

INSTABILITIES AND FLUX ANNIHILATION

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I would like to present a small argument to show why I think that flux annihilation is not as important in creating instabilities as is sometimes implied¹ and was mentioned this morning.

We look at a region inside the superconductor where flux lines in one direction (positive) are replacing flux lines in the opposite direction (negative), i.e., annihilate each other (such a situation is illustrated in Fig. 1a for a plane slab geometry). We must remember the following: From our basic knowledge of type II superconductors we have to conclude that a Meissner region will separate the regions between positive and negative flux. The Meissner region, of course, is bounded by London penetration surfaces which shield the interior completely from the adjacent magnetic field which has a value of H_{c1} at the boundary. We want to answer two questions: What is the mechanism by which this Meissner region travels inwards as the external field increases? How far apart are the boundaries or how thick ($= 2\rho$) is the region?

The superconductors under consideration here always show flux pinning, otherwise a field profile of the type given in Fig. 1 cannot arise. Pinning, being a defect-connected property is probably not very uniform and consequently we have to imagine that the boundary of the Meissner region may be quite rough and full of bumps and protrusions, etc.; this will not affect the basic ideas of the argument.

A fitting way of looking at forces connected with flux lines is in terms of the Maxwell tensor. It consists of a lateral pressure of $B^2/8\pi = n^2 \phi_0^2/8\pi$ (if we express B in terms of density n of flux lines) and a longitudinal tension also of $B^2/8\pi$. We experience these forces as repulsion between two equal magnet poles or as attraction between two opposite poles, and if we consider the forces transmitted through the median plane between the two poles we find the pressure between parallel flux lines in one case and the tension along flux lines in the other. The fluxoids then repel each other with a force inversely proportional to their distance and the resulting net force due to the gradient in fluxoid density (equivalent to dB/dx) is just large enough to overcome pinning and keep the fluxoids moving while the outside H increases with dH/dt . At the Meissner region boundary exists a pressure of $H_{c1}^2/8\pi$ which is countered by what has been called the London pressure; but of special interest here is the longitudinal tension of $n \phi_0^2/8\pi$ in a fluxoid,² which allows us to think of it as if it were a stretched rubber band. If such a band is bent around a radius of curvature ρ , a lateral pressure ($n \phi_0^2/8\pi\rho$) is created which also can overcome pinning.

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1. See C.R. Wischmeyer, Phys. Rev. 154, 323 (1967); the present argument was a private reply to a discussion of the Meissner region ("Interface") in this paper and in defense of a statement in the last paragraph of a paper by the author [Phys. Rev. 161, 404 (1967)] then in the process of publication.
 2. The total fluxoid line energy is given as $(\phi_0/4\pi\lambda)^2 \ln \lambda/\xi$ in P.G. deGennes and J. Matricon, Rev. Mod. Phys. 36, 45 (1964).

A whole sequence of two fluxoids annihilating each other is illustrated in Fig. 2, the various stages numbered 1-5:

1. The two opposing fluxoids are separated by the Meissner region which shields their mutual attraction.
2. With the outer fluxoid moving closer (because of dH/dt on the outside) the mutual attraction starts deforming the fluxoid pair in places where pinning has minima.
- 3 & 4. Further deformation leads to coalescence of the fluxoids under the formation of sections with radius of curvature ρ .
5. With ρ being small enough for the line tension to overcome pinning the curved sections move away from each other, thus annihilating two flux lines. The stage is set for a repeat of the same sequence.

The Meissner region moves deeper into the specimen by gradual redistribution of flux as indicated in Fig. 1b-d. Figure 1b shows the situation immediately after the annihilation process leaving the Meissner boundary at a slightly higher field than H_{c1} . Figure 1c shows what kind of flux movement corrects this situation. The left side admits flux through the surface, leaving H outside unchanged; the right side redistributes the flux, leaving the total flux inside the specimen constant. The result is a somewhat larger thickness ($> 2\rho$) of the Meissner region (corresponding to stage 1 of Fig. 2). Figure 1d: further rise in the outside field by ΔH will reduce the thickness to 2ρ , and by comparison with Fig. 1a the Meissner region has moved a small distance. The next annihilation takes place.

In this whole mechanism there are never more than one fluxoid pair involved. The energy dissipation due to annihilation will of course be released as heat in the Meissner region. This heat per unit volume may be somewhat different from the corresponding value in other parts of the shielding region where it comes from the ordinary pinning dissipation and this may constitute more or less of a disturbance of the pinning equilibrium. But on the whole the process is not so different in character from the ordinary growth of a shielding layer (without negative flux present) and the criterion for instability, i.e., the question whether the whole shielding region is in a stable or unstable equilibrium, may be marginally modified but remains unchanged in principle.

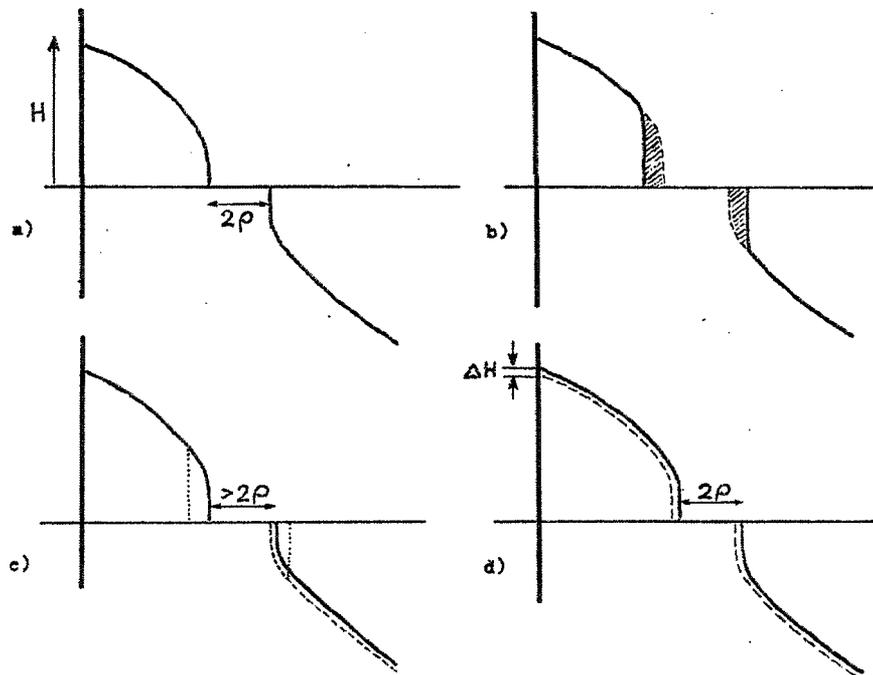


Fig. 1. Motion of the boundary of the Meissner region during flux annihilation.

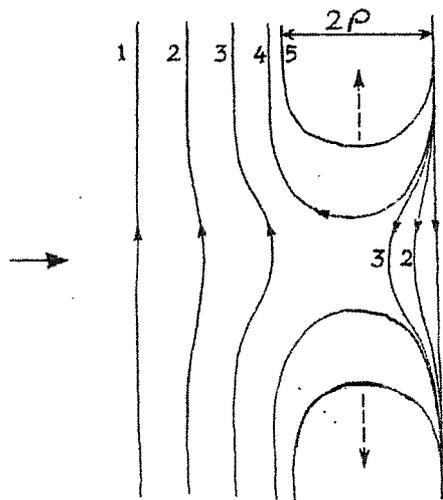


Fig. 2. The process of flux annihilation.