Beam Manipulation with an RF dipole†

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Abstract

Coherent betatron motion adiabatically excited by an RF dipole has been successfully employed to overcome strong intrinsic spin depolarization resonances in the AGS, while a solenoid partial snake has been used to correct imperfection spin resonances. The experimental results showed that a full spin flip was obtained in passing through an intrinsic spin resonance when all the beam particles were forced to oscillate coherently at a large amplitude without diluting the beam emittance. With this method, we have successfully accelerated polarized beam up to 23.5 GeV/c. A new type of second order spin resonances was also discovered. As a non-destructive manipulation, this method can also be used for nonlinear beam dynamics studies and beam diagnosis such as measuring phase advance and betatron amplitude function.

1 INTRODUCTION

For various purposes, one often needs to excite a coherent oscillation in an accelerator. However, the free coherent oscillation excited by a short pulsed dipole kicker can quickly decohere and cause beam emittance growth due to the tune spread arising from the chromatic and other effects. Alternatively, coherent oscillation can also be excited by driving beam with an RF dipole which provides a magnetic field oscillating at a frequency close to the beam free betatron oscillation frequency. If the excitation is made adiabatically by slowly energizing the RF dipole field, it can be well controlled and also keep beam emittance preserved. This nice feature is very useful for many applications which desire a non destructive beam manipulation. This method has been successfully applied in the Brookhaven National Laboratory AGS polarized proton acceleration experiment to maintain the beam polarization through strong intrinsic spin resonances. This paper discusses the application of RF dipole in spin manipulation as well as in beam diagnosis and non-linear beam dynamics studies.

As a driven oscillator, the beam coherent oscillation is determined by the RF dipole field strength and its frequency. Consider a particle driven by an RF dipole where the betatron amplitude function is β_z. Here, z is used to denote the vertical coordinate. The same discussion also applies to the horizontal plane. The RF dipole field ΔB oscillates as

$$\Delta B = \Delta B_{\rm m} \cos \nu_m \phi(s), \tag{1}$$

where $\Delta B_{\rm m}$ is the oscillating amplitude of the RF dipole field, $\nu_{\rm m}$ is the modulation tune defined as the ratio of the RF dipole oscillating frequency to the accelerator's revolution frequency, and $\phi(s)$ is azimuthal angle along the accelerator. In an accelerator with linear magnetic fields, the particle's equation of motion becomes ·

$$z'' + K_z(s)z = -\frac{\Delta B_{\rm m}(s)}{B\rho}\cos \nu_{\rm m}\phi(s), \tag{2}$$

where the prime is the derivative with respect to the longitudinal coordinate s and k_z is the focusing strength. Its Hamiltonian is

$$H = \frac{1}{2}z'^2 + \frac{1}{2}K_z z^2 + \frac{\Delta B_{\rm m}}{B\rho}\, z\, \cos \nu_{\rm m}\phi. \tag{3}$$

At resonance $\nu_{\rm m} = n \pm \nu_z$, the resonance term dominates the Hamiltonian. Here, n is an integer. Expressing the Hamiltonian in terms of action J and its conjugate variable ψ, the new Hamiltonian $H(J, \psi)$ becomes

$$H(J,\psi) \approx \nu_z J + \frac{1}{2}\sqrt{2J}|C_{\rm res}|\cos(\psi - n\phi + \nu_{\rm m}\phi), \tag{4}$$

where $|C_{\rm res}| = \frac{\Delta B \ell}{2\pi B\rho}\sqrt{\beta_z}$. Transferring this Hamiltonian into the resonant frame which rotates with the RF dipole driving frequency, we have

$$H(J,\psi) \approx \delta J + \frac{1}{2}\sqrt{2J}|C_{\rm res}|\cos\psi, \tag{5}$$

where the resonance proximity parameter $\delta = \nu_z - (n - \nu_{\rm m})$ is the distance of the external excitation to the resonance. In general, this is a very small number.

The stable fixed point in Eq. (5) is

$$J_{\rm SFP} = \frac{1}{2}(\frac{C_{\rm res}}{2\delta})^2, \tag{6}$$

Hence, the amplitude of the excited oscillation $Z_{\rm coh}$ is

$$Z_{\rm coh} = \sqrt{2\beta_z J} = \frac{\Delta B_{\rm m}\ell}{4\pi(B\rho)|\delta|}\,\beta_z, \tag{7}$$

where $B\rho$ is the momentum per charge. This shows that for an accelerator with linear magnetic fields, the RF dipole has to be slightly off resonance to maintain the beam stability. Eq. (7) can also be illustrated by the particle's phase space motion in the resonant frame [1].

This method was successfully tested at the Brookhaven AGS with Au^{+77} beam during the heavy ion physics run [1]. The experiment demonstrated that a large coherent oscillation can be adiabatically induced by slowly ramping the RF dipole field oscillation amplitude. Duration time of the excited oscillation is determined by the length of the RF dipole field pulse. Beam emittance was preserved after the

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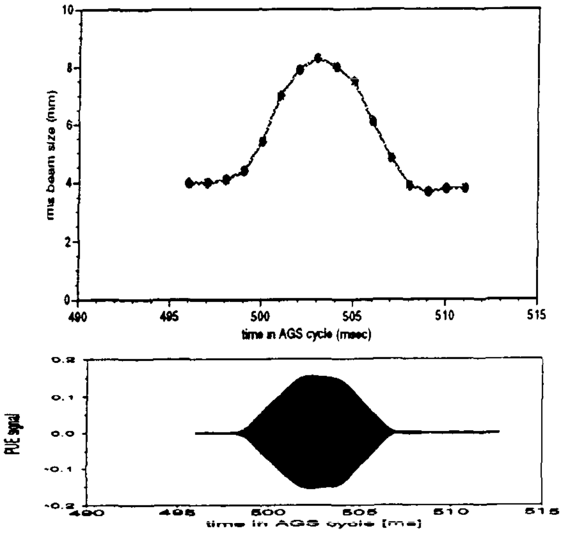

Figure 1: Measured transverse rms beam size versus time in an AGS acceleration cycle (top) and corresponding turn-by-turn beam position monitor data in the AGS cycle (bottom). The beam profiles are displayed in an mountain-range fashion.

manipulation. The top part of Fig. 1 is a typical example of the beam profile measurement and the corresponding beam coherent oscillation measured by a turn-by-turn beam position monitor . It clearly shows the beam emittance before and after the excitation remained constant. The beam profile was measured with the AGS IPM (Ionization Profile Measurement) system. The seemed bigger emittance during the excitation is because every IPM measurement takes 3 ms which is about 900 revolutions around the accelerator, and is the combination of the coherent motion and the actual beam size, i.e.

$$\sigma_{mea} = \sigma_0 \sqrt{1 + \frac{Z_{\rm coh}}{2\sigma_0}}, \tag{8}$$

where σ_{mea} and σ_0 are the measured beam size and actual beam size.

For an accelerator with non-linear magnetic fields, the RF dipole driven oscillation becomes more complicated and the simple relation of Eq. (7) is no longer valid. The most common non-linearity in an accelerator is detuning effect arising from the octupole magnetic fields and the second order effect of sextupoles. In the presence of those higher order multipole magnetic fields, the beam betatron oscillation tune is no longer independent of the size of the betatron oscillation. Assuming the coupling effect between the two transverse planes is negligible, the detuning effect then adds an additional term to the Hamiltonian in Eq. (9).

$$H(J, \psi) \approx \delta J + \frac{1}{2}\alpha_{zz} J^2 + \frac{1}{2}\sqrt{2J}|C_r es| \cos\psi \tag{9}$$

where α_{zz} is the detuning coefficient. Fig. 2 shows the corresponding Poincare when the RF dipole modulation tune is above a certain bifurcation tune.

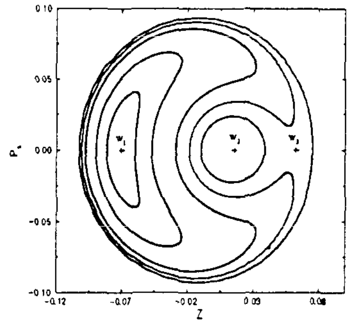

Figure 2: The Poincarer surface of section in the resonant frame which rotates with the RF dipole modulation frequency. w_1 and w_2 are the two stable fixed points. w_3 is the unstable fixed point.

2 APPLICATIONS OF BEAM MANIPULATION WITH AN RF DIPOLE

2.1 Overcoming intrinsic spin resonance

Intrinsic spin resonance is one of the common spin depolarization mechanisms in circular accelerators. It is driven by the quadrupole focusing magnetic fields due to the vertical betatron oscillation. For a perfect synchrotron with only the vertical magnetic guiding fields, the spin vector precesses about the vertical axis $G\gamma$ times per revolution, where $G = (g-2)/2 = 1.7928474$ is the proton anomalous magnetic g-factor, and γ is the relativistic Lorentz factor. $G\gamma$ is called the spin tune. Under the influence of the quadrupole focusing fields, the spin motion is perturbed. The perturbation is normally small. However, when the frequency at which the spin vector is perturbed coincides with the spin precession frequency, the spin vector is kicked away from the vertical direction constructively and a spin resonance occurs. Thus, the intrinsic spin resonance condition is $G\gamma = kP \pm \nu_z$, where k is an integer, P is the superperiods and ν_z is the vertical betatron tune [3, 4]. The other common spin depolarization mechanism is imperfection spin resonance which locates at $G\gamma = kP$ [3, 4].

The polarization after a beam crossing an isolated spin resonance is given by the Frossiart-Stora formula[5]

$$P_f = \left(2e^{-\pi|\epsilon_K|^2/2\alpha} - 1\right) P_i \tag{10}$$

where P_i is the beam polarization before crossing the spin resonance. ϵ_K is the resonance strength, α is resonance crossing rate given by

$$\alpha = \frac{d(G\gamma - kP \mp m\nu_z)}{d\theta}, \tag{11}$$

and θ is the orbiting angle in the synchrotron.

For an intrinsic spin resonance, the strength is proportional to the betatron oscillation amplitude. Normally in a beam, particles close to the core of the beam oscillate less than particles around the edge. Thus, the final polarization is an ensemble average of the Frossiart-Stora formula over the betatron amplitude of the beam particles. Using the Gaussian beam distribution model, the final polarization becomes

$$P_f = \left(\frac{1-\pi|\epsilon_{\rm rms}|^2/\alpha}{1+\pi|\epsilon_{\rm rms}|^2/\alpha}\right) P_i\ , \tag{12}$$

where $\epsilon_{\rm rms}$ is the spin resonance strength for a particle with an rms emittance. For a given intrinsic spin resonance, no polarization will be lost if the resonance is crossed very fast, i.e. $\frac{\pi|\epsilon_{\rm rms}|^2}{\alpha} \ll 1$. Although the fast-crossing resonance can be achieved by jumping the betatron tune with fast quadrupoles [6], it is a non-adiabatic manipulation and can cause beam emittance growth. On the other hand, the beam polarization can be preserved by crossing the resonance slowly so that the spin vector can be adiabatically flipped ($P_f = -P_i$). However, for strong spin resonance, this method is limited.

Alternatively, a full spin flip can also be obtained under the normal acceleration rate by enhancing the resonance strength. For intrinsic spin resonances, the effective resonance strength in Eq. (12) can be greatly strengthened in the presence of a large amplitude coherent oscillation which can be adiabatically excited by an RF dipole. Since the betatron coordinate can be expressed as the linear combination of the vertical betatron motion and the coherent betatron motion [?], the particles experience not only the intrinsic spin resonance, but also a coherent spin resonance at the driving frequency. The resulting polarization, in the limiting case that the driving frequency coincides with the free oscillation frequency, is given by [2]

$$\begin{aligned}\langle\frac{P_f}{P_i}\rangle &= \frac{2}{1+\pi|\epsilon_{\rm rms}|^2/\alpha} \\ &\quad \exp\left\{-\frac{(Z_{\rm coh}^2\hat\beta_z/2\beta_z\sigma_z^2)(\pi|\epsilon_{\rm rms}|^2/\alpha)}{1+\pi|\epsilon_{\rm rms}|^2/\alpha}\right\} - 1,\end{aligned} \tag{13}$$

and in the case that the two resonances are well separated, by

$$\begin{aligned}\langle\frac{P_f}{P_i}\rangle &= \frac{1-\pi|\epsilon_{\rm rms}|^2/\alpha}{1+\pi|\epsilon_{\rm rms}|^2/\alpha} \\ &\quad \left(2\exp\left\{-\frac{Z_{\rm coh}^2\hat\beta_z}{\beta_z\sigma_z^2}\frac{\pi|\epsilon_{\rm rms}|^2}{2\alpha}\right\} - 1\right).\end{aligned} \tag{14}$$

Here $\hat\beta_z$ is the maximum vertical betatron function in the accelerator, and σ_z is the rms beam size. Any case in between can produce rich interference patterns and the beam polarization is determined by both the relative strengths and phase of the two resonances [10].

Experimental results

This method has been successfully tested in the AGS polarized proton acceleration experiments [9] for attaining a full spin flip at intrinsic spin resonances. Acceleration of polarized protons in the AGS up to 25 GeV/c encounters 7 intrinsic spin resonances at $0+\nu_z, 12+\nu_z, 24-\nu_z, 36-\nu_z, 24+\nu_z, 48-\nu_z$ and $36+\nu_z$. Among them, $0+\nu_z$, $12+\nu_z$ and $36\pm\nu_z$ are strong ones. The beam polarization losses at the other three spin resonances are negligible under the AGS normal acceleration speed. The imperfection spin resonances during the polarized proton acceleration were corrected by the AGS 5% partial snake [7, 8].

Figure 3 shows the measured polarization at three intrinsic resonances vs the coherent oscillation amplitude which is proportional to the RF dipole strength. The data at spin resonance $12+\nu_z$ (in the middle plot) demonstrates that the spin was fully flipped at large coherent oscillations when the measured polarization saturated. The same result is also indicated from the data at $0+\nu_z$ (in the bottom plot) and $36-\nu_z$ (in the top plot) with the smallest resonance proximity parameter δ. The systematic error of the beam polarization was estimated to be 10%, and the statistical error was about ±3 %. The lines shown on the figure correspond to results obtained from numerical spin simulations of two spin resonances model.

Since the spin resonance at $12+\nu_z$ was relatively weak, the measured polarization depended smoothly on the dipole field strength shown in the middle plot of Fig. 3. On the other hand, since the intrinsic spin resonances at $0+\nu_z$ and $36-\nu_z$ were strong enough to partially flip the spin, they strongly interfered with the coherent spin resonance induced by the RF dipole. A significant interference pattern is shown in the top and the bottom plots of Fig. 3, where the degree of spin flip also depends on the relative phase. In agreement with the numerical simulation, the upper and lower plots of Fig. 3 show complicated interference patterns when the tune separation is large. Nevertheless, a full spin flip can eventually be obtained when the strength of the RF-induced spin resonance becomes strong.

2.2 *Measure betatron amplitude function*

Both the betatron amplitude function and phase advance are very important parameters for accelerators [?, ?, ?]. The coherent oscillation adiabatically excited by an RF dipole can be maintained for thousands of revolutions around the accelerator is very useful for measuring these two parameters in the machine. Especially, since this is a non-destructive method, it can also be used as a routine diagnostic tool for machine operation.

Assuming there are two beam position monitors (BPMs) which can measure the turn-by-turn betatron oscillation in the accelerator as shown Fig. 4, the coordinates at BPM2 are related with the coordinates at BPM1 through the transfer matrix between the two BPMs.

Therefore, x_1' can then be expressed by the positions at the two BPMs, i.e.

$$x_1' = \frac{x_2}{\sqrt{\beta_1\beta_2}\sin\psi_{21}} - \frac{\cot\psi_{21}+\alpha_1}{\beta_1}x_1. \tag{15}$$

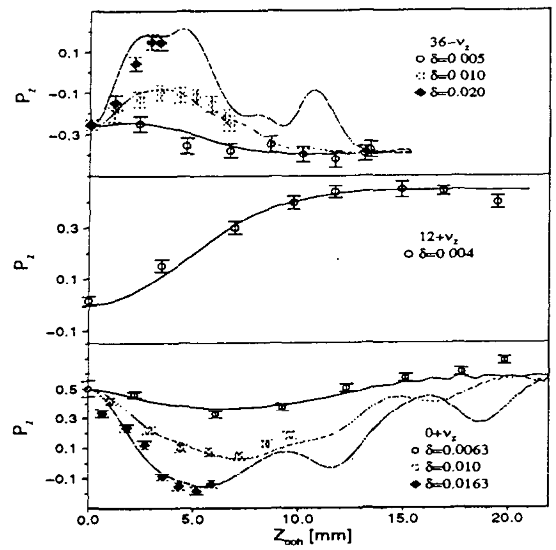

Figure 3: The measured proton polarization vs the coherent betatron oscillation amplitude (in mm) for different tune separations at spin depolarizing resonances $0 + \nu_z$ (bottom plot), $12+\nu_z$ (middle plot), and $36-\nu_z$ (upper plot) respectively. The error bars show only the statistical errors. The resonance strength of the coherent spin resonance due to the RF dipole is promotional to the coherent betatron amplitude. The lines are the results of multiparticle spin simulations based on the two nearby spin resonances model.

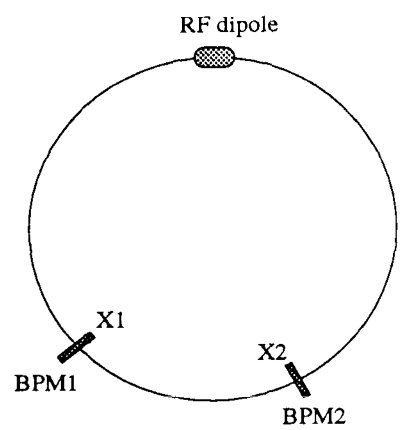

Figure 4: the schematic drawing of using an RF dipole and two BPMs to measure the linear optics.

where β_i and α_i, i=1,2, are the twiss parameters at BPM1 and BPM2, respectively. ψ_{21} is the phase advance between the two BPMs. Since

$$x_1^2 + (\beta_1 x_1' + \alpha_1 x_1)^2 = 2\beta_1 J, \tag{16}$$

x_1 and x_2 satisfy the elliptical equation

$$x_1^2 + (\sqrt{\frac{\beta_1}{\beta_2}} \frac{x_2}{\sin\psi_{21}} - \cot\psi_{21} x_1)^2 = 2\beta_1 J. \tag{17}$$

Hence, the ratio of the betatron functions $\sqrt{\frac{\beta_1}{\beta_2}}$, the phase advance between the two BPMs ψ_{21} and $\beta_1 J$ can be obtained by fitting the turn-by-turn data recorded at the two BPMs.

In the same spirit, this method can also be applied for the non-linear beam dynamics diagnosis and studies in accelerators [11]. Beside manipulating beam motion, an RF dipole can also be used as a spin flipper to change the spin vector by 180° degrees[12]. For this application, the RF dipole introduces an artificial spin resonance at its oscillating frequency. By slowly ramping its frequency through the spin precession frequency, a full spin flip can be obtained (See Eq. (10)).

Unlike the strength of the spin resonance due to the large amplitude coherent oscillation by operating an RF dipole close to the vertical betatron tune which is proportional to the amplitude of the oscillation (see Eq. (3.37)), the resonance strength of a spin flipper is independent of the lattice parameters and fully determined by the magnetic field strength of the spin flipper. Based on the Thomas-BMT equation, the resonance strength is given by

$$\epsilon_K = \frac{1 + G\gamma}{4\pi} \frac{\Delta B \Delta L}{B\rho}, \tag{18}$$

where $\Delta B \Delta L$ is the integrated field strength of the spin flipper. To achieve more than 99% spin flip, the spin resonance strength ϵ_K of the spin flipper should be greater than $1.84\sqrt{\alpha}$.

3 CONCLUSION

It was demonstrated that a sustained coherent oscillation with large amplitude can be adiabatically excited by an RF dipole while the beam emittance is preserved. This method has been successfully applied in the AGS polarized proton acceleration to overcome strong intrinsic spin depolarizing resonances. As a non-destructive method, it can also find other applications in beam diagnostics and dynamics studies, spin manipulations and etc.

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