Halo Particle Confinement in the VLHC Using Optical Stochastic Cooling

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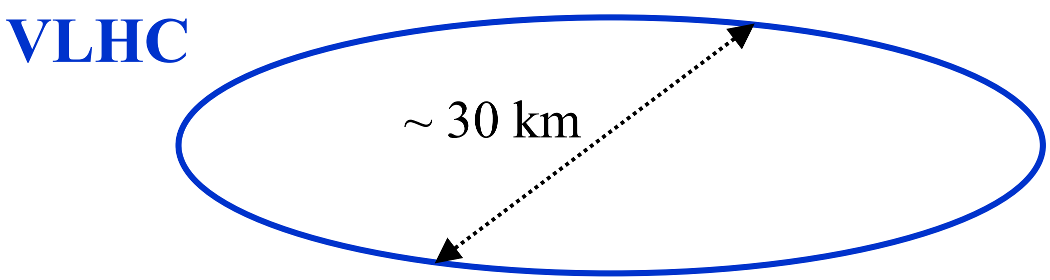
- **Goal**
- **Smith-Purcell radiation**
- **Optical Stochastic Cooling**
- **Results**

The VLHC parameter list (high field option)

• Beam energy, E_0	50 TeV
• Total number of protons	5×10^{14}
• Protons per bunch	1.5×10^{10}
• Normalized emittance	2.5×10^{-6} m-rad
• Energy spread, σ_e	1×10^{-4}
• Bunch length	6 cm
• Sychrotron radiation damping	1.3 hrs



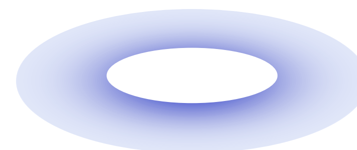
What Optical Stochastic Cooling can and can’t do ?

Can not affect beam emittance and energy spread - stochastic cooling of $5\text{x}10^{14}$ protons with a damping time < 1 hour is practically impossible.

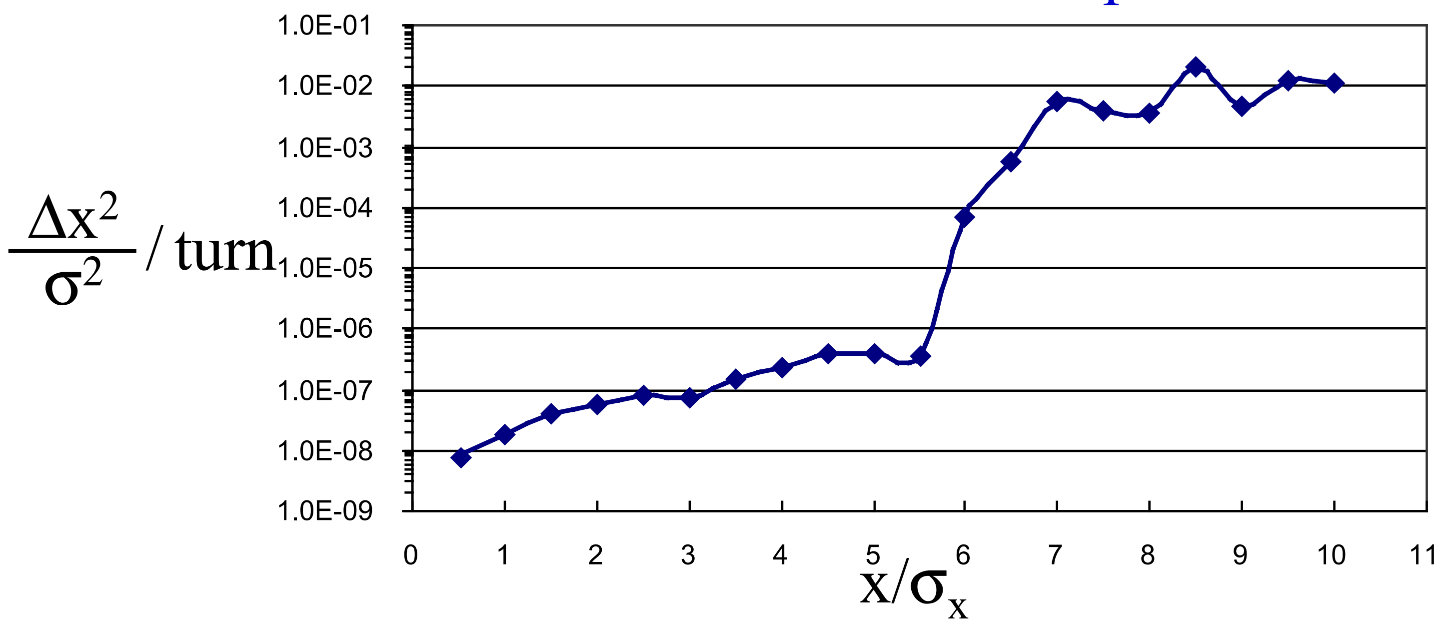
Can counterbalance a slow diffusion of particles to the aperture at large amplitudes by cooling of halo particles.

Instead of this beam

We want to deal with only ~ $5\text{x}10^{9}$ peripheral particles



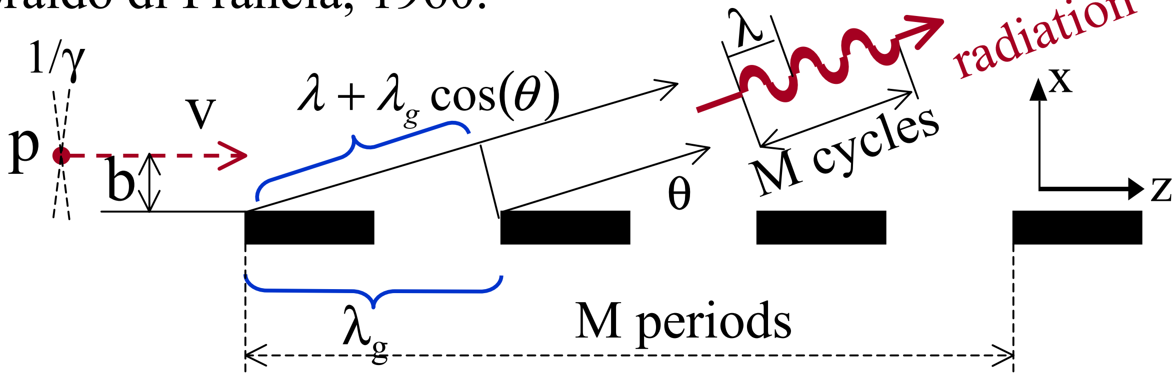
Diffusion versus the amplitude



Simulation for LHC by Papaphilippou and Zimmermann, 1999

Smith-Purcell radiation of a particle moving over the diffraction grating

Toraldo di Francia, 1960.



$$\frac{\lambda + \lambda_g \cos(\theta)}{c} = \frac{\lambda_g}{v}$$

or for $\gamma >> 1$ and $\theta << 1$

$$\lambda = \lambda_g \left(\frac{1}{2\gamma^2} + \frac{\theta^2}{2} \right)$$

wavelength of the radiation

SP radiation = diffraction of the evanescent waves

Bandwidth of the radiation signal: $\Delta\lambda / \lambda \approx 1/M$

Radiation emitted into the angle: $\theta \pm \Delta\theta$, where $\Delta\theta \approx 1/4M$

Diffraction limited size of the radiation source: $d \approx \lambda / 2\pi\theta$

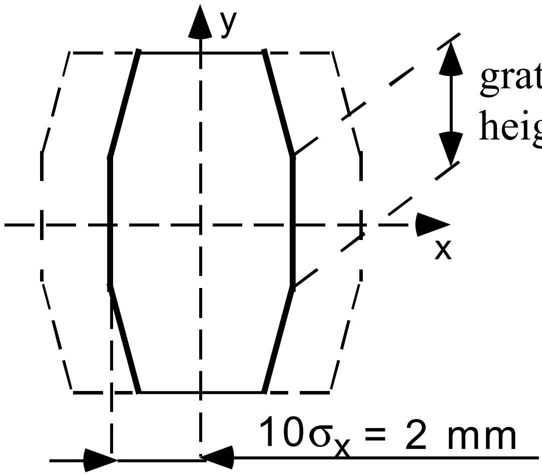
Number of radiated photons:

$$n_{ph} = \mu^2 \pi \alpha \frac{(\gamma\theta)^2}{1 + (\gamma\theta)^2} \exp\left\{ -\frac{4\pi b}{\lambda\gamma} \sqrt{1 + (\gamma\theta)^2 / 2} \right\}$$

grating efficiency   1/137

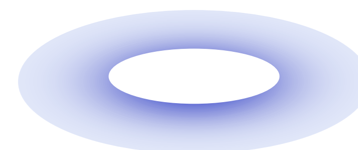
Coulomb field of the proton is represented by the superposition of evanescent waves attenuated in the x direction $E \sim exp\{-|x-b|/d_{dif}\}$, where d_{dif} is the size of the radiation source viewed in the far field at the wavelength λ. When the proton moves close to the grating these waves are diffracted by the grating and give rise to the propagating reflected waves.

A cross section of the vacuum chamber with the grating



dashed lines show grating positions at injection

We want : $\exp\left\{-\frac{4\pi\ 10\sigma_\perp}{\lambda\gamma}\sqrt{1+(\gamma\theta)^2/2}\right\}=10^{-5}$



For λ=800 nm and $10\sigma_\gamma$= 2 mm we get θ= 0.5 mrad

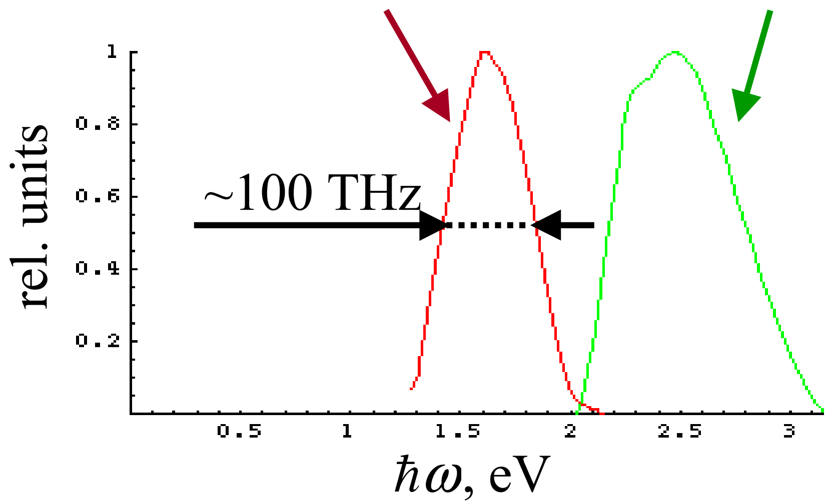
$$\lambda=\lambda_g\left(\frac{1}{2\gamma^2}+\frac{\theta^2}{2}\right)$$

grating period ~ 6m !

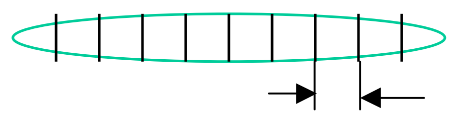
Optical Stochastic Cooling (OSC)

OSC obeys the same principles as the microwave stochastic cooling, but explores a superior bandwidth of optical amplifiers, ~ 10^{14} Hz

Fluorescence and absorption spectra of Ti:sapphire

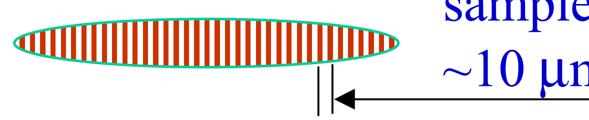


“microwave” sampling



$$N_s = N \frac{\Delta \ell}{\ell_b}$$

“optical” sampling



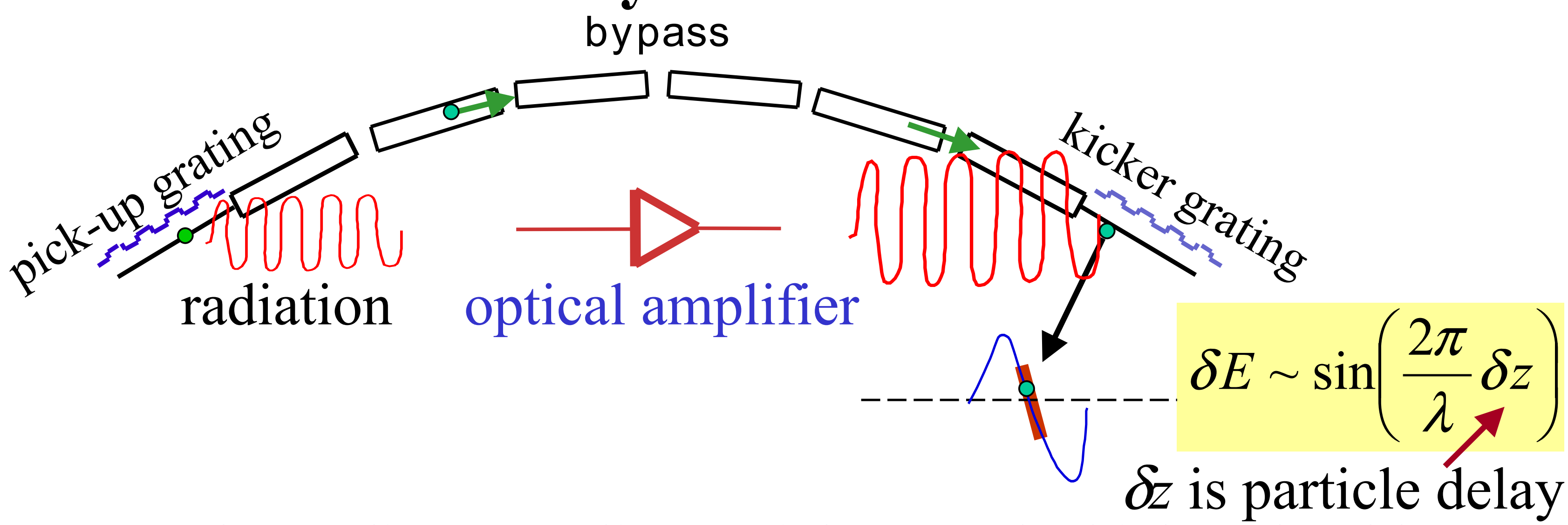
sample length ~10 μm

Damping time expressed in a number of orbit turns:

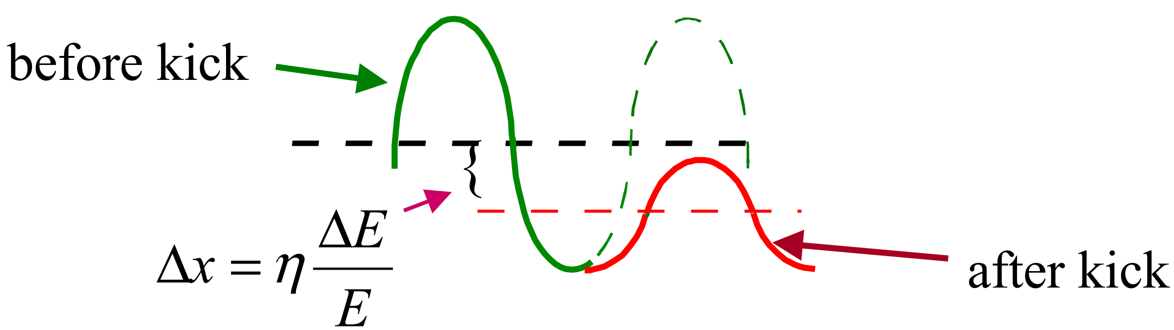
$$n_d \approx N_s$$

In the case of the VLHC the damping is defined by the available power of optical amplifiers

A schematic of the OSC system:



For energy and coordinate cooling a pick-up and a kicker should be installed in a position with a nonzero dispersion function (*similar to the Palmer's method of the momentum cooling*).



- Coupling is used to share dumping between vertical and horizontal coordinates

Calculation of the energy kick

in the far field region → $E(\omega,r)=E_A(\omega,r)+E_R(\omega,r)$

Amplified field (E_A) Field of spontaneous radiation (E_R)

$|E_A| = g\,|E_R|$, where g is the amplitude gain of the amplifier

field energy → $\mathrm{A} = \iint |E|^2 \mathrm{d}S\mathrm{d}\omega$

$$= \iint |E_A|^2 \mathrm{d}S\mathrm{d}\omega + \iint |E_R|^2 \mathrm{d}S\mathrm{d}\omega + 2\iint |E_A E_R| \mathrm{d}S\mathrm{d}\omega$$

$$= \mathrm{A}_A + \mathrm{A}_R + \underbrace{2\iint |E_A(\omega,r)E_R(\omega,r)| \mathrm{d}S\mathrm{d}\omega}_{\text{Energy exchange}}$$

energy gain

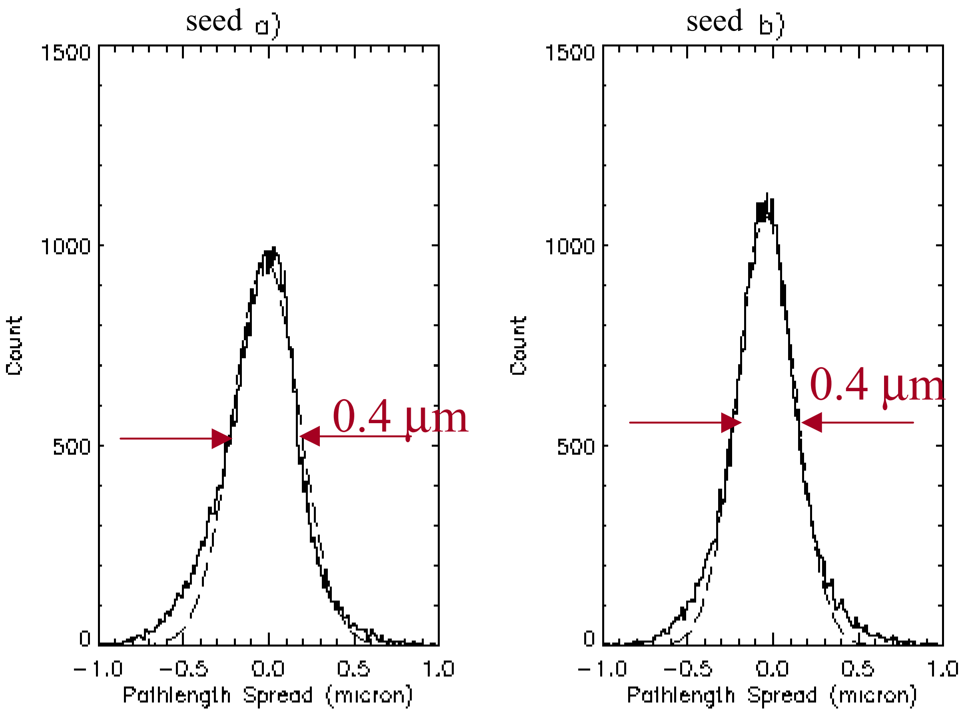
$$\delta E = 2\eta\sqrt{\mathrm{A}_R\mathrm{A}_A}\,\sin\left(\frac{2\pi}{\lambda}\delta z\right) = 2\eta\sqrt{\mathrm{n}_{\mathrm{ph}}\hbar\omega\mathrm{A}_A}\,\sin\left(\frac{2\pi}{\lambda}\delta z\right)$$

efficiency of the field matching (η)

Time-of-flight parameters of the bypass lattice

Errors:

Quadrupole gradient : $\Delta G/G=1x10^{-3}$
Bending field: $\Delta B/B=1x10^{-3}$
Sextupole gradient: $\Delta S/S=1x10^{-3}$
Tilt angle: 0.2 mrad
Misalignment: 150 µm
Multipoles: $\Delta G/G=1x10^{-4}$ at r=3cm
$\Delta B/B=1x10^{-4}$ at r=3cm
Power supply ripple: $1x10^{-4}$



Histograms showing a spread of the pathlengths due high order aberrations and all kind of errors

$$N_s \approx 10^{-5} \times 1.5\,10^{10} \times \frac{2\,10^{-3}}{6} = 50$$

number of protons in the sample

suppression factor

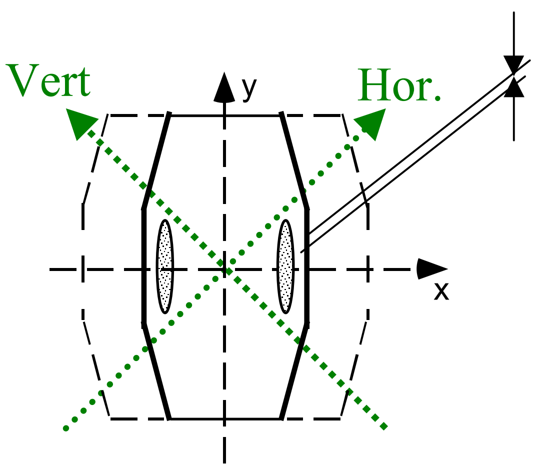
protons per bunch

(sample length)/(bunch length)

Thus, damping time can be estimated as:

average power of amplifiers

$$\frac{1}{\mathrm{n}_\mathrm{d}^2} \cong \frac{\delta E^2}{\sigma_e^2 E_0^2} = 4\pi\,\eta^2 \mu^2 \frac{\alpha\hbar\omega P}{N f_0 \sigma_e^2 E_0^2} \exp\{-\frac{4\pi b}{\lambda\gamma}\sqrt{1+(\gamma\theta)^2}\}$$



d= 0.25mm, the diffraction limited size of the source

five sources on each side

10 amplifiers, each 20 W

For $\eta = 0.5$, $\mu^2 = 1$, and $N = 5 \times 10^9$:

$$n_d(x) \approx 3\times 10^5 \exp\{0.57(10 - x/\sigma_\perp)\} \text{ turns}$$

→ ~ 100 sec at $x = 10\sigma_\gamma$

Amplitude evolution due to the damping and diffusion

$$\frac{d\,x^2}{dt} = -\frac{x^2}{\tau_d} + D(x^2)$$

Critical diffusion when: $\frac{d\,x^2}{dt} = 0$

Then:

$$\frac{\Delta x^2}{\sigma_\perp^2}\left(\frac{1}{\text{turn}}\right) = \left(\frac{x}{\sigma_\perp}\right)^2 \left[\frac{1}{n_d(x)} + \frac{1}{n_{SR}}\right]$$

Synchrotron radiation damping = 1.5×10^7 turns

Plots of the critical diffusion for VLHC and LHC