HOW TO FIND THE QCD CRITICAL POINT AT RHIC,

IF IT IS AT $\mu_B \leq 400$ MeV

KRISHNA RAJAGOPAL (MIT)

Talks and discussions at:
  INT program, August 2008
  CPOD conference, June 2009
and reviews:
Koch 0810.2520 ; Lombardo 0808.3101;
  Philipsen 0808.0672; Karsch 0711.0656;
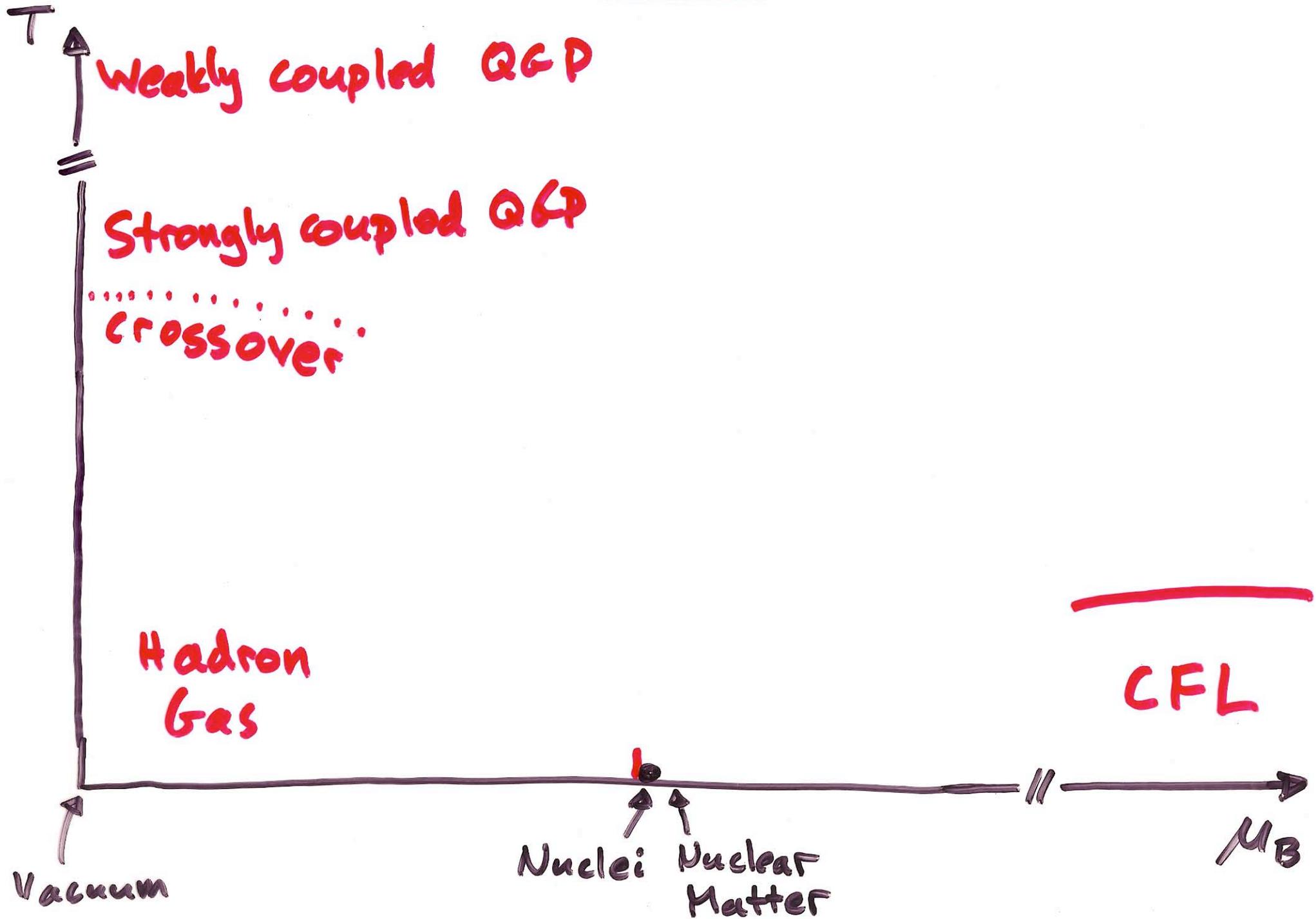 Stephanov hep-lat/0701002
have all been very helpful as I prepared
this talk.

This talk is my attempt to describe a suite of measurements that RHIC can perform that will either find experimental evidence for the QCD critical point or demonstrate convincingly that it is not at $\mu \leq 400$ MeV.
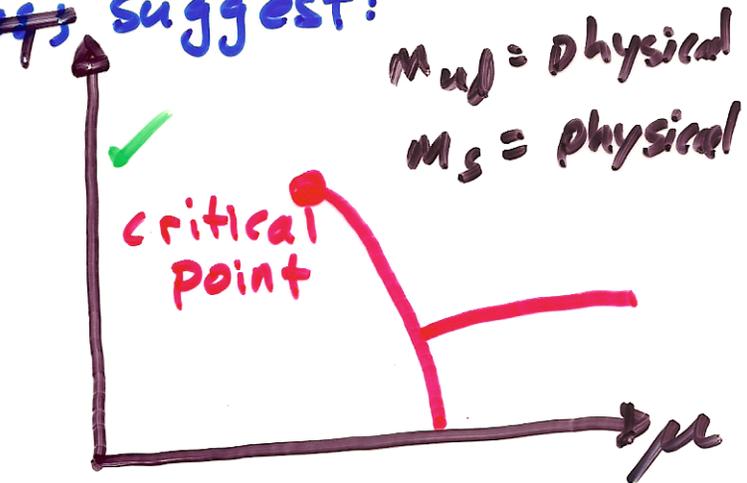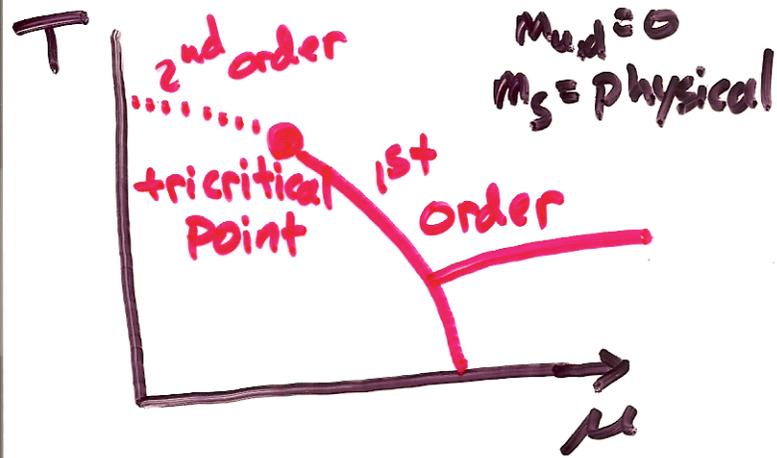
EXPLORING *the* PHASES *of* QCD
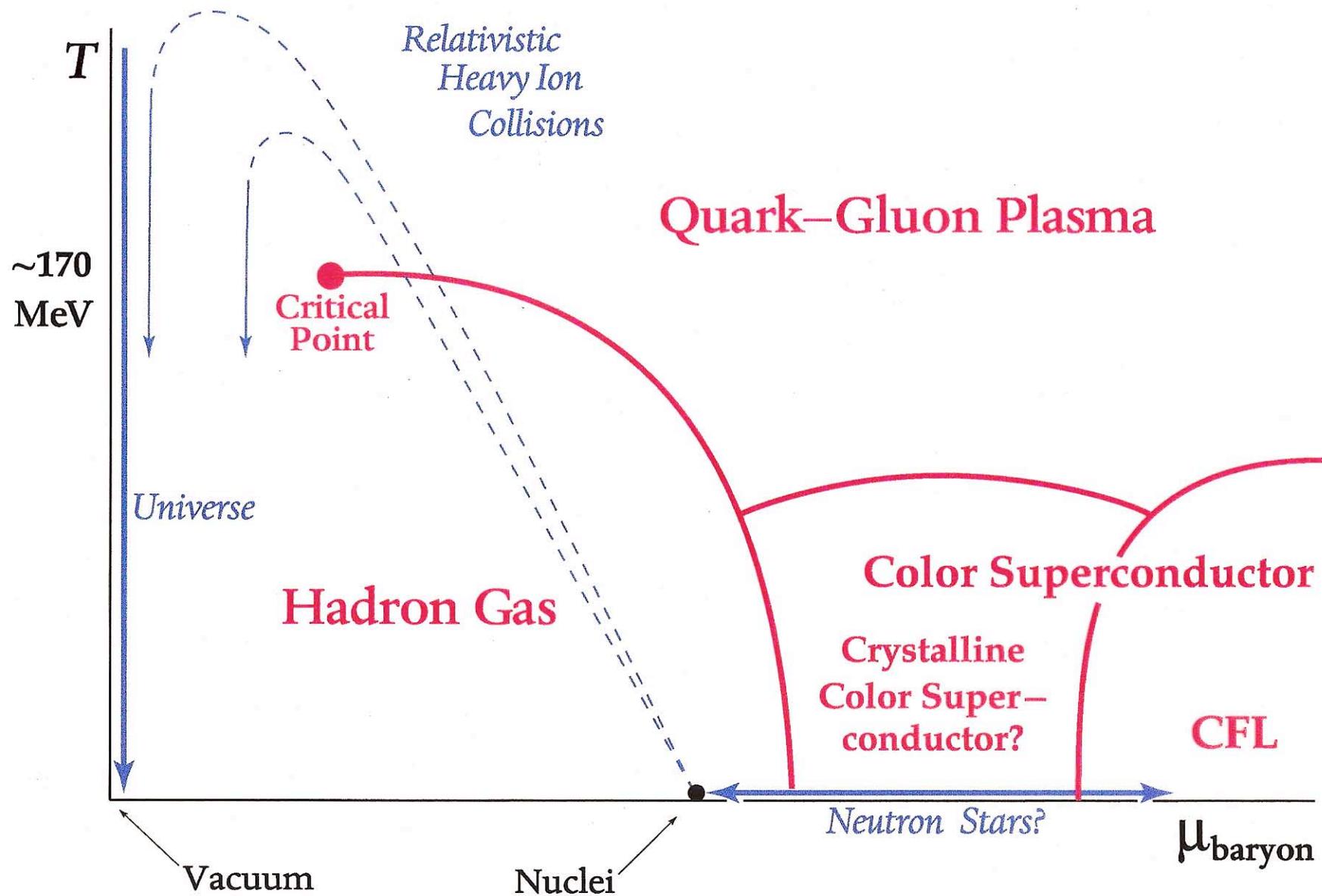
WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu = 0$ with varying quark masses, suggest:

$M_{u,d} = 0$
$M_S = \text{physical}$

$M_{u,d} = \text{physical}$
$M_S = \text{physical}$

$M_{u,d} = \text{physical}$
$M_S < \text{physical}$

$M_{u,d} = \text{physical}$
$M_S \ll \text{physical}$
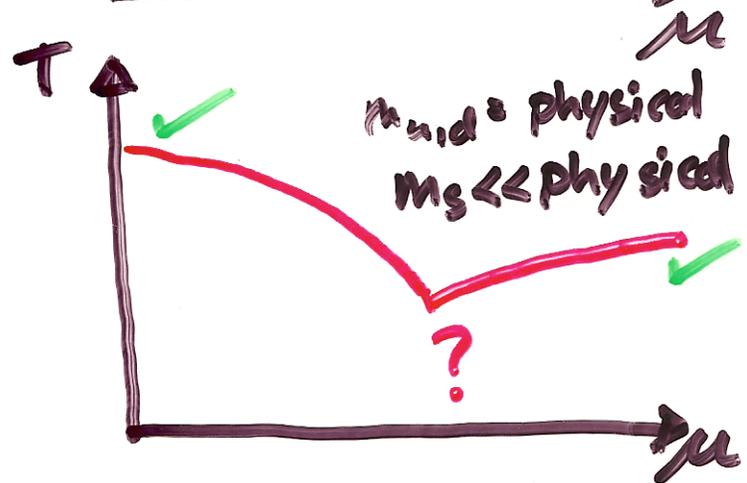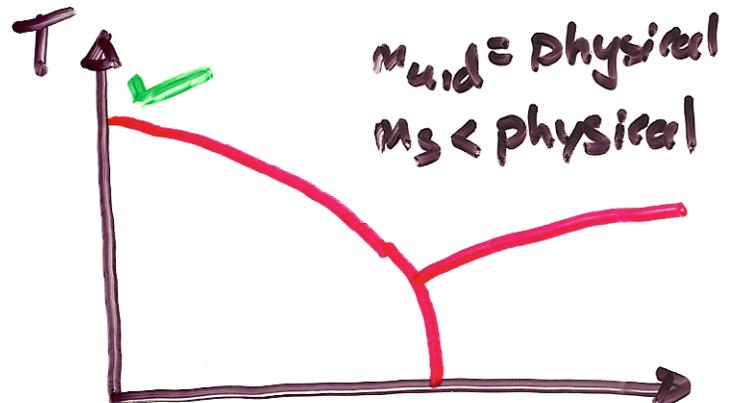
✓ : known

- Universality class of the QCD critical point is known. (ISING)
- Experiments, and lattice calculations with $T \neq 0$, $\mu \neq 0$, needed to locate it.

WARNING

Nothing we know precludes pushing the critical point so far to the right that:

although models and <u>some</u> lattice calculations favor
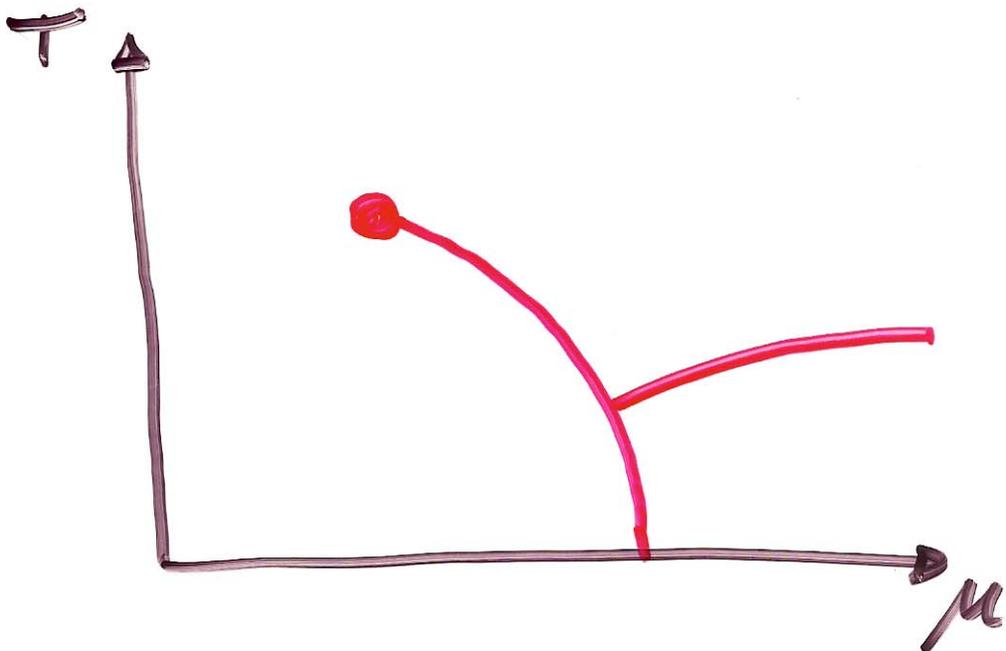
LOCATING THE CRITICAL POINT....

... either via lattice calculations,
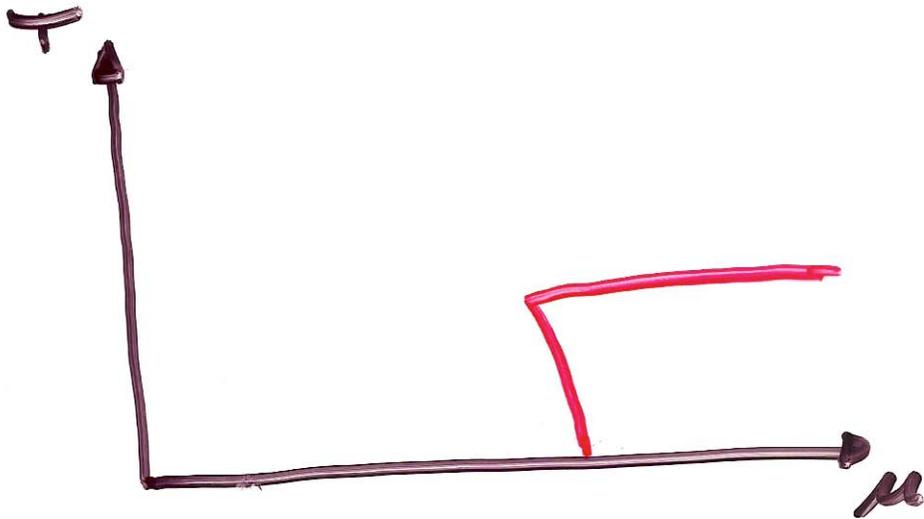or via detection of its signatures
in heavy ion collision experiments,

would add a point and a line
to the known QCD phase diagram.

- A qualitative leap in our
  understanding of QCD in the
  interior of its phase diagram,
  currently terra incognita.

- An opportunity for RHIC to write
  another new chapter in any
  future book on QCD.

LATTICE QCD WITH T≠0, μ≠0, μ/T NOT LARGE

- $\mu \neq 0 \rightarrow$ complex Euclidean action
    - $\rightarrow$ sign problem
    - $\rightarrow$ difficulty of standard monte carlo $\sim \exp V$

- Several lattice methods now in use
  rely on smallness of $M_q/T = \mu_B/3T$

  to control the sign problem:
  - reweighting (Fodor + Katz)
  - continue from imaginary μ
      (de Forcrand & Philipsen; D'Elia & Lombardo)
  - Taylor expansion of P; radius of convergence
      (RBC-Bielefeld; Gavai & Gupta)

  Uncertainties still dominated by systematics;
      (different systematics for different methods,
      but in all cases includes coarseness of
      lattice spacing)

  Steady progress; "crawling towards the
      continuum limit"

- Several groups exploring calculations at
  fixed n_B, instead of μ. (de Forcrand +
  Kratochvila; Li, Alexandru & Liu; ...)

LATTICE RESULTS

- via reweighting (Fodor & Katz)

$$\mu_0 = 360 \pm 40 \,(\text{stat}) \text{ MeV}$$

- via calculating $dm_c/d\mu$ (de Forcrand + Philipsen)

$$\frac{\mu_0}{T_0} > 3 \rightarrow \mu_0 > 500 \text{ MeV}$$

- via radius of convergence of Taylor expansion

$$\frac{\mu_0}{T_c(\mu=0)} = 1.7 \pm .1 \,(\text{stat}) \quad (\text{Gavai} \& \text{Gupta})$$

$$\rightarrow 250 < \mu_0 < 400 \text{ MeV}$$

(with a "very naive" estimate of systematics)

$$\mu_0/T_c(\mu=0) > 1.5 \quad (\text{RBC - Bielefeld})$$

- STILL SYSTEMATICS DOMINATED
   Nevertheless,

ONE CLEAR LESSON

Lattice calculations provide strong indications, via <u>all</u> algorithms employed to date, that:

$$\boxed{\mu_0 > 200 \text{ MeV}}$$

In the race between lattice calculations and experimental searches to locate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, lets turn to experimental searches ....

HOW EXPERIMENTS CAN LOCATE •

① At large $\sqrt{s}$, ie small μ, Collisions
   equilibrate well above the crossover....

② Decrease $\sqrt{s}$ in steps, moving freezeout
   to larger and larger μ.

③ Look for signatures:

a) Of the "lumpy" non-equilibrium final
   state expected after cooling through a
   first order transition. (Mishustin; Dumitru
   Paech Stöcker; Randrup; Koch Mojumder Randrup;...)
   - proton-proton angular correlations $(\Delta\phi, \Delta\gamma)$.
     [fbw turns lumps into clustering in angle]
                                    Mocsy & Sorensen

   - enhanced event-by-event V_2 fluctuations?

b) Of the critical point itself. I.e.
   signatures of the long wavelength
   fluctuations occurring only near •.
   Rise and then fall as $\mu\uparrow$, $\sqrt{s}\downarrow$.
   I shall describe several such analogues
   of critical opalescence.

SEARCHING FOR THE CRITICAL POINT
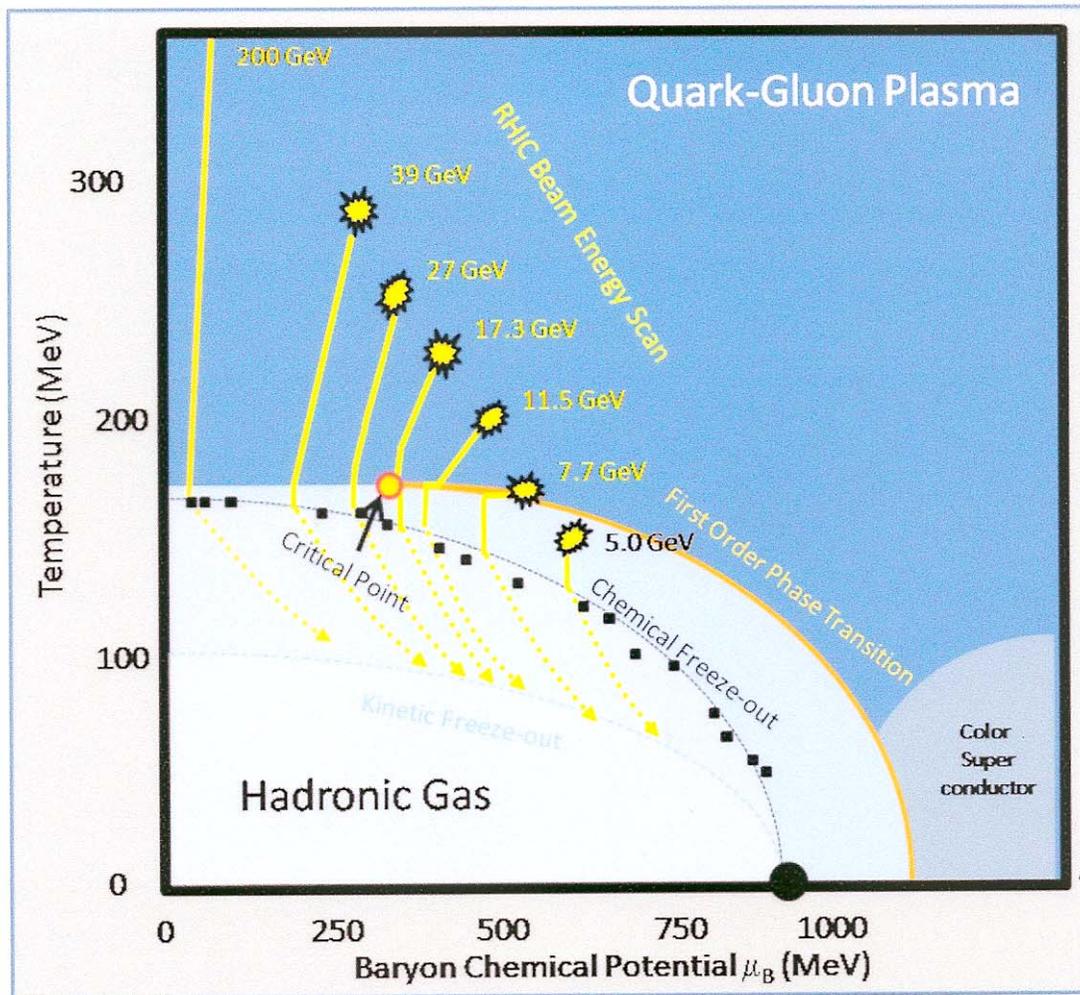
Decreasing $\sqrt{s}$: decreases T and increases μ at which collision equilibrates, "landing on the phase diagram":

$\Rightarrow$ increases μ at which the trajectory followed by the cooling plasma crosses the transition or crossover.

NB: location of 🔴 in fig. is merely illustrative — we don't know where 🔴 is !

CHEMICAL FREEZEOUT T and μ
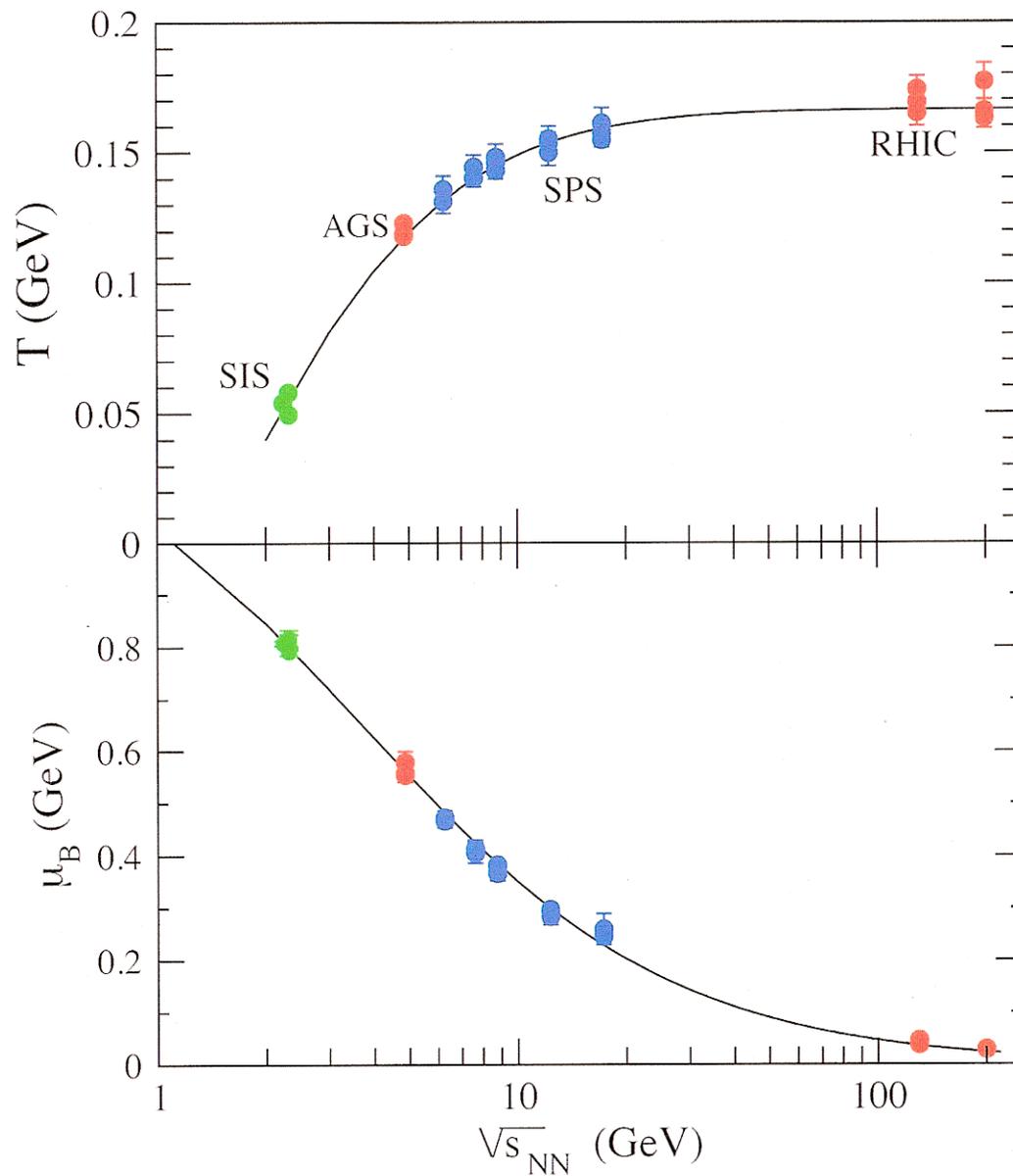
Parametrization from Cleymans et al, 2005
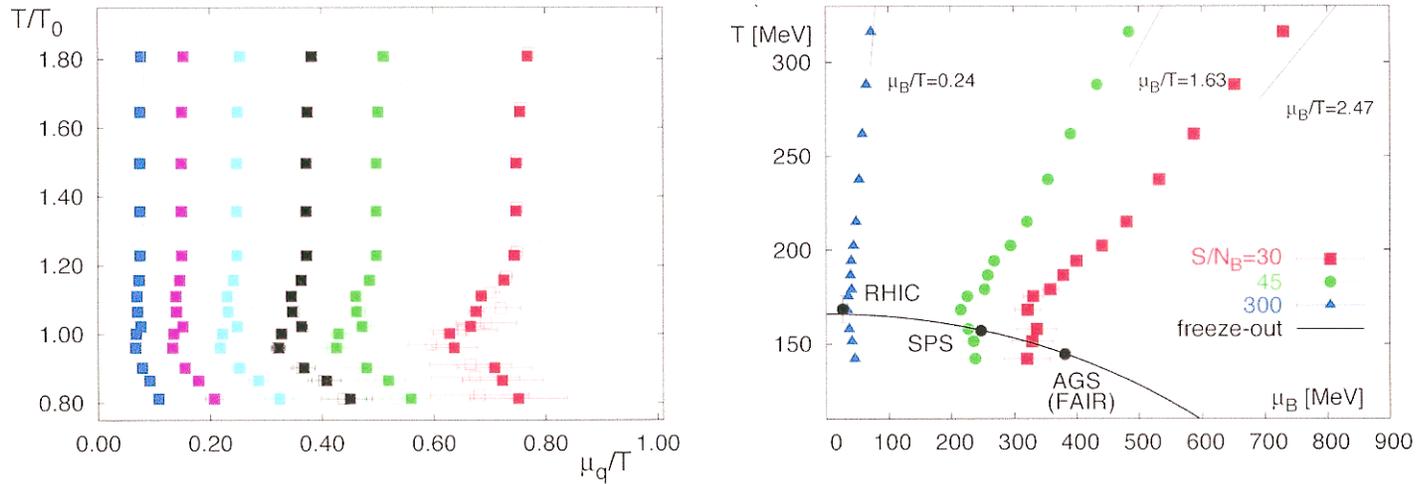
ISENTROPIC TRAJECTORIES

FIG. 3: Lines of constant entropy per quark number versus μ_q/T (left) and in physical units using $T_0 = 175$ MeV to set the scales (right). In the left hand figure we show results obtained using a 4th (full symbols) and 6th (open symbols) order Taylor expansion of the pressure, respectively. Data points correspond to $S/N_B = 300$, 150, 90, 60, 45, 30 (from left to right). The vertical lines indicate the corresponding ideal gas results, $\mu_q/T = 0.08$, 0.16, 0.27, 0.41, 0.54 and 0.82 in decreasing order of values for S/N_B. For a detailed description of the right hand figure see the discussion given in the text.
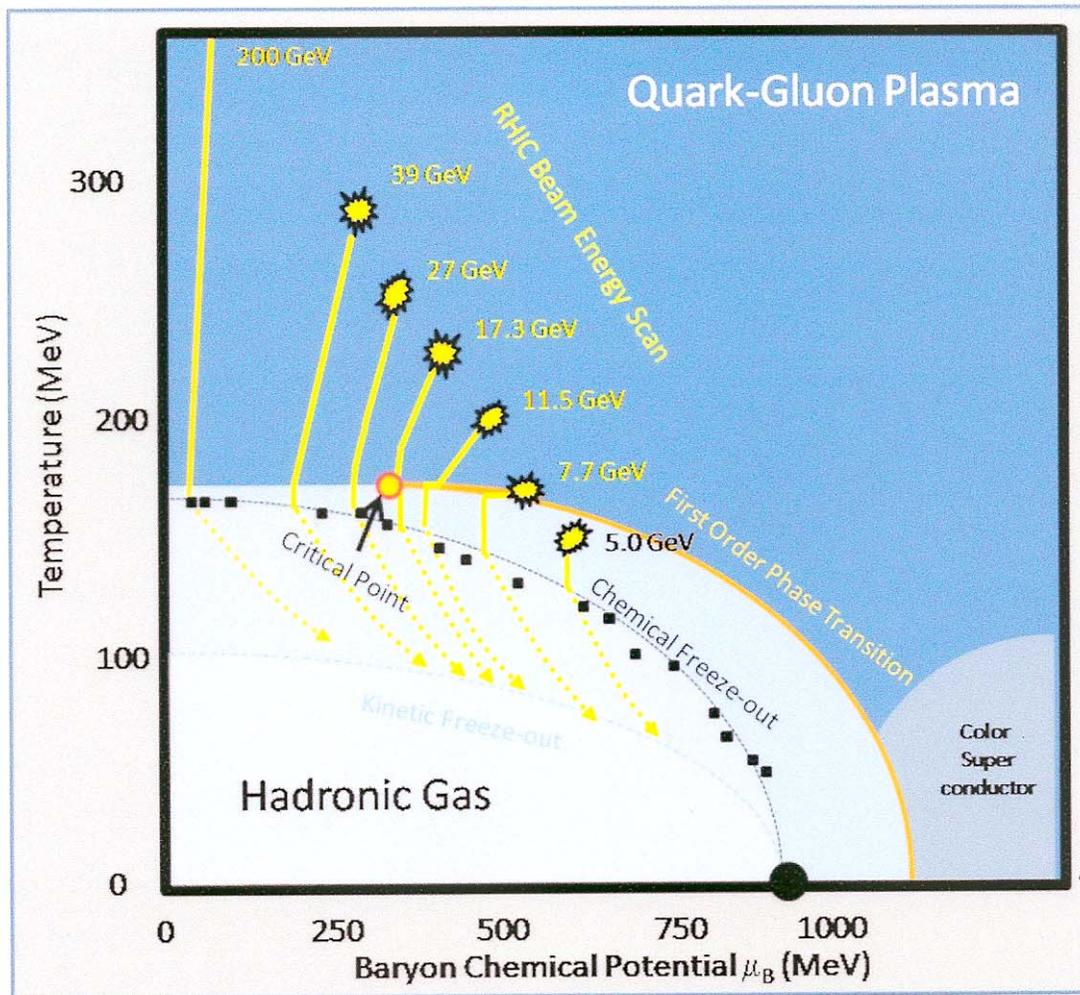
Ejiri Karsch Laermann Schmidt

- Shape of isentropic trajectories in QGP phase and in crossover region is known from lattice calculations
- isentropic trajectories zigzag as they cross first order line

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TNB: location of ● in fig. is merely illustrative — we don't know where ● is !

HOW LOW TO GO ?

Down to what $\sqrt{s}$ should we look?

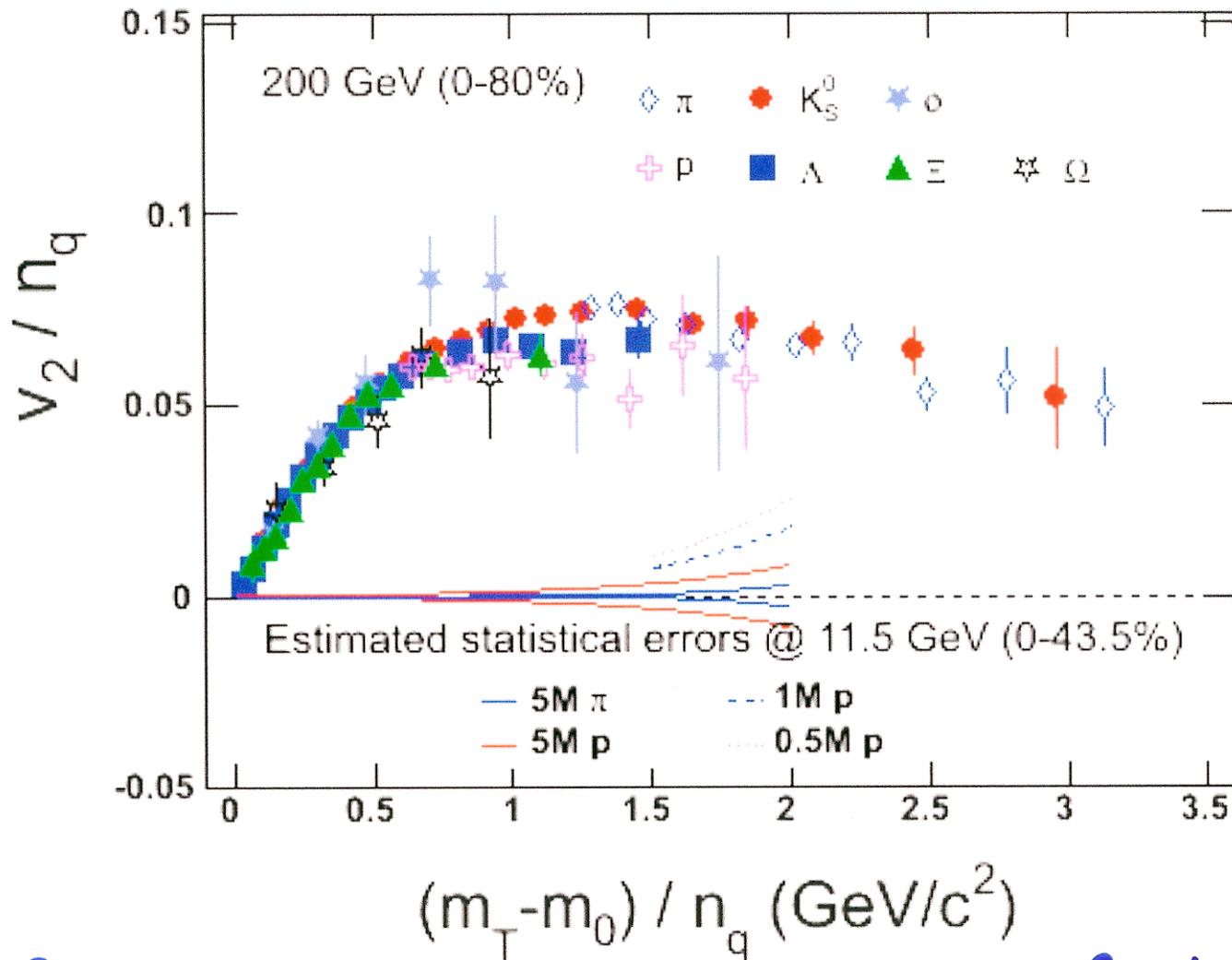⇒ Up to what μ can we look?

This question should be answered experimentally.

Need an effect that is:

- well-measured at $\sqrt{s} = 200$ GeV
- expected only in collisions that do begin above the crossover/transition
- expected at lower $\sqrt{s}$, as long as collisions do begin above crossover.
  ⌈i.e. jet quenching won't do since that can turn off due to absence of jets⌋

Here are two suggestions....

n_q - SCALING OF V_2

V_2 / n_q versus $(m_T - m_0) / n_q$ (GeV/c^2)

200 GeV (0-80%) ◇ π   ● K_S^0 ✦ ○   ✚ p   ■ Λ   ▲ Ξ   ✧ Ω

Estimated statistical errors @ 11.5 GeV (0-43.5%)

— 5M π   --- 1M p
— 5M p   ⋯ 0.5M p
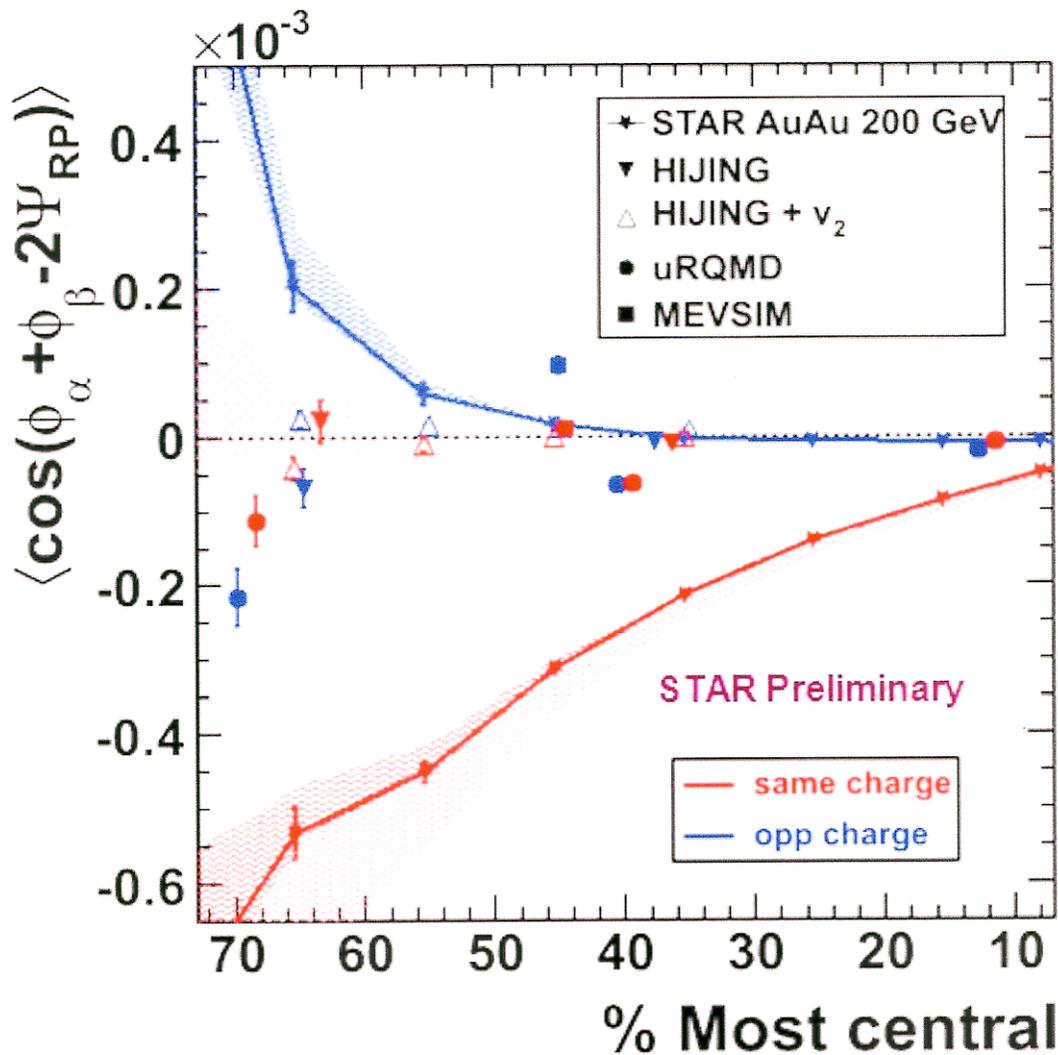
STAR B.U.R.

- V_2 same for mesons of varying mass; same for baryons of varying mass ⟹ V_2 developed before hadrons formed
- Measurement can be done for π/K/p/Λ with 5M min. bias events @√s = 11.5 GeV with errors shown

PARITY VIOLATING FLUCTUATIONS

Need 5M min. bias events per √s to measure. (STAR B.U.R.)

STAR AuAu 200 GeV
HIJING
HIJING + v₂
uRQMD
MEVSIM

STAR Preliminary

same charge
opp charge

% Most central

- Data ⟹ charge separation!
  ⟹ an electric field (⊥ to reaction plane, parallel or antiparallel to L⃗ & B⃗) that is coherent over a volume corresponding to many charged particles
- No hadronic explanation of data
- Khartzev's explanation requires deconfinement

So.....

1. Decrease $\sqrt{s}$ in steps

2. Measure the $\sqrt{s}$ at which
    - n_q scaling of v_2
    - parity violating fluctuations
        (charge separation)
   turn off.

3. You can only look for
   signatures of ● down to, or
   perhaps slightly below, that $\sqrt{s}$.

4. In this way, learn up to what
   μ heavy ion collision experiments
   can find ● .

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (ie fluctuations couple to $\pi\pi$ and pp) and baryon number.

Fujii Ohtani; Son Stephanov

The longer the correlation length ξ gets, the bigger the signatures.

Signatures are event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ. Stephanov KR Shuryak

- Vary μ by varying $\sqrt{s}$
- Search for enhancement of these fluctuations in a window in $\sqrt{s}$, ie μ
- Analogue of critical opalescence
- Long wavelength fluctuations $\Rightarrow$ effects greatest at low $P_\perp$.

Examples ......   But, first :

HOW LARGE CAN ξ GET?
HOW CLOSE TO ● NEED WE BE?

- Obviously ξ limited by finite size of system. But, turns out that finite time is a more severe limitation.

  Berdnikov KR; Asakawa Nonaka

- Finite time spent in critical region means that even if equilibrium value of ξ is much larger, actual ξ won't grow bigger than 2-3 fm.

- Means no need to hit ● precisely.

$\xi_{eqbm} \sim 2-3\ fm$ } $\xi_{actual} \sim 2-3\ fm$

$\xi_{eqbm} \sim \infty$
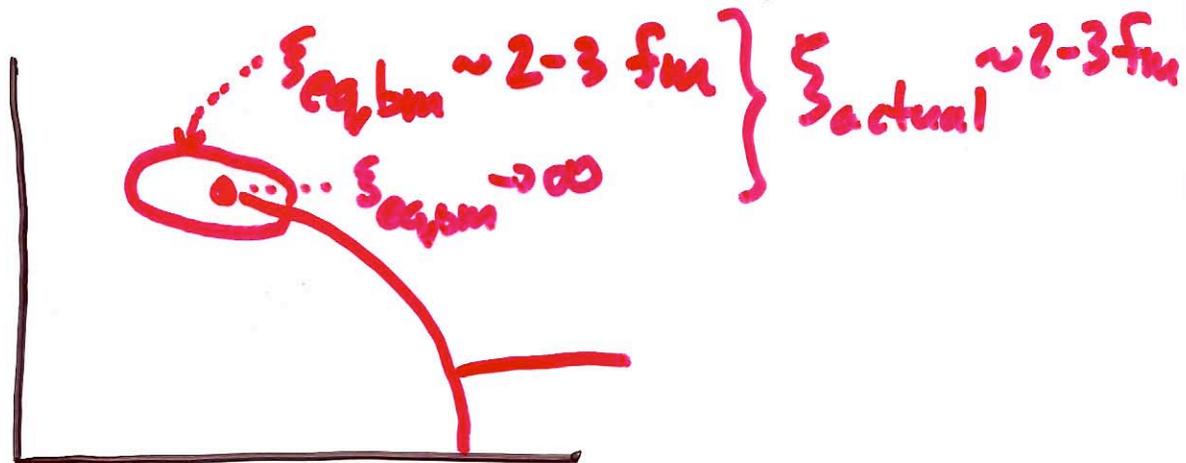
Signatures will be just as big if you pass anywhere in ⬭. No bigger, even if you hit ●.

- Hatta + Ikeda calculated "$\bigcirc$'s" in a model, but did so with contours of χ_B rather than ξ. → Figs. The robust point is that the extent of these $\bigcirc$'s in μ_B is not small. Width in μ_B is ~100 MeV, an estimate that is both crude and uncertain. Can this be obtained on lattice??

- NB also: since ξ cannot be > 2-3 fm, heavy ion collision experiments can never be used to measure the critical exponents of the 2nd order critical point. That's OK: we know it is Ising. What we don't know, and need experiments for, is <u>where</u> it is located.
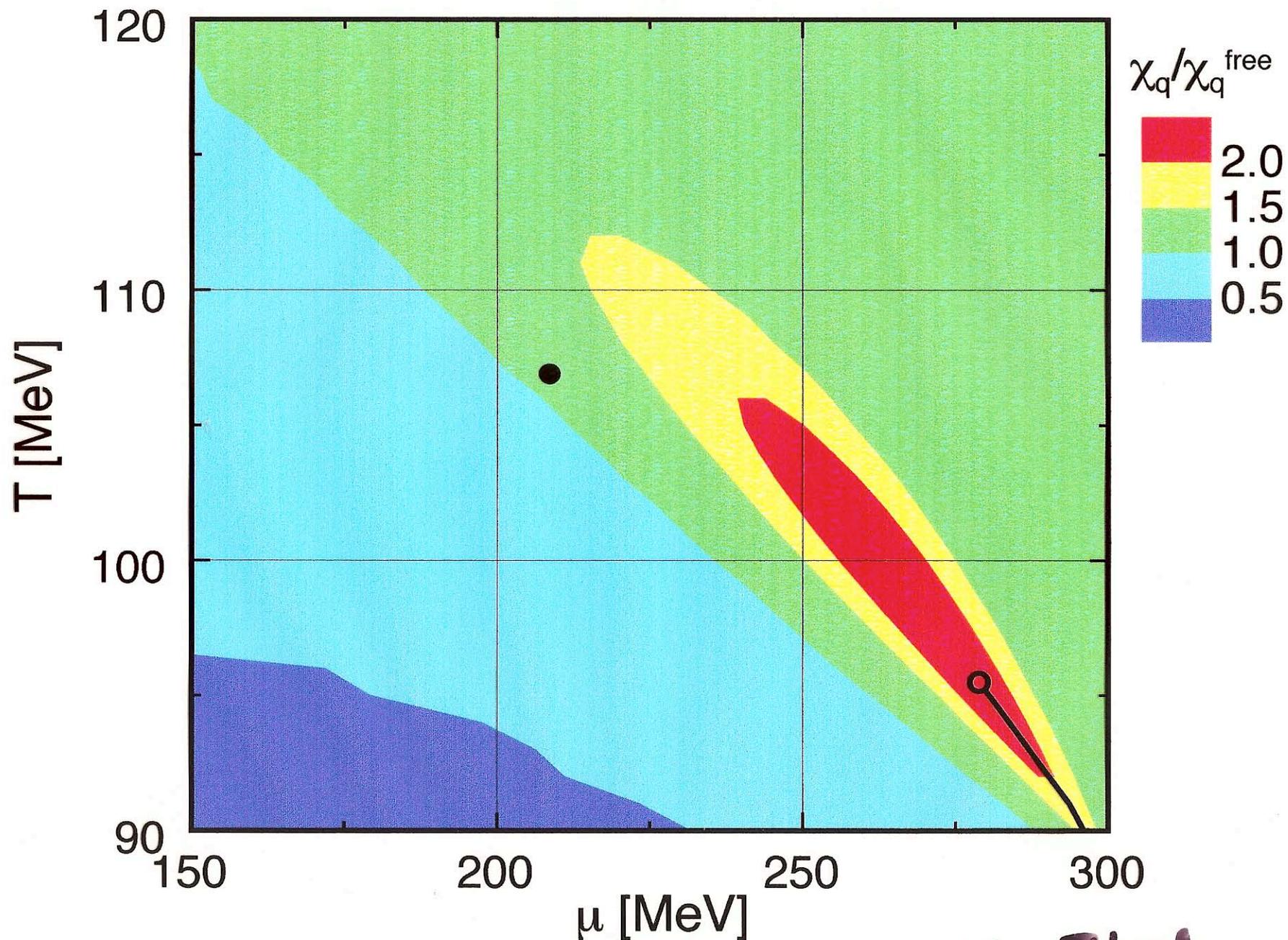
SIGNATURES OF CRITICAL POINT

- Decreasing $\sqrt{s}$ → Increasing μ

- Vary $\sqrt{s}$, and hence μ, and look for <u>nonmonotonic</u> enhancement (rise and then fall) of :

  i) Event-by-event <u>fluctuations</u> of <u>mean P_T</u> of low P_T pions

  ii) Event-by-event <u>fluctuations</u> of <u>net proton number</u> ($N_p - N_{\bar{p}}$)

  iii) Event-by-event <u>fluctuations</u> of <u>particle ratios</u> involving pions and/or protons

  iv) <u>Kurtosis</u> of the N_p or ($N_p - N_{\bar{p}}$) event-by-event distribution

  ⋮

In all cases, the <u>enhancement should be greater at lower P_T</u>.

MEAN P_T OF LOW P_T PIONS

Stephanov KR Shuryak (1999)

First example of a <u>quantitative</u> connection between long wavelength fluctuations of the chiral order parameter with correlation length ξ and magnitude of event-by-event fluctuations of an experimental observable.
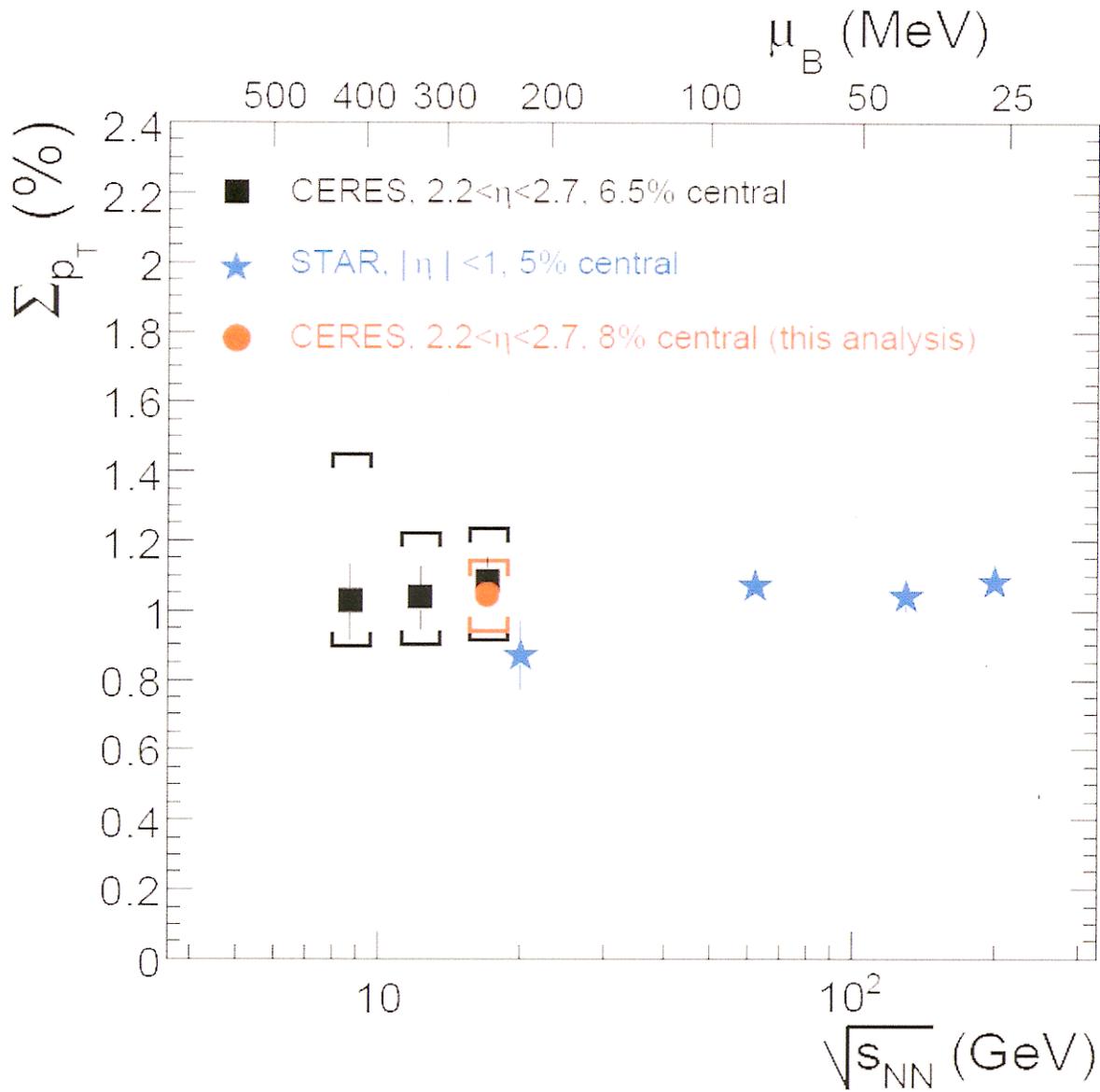
DISADVANTAGES:

- Effect predicted is not large for $\xi = 3$ fm
- Will fluctuations in P_T survive the late time hadronic gas? Will they get washed out between chemical and kinetic freezeout?
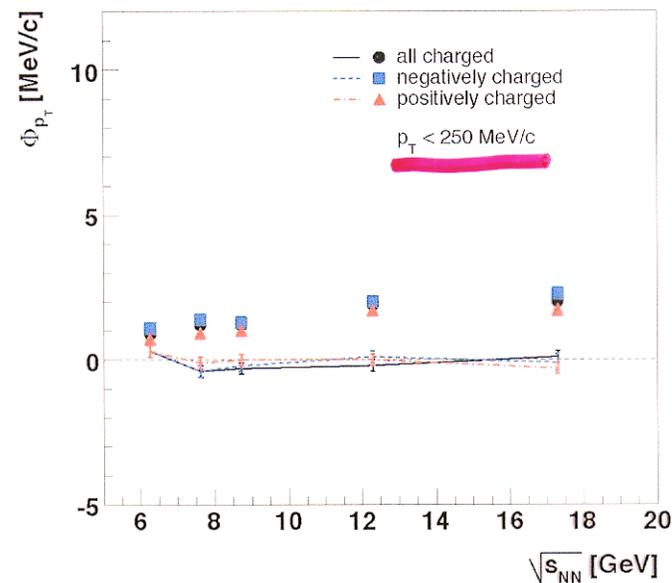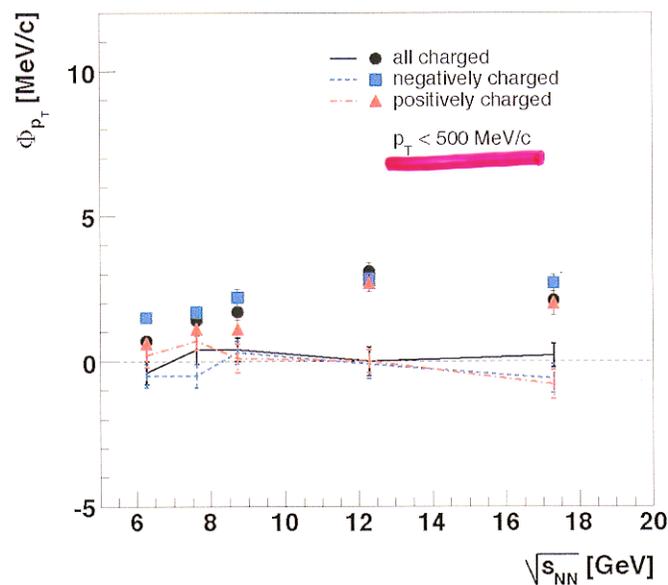
RESULT:

NA49 has done a beautiful analysis and sees no $\sqrt{s}$ dependence....
CERES has done a beautiful analysis and sees no $\sqrt{s}$ dependence....

EVENT-BY-EVENT FLUCTUATIONS OF MEAN P_T

CERES, Nucl Phys A, 2008

EVENT-BY-EVENT FLUCTUATIONS OF MEAN P_T OF PARTICLES WITH P_T BELOW A CUT
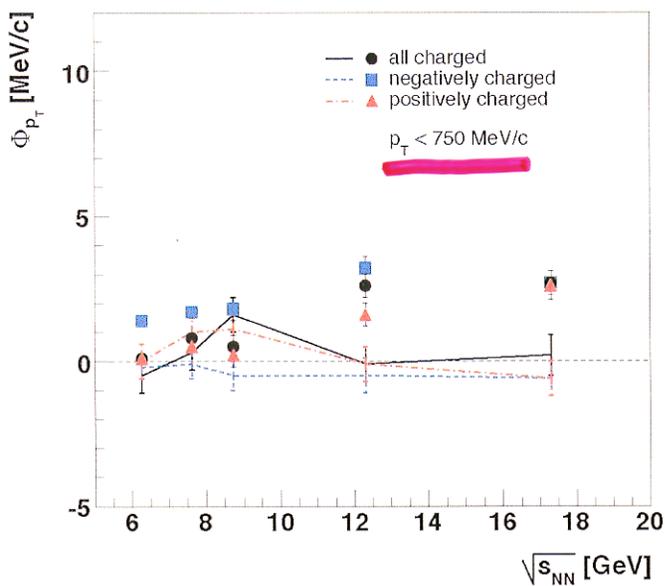
NA49, 2008

No enhancement at low P_T.

MEAN P_T FLUCTUATIONS, vs. μ

NA49, 2008

Height of solid (dashed) curve is magnitude of effect predicted for $\xi = 3$ fm ($\xi = 6$ fm).

POSSIBLE CONCLUSIONS

- $\mu_\bullet > 470$ MeV ?

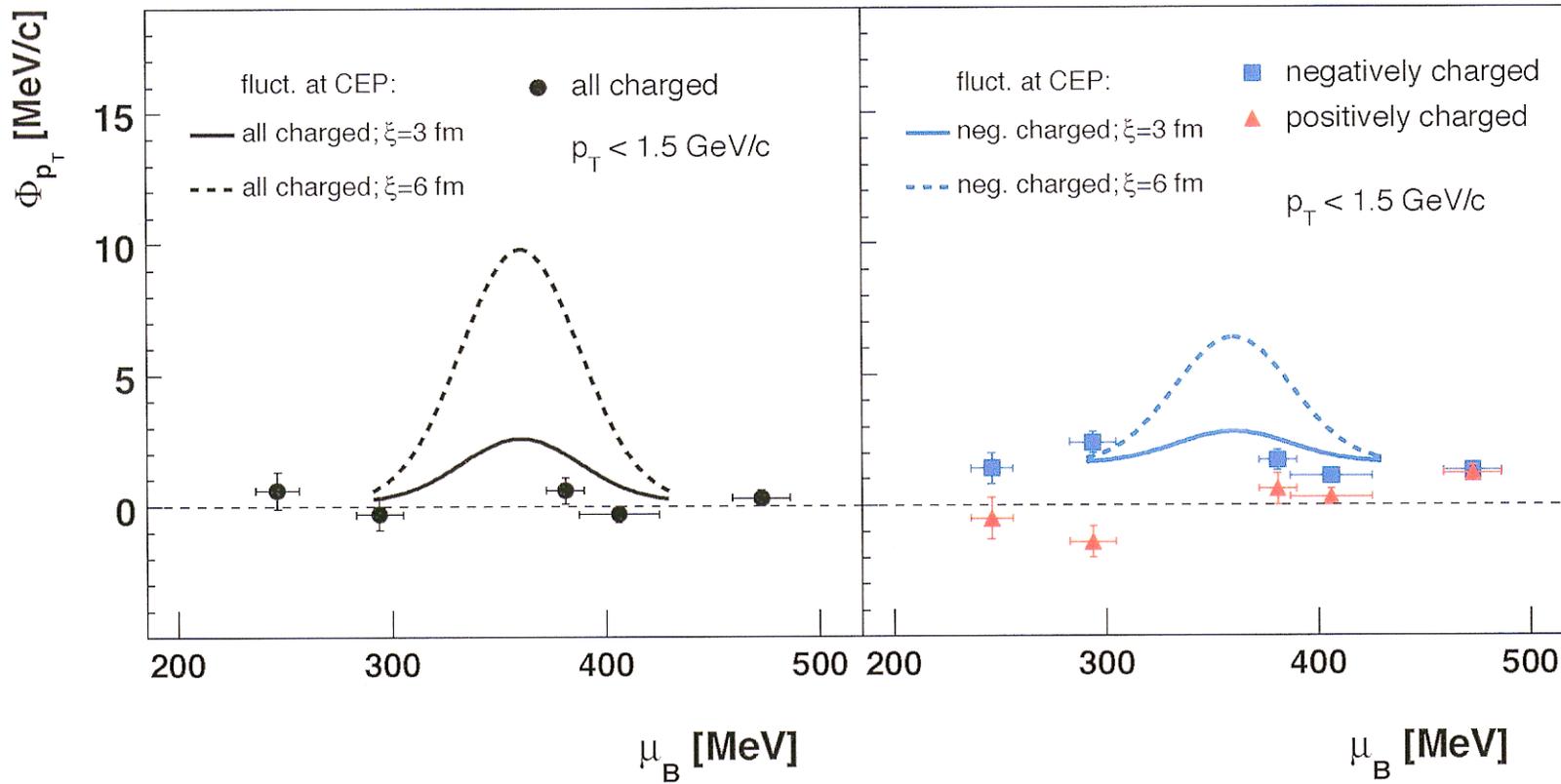- P_T fluctuations washed out.
  Predicted effect was not large, and
  was susceptible to being erased
  after chemical freezeout.

$\Rightarrow$ Look for event-by-event
  fluctuations of other observables
  a) for which the predicted effect
  of proximity to the critical
  point is larger

  b) which cannot be washed out
  after chemical freezeout
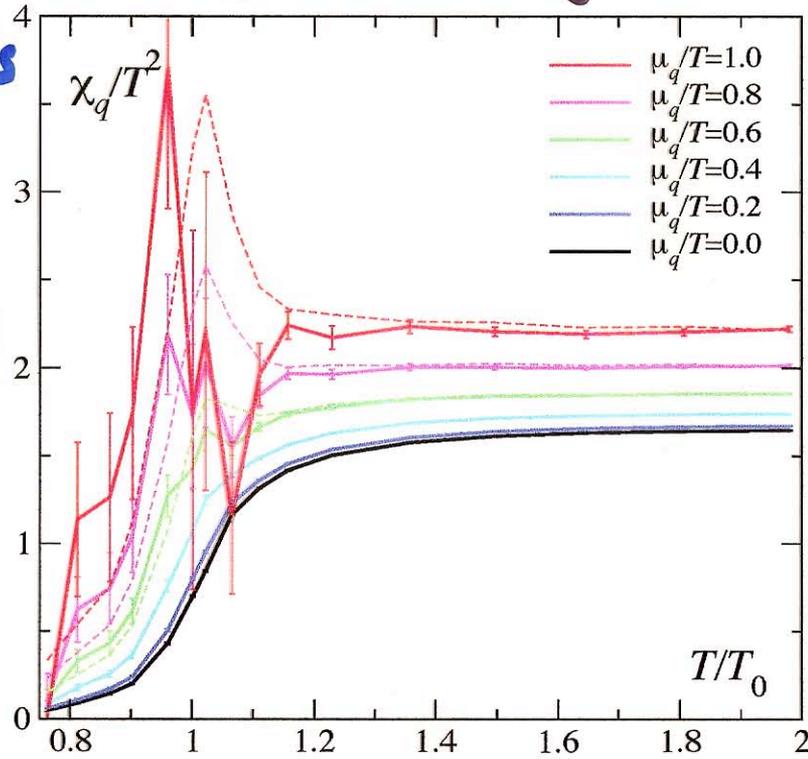
BARYON, AND PROTON, NUMBER FLUCTUATIONS

Hatta Ikeda; Hatta Stephanov

- Seen on the lattice → FIG

- should be looked for in experimental data

$$\frac{\partial^2 \Omega}{\partial \mu_B^2} \rightarrow B\text{-fluctuations} \sim \xi^2 + \text{nonsingular}$$

$$\frac{\partial^2 \Omega}{\partial \mu_I^2} \rightarrow \text{no enhanced } (u\text{-}d) \text{ fluctuations} \sim \text{nonsingular}$$

Suggests $\frac{\mu_q}{T} \sim 1$ getting close to ⬤.
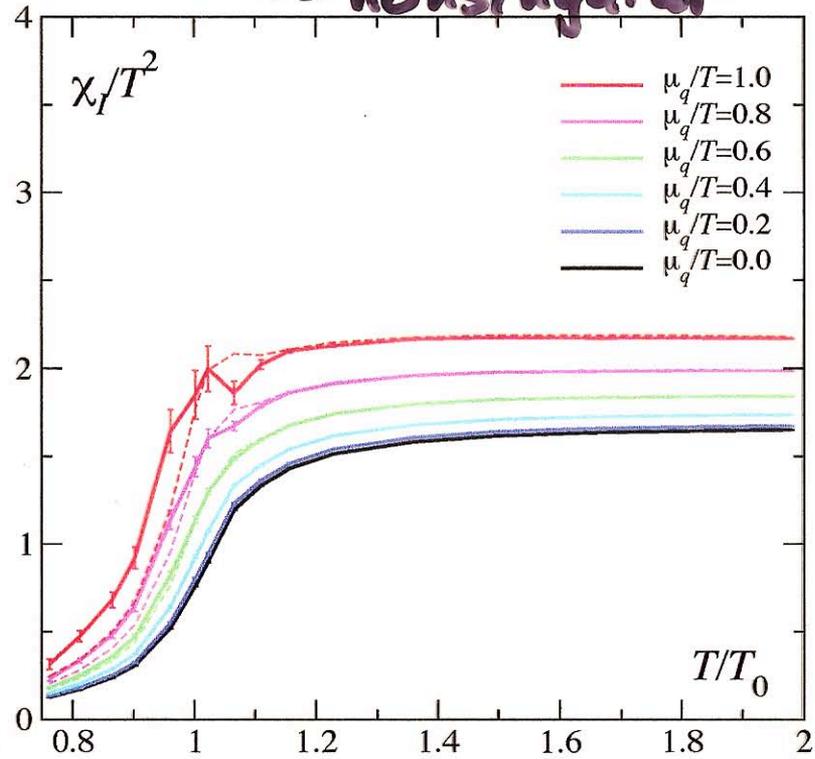
Figure 3.3: The quark number susceptibility χ_q/T^2 (left) and isovector susceptibility χ_I/T^2 (right) as functions of T/T_0 for various μ_q/T ranging from $\mu_q/T = 0$ (lowest curve) rising in steps of 0.2 to $\mu_q/T = 1$, calculated from a Taylor series in 6th order. Also shown as dashed lines are results from a 4th order expansion in μ_q/T.

(Because B fluctuates while isospin does not, proton fluctuations $\sim$ B fluctuations)
Hatta Stephanov

Ejiri et al

PARTICLE RATIOS

- Originally motivated by peak in

  $\langle K \rangle / \langle \pi \rangle$ at $\sqrt{s} = 7.6$ GeV.

  To better understand this, look at fluctuations of K/π ratio.

- Now motivated by observation that these fluctuations will better survive the late time hadron gas.

RESULT:

- Large K/π fluctuations at

  $\mu_B \sim 400 - 450$ MeV

- Why no P/π fluctuations ???
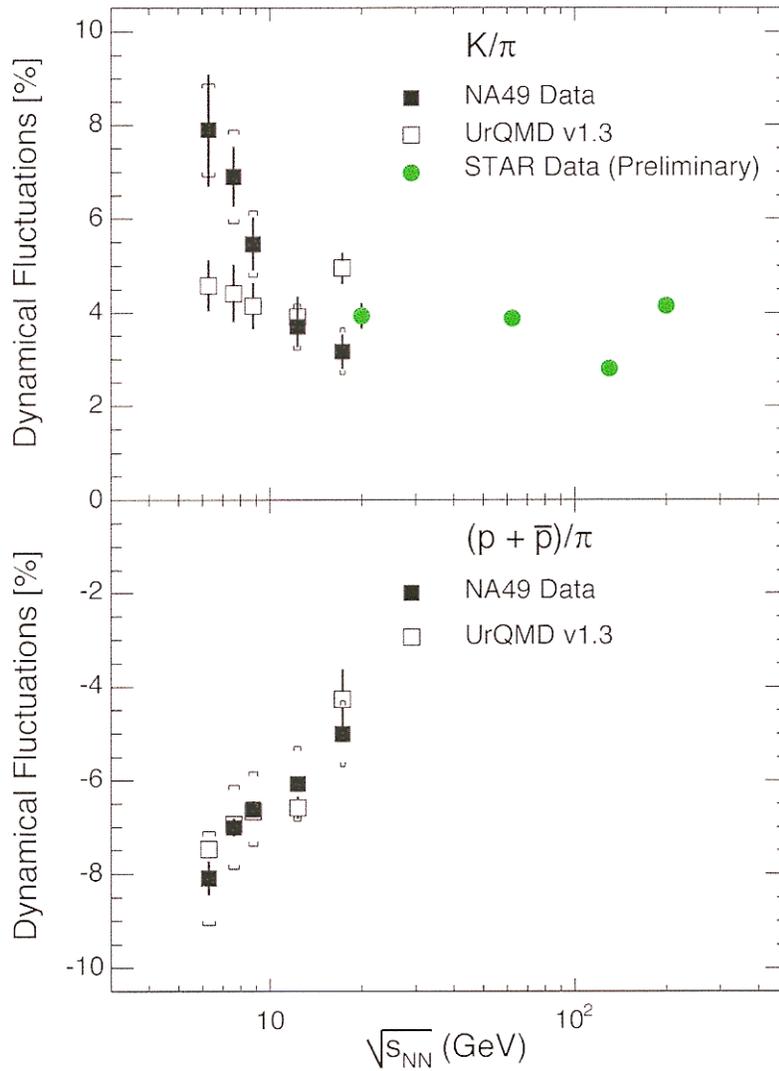
FIG. 8: Energy dependence of the event-by-event non-statistical fluctuations of the K/π ratio (top panel) and the $(p + \bar{p})/\pi$ ratio (bottom panel). Filled symbols show data, open symbols show calculations with the UrQMD transport code, using NA49 acceptance tables. Systematic uncertainties are shown as brackets.

NA49, 2008

- Large event-by-event fluctuations in K/π ratio at $\mu \sim 350-450$ MeV

- Intriguing, but puzzling
  - error bars still substantial
  - interpretation is complicated by change in # of accepted kaons (& pions) as $\sqrt{s}$ changes. Koch Schuster Westfall
  - why no enhancement in p/π fluctuations?
    → K/π fluctuations apparently not driven by low P_T pions

STAR has used this observable as a case study to see how they can improve on this measurement with a beam energy scan at RHIC.

A collider has advantages...

RHIC Critical Point Search Program - Advantage

Hadron Mass

Beam Energy
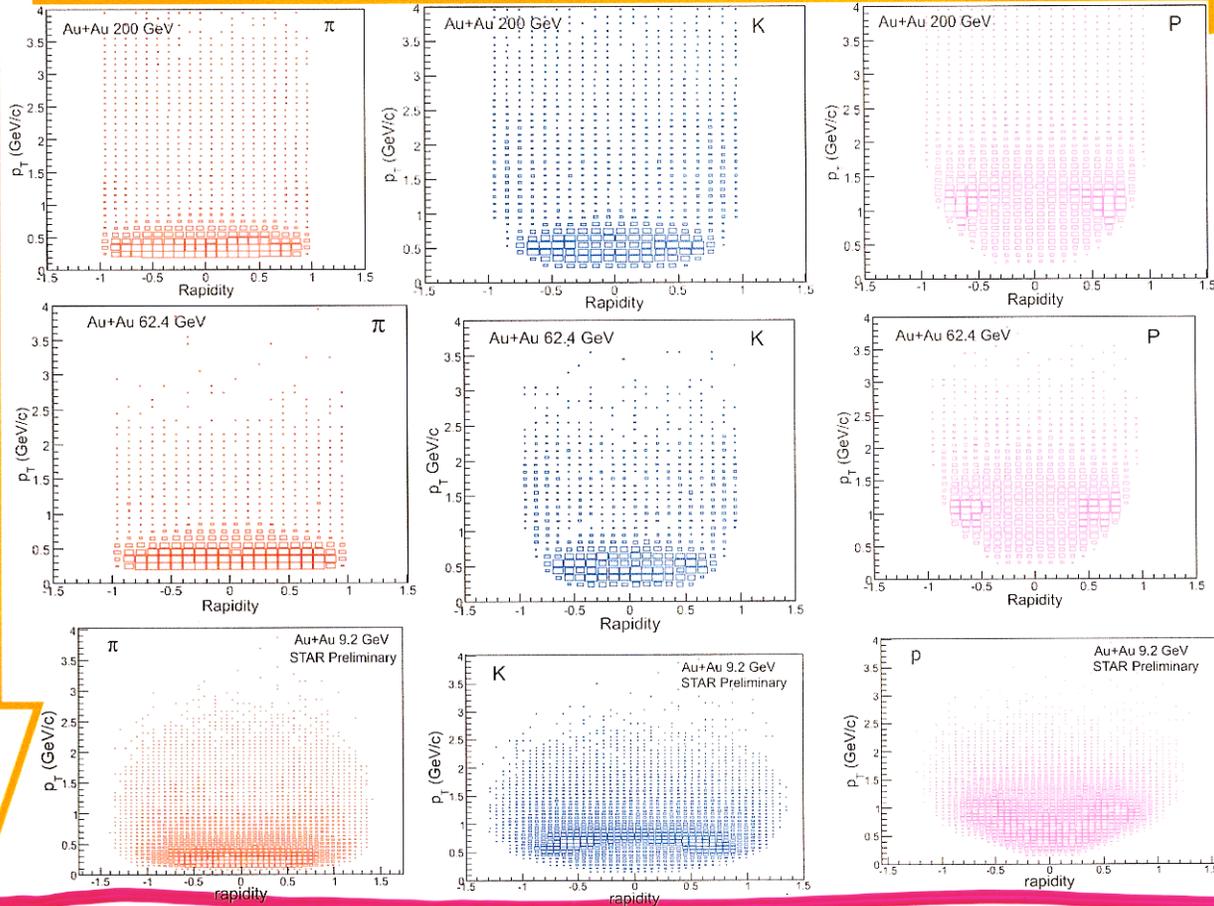
200 GeV

62.4 GeV

9.2 GeV

Uniform acceptance for different particle species and for different beam energies in the same experimental setup (advantage over fixed traget expt.)[36]

Mohanty, QM 09

Old Program : Fixed Target

Hadron Mass
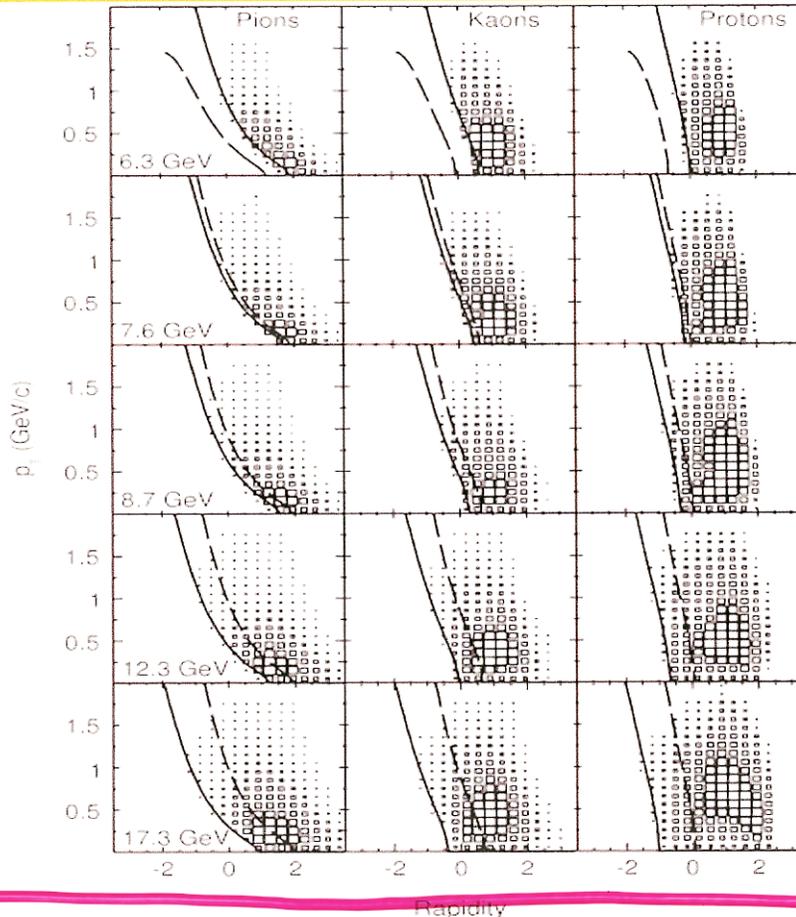
Beam Energy

NA49 : arXiv : 0808.1237

6.3 GeV

7.6 GeV

8.7 GeV

12.3 GeV

17.3 GeV

Non-Uniform acceptance for different particle species
and for different beam energies in the same experimental setup

48

Mohanty,
QM09

G. Roland

Collider experiment : Variation of particle density with beam energy slower. Occupancy in detectors reasonable compared to fixed target experiments at similar collision energy   37

Mohanty, QM09

K/π FLUCTUATIONS WITH A RHIC BEAM ENERGY SCAN

Fig. 3-7: Estimate of the error in σ_{dyn} for charge-integrated K/π fluctuations, based on 100K central events analyzed in the STAR detector (with the newly completed ToF). Shown for comparison are the current measurements from NA49 and STAR.

STAR B.U.R.

Error bars with 1M min bias events per energy

KURTOSIS OF EVENT-BY-EVENT DISTRIBUTION OF (NET) PROTON NUMBER
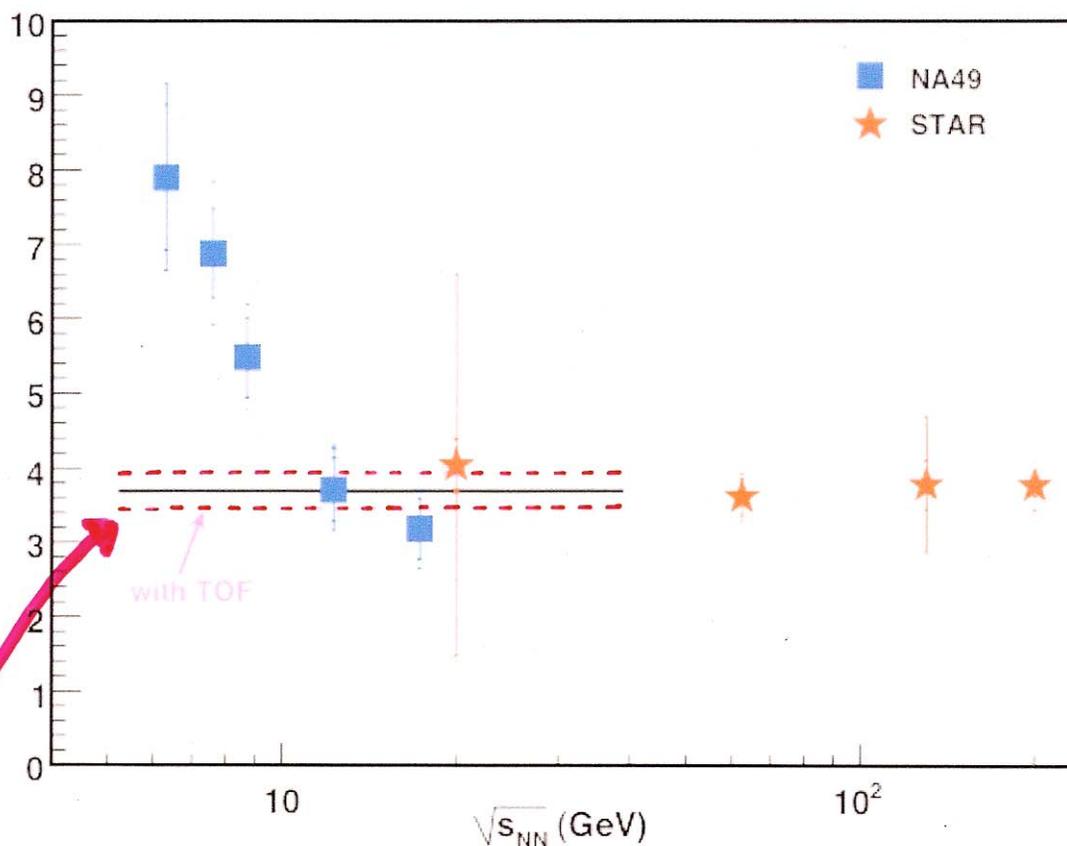
Stephanov; "a direct consequence of discussions" at Aug 2008 INT workshop

Critical fluctuations couple to $\pi\pi$, PP
→ event-by-event fluctuations in their multiplicities, multiplicity ratios, P_T, that are $\propto \xi^2$

Higher moments of the event-by-event distributions receive effects that are more sensitive to ξ.

$\qquad$ Skewness $\propto \xi^{4.5}$

$\qquad$ Kurtosis $\propto \xi^7$!!!

The prefactors work out particularly nicely for kurtosis of proton distribution, but Stephanov makes predictions for π & p, skewness & kurtosis.

DEFINITIONS

N = # of protons (or π; or $p-\bar{p}$) in 1 event

$\bar{N} \equiv \langle N \rangle$ = mean

$\delta N \equiv N - \bar{N}$ in one event

$\langle (\delta N)^2 \rangle$ = variance

$$K^{eff} \equiv K \langle (\delta N)^2 \rangle \equiv \frac{\langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2}{\langle (\delta N)^2 \rangle}$$

Kurtosis $\sim \frac{1}{N}$ Variance $\sim \bar{N}$

⌈In lattice QCD literature, K^{eff} is called κ_4/κ_2 or χ_4/χ_2. It is calculated for N = baryon number, & N = charge.⌉

$K > 0$

$K < 0$
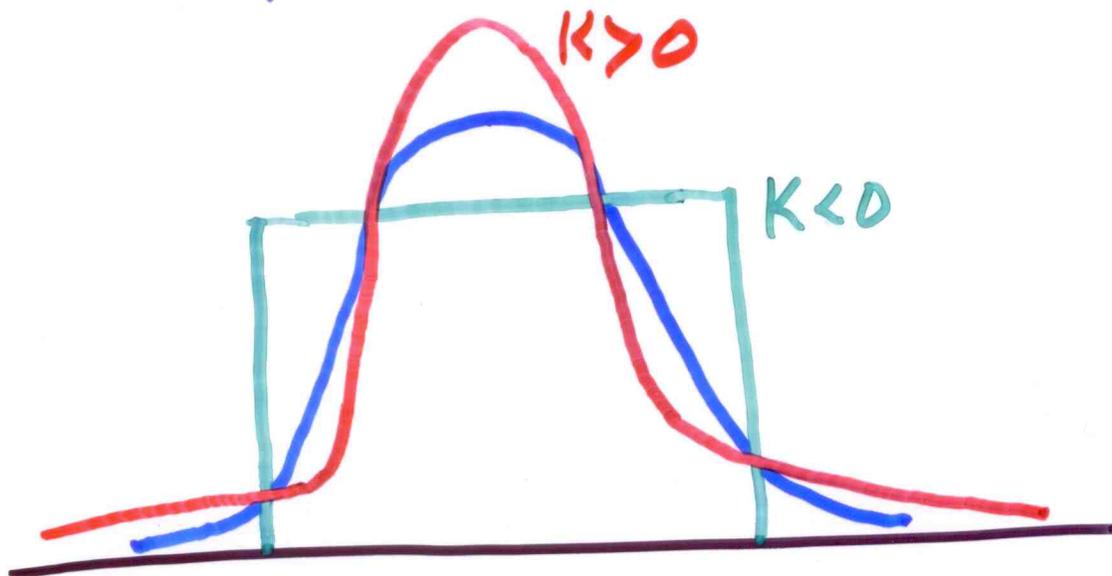
Gaussian: $K = 0$

EFFECT OF CRITICAL FLUCTUATIONS

Stephanov

T → μ

freezeout point with max ξ

chemical freezeout vs $\sqrt{s}$

contours of equal ξ

$$K^{eff}\underbrace{\left(\frac{\bar{N}}{\langle(\delta N)^2\rangle}\right)}_{\simeq 1} = 23 \underbrace{\left(\frac{2\tilde{\chi}_3^2 - \tilde{\chi}_4}{50}\right)\left(\frac{g}{10}\right)^4}_{\theta(1)}\underbrace{\left(\frac{\xi}{1\,fm}\right)^7}_{!!!!}$$
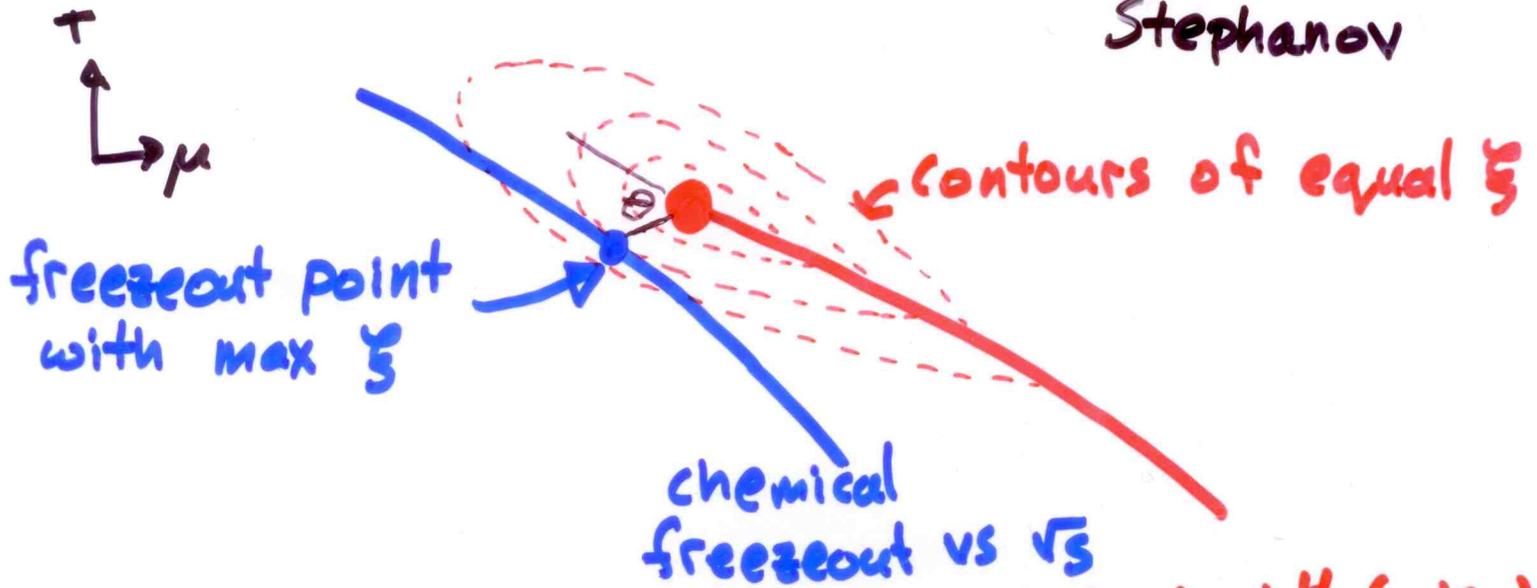
$\tilde{\chi}_3, \tilde{\chi}_4$: Obscure but universal constants that depend on θ. Known for Ising ●.

g : σPP coupling. $g \sim m_P/f_\pi \sim 10$

So, how big is the background? Theory suggests $\theta(1)$, but better to determine it experimentally ....

K^{eff} AT $\sqrt{s}$ = 200 GeV AND 62 GeV

ie at very low μ, far from ●
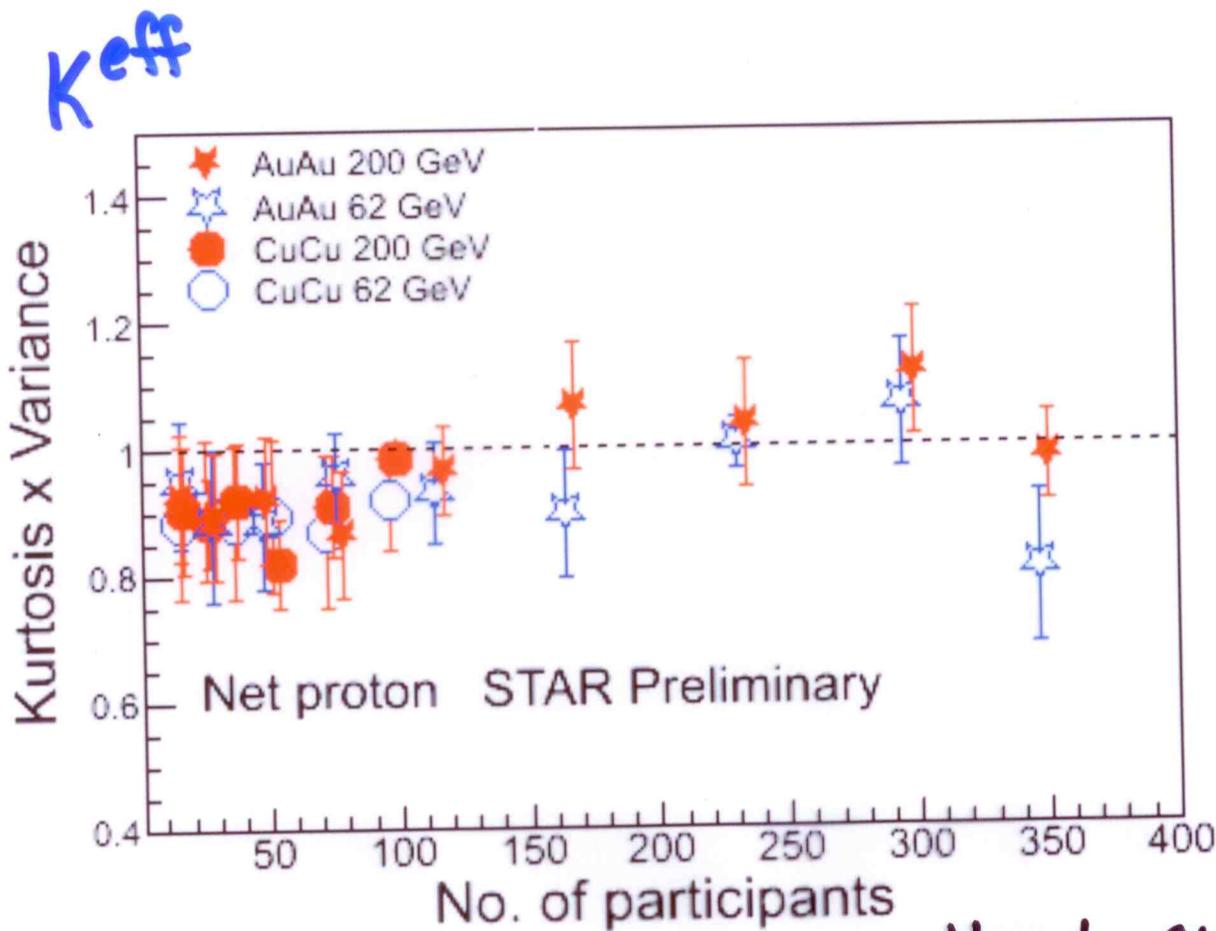
K^{eff}

Nayak, QM 09

- Background K^{eff} measured, $\mathcal{O}(1)$.

- Error bars $\simeq \pm .1$.

PROTON NUMBER KURTOSIS

- Predicted effect of proximity to 🔴 is:
  - <u>large</u>
  - strongly ξ-dependent, ∴ <u>$\sqrt{s}$-dependent</u>
  - <u>larger</u> at lower P_T
    - → How you know its 🔴 if you see it.

- Cannot be washed out between chemical & kinetic freezeout

- Background measured, and small, at $\sqrt{s} = 62$ & 200.

- Error bars $\sim \pm .1$ with 5M events STAR BUL

- Calculations done for $\bar{N} \to \infty$. Finite $\bar{N}$ and acceptance corrections need to be assessed. (Latter minimal for STAR)

- Analogous analyses can be done for skewness, and for pions. Many signatures will be in play if/when 🔴 found.

A BEAM ENERGY SCAN TO SEARCH FOR ●

$\sqrt{s}/A$ (GeV)	μ (MeV) [*]	8 hr days per 5M events [†]
5	550	
6.2	485	
7.7	420	56
9.8	355	30
12.7	290	13
17.3	230	5
27	155	2
39	110	1
200	25	
		107 days

[*]: from Cleymans et al's 2005 empirical fit to compilation of data

[†]: from STAR B.U.R.

WHAT NEED BE MEASURED AT EACH $\sqrt{s}$

- Enough $\langle$particle ratios$\rangle$ to evaluate μ.
  So you know where on phase diagram you are freezing out.

- n_q scaling of v_2 for $\pi/K/P/\Lambda$
  parity violating fluctuations
  So you know whether collision got above the crossover/transition

- Event-by-event fluctuations of K/π and P/π ratios with significantly smaller error bars than in NA49 data

- Variance, skewness & kurtosis of event-by-event distributions of N_P, $N_P - N_{\bar{P}}$, N_π.

- **All can be done with 5M min bias events per $\sqrt{s}$.** STAR B.U.R.

CAN WE DISCOVER THE QCD CRITICAL POINT IN RUN 10 AT RHIC?

YES, IF:

Nature is kind, and puts $\mu_\bullet \lesssim 420\, MeV$

IF YES:

- <u>The</u> landmark discovered. Our map of the QCD phase diagram then anchored by experiment.

- Assuming reasonable progress in lattice QCD, quantitative comparison between theory & experiment for $\mu_\bullet$ will come.

- RHIC (and FAIR) can study the first order phase transition. RHIC can further probe the region around $\bullet$.

IF NO:

- We learn that $\mu_\bullet > 420\, MeV$

- We will be able to make a data-driven decision about whether to run at $\sqrt{s} = 6.2$ and 5 in a future year.

	Collision Energies (GeV)	5	7.7	11.5	17.3	27	39
Section	Observables	Millions of Events Needed					
A1	v_2 (up to ~1.5 GeV/c)	0.3	0.2	0.1	0.1	0.1	0.1
A1	v_1	0.5	0.5	0.5	0.5	0.5	0.5
A2	Azimuthally sensitive HBT	4	4	3.5	3.5	3	3
A3	PID fluctuations (K/π)	1	1	1	1	1	1
A3	net-proton kurtosis	5	5	5	5	5	5
A3	differential corr & fluct vs. centrality	4	5	5	5	5	5
A3	integrated p_T fluct (T fluct)						
B1	n_q scaling π/K/p/Λ $(m_T-m_0)/n<2$GeV		6	5	5	4.5	4.5
B1	φ/Ω up to $p_T/n_q=2$ GeV/c		56	25	18	13	12
B2	R_{CP} up to p_T ~4.5 GeV/c (at 17.3) 5.5 (at 27) & 6 GeV/c (at 39)				15	33	24
B3	untriggered ridge correlations		27	13	8	6	6
B4	parity violation		5	5	5	5	5
See[1]:	charge-photon fluctuations (DCC)	1	1	1	1	1	1
	kink/step/horn	0.1	0.1	0.1	0.1	0.1	0.1
	v_2 fluctuations	0.5	0.5	0.5	0.5	0.5	0.5
	HBT (R_l, R_o/R_s)	0.8	0.8	0.5	0.5	0.5	0.5
	Jet/ridge 2<trig<4, 1<assoc<trig				30	8.8	4.5
	Jet/ridge 3<trig<6, 1.5<assoc<trig					53	24
	Baryon-Strangeness cor (hypernuc)						50
	Forward π⁻ yield (rapidity scaling)						
	Forw. γ(π⁰) yield (rapidity scaling)						
	Long-range forward-backward corr.						
	Other PID fluctuations (esp. K/p)						
	Particle ratios (many examples)						
	p_T spectra						
	Prod. of light nuclei & antinuclei						
	Yields of species & stat model fits						

Table 3-2: Observables and statistics needed for the first BES run. The observables in the yellow-shaded area relate to the search for a phase transition or critical point (see section A), while observables in the blue-shaded area search for turn-off of new phenomena already established at higher RHIC energies (see section B). The numbers listed in boldface above are all within reach (nominally require no more than 1.5 times the proposed statistics) in the first BES run plan as set out in Table 3-1. The remaining numbers (not boldface) will need to wait for higher

MULTIPLICITY FLUCTUATIONS

Also coupled to order parameter fluctuations. In fact their effect on multiplicity fluctuations is greater than on P_T fluctuations.

Stephanov KR Shuryak

BUT: large "background", due to impact parameter fluctuations.

Still, nonmonotonic variation with $\sqrt{s}$ would be suggestive.

Order of Phase Transition for $\mu_B \sim 0$
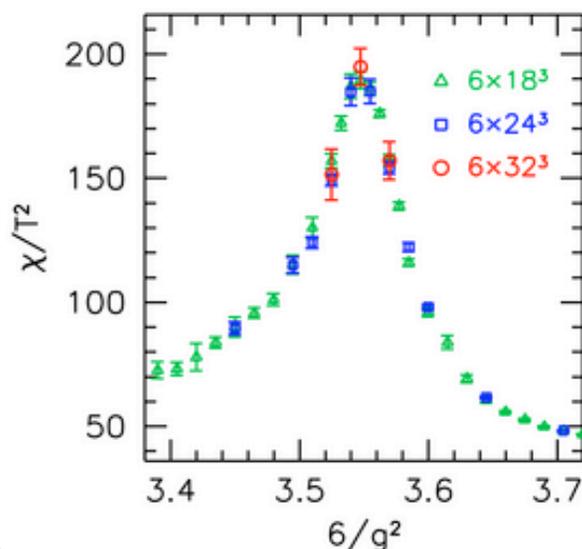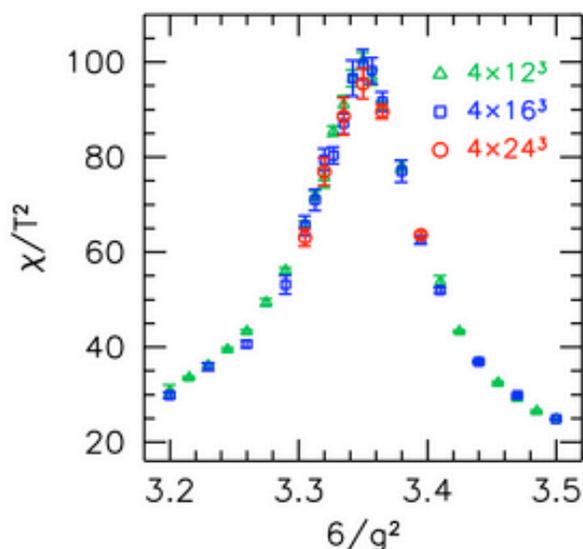
Physical quark masses
Continuum limit
Simulations along Lines of Constant Physics
$m_K/m_\pi = 3.689$; $f_K/m_\pi = 1.185$
Staggered fermionic action

$$\chi(N_s, N_t) = \partial^2/(\partial m_{ud}^2)(T/V) \cdot \log Z$$

Y. Aoki et al., Nature 443:675-678, 2006

T grows with $6/g^2$, g : gauge coupling

No significant volume dependence (8 times difference in volumes)

Phase transition at high T and $\mu_B = 0$ is a cross over

1st order :
    Peak height $\sim V$
    Peak width $\sim 1/V$
Cross over :
    Peak height $\sim$ const.
    Peak width $\sim$ const.
2nd order :
    Peak height $\sim V^\alpha$

Relevant to LHC and current RHIC regimes

Lattice results on electroweak transition in standard model is an analytic cross-over for large Higgs mass

K. Kajantie et al., PRL 77, 2887-2890, 2006

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Method to determine the CEP:

- find largest temperature were all expansion coefficients are positive ➔ T^{CEP}
- determine the radius of convergence at that temperature ➔ μ^{CEP}

all expansion coefficients positive: singularity on the real axis!

c_2

c_4

c_8

c_6

T^{CEP}

T_{10}^{CEP}

T_8^{CEP}

T^{RW}
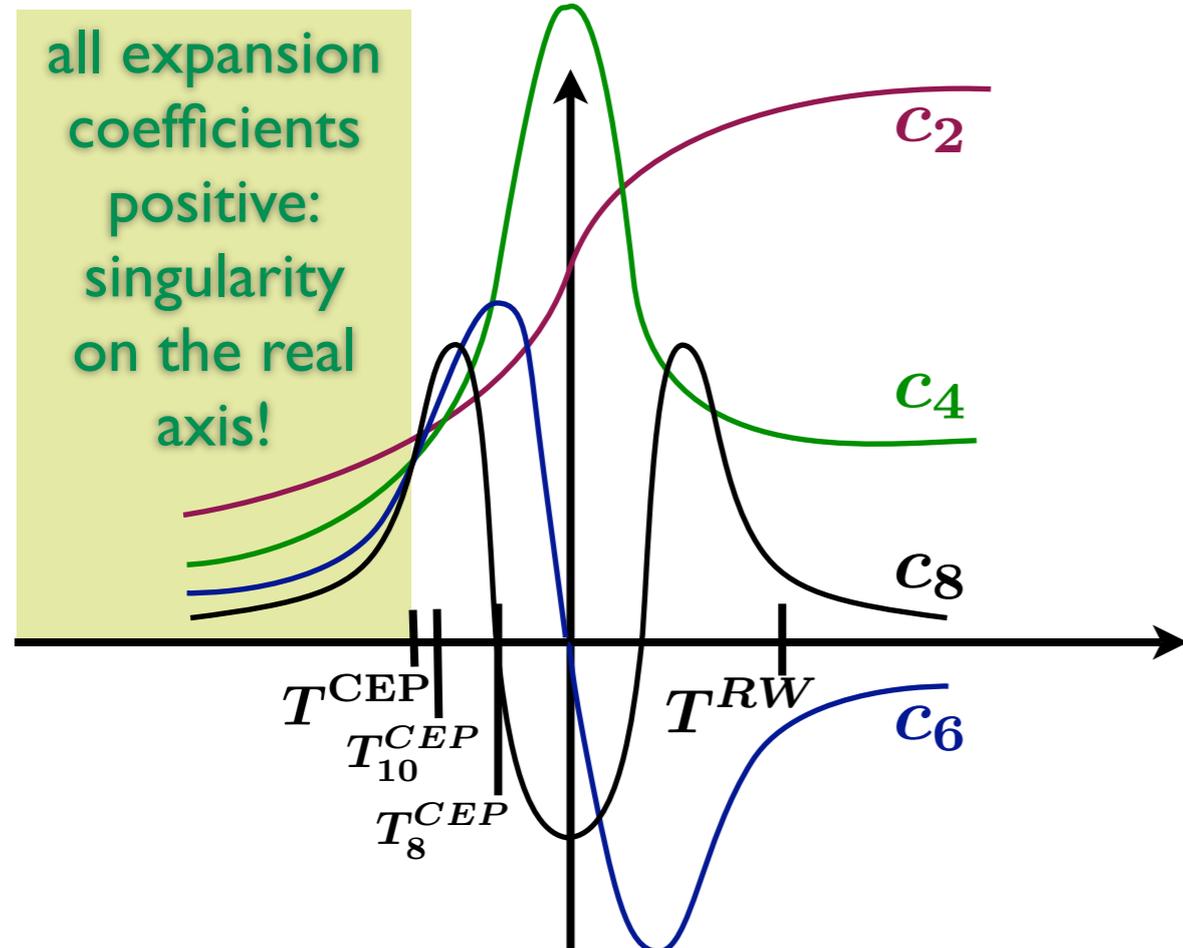
➔ first non-trivial estimate of T^{CEP} from c_8
second non-trivial estimate of T^{CEP} from c_{10}

$T / T_c(0)$

r_2

r_4

Gavai, Gupta
Nt=4   Nt=6

Fodor, Katz
Nt=4

ρ_2

ρ_4

$\mu_B / T_c(0)$
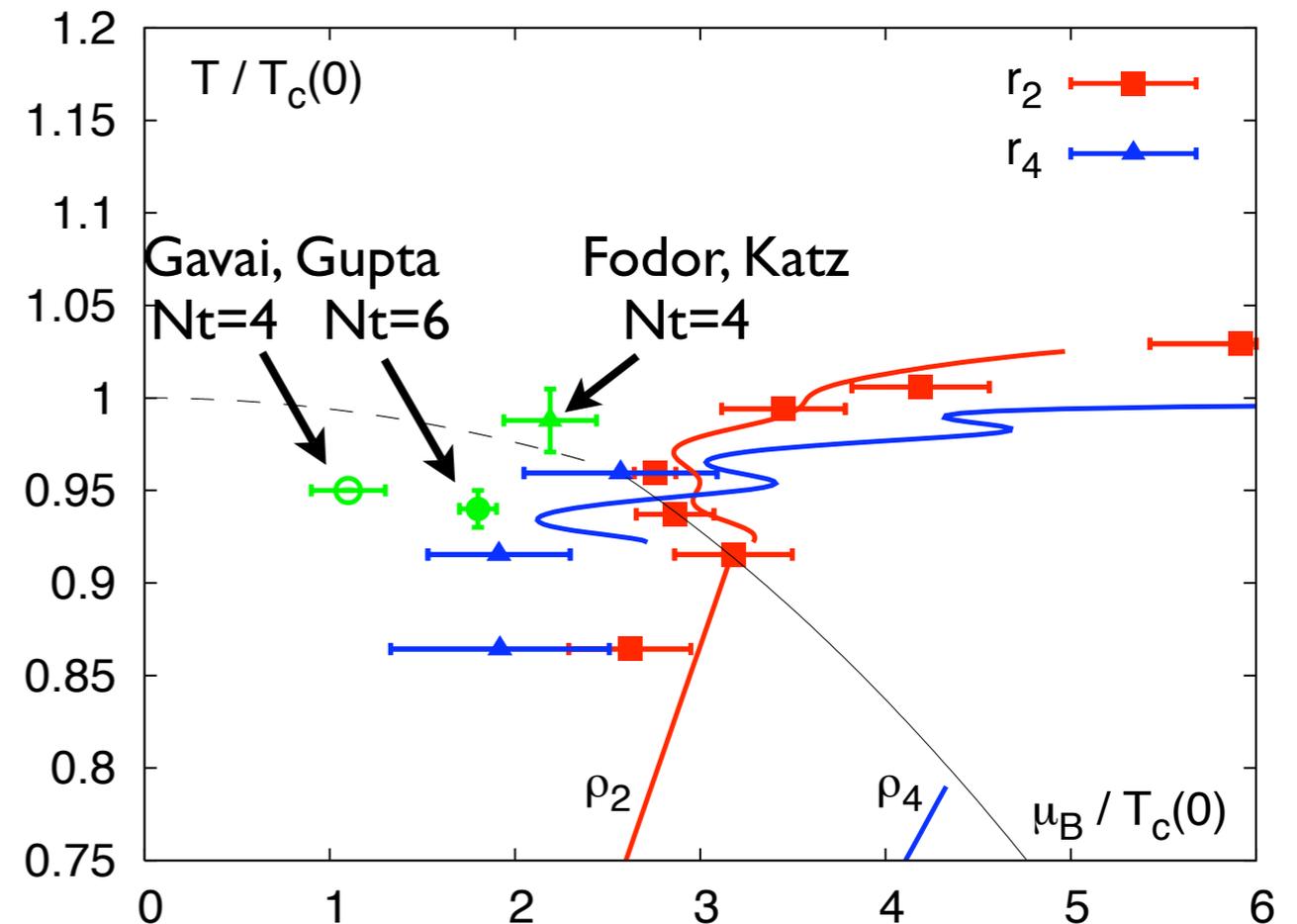
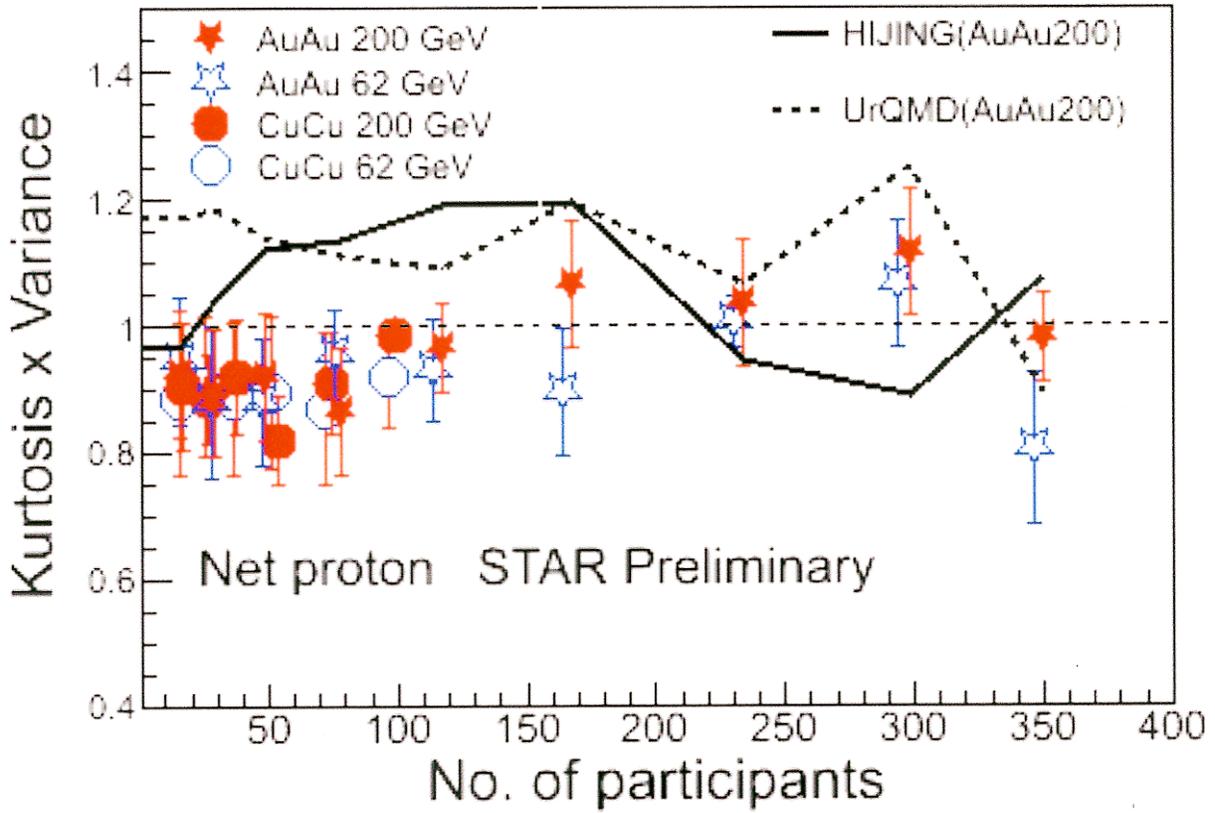$$\mu^{\mathrm{CEP}} = \lim_{n \to \infty} \mu_n^{\mathrm{CEP}}$$

$$\mu_n^{\mathrm{CEP}} = T^{\mathrm{CEP}} \sqrt{c_n/c_{n+2}}$$

The Resonance gas limit:

$$\frac{p}{T^4} = G(T) + F(T) \cosh\left(\frac{\mu_B}{T}\right)$$

$$\longrightarrow \quad \rho_n = \sqrt{(n+2)(n+1)}$$

K eff

Legend:
- ★ AuAu 200 GeV
- ☆ AuAu 62 GeV
- ● CuCu 200 GeV
- ○ CuCu 62 GeV
- — HIJING(AuAu200)
- --- UrQMD(AuAu200)

Y-axis: Kurtosis x Variance
X-axis: No. of participants

Net proton   STAR Preliminary

Nayak, QM09

MODEL ANALYSIS OF EXTENT OF CRITICAL REGION

$m_u = m_d = 0$

χ_q / χ_q^{free}

$\chi_B \sim \xi^2 +$ non-singular

$\chi_B \equiv \frac{\partial^2 \Omega}{\partial \mu_B^2}$

μ [MeV] $= \mu_B / 3$

Hatta Ikeda