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Nucleon (Spin) Structure from the Lattice

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Thanks to Kostas Orginos

Outline

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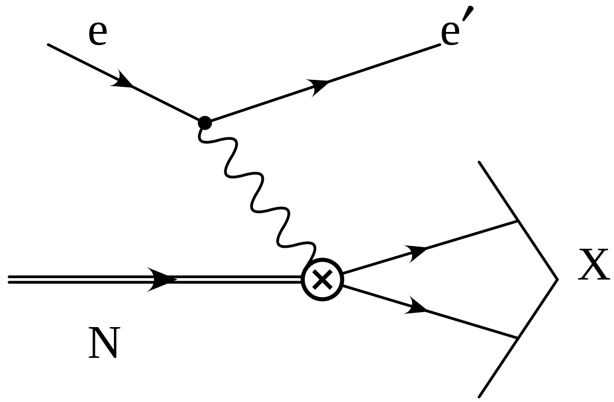
   (a) Introduction
   (b) Lattice operators
   (c) Matrix Elements
   (d) Comparison to experiment
2. Generalized Parton Distributions
   (a) Brief overview
   (b) Lattice results
   (c) Gluon total angular momentum

Part I

Introduction



$$\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$

$$W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$$

$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot SP_\sigma}{\nu^2} g_2(x,Q^2)\right)$$

with $\nu = q \cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$

Calculate **non-perturbatively** nucleon structure functions:

- **Unpolarized:** $F_1(x,Q^2)$, $F_2(x,Q^2)$
- **Polarized:** $g_1(x,Q^2)$, $g_2(x,Q^2)$, ($h_1(x,Q^2)$)

Moments of Structure Functions

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2), \\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - \\
&- 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2 and e_1, e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and d_n are forward nucleon matrix elements of certain local operators $\mathcal{O}$.

Lattice Operators

Broken Lorentz symmetry: $O(4) \rightarrow H(4)$

Operators in irreducible representations of $O(4)$ transform *reducibly* under the lattice Hyper-cubic group.

Consequence: higher moment operators mix with **lower** (space-time) dimensional operators.

The lower dimensional operators mix with **power divergent** coefficients

In practice, lattice is limited to the lowest few moments if lower dimensional operator mixing is to be avoided. (Non-perturbative lower dimesional operator subtraction is possible, *c.f.* Kaon physics)

Unpolarized (F_1 and F_2):

$$\frac{1}{2}\sum_{s}\langle P,S|\mathcal{O}^{q}_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - trace]$$

$$\mathcal{O}^{q}_{\mu_1\mu_2\cdots\mu_n} = \overline{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - trace\right]q$$

On the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$

Only $\langle x\rangle_q$ can be measured with $\vec{P} = 0$

Polarized (g_1 and g_2):

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots - traces]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \overline{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_\sigma \overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - traces\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots - traces]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \overline{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma} \overset{\leftrightarrow}{D}_{\mu_1]}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - traces\right]q$$

On the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2.

Only $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1 can be measured with $\vec{P}=0$

Transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - traces]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \overline{q}[\left(\frac{i}{2}\right)^n \gamma_5\sigma_{\rho\nu}\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - traces]q$$

On the lattice we can measure $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$

Only $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$

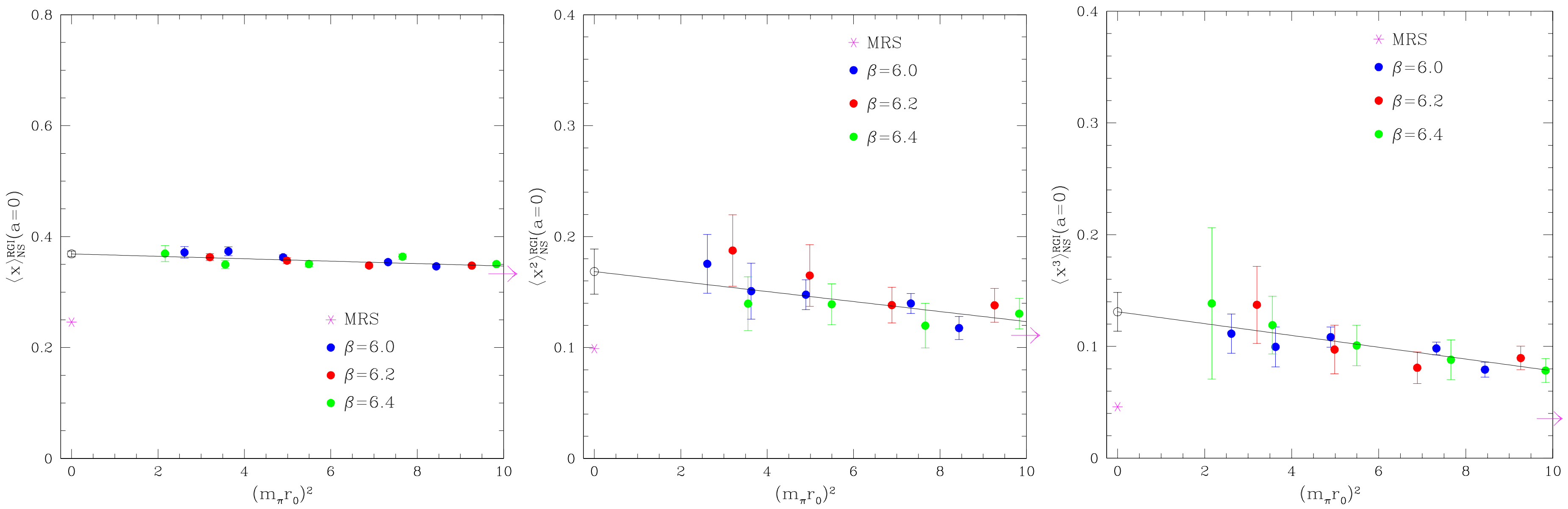
Status of lattice calculations

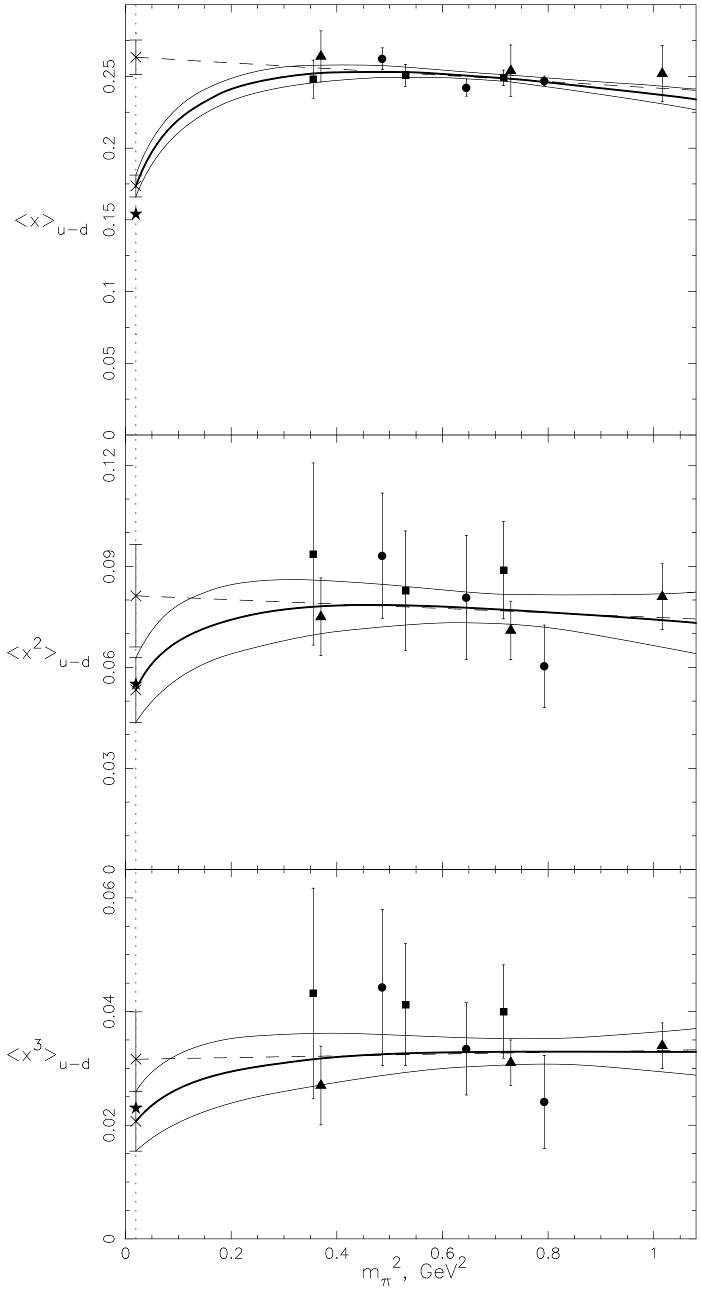
- **QCDSF** [Phys.Rev.D53 1996, Phys.Rev.D63 074506 2001, hep-ph/0304249]
  - Structure func./Generalized PD and Form Factors
  - Wilson fermions improved and unimproved
  - quenched and $N_f = 2$ dynamical (with **UKQCD**).
- **LHPC - SESAM** [hep-lat/0201021, hep-lat/0312014]
  - Structure func./Generalized PD and Form Factors
  - unimproved Wilson fermions
  - quenched and $N_f = 2$ dynamical.
- **RBC** (preliminary results) [hep-lat/0209137, hep-lat/0309113]
  - Structure func. and Form Factors
  - Domain wall fermions
  - quenched and $N_f = 2$ dynamical

Unpolarized moments for the proton

QCDSF:

Quenched results, $2 \leq a^{-1} \leq 4$ GeV. $\mathcal{O}(a)$ improved Wilson fermions. Volume $\sim$ (1.5-3 fm)^3, $m_\pi > 600$ MeV. Nonperturbative renormalization





LHPC-SESAM:

diamonds - quenched,
squares - dynamical

Roughly the same masses, as QCDSF
$a^{-1} \sim 2$ GeV
Volume only $(1.5\ \mathrm{fm})^3$
Perturbative renormalization

QCDSF:

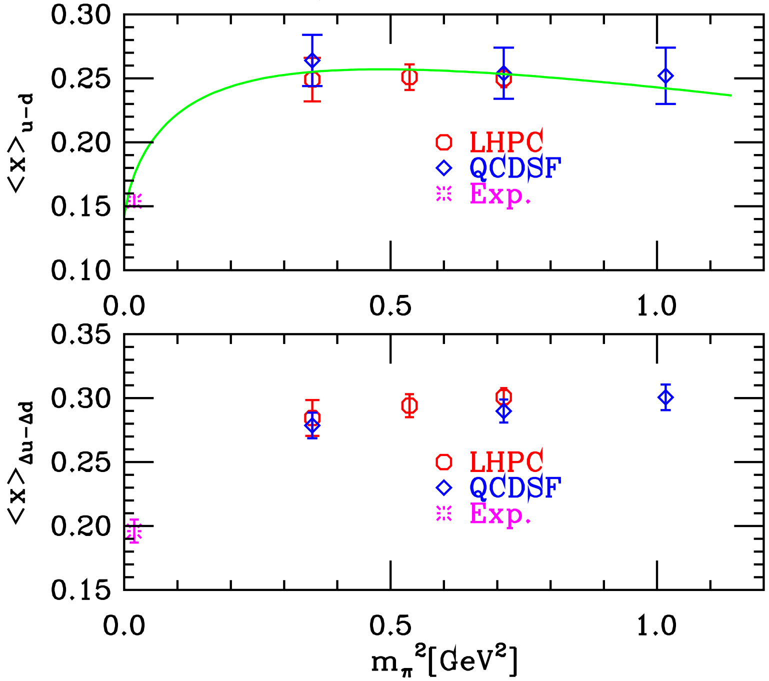
quenched - triangles

[hep-lat/0201021]

What happens at the chiral limit?

Existing calculations (quenched and dynamical) at relatively heavy quark masses seem to disagree with experiment.

$\beta = 6.0 \; 16^3 \times 32$



[QCDSF: Phys.Rev.D53 1996]
[LHPC-SESAM: hep-lat/0201021]
[Detmold et.al. Phys.Rev.Lett.87 2001]

Possible resolution(?):

- Finite lattice spacing
- Finite volume(?)(see g_A)
- Chiral logs

$$V(m_\pi^2) = V_c \left[1 + C_\chi \, m_\pi^2 \, ln \frac{m_\pi^2}{\mu_\chi^2} \right]$$

Assume:

$$V(m_\pi^2) = V_c \left[1 + C_\chi \, m_\pi^2 \, ln \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right] + B \, m_\pi^2$$

C_χ calculated in χ-PT,
depends of f_π and g_A only
μ^2 phenomenological parameter
$\mu \sim$ 550MeV.

Detmold et.al.
Arndt&Savage
Chen&Ji, Chen&Savage

Simulation parameters (quenched)

- Gauge Action: DBW2

$$S_g = \frac{\beta}{3}\mathrm{Re\,Tr}\left[(1-8c_1)\left\langle 1- \Box \right\rangle + 2c_1\left\langle 1- \Box\!\Box \right\rangle\right]$$

With $c_1 = -1.4067$ computed by non-perturbative RG blocking.
[Takaishi Phys.Rev. D54 (1996)]

- $\beta = 0.870$ or $a^{-1} = 1.3$GeV, Volume: $16^3 \times 32 \sim 2.4^3 fm^3$ box.
- Fermion Action: Domain wall fermions $L_s = 16 \rightarrow m_{\mathrm{res}} \sim .7$MeV
- quark mass: as light as $1/4 \times m_{strange}$
- Statistics: 416 Lattices QCDSP 300Gflops for 4 months

Simulation parameters ($n_f = 2$ dynamical)

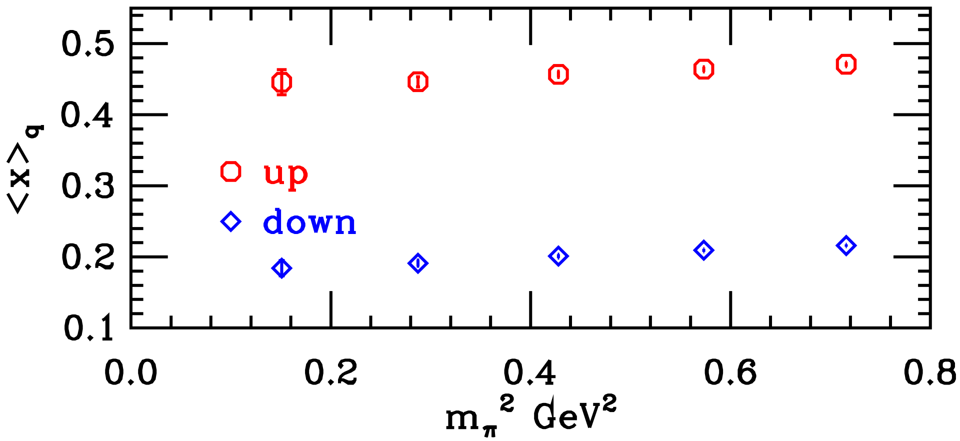
- Gauge Action: DBW2

$$S_g = \frac{\beta}{3} \mathrm{Re\,Tr} \left[(1 - 8c_1) \left\langle 1 - \square \right\rangle + 2c_1 \left\langle 1 - \square\!\square \right\rangle \right]$$

With $c_1 = -1.4067$ computed by non-perturbative RG blocking.

[Takaishi Phys.Rev. D54 (1996)]

- $\beta = 0.800$ or $a^{-1} = 1.7$GeV, Volume: $16^3 \times 32 \sim 2.0^3 fm^3$ box.
- Fermion Action: Domain wall fermions $L_s = 12 \rightarrow m_{\mathrm{res}} \sim 2.5$MeV
- quark mass: as light as 1/2 $\times m_{strange}$
- Statistics: about 50 Lattices
- Status: ON GOING

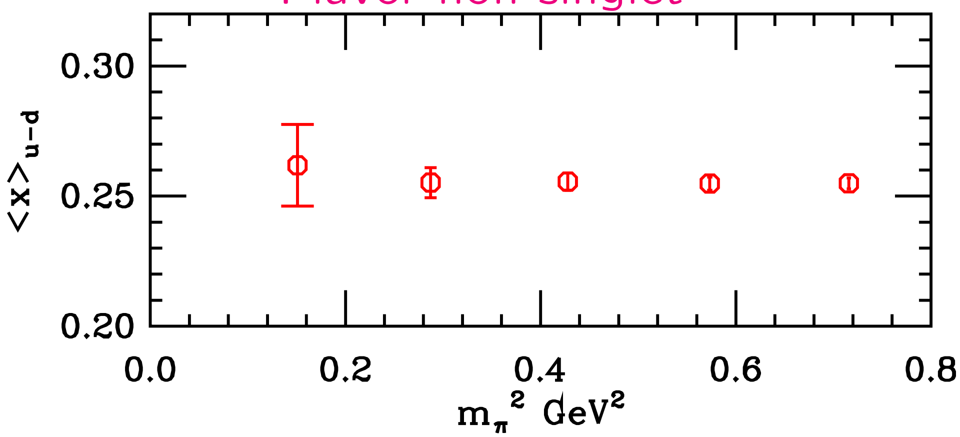


Quark density

$$\mathcal{O}_{44}^{q} = \overline{q}\left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k\right] q$$

- Hypercubic group rep. $\mathbf{3}_1^+$
- Momentum: $\vec{P} = 0$
- Renormalization: Multiplicative

Flavor non-singlet



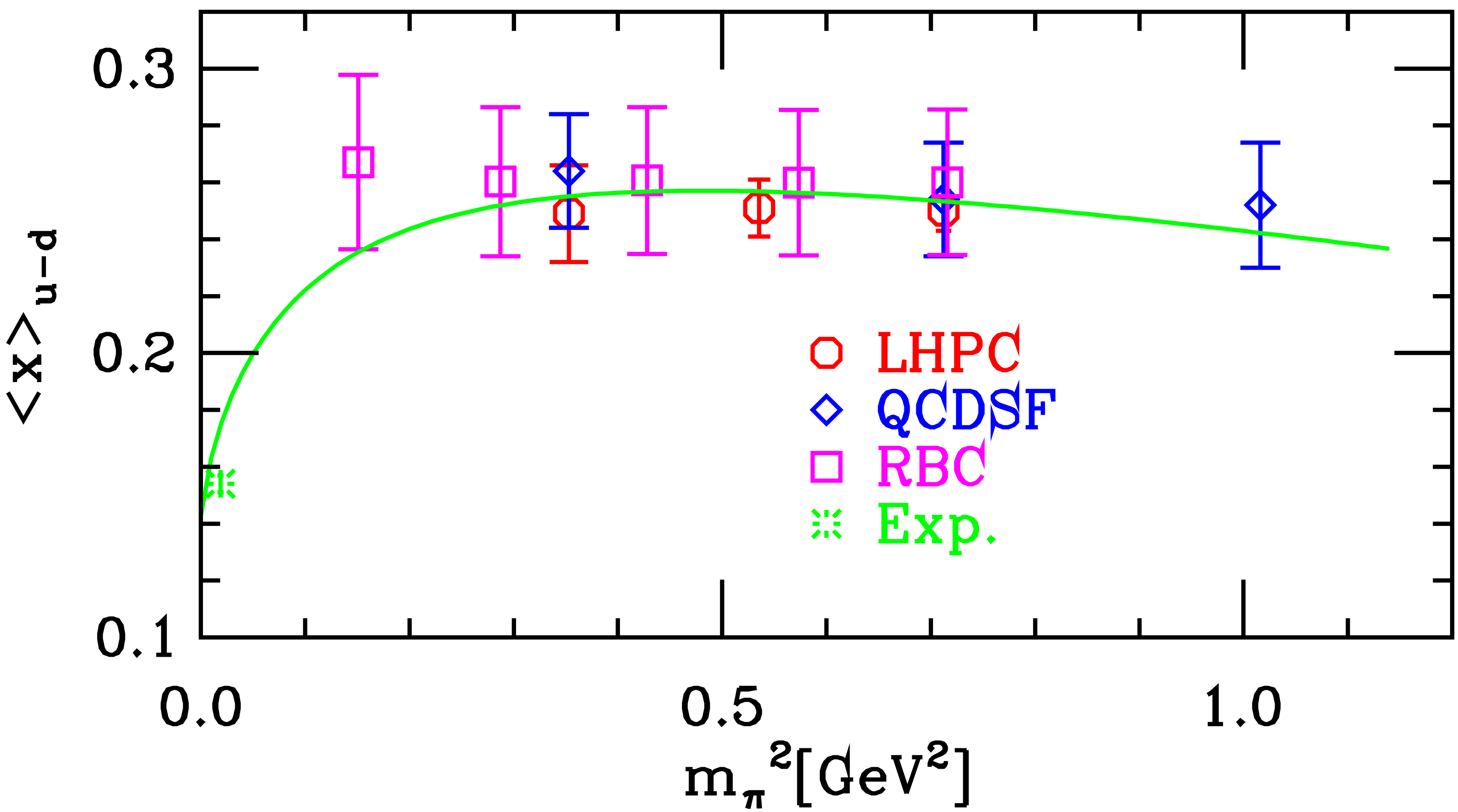
Note:

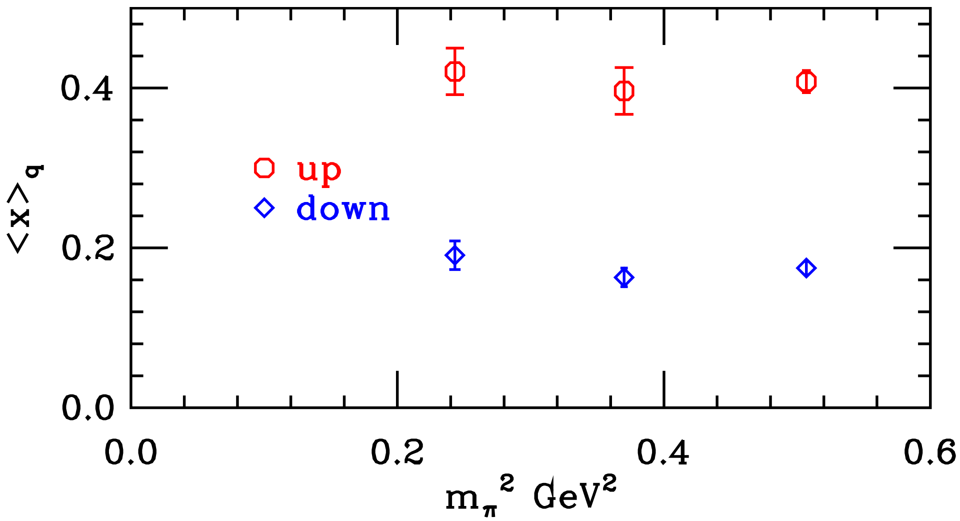
- Renormalization:

  $Z = 1.02(10)$

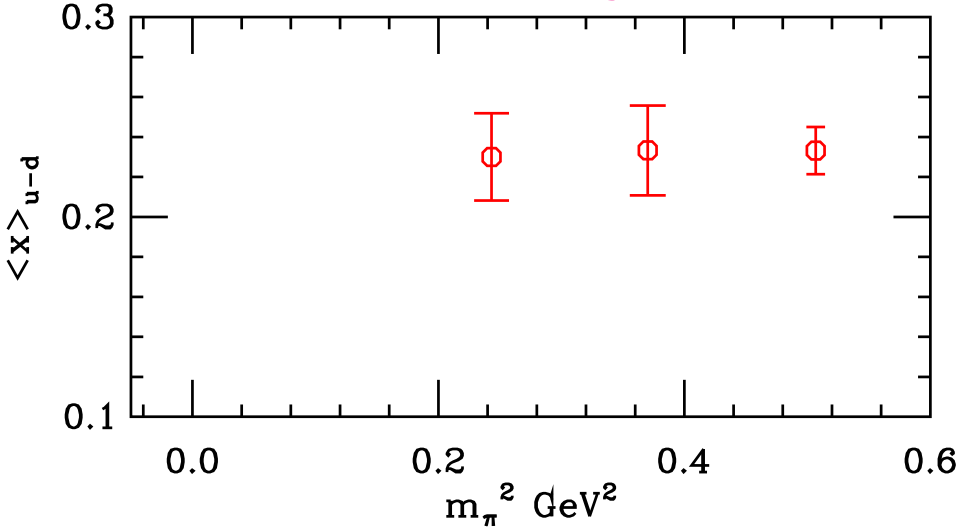
  $\overline{MS} = 2GeV$ 2-loop runnig
- No Curvature in the chiral limit

$\frac{\langle x\rangle_u}{\langle x\rangle_d} = 2.41(4)$ at the chiral limit


0.3
0.2
0.1
0.0
0.5
1.0
$\langle x \rangle_{u-d}$
m_π^2[GeV^2]
LHPC
QCDSF
RBC
Exp.



Flavor non-singlet



Quark density: (dynamical)

$$\mathcal{O}^q_{44} = \overline{q}\left[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k\right] q$$

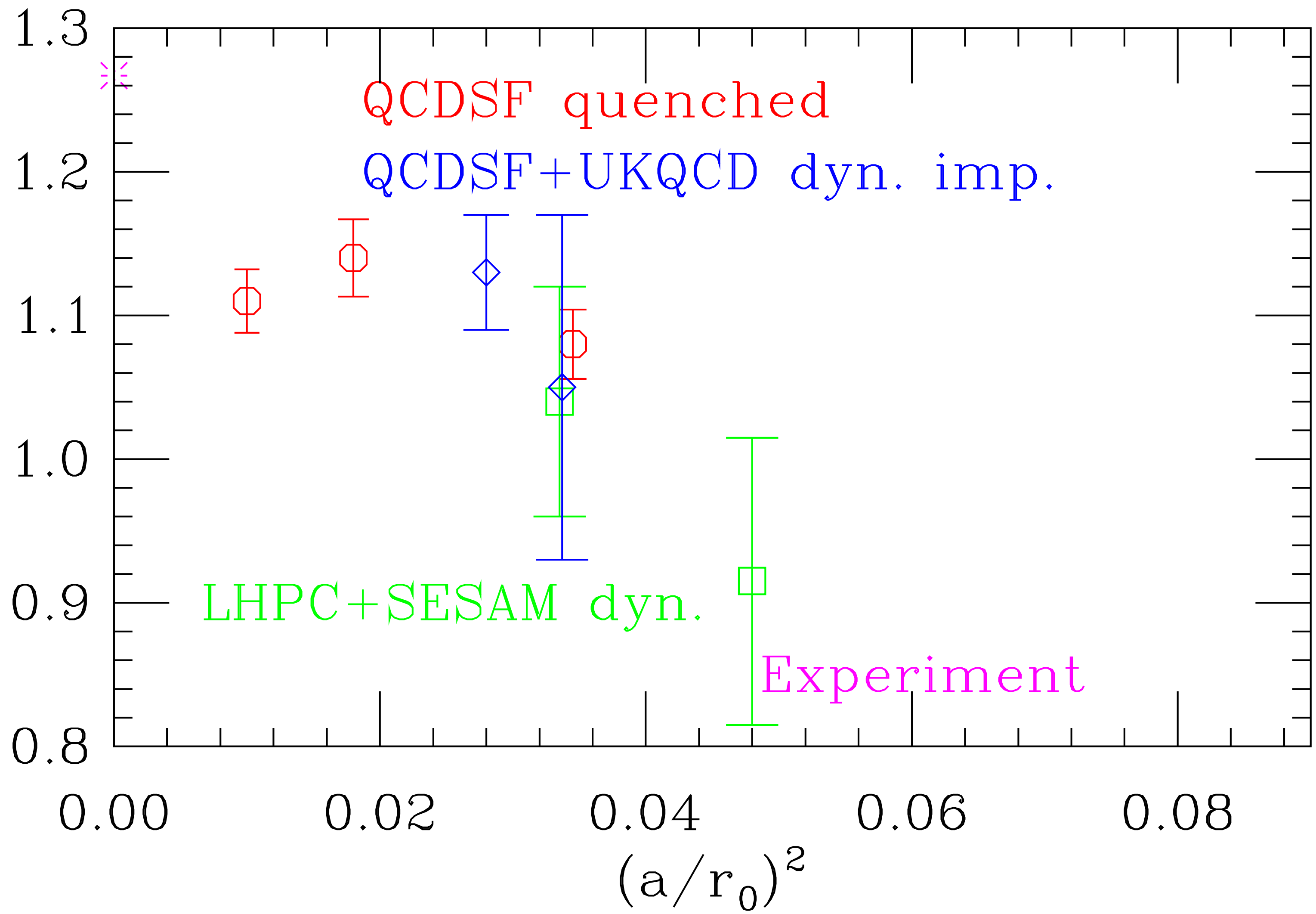
- Hypercubic group rep. $\mathbf{3}_1^+$
- Momentum: $\vec{P} = 0$
- Renormalization: Multiplicative

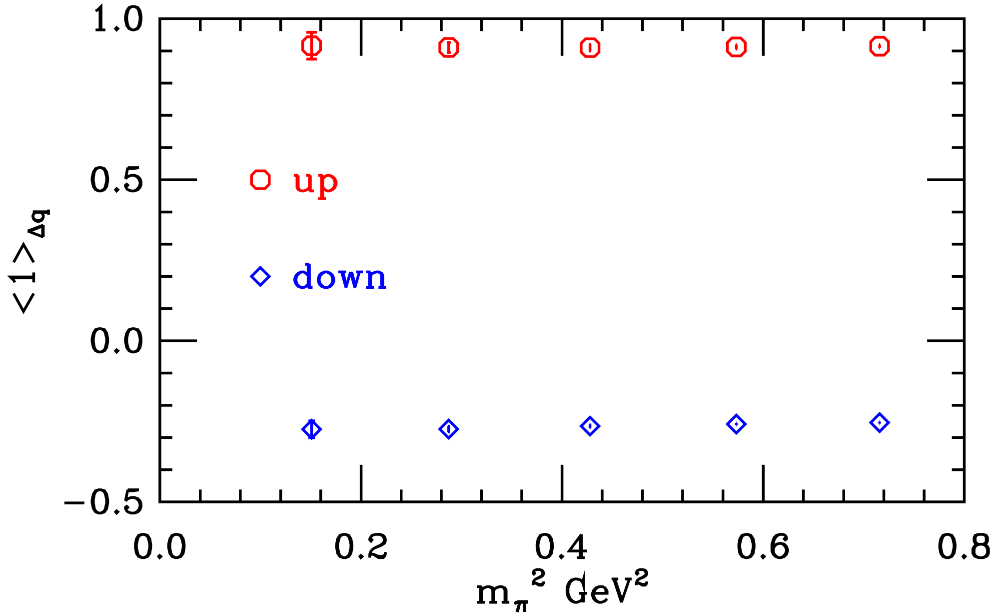
Note:

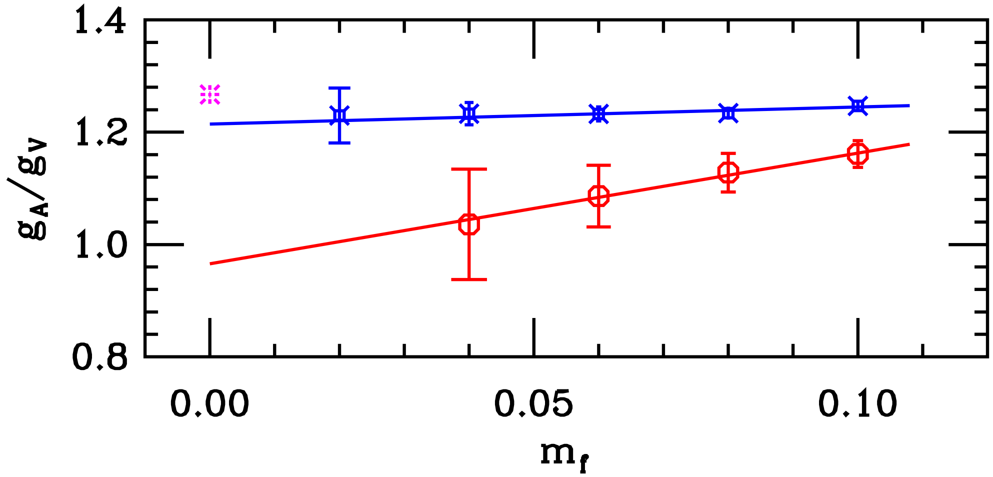
- Unrenormalized

Axial Charge

$\langle 1 \rangle_{\Delta q}$ (g_A : Axial charge) Plot: Horsley review, Lattice 2002, Boston







Axial Charge

- Renormalization:

$$\langle \mathcal{A}^{cons}\overline{q}\gamma_5 q\rangle = Z_A \langle A^{loc}\overline{q}\gamma_5 q\rangle$$

[Y. Aoki LAT01, hep-lat/0201021]

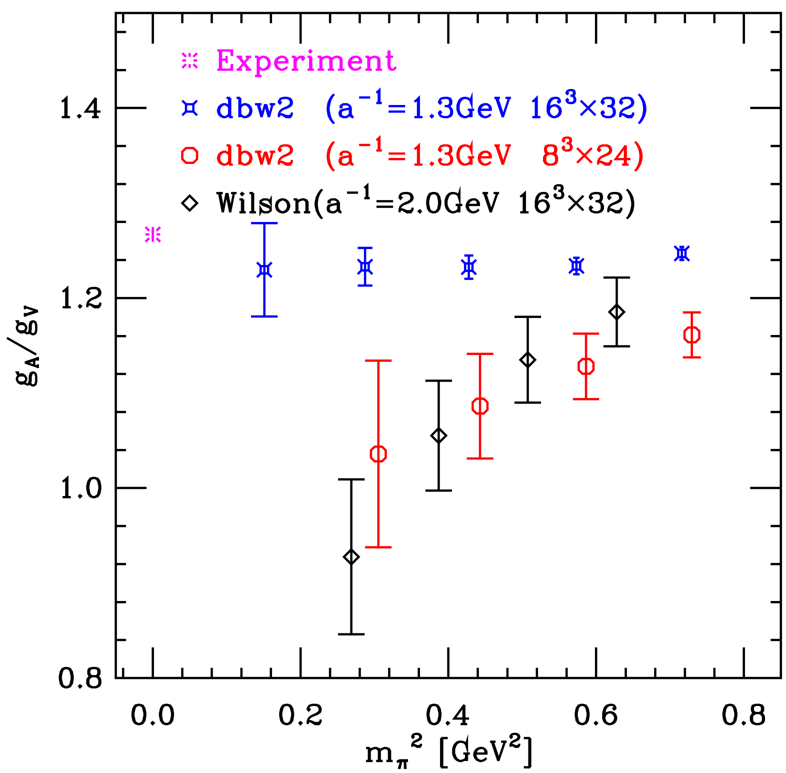
$$Z_A = 0.77759(45)$$

- Chiral limit (Linear fit):

$$g_A = 1.212 \pm 0.027_{stat} \pm 0.024_{norm}$$

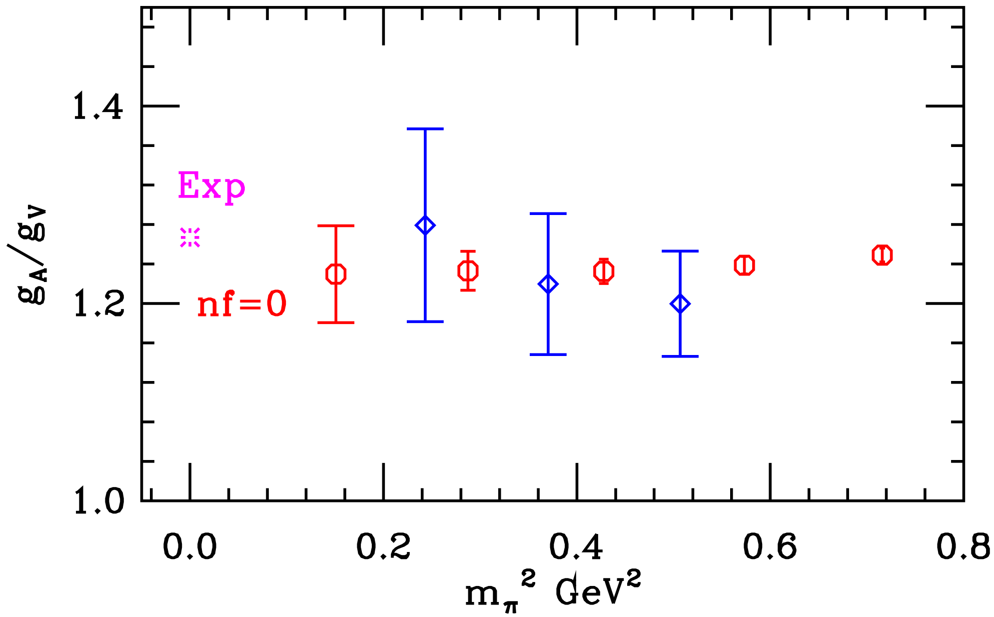
See Sasaki, Orginos, Ohta, Blum
Phys. Rev. D68: 054509, 2003

Finite volume effect for g_A



Previous RBC study:
[Blum,Ohta,Sasaki] 1.6fm box
New RBC study:
2.4fm and 1.2fm box
Clear finite volume effect

For dwf (chiral symmetry): $Z_A = Z_V = 1/g_V$



Axial Charge (dynamical)

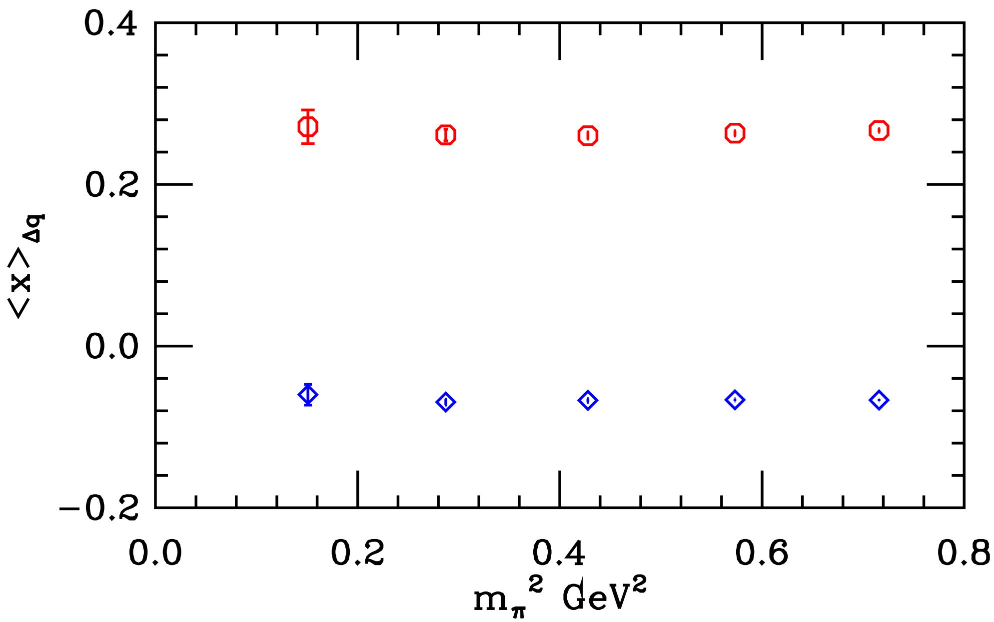
- Renormalization:

$$\langle \mathcal{A}^{cons} \bar{q}\gamma_5 q\rangle = Z_A \langle A^{loc} \bar{q}\gamma_5 q\rangle$$

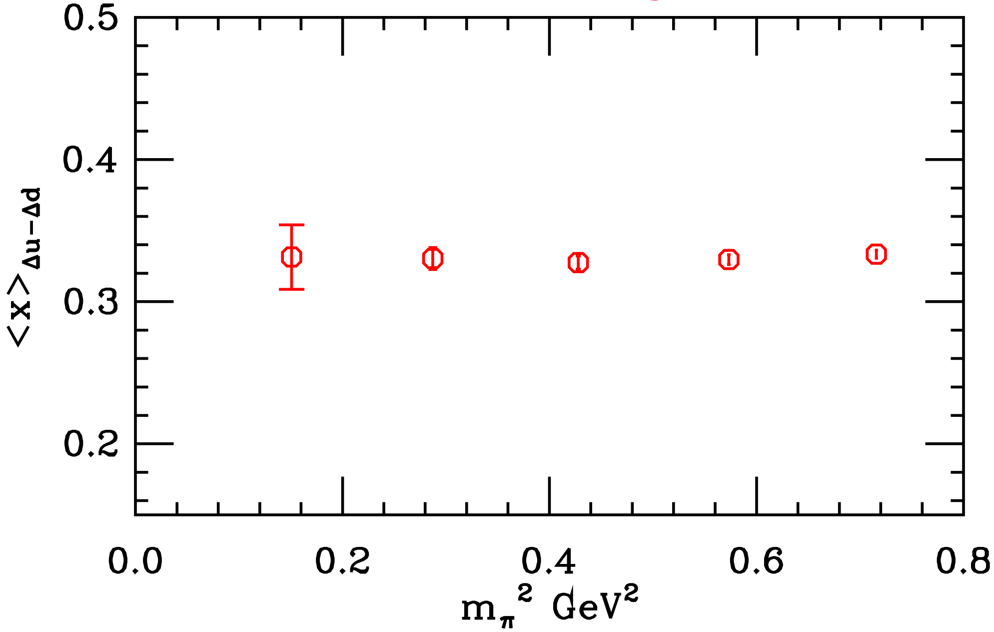
[C. Dawson LAT03]

$$Z_A = 0.75765(45)$$

See Orginos, Ohta Lattice 2003



Flavor non-singlet



Measure:

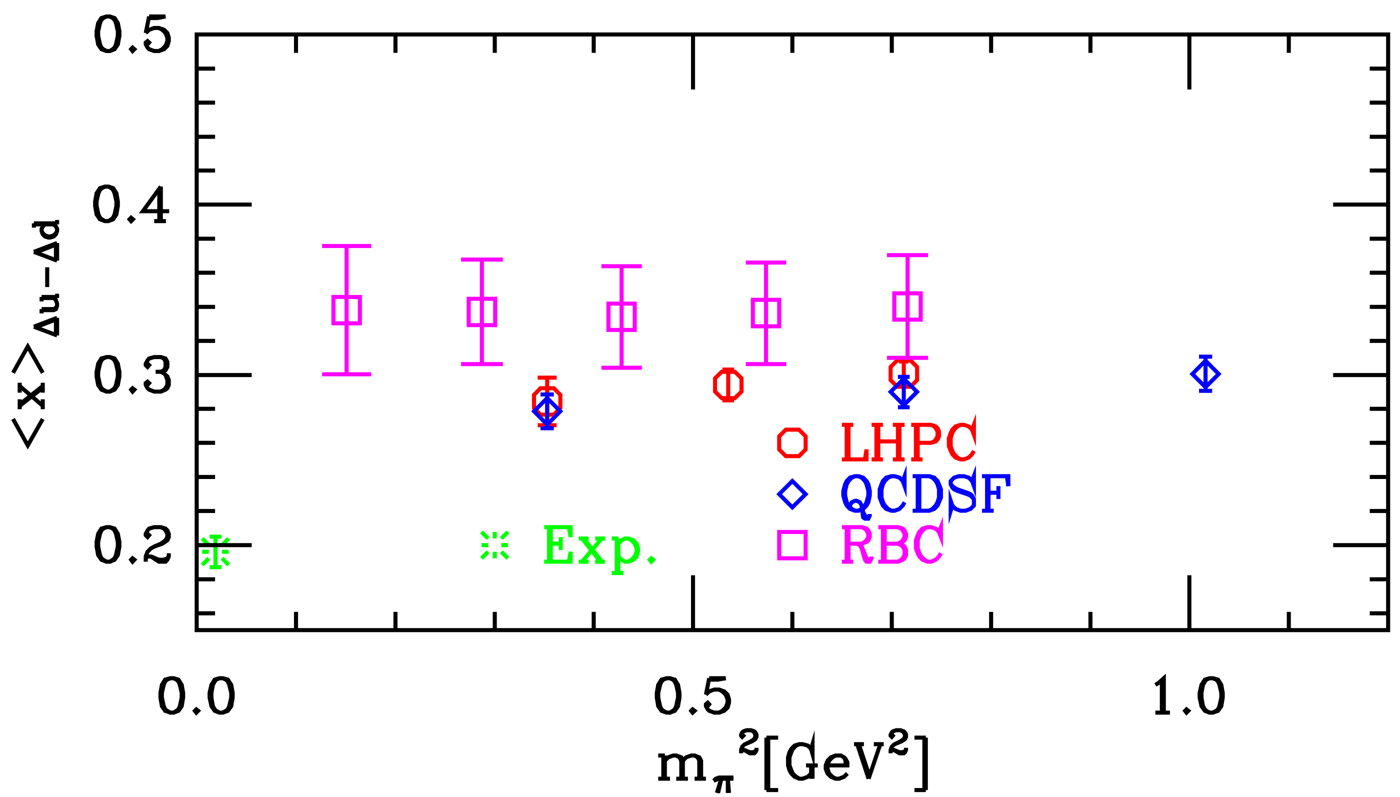
$$\mathcal{O}_{34}^{5q} = \frac{1}{4}\bar{q}\gamma_5\left[\gamma_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_3\right] q \quad \rightarrow \quad \langle x \rangle_{\Delta}$$

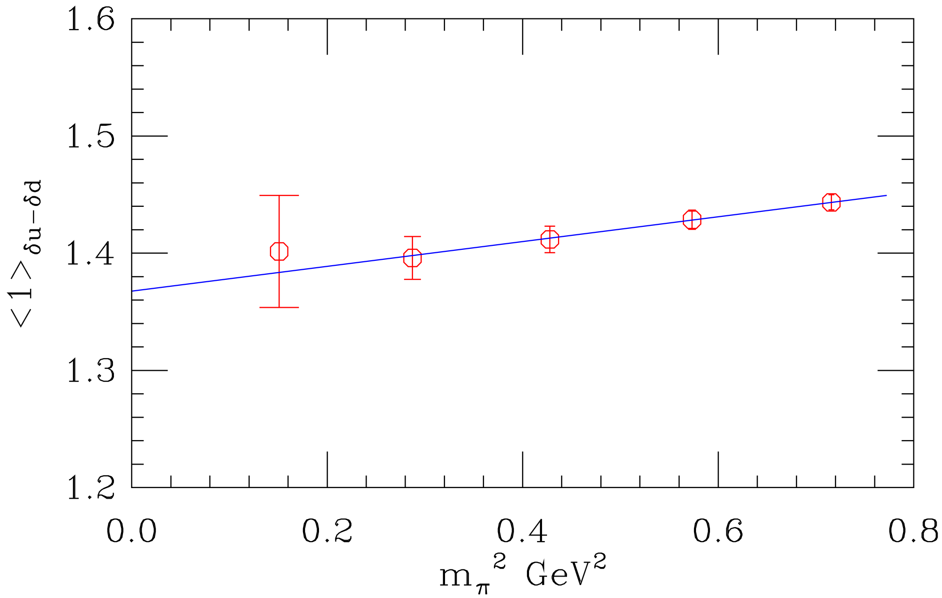
- Hypercubic group rep.: $\mathbf{6}_3^-$
- Momentum: $\vec{P} = 0$
- Renorm.: Multiplicative

Note:

- Renormalization:
  $Z = 1.02(9)$
  $\overline{MS} = 2GeV$ 2-loop runnig
- No curvature in the chiral limit
- Light mass needs more statistics

Large discrepancy here as well





Transversity:

$$\mathcal{O}_{34}^{\sigma q} = \bar{q}\gamma_5\sigma_{34}q \quad \longrightarrow \quad \langle 1 \rangle_{\delta q}$$

- Hyper-cubic group representation: $\mathbf{6}_1^+$
- Momentum: $\vec{P} = 0$
- Renorm.: Multiplicative

Note:

- Renormalization (NPR)

[C. Dawson LAT02]

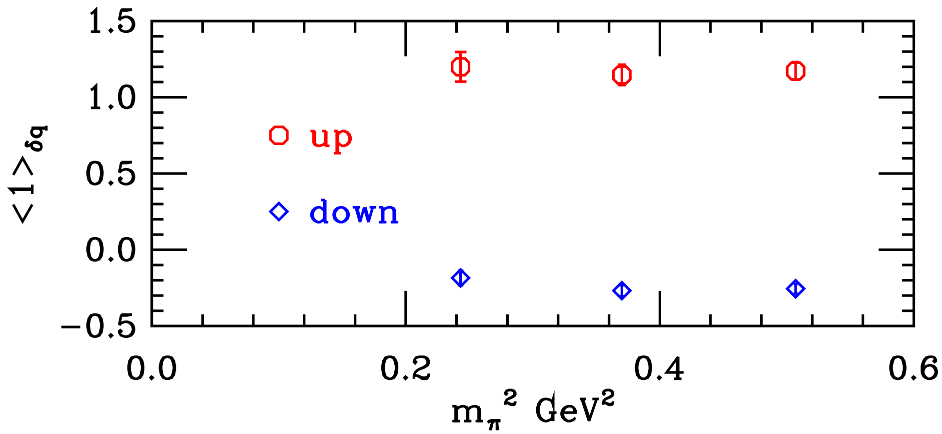
$$\langle 1 \rangle_{\delta u - \delta d} = 1.193(30)$$

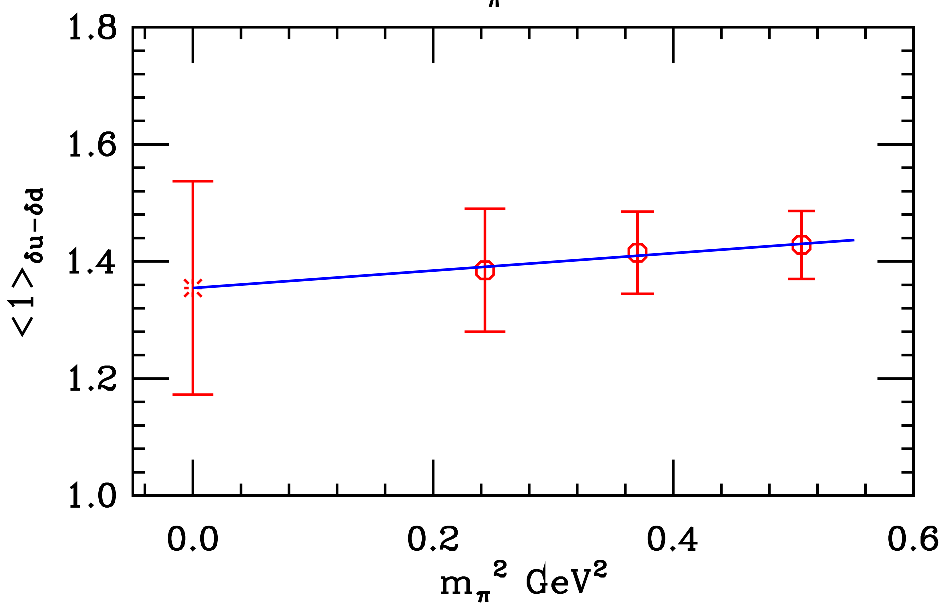
$\overline{MS}(2GeV)$ 2-loop running

QCDSF(quenched continuum):

$$\langle 1 \rangle_{\delta u - \delta d} = 1.214(40)$$

$\overline{MS}(2GeV)$ 1-loop perturbative





Transversity (dynamical):

$$\mathcal{O}_{34}^{\sigma q} = \bar{q}\gamma_5\sigma_{34}q \quad \rightarrow \quad \langle 1 \rangle_{\delta q}$$

- Hyper-cubic group representation: $\mathbf{6}_1^+$
- Momentum: $\vec{P} = 0$
- Renorm.: Multiplicative

Note:

- Unrenormalized (will do NPR)

The g_2 structure function

$$2\int_0^1 dx x^n g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d}[e^q_{2,n}(\mu^2/Q^2, g(\mu)) d^q_n(\mu) - 2e^q_{1,n}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\triangle q}(\mu)],$$

$$\begin{aligned} \langle x^n\rangle_{\triangle q}(\mu) &\rightarrow \text{Twist 2} \\ d^q_n(\mu) &\rightarrow \text{Twist 3} \end{aligned}$$

$d^q_n(\mu)$ estimations:

- Negligible $\Longrightarrow$ Wandzura - Wilczek relation of g_1 and g_2
- Need not be small in a confining theory [Jaffe and Ji Phys.Rev.D43,91].

Twist Three

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1} d_n^q(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - traces]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \overline{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma} \overset{\leftrightarrow}{D}_{\mu_1]}\cdots \overset{\leftrightarrow}{D}_{\mu_n} - traces\right] q$$

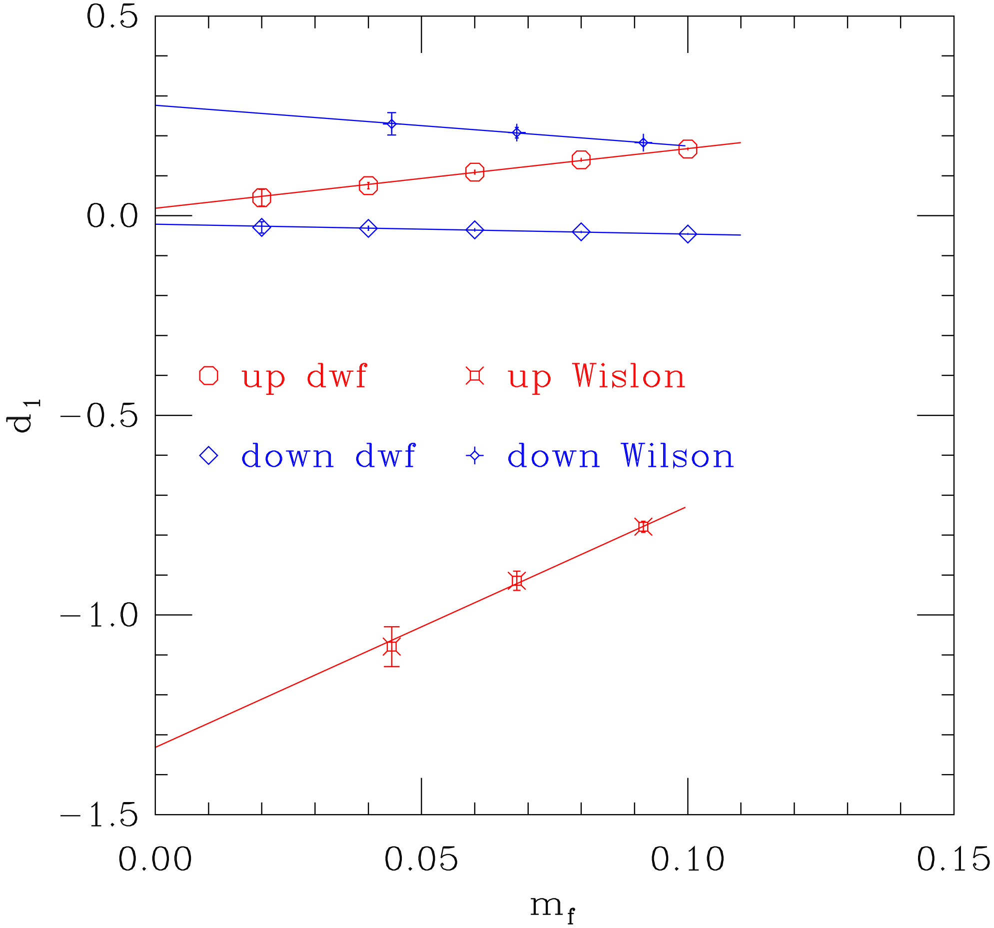
Measure:

$$\mathcal{O}^{[5]q}_{34} = \frac{1}{4}\overline{q}\gamma_5\left[\gamma_3 \overset{\leftrightarrow}{D}_4 - \gamma_4 \overset{\leftrightarrow}{D}_3\right] q \quad \rightarrow \quad d_1^q$$

- Hyper-cubic group representation: $\mathbf{6}_1^+$
- Momentum: $\vec{P} = 0$
- Renormalization: Multiplicative (DWF Chiral symmetry)

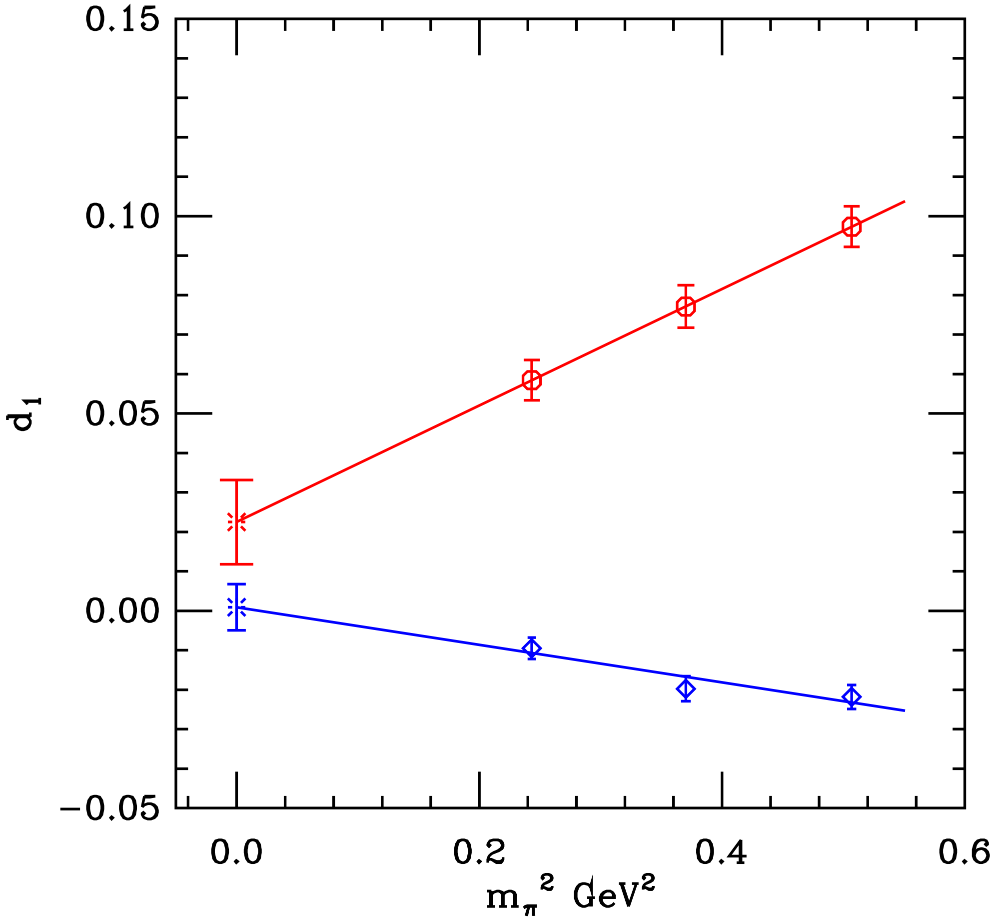
chiral symmetry breaking causes mixing with

$$\mathcal{O}^{\sigma q}_{34} = \overline{q}\gamma_5\sigma_{34}q$$



Note:

- Unrenormalized
- Disagreement with the Wilson results
  Power divergent mixing
  [LHPC-SESAM: hep-lat/0201021]
- Small at chiral limit



dynamical

- Unrenormalized
- Chiral symmetry is good
- Small at chiral limit

- Lattice QCD can compute non-perturbatively certain low moments of structure functions.
- Several systematic errors still need careful study:
  - Finite lattice spacing, Finite volume
  - Chiral limit
  - Quenching
- Started the calculation of Nucleon matrix elements with
  Domain wall fermions ... improved chirality,scaling
- Preliminary results look promising
  Simulation at light quark masses possible
- g_A : $1.212 \pm 0.027_{stat} \pm 0.024_{norm}$ ... finite volume (quenched)
  dynamical : consistent/need better statistics
- $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$: No curvature down to 390MeV pion mass
- d_1: Absence of power divergent mixing (chiral symmetry)
  Small at the chiral limit

Part II: Generalized parton distributions

Introduction (follow LHPC-SESAM, QCDSF, Ji)

Start with the light-cone operator $\mathcal{O}_q(x)$ that describes parton-parton *correlations*:

$$\mathcal{O}_q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \overline{q}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig \int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} q(\frac{\lambda}{2}n).$$

Its forward matrix element is the usual parton distribution:

$$q(x) = \langle P|\mathcal{O}_q(x)|P\rangle$$

Expand in terms of local operators (OPE)

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{q}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} q\,,$$

and, as before

$$\langle x^n\rangle_q(\mu) = \int dx x^{n-1} q(x)$$

Off-forward matrix element:

$$\langle P'|\mathcal{O}(x)|P\rangle = \bar{u}(p')\left(\not{n}H(x,\xi,t)+\frac{i\Delta_\nu}{2m}\sigma^{\mu\nu}n_\mu E(x,\xi,t),\right)u(p)$$

defines GPD's: $H(x,\xi,t)$, $E(x,\xi,t)$

$t=\Delta^2$, $\xi=-n\cdot\Delta/2$, $\Delta=P'-P$

Insert $\gamma_5 \rightarrow$ spin dependent GPD's: $\tilde{H}(x,\xi,t)$, $\tilde{E}(x,\xi,t)$

Generalized Form Factors

The moments of the GPD's give (generalized) form factors.

The lowest give Dirac and Pauli form factors:

$$\begin{aligned} \int_{-1}^{1} \mathrm{d}x\, H_q(x,\xi,\Delta^2) &= F_1^q(\Delta^2)\,, \\ \int_{-1}^{1} \mathrm{d}x\, E_q(x,\xi,\Delta^2) &= F_2^q(\Delta^2)\,. \end{aligned}$$

$$\begin{aligned} \int_{-1}^{1} \mathrm{d}x\, x\, H_q(x,\xi,\Delta^2) &= A_2^q(\Delta^2) + \xi^2 C_2^q(\Delta^2)\,, \\ \int_{-1}^{1} \mathrm{d}x\, x\, E_q(x,\xi,\Delta^2) &= B_2^q(\Delta^2) - \xi^2 C_2^q(\Delta^2)\,, \end{aligned}$$

nucleon matrix elements of the energy-momentum tensor (EMT):

$$\begin{aligned}
\langle p'|\mathcal{O}^q_{\{\mu\nu\}}|p\rangle &\equiv \frac{\mathrm{i}}{2}\langle p'|\bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}q|p\rangle \\
&= A^q_2(\Delta^2)\,\bar{u}(p')\gamma_{\{\mu}\bar{p}_{\nu\}}u(p) \\
&- B^q_2(\Delta^2)\,\frac{\mathrm{i}}{2m_N}\bar{u}(p')\Delta^\alpha\sigma_{\alpha\{\mu}\bar{p}_{\nu\}}u(p) \\
&+ C^q_2(\Delta^2)\,\frac{1}{m_N}\bar{u}(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}\,.
\end{aligned}$$

In the forward limit, $\Delta^2 \to 0$,

$$A^q_2(0) = \langle x_q\rangle$$

and

$$\frac{1}{2}\big(A^q_2(0) + B^q_2(0)\big) = J_q\,,$$

The angular momentum is given by the *gauge invariant* pieces (Ji 1997)

$$J_q = L_q + S_q$$

One calculates the spin S_q separately, then determines the orbital piece L_q

Lattice results

QCDSF (Phys. Rev. Lett. 92:042002,2004):

- quenched
- non-pert improved Wilson fermions
- Wilson gauge action, $a^{-1} \sim 2$ GeV
- $m_\pi \sim 600 - 1000$ MeV
- $L \sim 1.6$ fm ($16^3 \times 32$)

For the EMT, use two sets of operators

$$\frac{1}{\sqrt{2}}\big(\mathcal{O}_{\mu\nu}+\mathcal{O}_{\nu\mu}\big)\,,\ 1\leq\mu<\nu\leq 4$$

and

$$\frac{1}{2}\big(\mathcal{O}_{11}+\mathcal{O}_{22}-\mathcal{O}_{33}-\mathcal{O}_{44}\big)\,,$$

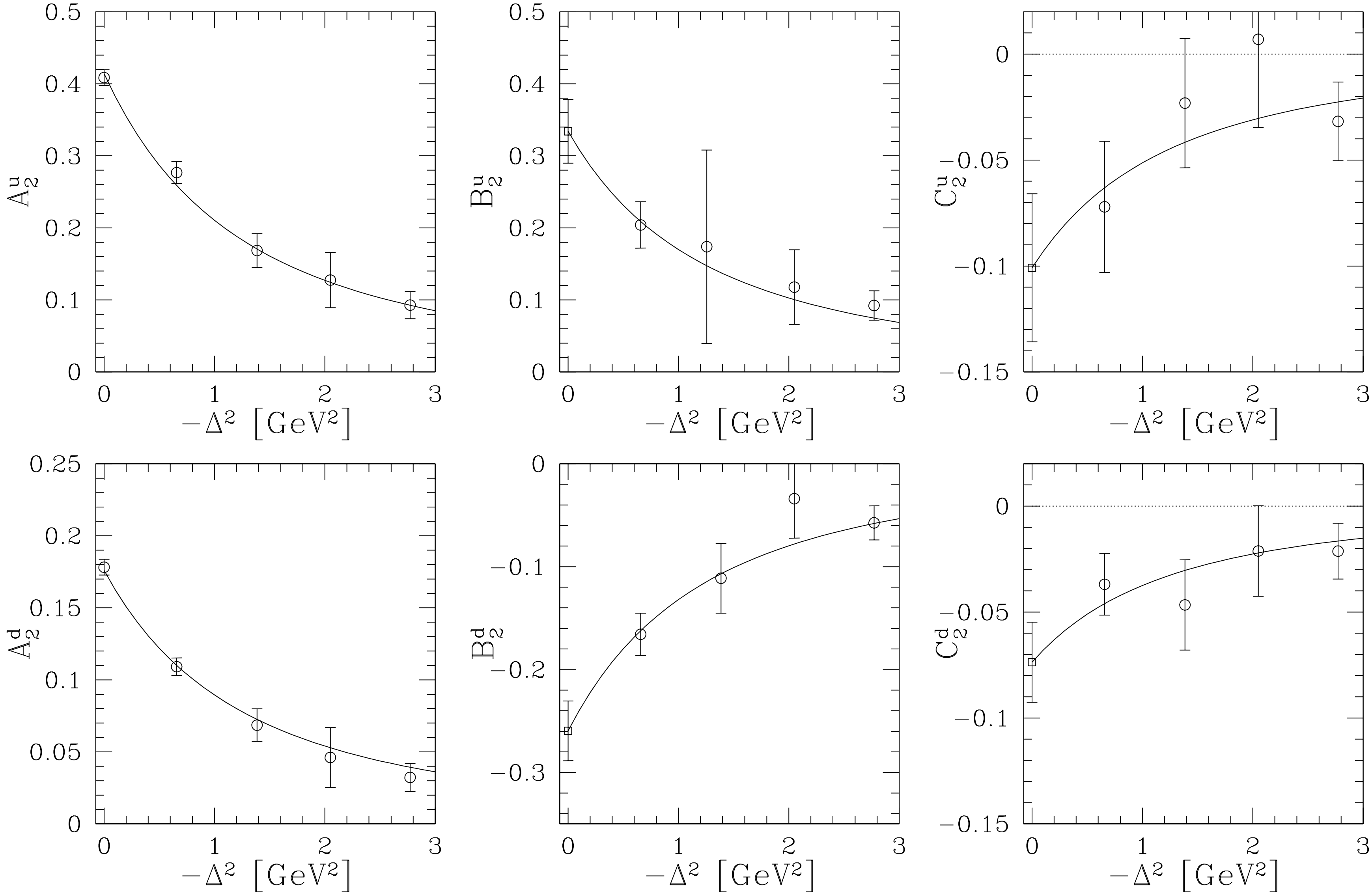
$$\frac{1}{\sqrt{2}}\big(\mathcal{O}_{33}-\mathcal{O}_{44}\big)\,,\ \frac{1}{\sqrt{2}}\big(\mathcal{O}_{11}-\mathcal{O}_{22}\big)\,.$$

Each set transforms irreducibly under the hypercubic group.

The operators are renormalized at 2 GeV in the $\overline{MS}$ scheme:

$Z^{\overline{MS}}_{v_{2a}}(2\,\mathrm{GeV})=1.10$ and $Z^{\overline{MS}}_{v_{2b}}(2\,\mathrm{GeV})=1.09$.

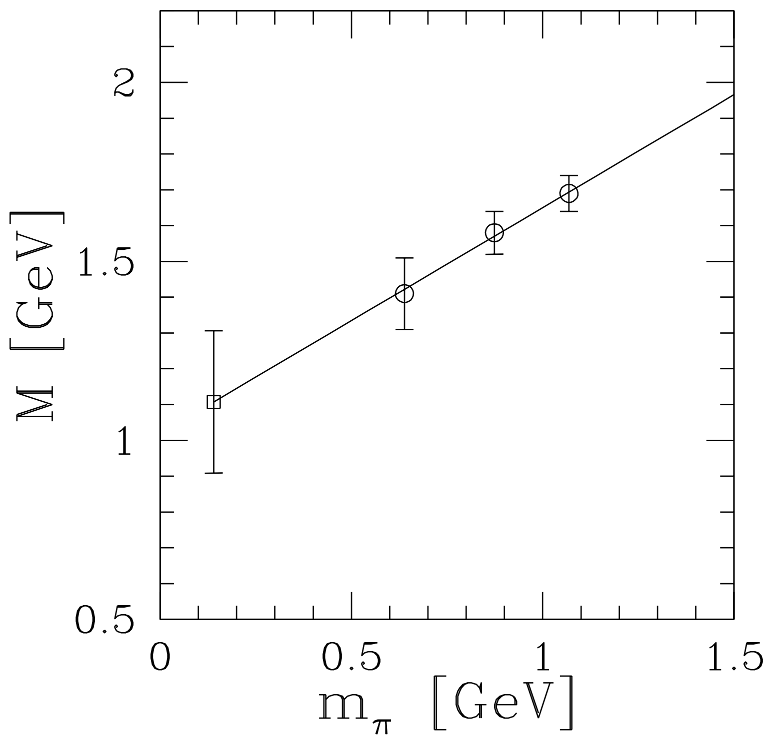
($\kappa = .1333$ results, $m_\pi \sim 800$ MeV)



Dipole Fit

$$A_2^q(\Delta^2) \;=\; A_2^q(0)/(1-\Delta^2/M^2)^2$$
$$M \;=\; 1.1(2)\,(\mathrm{GeV})$$

Close to f_2 and a_2 meson masses (lattice systematics ?)

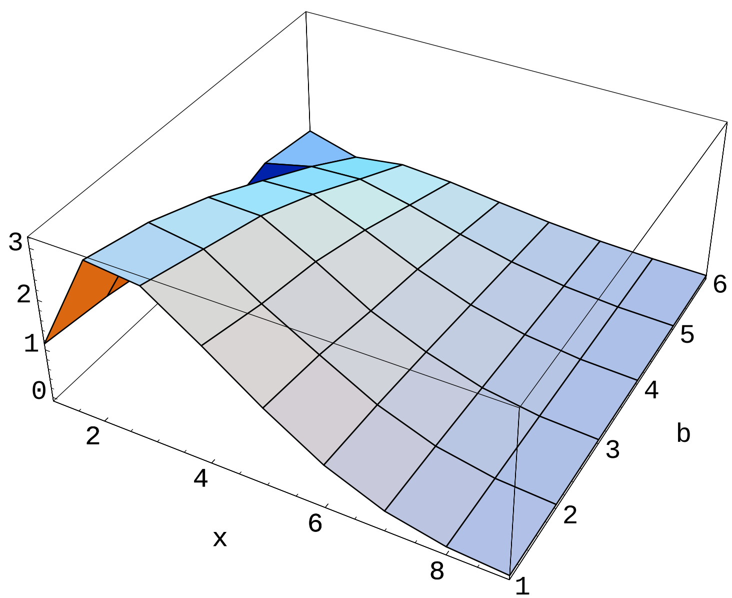


Assuming dipole (Regge) behavior holds for higher moments,

$$\int_{-1}^{1} \mathrm{d}x\, x^n\, H_q(x, 0, \Delta^2) = \langle x_q^n \rangle / (1 - \Delta^2 / M_{n+1}^2)^2\,,$$

Then from inverse Mellin transform (using previous $\langle x_q^n \rangle$, $0 \leq n \leq 3$)

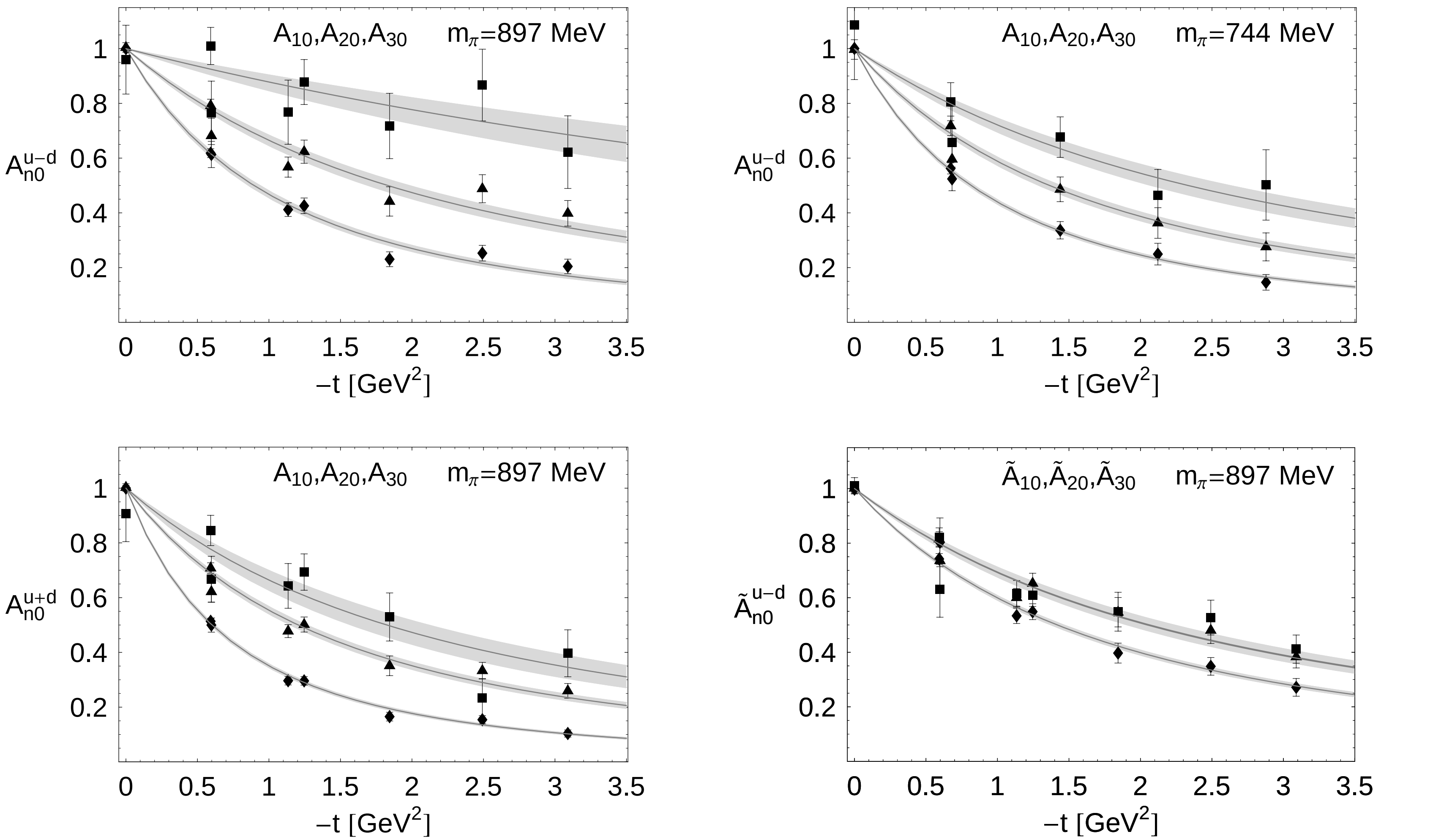
$$H_q(x, 0, \mathbf{b}_\perp^2) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{\Delta}_\perp\, \mathrm{e}^{i \mathbf{b}_\perp \mathbf{\Delta}_\perp} H_q(x, 0, \mathbf{\Delta}_\perp^2)$$ [M.Burkhardt (2000)]



probability of quark with momentum x
at transverse impact parameter
(Schierholz, *et al.*, hep-ph/0312104)

LHPC-SESAM (hep-lat/0312014)

- $N_f = 2$
- unimproved Wilson fermions
- Wilson gauge action, $a^{-1} \sim 2$ GeV
- $\kappa = 0.1570$, 0.1560: $m_\pi \sim 750, 900$ MeV
- $L \sim 1.6$ fm ($16^3 \times 32$)



A_{10}, A_{20}, A_{30} m_π=897 MeV
A_{n0}^{u-d}
$-t$ [GeV^2]
A_{10}, A_{20}, A_{30} m_π=744 MeV
A_{n0}^{u-d}
$-t$ [GeV^2]
A_{10}, A_{20}, A_{30} m_π=897 MeV
A_{n0}^{u+d}
$-t$ [GeV^2]
$\tilde{A}_{10}, \tilde{A}_{20}, \tilde{A}_{30}$ m_π=897 MeV
$\tilde{A}_{n0}^{u-d}$
$-t$ [GeV^2]

Dipole fits

$n = 2$ moments consistent with QCDSF (quenching effects?)

Slope at origin $\sim$ transverse size, decreases as n increases

Weaker dependence at lighter mass

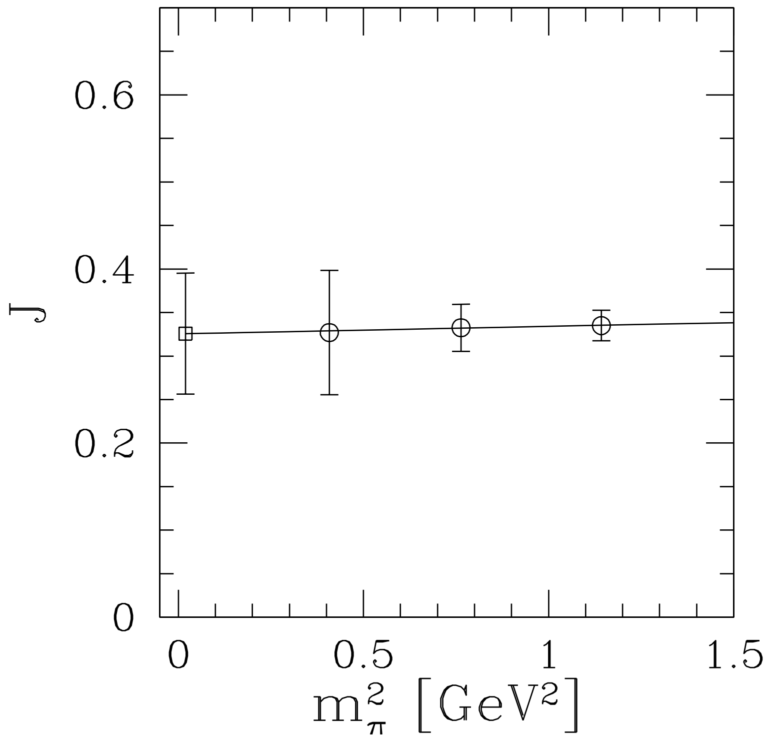
Flavor dependence

Spin dependent case shows markedly weaker dependence on n

Nucleon spin carried by the quarks

Ji (1997): Total angular momentum from EMT:

$$
\begin{aligned}
J &= \frac{1}{2}\sum_q \left(A_2^q(0) + B_2^q(0)\right) \\
&= \sum_q S_q + \sum L_q \\
&= S + L
\end{aligned}
$$



Spin from usual moments Δ^q

	J	J_u	J_d	S_u	S_d	S	L
QCDSF(2003)	0.33(7)	0.37(6)	-0.04(4)	0.42(1)	-0.12(1)		0.03(7)
Kentucky(2000)	0.30(7)					0.13(6)	0.17(6)

30-40% (?) Carried by the gluons

The gluon total angular momentum (Ji 1997)

Need nucleon matrix element of $T^{g}_{\mu\nu}$ (same as before)

$$\begin{aligned}\langle p'|T^{q}_{\mu\nu}|p\rangle &\equiv \langle p'|\frac{1}{2}\left(\frac{1}{4}\delta_{\mu\nu}F^2 + F_{\mu\sigma}\,F_{\sigma\nu}\right)|p\rangle \\ &= A_2^q(\Delta^2)\,\bar{u}(p')\gamma_{\{\mu}\bar{p}_{\nu\}}u(p) \\ &- B_2^q(\Delta^2)\,\frac{\mathrm{i}}{2m_N}\bar{u}(p')\Delta^{\alpha}\sigma_{\alpha\{\mu}\bar{p}_{\nu\}}u(p) \\ &+ C_2^q(\Delta^2)\,\frac{1}{m_N}\bar{u}(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}\,.\end{aligned}$$

On the lattice, $T_{\mu\nu}$ is given by the plaquette $U(x)_{\mu\nu}$ (Karsch and Wyld (1987)):

$$\begin{aligned}T_{\mu\mu} &= \frac{2}{g^2}\left(-\sum_{\nu\neq\mu}\mathrm{Tr}U(x)_{\mu\nu} + \sum_{\sigma\nu\neq\mu,\sigma>\nu}\mathrm{Tr}U(x)_{\mu\nu}\right) + \mathcal{O}(g^2) \\ T_{\mu\nu} &= \frac{2}{g^2}\mathrm{Tr}\tilde{U}(x)_{\mu\sigma}\tilde{U}(x)_{\nu\sigma} \\ \tilde{U}(x)_{\mu\nu} &\equiv -\frac{i}{2}\left(U_{\mu\nu} - U_{\nu\mu}\right)_{\mathrm{traceless}}\end{aligned}$$

The above yields J^g.

Likely very noisy: non-zero momentum nucleons, gluonic operators make life difficult.

Worth a try?

Can we get S^g and L^g separately? Probably not.

non-local operator $\mathcal{O}$ matrix element yields (lowest moment of) Δg.

$\mathcal{O}$ corresponds to S^g in the $A^+ = 0$ gauge in the infinite momentum frame. Not accesible to a Euclidean lattice calculation.

Summary

- Lattice calculation of GPD's just getting started
- interesting qualitative results
- Systematics not yet under control