&D will have as a second stage goal to achieve 17 MV/m. At this E-field strength the ring will be smaller and hence it will be cheaper to build.
3) The last stage of the R&D phase will include pushing the E-field strength higher by about 20%, i.e. to ~20 MV/m. This was the method followed by the Fermilab electrostatic separator group to estimate, by extrapolation, the expected number of sparks within a year (they had a similar total surface area E-field plates, even

though they had much larger plate separation). During the R&D period we will determine the operating voltage value for a reliable operation taking into account additional factors such as cost. It turns out that the Fermilab electrostatic separators used in the Tevatron to separate the protons from the anti-protons are suitable to be used in the proton EDM experiment. We have inquired about obtaining the modules after the Tevatron ceases operations. They are using 24 modules in the Tevatron ring and have two spares. Each one is ~2.5m long and has operated up to ±180KV for 5 cm plate separation. The plates are ~0.2 m high. More on the Tevatron electrostatic separators see in the E & B-field Appendix.

3.2.2 Spin coherence time

From equation (3) of section 3 we can see that the g-2 precession frequency will vary with momentum and if the proton beam has non-zero momentum dispersion there is going to be a spread in the longitudinal spin direction (phase) that will grow with time. To estimate the order of the spin coherence time we take the derivative as a function of momentum:

$$\frac{d\vec{\omega}_a}{dp} = \frac{e}{m}\left[2\frac{m^2}{p^3}\right]\frac{\vec{\beta}\times\vec{E}}{c} + \ldots \Rightarrow d\omega_a = 2\frac{e}{mc^2}\left[\left(\frac{m}{p}\right)^2\right]\beta c E\frac{dp}{p} \Rightarrow$$

$$d\omega_a = 2\frac{e}{938\text{M}e\text{V}}\left[\left(\frac{0.938}{0.7}\right)^2\right]0.6\times3\times10^8\,\text{m/s}\times17\text{MV}\frac{dp}{p} \Rightarrow \tag{10}$$

$$d\omega_a = \frac{dp}{p}\times10^7\,\text{rad/s}$$

and for rms dp/p=2.5×10^{-4} we get

$$d\omega_a = 2.5\times10^{-4}\times10^7\,\text{rad/s}=2.5\times10^3\,\text{rad/s} \tag{11}$$

meaning that the SCT is about 0.4 ms if we do nothing to improve it. We are planning to use an RF-cavity to cancel the first order effect on the momentum dispersion. The second order effect will still be there and is estimated to be of order

$$d^2\omega_a = \left(\frac{dp}{p}\right)^2\frac{3}{2}\times10^7\,\text{rad/s=1 rad/s} \tag{12}$$

implying ~1 second of SCT. It is estimated [9] that the SCT can be extended to ~10^{3}s by using sextupole magnets placed at specific locations around the ring. Actually dp/p is only one limiting factor for the SCT. The horizontal and vertical betatron oscillations are also very important and their effect needs to be cancelled similarly by sextupoles [9,10].

The effect of the sextupole is estimated analytically and by simulation and is given in detail in the SCT section.

3.2.3 Polarimeter efficiency and analyzing power

We have estimated the maximum early-to-late change in the vertical spin component to be of order of 5 ppm of the value of the longitudinal component at the time of injection. The best way to probe and monitor this change as a function of storage time is to use nuclear elastic scattering from a solid carbon target. To the extent that this interaction is parity conserving, there is only sensitivity to transverse polarization and there is no asymmetry for scattered protons that are longitudinally polarized.

The current plan is to slowly extract the proton beam onto a solid carbon target (see Fig. 3) acting as the limiting aperture in the ring using electric field kicks from striplines running at random frequencies. The random kicks will cause a random walk on the particle betatron oscillation amplitudes and effectively it will increase the horizontal and/or vertical phase space with a rate that is controlled by the amplitude of the kicks. The scattered protons will be captured by the detector and sorted into functional groups labeled L, R, U, and D with the combination

$$\varepsilon(t) = \frac{L-R}{L+R}(t) \tag{13}$$

giving information on the proton EDM. The optimum statistical sensitivity is obtained when the spin is kept along the momentum vector for the duration of the storage time. Eq. (13) is then fit to the integral of the longitudinal polarization as a function of time and a statistically significant non-zero value for the change from early to late in the store will be an indication of an EDM signal. Any *dc* factor in the signal is just the dc-offset in the L-R asymmetry at the beginning of the store. For further details, see the Polarimeter Appendix.

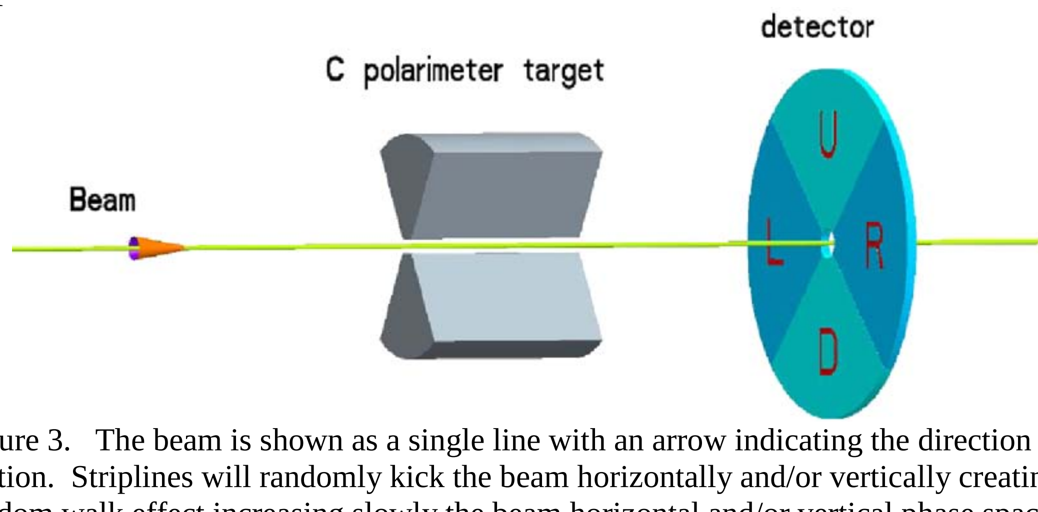

Figure 3. The beam is shown as a single line with an arrow indicating the direction of motion. Striplines will randomly kick the beam horizontally and/or vertically creating a random walk effect increasing slowly the beam horizontal and/or vertical phase space. The particles eventually scatter on the aperture limiting carbon solid target and the detector collects the scattered protons between 5-20 degrees. The solid target is shown as two wedges. They will be movable and they will be acting as the limiting aperture of the stored beam.

The corresponding combination for up-down counting will be

$$\zeta(t) = \frac{D-U}{D+U}(t) \tag{14}$$

describing the g-2 (in-plane) precession.

There are three comments in order here:

1) First order contributions of the systematic errors to the left-right asymmetry can be removed by using detection on both sides of the beam in combination with beam bunches with opposite polarizations, all combined into a cross ratio asymmetry [11].
2) The second order effects of the above systematic errors are cancelled by using a more sophisticated combination of ratios described in the Polarimeter appendix in combination with a calibration of sensitivity of the polarimeter to systematic errors. The proof of the efficacy of this method of canceling the systematic errors beyond first order is a major part of our on-going systematic error studies program at COSY-Germany using polarized stored beams extracted onto a polarimeter.
3) The (L-R)/(L+R) counting rate as a function of time is the EDM signal. However, if the detector, as shown in Fig. 4, is slightly tilted and the spin precesses horizontally there is going to be a false EDM signal. Therefore there is a need to establish the L/R and U/D planes independently of the EDM signal. We will either use 5% of the injections for short runs to establish periodically the detector planes (slightly detuning the RF-frequency above or below the "magic" momentum).

It turns out the average polarimeter asymmetry of protons at the "magic" momentum is very high (more than 50%) making the use of "magic" momentum protons even more appealing. This asymmetry can be increased by applying certain cuts that reduce the efficiency but overall the statistical power (a.k.a. "figure of merit") is improved. The estimated average polarimeter asymmetry for 0.7 GeV/c protons is about 0.6 (i.e. 60%) for an average detection efficiency of 1.1%.

The above points will be addressed in more detail in the polarimeter section.

3.2.4 Statistical error

The statistical error has been estimated using both analytical calculations and numerically with a MC program. The statistical error is given by

$$\sigma_{d(p)} = \frac{3\hbar}{PAE\sqrt{N_{\text{tot,c}} f T_{\text{tot}} \tau_p}} \tag{15}$$

which holds for an extraction that keeps the same number of detected particles as a function of time. Assuming the following parameters $P = 0.8$, $A = 0.6$, $E = 17$ MV/m, $N_{\text{tot,c}} = 2\times10^{10}$ particles/storage, an effective detection efficiency of $f = 0.011/2$, total running time $T_{\text{tot}} = 10^7$ s/year, and SCT of $\tau_p = 10^3$ s we get a statistical error of $\sim 2\times10^{-29}$ e·cm per year. The E-field azimuthal coverage is only 65% of the circumference increasing the statistical error to ~3.5×10^{-29}e·cm/year or ~1.8×10^{-29}e·cm/4-years.

During the R&D stage we will be working improving the SCT, the E-field we can safely apply between the plates, and the polarimeter parameters. In addition we will work towards reducing the intra-beam-scattering (IBS), which is limiting the maximum beam intensity we can store, by optimizing the ring lattice.

3.3 Systematic errors of the proton EDM experiment

The main systematic errors are

1) Vertical forces other than magnetic acting on the stored beams. Those forces are balanced by radial B-fields which are acting on the particle spins and imitating an EDM effect. The proposed solution to this effect is to store the beams clock-wise (CW) and counter-clock-wise (CCW) and feel the same fields in both directions. The EDM effect and the systematic errors have opposite sign between CW and CCW storage.
2) The polarimeter detector can be sensitive to different scattered particles fluxes from early to late times. In addition the beam itself could be moving on the target from early to late times.
3) The presence of horizontal B-fields can create the so-called geometrical phase effect which arises from the fact that spin rotations do not commute in three dimensions. Below we will give a rough estimation of the effect and indicate the limits under which there is no issue. For the simultaneous CW & CCW beam storage this effect is negligible.
4) Beam and/or spin dynamics resonances. We need to map out the resonances in the neighborhood of the final n-value and will reduce the field multipoles below the needed level by artificially making the multipoles large and observing their effect. We will study this effect using particle momentum and spin tracking software. Items 1 and 3 are a specific class of spin resonances, but since we expect them to be the dominant ones we are going to study them separately.
5) Wake fields due to the lattice structure. Preliminary estimations (shown in the E-field section) indicate they are manageable. However, we plan to replicate the estimations including the use of software packages used by the experts in the field.

3.3.1 Vertical forces acting on the stored particles, uniformity, and stability.

For a stored beam away from beam dynamics resonances in a ring where there are only magnetic vertical forces, the average radial B-field acting on the beam is zero. However, in the presence of vertical forces, like, e.g. gravity, the net radial B-field is not zero. The total vertical force acting on the particles is zero as shown in equation 16 below

$$\vec{F} = mg\hat{y} + q\vec{v}\times\vec{B} = 0 \Rightarrow B_r = \frac{mg}{v} = \frac{1.7\times10^{-27}\,\text{Kg}\times10\text{m/s}^2}{1.6\times10^{-19}\,\text{C}\times0.6\times3\times10^8\,\text{m/s}}$$
$$\Rightarrow B_r = 6\times10^{-16}\,\text{T} \tag{16}$$

This radial B-field will cause a spin precession given by

$$\frac{d\vec{s}}{dt} = \vec{\mu}\times\vec{B} \Rightarrow \omega = \frac{ge}{2m}B_r = \frac{5.6\times1.6\times10^{-19}\,\text{C}}{2\times1.7\times10^{-27}\,\text{Kg}}6\times10^{-16}\,\text{T} \Rightarrow$$
$$\omega = 1.6\times10^{-7}\,\text{rad/s} \tag{17}$$

which is about a factor of 35 larger than the expected EDM effect. Fortunately the force of gravity is stable and expected to be the same CW and CCW. The EDM effect has opposite sign going from CW to CCW and it will cancel after subtracting the effects from the two data sets[2].

Other vertical forces include the out of plane E-field component in the electric field region and vertical electric field forces due to image charges. In order to eliminate the vertical forces due to image charges we are planning to use long vertical E-field plates in the bending sections and long vertical E-field plates in the straight sections as well. The plates will be 2 cm apart and completely shield the beams within to the required levels (see appendix WF "Image and Wake Fields on the Proton Beam" in the E-field section).

Let's examine the vertical E-field effect first. Since the particles are stored and ignoring the force of gravity, the total force acting on the particles are summed to zero:

$$\vec{F} = q\left(\vec{E}_v + \vec{v}\times\vec{B}\right) = 0 \Rightarrow B_r = \frac{E\theta}{v} \tag{18}$$

Comparing the vertical spin precession rate due to an EDM of 1×10^{-29} e·cm and the same precession due to the radial B-field from eq. (18) we have

$$\omega = \frac{ge}{2m}\left(B_r + vE\theta\right) \le \frac{2dE}{\hbar} \Rightarrow \frac{ge}{2m}\frac{E\theta}{v\gamma^2} \le \frac{2dE}{\hbar} \Rightarrow$$
$$\theta \le \frac{4d}{ge}\frac{\beta\gamma^2 mc^2}{\hbar c} = \frac{4\times10^{-31}\,\text{e}\cdot\text{m}}{5.6\times\text{e}}\frac{0.6\times1.6\times938\text{MeV}}{200\text{MeV fm}} \Rightarrow \tag{19}$$
$$\theta \le 3\times10^{-16}\,\text{rad}$$

From eq. (18) we estimate the radial B-field to be about 0.2 pG, which is similar to the systematic error sensitivity level of the SNS nEDM experiment. In our case, however, when we use only magnetic focusing, any stray radial B-field is shielded (i.e. self canceling) by the focusing system (see next section).

[2] The radial E-field remains stable in the same direction going from CW to CCW, however, the particle spins change direction and the EDM effect is opposite. The force of gravity is also stable going from CW to CCW, but the systematic error rises due to the compensating radial B-field which does change sign. This sign change coupled with the spin direction results to preserving the sign of the systematic error effect.

3.3.1.1 Canceling the out of plane E-field component.

The requirement on the average E-field alignment is of course impossible to achieve. The plan is to store the beam clock-wise (CW) and counter-clock-wise (CCW). There are different options of achieving this goal, each with pros and cons. After studying several options we believe the best way to achieve this is to store the proton beams CW and CCW beams simultaneously in the same place (one ring option). This is possible to achieve for protons at their magic momentum of 0.7 GeV/c since the ring includes only electric bending and no dipole magnetic fields. The focusing can be either electric or magnetic, see table 3 below.

The two focusing options have different systematic errors and different requirements on the BPMs. Option #1 requires beam position monitor (BPM) sensitivity of 1 pm, which we should be able to achieve with resonant cavities in less than a day of running. The concern is to minimize any vertical E-field that is not common to both beams. Option #2 eliminates the effect of the vertical E-field. A radial B-field would split the two beams vertically which can be detected by the BPMs. A BPM resolution of 0.1 pm would bring this background below the EDM effect.

Both of these options require self-fields or the fields from the beam-beam interactions to be small. This can be accomplished by reducing the horizontal to vertical coupling to below 10^{-6} rad, which we should be able to achieve.

Table 3. Different options of storing the proton beams CW and CCW (only the one ring option is considered here, a.k.a. Colliding BeamS (CBS))

Option	**#1**	**#2**
Description	2 counter-rotating beams in one ring with ***magnetic*** focusing.	2 counter-rotating beams in one ring with ***electric*** focusing.
Comments	The two beams **probe the *same* fields at the *same* time**. The two beams need to be vertically aligned, on average, to better than ~1 pm.	The two beams **probe the *same* fields at the *same* time**. The two beams need to be vertically aligned, on average, to better than ~0.1 pm.
Advantages	Eliminates the direct radial B-field problem	Eliminates the vertical E-field problem.
Issues	Need to pay attention to vertical E-fields not common to both beams as well as to combinatorial effects due to horizontal B-fields.	Need to pay attention to radial B-fields.

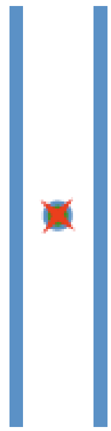

Figure 4. The vertical lines represent vertical electric field plates. The counter-rotating beams are indicated in the middle between the plates. Either electric or magnetic focusing systems are under consideration.

This option obviously offers the advantage of probing the same electric and magnetic fields at the same time. Most systematic errors drop out. The issues in this case are beam-beam interactions (which have been estimated by the experts to be small) and the vertical offset between the two beams, which creates vertical E-fields acting in opposite directions on the two beams and thus they do not cancel. The last one requires state of the art BPMs that have ~10 nm resolution with 1 Hz BW not in absolute but only in *relative position* between the two beams. We believe that it is possible to develop such a system and we are planning to produce it during the R&D period.

3.3.2 Magnetic or electric focusing?

This is an important question since applying an all-electric focusing eliminates completely the vertical E-field problem. Instead it generates a radial B-field problem. The limit on the B-field is at the 0.1 pG level on average around the ring. The neutron EDM experiments at SNS of Oak Ridge and ILL/Grenoble need to be shielded against any potential B-field source at a similar level. It may be possible to shield against that magnetic field by monitoring the average vertical beam positions around the ring using resonant cavities as BPMs. The needed resolution is of order 0.1 pm depending on the strength of the focusing system in the vertical direction.

If we use magnetic focusing instead, then the average radial B-field integrated around the ring is zero unless there is a vertical force of different origin compensating for it. An average vertical electric field of 0.5×10^{-8}V/m is the maximum vertical E-field allowed in this case. With both counter-rotating beams most effects from non-magnetic vertical forces cancel except for a few of them as described in section 3.4. One such example is the vertical E-field due to an offset between the two beams. The average offset over the course of the experiment is required needs to be better than 1 pm for this effect to be below the EDM sensitivity.

Obviously the two focusing options are interesting in that they have very different systematic errors. We plan to study both options during the R&D period and decide which one to pursue first.

3.3.3 Polarimeter systematic errors

The main polarimeter systematic errors are:

1) Beam motion on the target coupling through detector acceptance difference gradients as a function of beam position and angle.
2) The detectors may exhibit early to late rate dependent effects such as pile-up or gain/threshold changes.

Those effects are currently under investigation by our group running with polarized beams in the COSY storage ring at Jülich/Germany and they are discussed in detail in the Polarimeter appendix.

3.3.4 Horizontal B-fields

The magic proton EDM ring in the Colliding BeamS (CBS) version and magnetic focusing will be sensitive to magnetic quadrupole position instabilities. The two counter-rotating beams will receive opposite kicks for the same quad misplacement. The particle position is given by

$$\Delta_i(s) = \frac{\sqrt{\beta_i(s)}}{2\sin\pi\nu_i}\ \theta_i\ \sqrt{\beta_i(s_i)}\ \cos\left[\left|\Phi(s)-\Phi(s_i)\right|-\pi\nu_i\right] \qquad (20)$$

As shown in the note, s is the particle azimuth, s_i is the azimuth of the kick, $\Delta_i(s)$ is the radial or vertical orbit deviation at azimuth s. At the start of the experiment we plan to map the kicks at different locations around the ring and their effects on the radial and vertical displacements of the beams. We expect to have the two counter-rotating beams at very small relative offset at each location around the ring. If we hope to have the average offset of the two beams at the level of 1 pm, then we need to have the two beams at least within 2 nm with a BW of 1 Hz. At this level they practically will be subjected to the same external fields to very high accuracy. Therefore the horizontal B-field effect should cancel by the two counter-rotating beams. Analytical estimations by Yuri Orlov show that for the same fields the horizontal field effect cancels for CW and CCW injections. Elementary tracking using consecutive spin rotations has confirmed this finding. The horizontal B-field effect is a very low risk for the experiment because:

1) The magnets will be aligned to eliminate (as far as possible) any effects; problems will come from magnet regulation and stray field issues that are expected to vary randomly with time.

2) This problem is eliminated anyway in the CW/CCW cancellation; we are making it small from the start.

Nonetheless it is good to know the level of the effect for a single ring in order to understand the requirements on the quadrupole stability and the earth's remnant B-field. Spin rotations do not commute in three dimensions resulting to a vertical component if the beam encounters the wrong combination of fields. In general higher order horizontal fields of the same amplitude contribute less to the final vertical spin rotation.

A rough estimation of the horizontal B-field needed to produce a vertical spin rotation is given here:

The EDM signal is a vertical spin component changing at a rate of 5 nrad/s or 5.5×10^{-15} rad/revolution and the systematic error effect per revolution needs to be kept below that level. Of course there are effects that cause a larger vertical spin growth than that, like, e.g., vertical betatron oscillations but those oscillations average to zero since they don't have a specific (fixed) handedness during storage. Spin rotations in three dimensions do not commute, i.e. consecutive spin rotations with axes in two dimensions result to a net rotation of the spin with respect to the third axis. Therefore if there is a *local* spin rotation with respect to a vertical axis and then with respect to a radial axis, the net rotation will be with respect to a longitudinal axis [12]. Actually an imaginary detector distributed uniformly in the azimuth around the ring would see a zero effect. However, our polarimeters are local and at those locations it is possible to have an EDM-like signal even if at a different location it may give a signal with opposite sign. Figure 5 below shows a combination of fields around the ring. Even though the integrated B-field around the ring is zero the net effect on a local polarimeter may not be zero. One way to get around this problem is to have polarimeters at different locations around the ring and compare their signals.

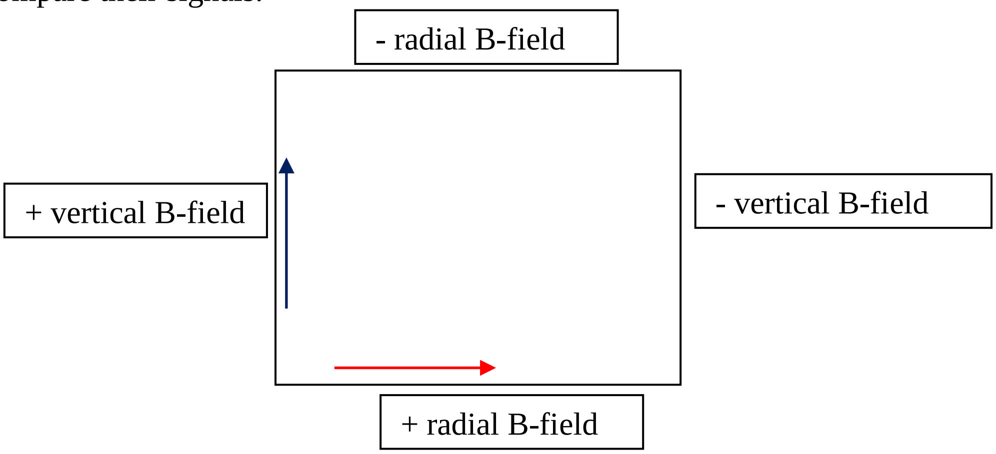

Figure 5. The B-fields integrated around the ring can be zero but their effects on the particle spin as observed by a local polarimeter can be non-zero. An imaginary polarimeter with a uniform sensitivity around the ring to the proton spin would also show zero effect. On local polarimeters CW and CCW storage gives an effect that has opposite sign from that of the EDM signal and hence it will cancel.

The global horizontal and vertical spin precession will be monitored using the polarimeter with very high accuracy to better than 10^{-9} rad/s since the resulting large up/down asymmetries will immediately be evident.

Two comments are in order here:

1) Of course if the spin is always kept along the longitudinal axis the effect would be zero on average. However, the analyzing power of the polarimeter at zero degrees is zero (no parity violating effects are present here) and we will probably need to keep the spin within ±30° of zero. We will conservatively assume that the average longitudinal component is within ±1° of zero. This point can only be used for the B-fields originating from the quadrupoles and not for the stray magnetic fields (e.g. earth's B-fields) whose directions are arbitrary with respect to the particle momentum.
2) The only way the local horizontal spin precession can be significant is due to a vertical B-field in the focusing system (at the magic momentum the E-field has no effect on the horizontal spin precession).

The magnetic quadrupoles have strength of order ~10 T/m and they are ~0.15 m long. A radial misalignment of the quads by 10 μm means that the particles will go through a vertical dipole B-field of order 1 G. This field will cause a local spin precession of order 10 μrad/turn. With 10^4 CW&CCW injections we expect the average to go down by a factor of 10^2, i.e. to ~100 nrad/turn. If a similar quadrupole offset is there in the vertical direction then the spin rotation with respect to the horizontal axis will be of order 20 μrad/turn (the rotation is larger in the vertical direction since it is proportional to *g* and not (*g*-2)/2 as it is in the horizontal direction). Again for 10^4 CW&CCW injections the average should be 10^2 times smaller, i.e. 200 nrad/turn.

However, since this so called geometrical effect is due to a combination of spin rotations in two dimensions, this rotation is minimum when the spin is along one of the rotation axis, e.g. along the longitudinal direction. Since we expect the average spin angle to be within ±1° of zero we expect the rotation to be proportional to sin1°~0.02. Therefore, on average, the maximum spin rotation per turn is

$$\delta_{\mathrm{HF}} = 100 \ \frac{\mathrm{nrad}}{\mathrm{turn}} \times 200 \ \frac{\mathrm{nrad}}{\mathrm{turn}} \times \sin 1^{\circ} = 4 \times 10^{-16}\,\mathrm{rad/turn} \tag{21}$$

which is more than 10 times smaller than the EDM signal per revolution even for a single ring.

We are planning to align the quads to their nominal position by beam-based alignment to better than 1μm completely eliminating this effect. In this case the key factor is quad position stability at the 1μm level.

The above treatment assumed only one quadrupole was misaligned in the horizontal plane and only one in the vertical. More elaborate study is needed to determine the quadrupole stability specifications and the residual field limits from the earth's B-field.

Only a limited number of stray magnetic field cases have been investigated so far. We plan to complete the first phase of this study in the near future, while the complete study with realistic fields we are going to finish during the R&D period. The development of the program capable of doing this with both high accuracy and speed is under way.

For the CBS option we expect this effect to be of very low risk since the two beams are exposed to the same fields to a very high accuracy.

3.4 Fields that don't directly cancel in the CBS version of the magic proton ring

In the two beams in one ring a.k.a. CBS version of the magic proton EDM ring the two beams are expected to observe the same fields on average going CW and CCW. Their effects have opposite sign than the EDM and hence cancel.

There are some fields that don't cancel:

1) Self-fields. In this case most of the fields would cancel since, on average, the particle oscillations will be around a common radius and vertical position. However, if there is horizontal to vertical coupling (H2VC) due to, e.g., a rotation of a quadrupole with respect to a longitudinal axis, particles with larger horizontal amplitudes will have (in the presence of sextupoles) an average vertical position that is higher or lower than the particles with smaller betatron oscillation amplitudes.
2) Fields from the counter-rotating beam. If the two beams are vertically offset they will feel a vertical E-field as well as a radial B-field.
3) Field from the RF cavity, which is 180° out of phase for the two counter-rotating beams. This would arise in case, e.g. of energy losses.

   The effects will have a small dependence on whether we use magnetic or electric focusing.

Magnetic focusing:

1. **Self fields.**

To prolong the SCT we plan using sextupole magnets that change the equilibrium radius of the particles by an amount of order 1 μm. If there is a H2VC of order of 1 μrad then the particles with large betatron oscillations or large dp/p will have a vertical equilibrium position that differs from their "cooler" particles by about 1 pm. Since the particles with large betatron amplitudes are more likely to be extracted earlier this effect has the potential to be a significant systematic error. If the extraction is smooth (e.g. $5mm/10^9$ rotations = 5pm/rotation) then those particles would get out early whereas the other ones later. This effect would have the potential to be at the level of our systematic error limit. However, since our effective extraction step is larger than the estimated offset, it becomes ~5 times smaller. The H2VC on average over the storage time and the course of the experiment needs to be of the order of 1 μrad or less. This level we should be able to achieve using beam-based ring alignment.

2. Fields from the counter-rotating beams.

With magnetic focusing the relevant field (source of systematic error) is the vertical E-field. If the two beams don't overlap completely they will feel, on average, a vertical component, which is opposite in direction for the two counter-rotating beams and will depend on beam intensity.

The vertical E-field influencing each other is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{a^2}r_0 = \frac{1}{2\pi \cdot 8.8\times10^{-12}\,\mathrm{F/m}}\frac{2\times10^{10}\,\mathrm{p}\times1.6\times10^{-19}\,(\mathrm{C/p})/240\mathrm{m}}{0.005^2\,\mathrm{m}^2}10^{-12}\,\mathrm{m} \Rightarrow$$
$$E \approx 10^{-8}\,\mathrm{V/m} \tag{22}$$

where we have used as the average offset between the centers of the two beams r_0 = 1pm, an rms beam size of a = 5 mm, and 2×10^{10} p the number of particles per stored beam in the ~240 m ring circumference.
Our requirements are <5 nV/m and they are obviously satisfied by eq. (22) if the average distance between the two beams is less than 0.5 pm. We plan achieving this resolution or better over the course of the experiment using resonant cavity BPMs and/or striplines.

3. Fields due to E-field in the RF-cavity.

This would be a concern if there is energy loss around the ring and it is compensated by the E-field in the RF-cavity. Let's say we have 1 stripline detector around the ring in which case the longitudinal impedance will be 25 Ω. Then for our maximum beam current of 3 mA the total energy loss, per particle and per rotation, will be ~0.1 eV. Each proton has a kinetic energy of 232 MeV so 0.1 eV energy loss corresponds to a negligible level (below 10^{-9}). However this energy loss is compensated by the electric field in the RF-cavity. If the RF-cavity is rotated with respect to a radial axis then a net vertical E-field can be present. This vertical E-field will have opposite sign for the counter-rotating beams since they have a phase difference of 180°, in which case they don't cancel. The level of the systematic error is irrespective of the length of the cavity, it only depends on the energy loss. The vertical E-field can be estimated:

$$\langle E_V \rangle = E_{rf}\cdot l\cdot\vartheta\,/(\text{total E-field plate length equal to ~150 m})=$$
$$0.075\ \mathrm{V}\cdot\vartheta/150\ \mathrm{m}=5\times10^{-4}\vartheta\ \mathrm{V/m} \tag{23}$$

Since our limit for the vertical E-field systematic error is <5 nV/m then ϑ < 10 μrad. We will make the impedance 10 times worse and align the RF-cavity and then run the experiment there with as low longitudinal impedance as possible. The goal for the total longitudinal impedance should be to keep it below 10-100 kΩ.

When electric focusing is used it is the magnetic field that becomes important and needs to be minimized. Since the proton velocity $\beta = 0.6$ the equivalent systematic errors are about a factor of two less than when magnetic focusing is used. One effect is stricter in this case: the vertical offset between the two beams due to a radial B-field is smaller than when magnetic focusing is used and depends on the strength of the vertical focusing. All these effects need to be analyzed more precisely during the R&D period.

3.5 Present state of the art and R&D goals

The out of plane E-field systematic error is canceled by simultaneously storing two beams in the same location probing the same electric and magnetic fields (shown in Figure 4 above). In this case the systematic errors and their relation to the current state of the art are given here:

1) The vertical offset between the two beams need to be very small, of order ~1pm averaged over the duration of the experiment. This requires BPMs with a relative position resolution of order ~10 nm and BW of 1 Hz. The state of the art today is ~10 nm for a single beam and *single pass* with 10^{10} particles, using resonant cavities. Most of this development was accomplished as part of the ILC R&D [13].
2) SCT of order 10^3s or about 10^9 turns. The state of the art was achieved at Novosibirsk for ~10^7 turns for an electron/positron machine [9].
3) Internal polarimeter with small systematic errors. Similar systems have been developed but never for a storage ring. We expect the stability of the beam position will completely eliminate this error. Our recent work at COSY demonstrated that we could achieve the polarimeter systematic error goals.
4) The electric field gradient for large surface areas. A similar system is in operation at Fermilab as part of the Tevatron where they have applied up to ±180 KV for 5 cm plate separation. We expect to reach a similar voltage for 2 cm plate separation using high-pressure water rinsing which was shown to give the needed improvement. The aim is to reduce the cost of the EDM ring.
5) Beam-beam interactions from the beam and spin dynamics points of view. Our effective luminosity is moderate (a few × 10^{26}/cm^2-s) where the beam-beam, and spin-spin effects are negligible.

We believe we can develop this system within the two year R&D period. The total requested amount is $3M and the breakdown per system is shown in table 4. The details of the requested support are given in the corresponding appendices. For SCT and polarimeter the request is for the total of 3 years support.

Table 4. The total amount requested to develop the system ready for deployment

System	Total amount requested	Comments
Spin Coherence Time	\$1M	We need to improve over the current state of the art by a factor of $\sim 10^2$
Beam Position Monitors	\$0.45M	Critical achieving 10 nm position resolution with 1 Hz BW or better. This resolution will result to better than 1 pm resolution over the course of the experiment.
E-field	\$0.55K	The higher the E-field the higher the experimental sensitivity and the cheaper the ring.
Polarimeter	\$1M	Internal polarimeter development for the pEDM experiment.

3.6 Run plan during the first (engineering) run of the experiment

The three pillars of the experiment: **Ring alignment**, **Stability**, and **E-field uniformity** define the path to success. The main aspects of our engineering run are given below:

1) The placement of the ring elements will be done with ~0.1 mm absolute position resolution. We plan to achieve the required ring alignment after storing beam using beam-based alignment, expecting to get 1-10 μm in the quadrupoles. Beam position repeatability is expected to be on average 1-10 μm.
2) Commissioning of the kickers, solenoid, beam injection, polarization preparation, stripline extraction and polarimeters will take most of the engineering run during the first year.
3) We will establish the transfer function of beam motion when the lattice elements move in the horizontal and vertical plane. We will be moving one element at a time and observing the BPM readout around the ring to establish this transfer function.
4) We will be moving the beam in the radial and vertical direction to establish the average E-field multipoles integrated around the ring. We will establish the level of stability and we will remove the multipoles using trim electric fields.
5) We will establish the level of systematic error sensitivity of the polarimeter to beam position and angle changes on the target by artificially moving the beam. The polarization sensitivity will be calibrated using beams of known polarization.
6) We will establish the beam intensity effect on the systematic errors.
7) We will take EDM sensitive data for analysis. The statistical sensitivity per day will be of order 10^{-28} e·cm.

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