DVCS +

EXCLUSIVE PROCESSES

M. DIEHL (DESY)

SOME LITERATURE *)

X. JI	hep-ph/9603249	SPIN SUM RULE
M. BURKARDT	0005108	SPATIAL INTERPRETATION
M.D. et al	9706344	DVCS
J. COLLINS	9907513	FACTORIZATION
D. MÜLLER et al	9812448	EVOLUTION
A. RADYUSHKIN	9704207	

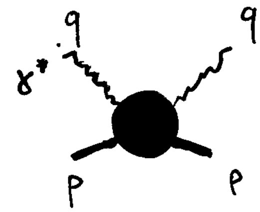
REVIEWS

X. JI	9807358	
A. RADYUSHKIN	0101225	DOUBLE DISTRIBUTIONS
M. BURKARDT	0207047	SPATIAL INTERPRETATION
K. GOEKE et al	0106012	
M.D.	0307382	

*) A SELECTIVE, PERSONALLY BIASED LIST

1. COMPTON SCATTERING AND GENERALIZED PARTON DISTRIBUTIONS

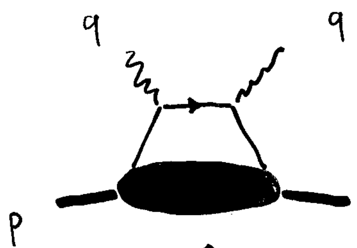
INCLUSIVE DIS :



$$\sigma_{tot}(\gamma^* p) \propto \mathrm{Im}\, \mathcal{A}(\gamma^* p \to \gamma^* p)$$

BJ LIMIT

$$Q^2 = -q^2 \to \infty$$
$$W^2 = (p+q)^2 \to \infty$$
$$x_B = \frac{Q^2}{2pq} \quad \text{FIXED}$$



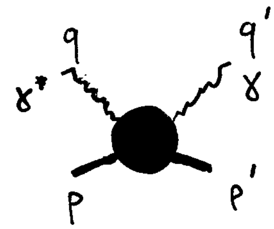
$+ \; O(\alpha_s)$

PARTON DISTRIBUTION

1. COMPTON SCATTERING AND GENERALIZED PARTON DISTRIBUTIONS

~~INCLUSIVE DIS~~ :

COMPTON AMPLITUDE



IN

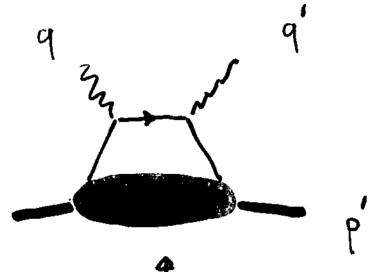
$ep \to ep\gamma$

DEEPLY VIRTUAL COMPTON SCATTERING

$$\cancel{\sigma_{tot}(\gamma^* p) \propto \mathrm{Im}\, A(\gamma^* p \to \gamma^* p)}$$

BJ LIMIT

$$Q^2 = -q^2 \to \infty$$
$$W^2 = (p+q)^2 \to \infty$$
$$x_B = \frac{Q^2}{2pq} \quad \text{FIXED}$$
$$t = (p-p')^2 \quad \text{FIXED}$$



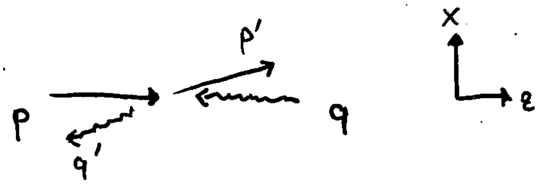
$+ \; O(\alpha_s)$

GENERALIZED PARTON DISTRIBUTION

GPD

IN MORE DETAIL :

$\gamma^*(q) + p(p) \rightarrow \gamma(q') + p(p')$ IN c.m.



LARGE W^2 $\rightarrow$ p AND p' MOVE FAST TO THE RIGHT

LIGHT - CONE COORDINATES (*)

$p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3)$; p'^+ LARGE $\sim W \sim Q$

$\vec{p}\,'_T$ SMALL $\sim \sqrt{-t}$

p^- ; $p'^- = \dfrac{m^2 + \vec{p}_T'^{\,2}}{2p'^+}$ VERY SMALL

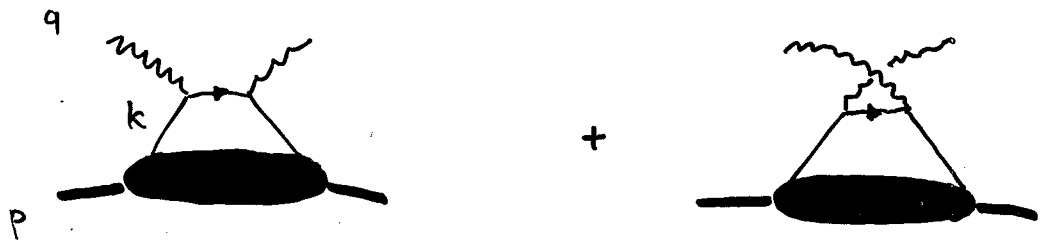
LARGE Q^2 $\rightarrow$

q^+ ; q^- LARGE $\sim Q$

(*) $v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$ $\qquad v^2 = 2v^+v^- - \vec{v}_T^{\,2}$

EVALUATING THE HANDBAG DIAGRAMS

(VERY SCHEMATICALLY)



$$\mathcal{A} \sim \int d^4k \ \text{DIRAC MATRICES} \times \frac{1}{(k+q)^2 + i\epsilon}$$

$$\times \int d^4z \ e^{izk} \ \langle p' | \bar{q}_\alpha(0) \ q_\beta(z) | p \rangle$$

$$+ \text{ 2nd GRAPH}$$

- QUARK MOMENTUM k ~ ALONG p

  $\Rightarrow$ $k^+ \sim p^+$ BIG, $\vec{k}_T$ SMALL, $k^- \sim p^-$ YET SMALLER

- IN QUARK PROPAGATOR + DIRAC MATRICES

  NEGLECT k^- ($\ll |q^-| \sim Q$) AND $\vec{k}_T$

  COLLINEAR APPROXIMATION

$$\Rightarrow \mathcal{A} \sim \int dk^+ \ \text{DIRAC MATRICES} \times \frac{1}{2(k^+ + q^+)q^- + i\epsilon}$$

$$\times \int dk^- \, d^2k_T \int dz^+ \, d^2z_T \, dz^- \ e^{iz^+k^- + iz^-k^+ - i\vec{z}_T\vec{k}_T}$$

$$\times \langle p' | \bar{q}_\alpha(0) \ q_\beta(z) | p \rangle \quad + \text{ 2nd GRAPH}$$

- INTEGRATION OVER k^-, k_T $\Rightarrow$

  z BECOMES LIGHT-LIKE

AFTER SIMPLIFICATION OF DIRAC MATRICES AND CHOOSING γ^* RIGHT-HANDED

$$\mathcal{A} \sim \int dk^+ \frac{1}{k^+ + q^+ - i\epsilon}$$

$$\times \int dz^- \, e^{i z^- k^+} \langle p' | \bar{q}(0) (\gamma^+ + \gamma^+ \gamma_5) q(z) | p \rangle_{\substack{z^+ = 0 \\ \vec{z}_T = 0}}$$

$$+ \text{ 2nd GRAPH}$$

$q^+ = - x_B p^+$ (EXERCISE)

$k^+ = x \, p^+$ (DEFINITION)

$$\mathcal{A} = \sum_q e_q^2 e^2 \int dx \frac{1}{x - x_B - i\epsilon}$$

$$\times \boxed{\int \frac{dz^-}{4\pi} e^{i x z^- p^+} \langle p' | \bar{q}(0) (\gamma^+ + \gamma^+ \gamma_5) q(z) | p \rangle_{\substack{z^+ = 0 \\ z_T = 0}}}$$

$$+ \text{ 2nd GRAPH}$$

$$= \sum_q e_q^2 e^2 \left[\text{P.V.} \int dx \frac{1}{x - x_B} \times \text{2nd LINE } (x) \right.$$

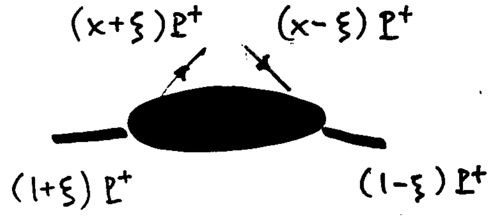
$$\left. + \, i\pi \times \text{2nd LINE } (x_B) \right]$$

PARTON DISTRIB'S

$$+ \text{ 2nd GRAPH}$$

DEFINITION OF NUCLEON GPDs

TAKE DIFFERENT DEF. OF x



$$P = \frac{1}{2}(p+p')$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$F^q = \int \frac{dz^-}{4\pi}\, e^{ixz^-P^+} \langle p', s' |\, \bar{q}(-\tfrac{z}{2})\, \gamma^+\, q(\tfrac{z}{2})\, | p, s\rangle \Big|_{\substack{z^+=0\\ z_T=0}}$$

$$= \frac{1}{2P^+}\, \bar{u}(p', s')\, \gamma^+ u(p,s)\; H^q(x, \xi, t)$$

$$+ \frac{1}{2P^+}\, \bar{u}(p', s')\, \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m}\, u(p,s)\; E^q(x,\xi,t)$$

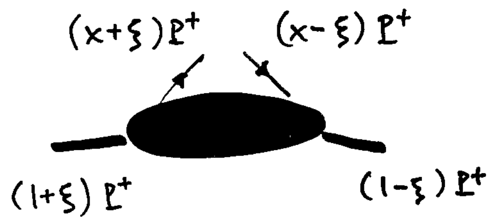
$$\tilde{F}^q = (\;\dots\; \gamma^+ \to \gamma^+\gamma_5\;)$$

$$= (\dots \gamma^+ \to \gamma^+\gamma_5\,)\cdot \tilde{H}^q(x,\xi,t) + \frac{1}{2P^+}\, \bar{u}(p', s')\, \frac{(p'-p)^+}{2m}\, u(p,s)$$

$$\times\; \tilde{E}(x,\xi,t)$$

DEFINITION OF NUCLEON GPDs

TAKE DIFFERENT DEF. OF x



$$P = \frac{1}{2}(p+p')$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$F^q = \int \frac{dz^-}{4\pi} e^{ixz^-P^+} \langle p',s'|\, \bar{q}(-\tfrac{z}{2})\, \gamma^+\, q(\tfrac{z}{2})\, |p,s\rangle \Big|_{\substack{z^+=0\\ z_T=0}}$$

$$= \frac{1}{2P^+} \bar{u}(p',s')\, \gamma^+\, u(p,s)\; H^q(x,\xi,t)$$

$$+ \frac{1}{2P^+} \bar{u}(p',s') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m} u(p,s)\; E^q(x,\xi,t)$$

$$\tilde{F}^q = (\;\dots\; \gamma^+ \to \gamma^+\gamma_5\;)$$

$$= (\dots\; \gamma^+ \to \gamma^+\gamma_5\;)\; \tilde{H}^q(x,\xi,t) + \frac{1}{2P^+} \bar{u}(p',s') \frac{(p'-p)^+}{2m} u(p,s)$$

$$\times\; \tilde{E}(x,\xi,t)$$

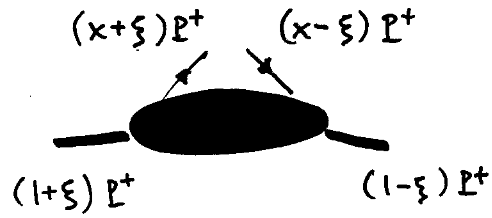
DVCS $\qquad \xi = \dfrac{x_B}{2-x_B}$

$$A(\gamma^*(+)\, p \to \gamma(+)\, p) = \sum_q (e e_q)^2 \Big[\int dx\; F^q \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

$$+ \int dx\; \tilde{F}^q \left(\frac{1}{\xi - x - i\epsilon} + \frac{1}{\xi + x - i\epsilon} \right) \Big]$$

DEFINITION OF NUCLEON GPDs

TAKE DIFFERENT DEF. OF x



$$P = \tfrac{1}{2}(p+p')$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$F^q = \int \frac{dz^-}{4\pi}\, e^{\,i x z^- P^+} \langle p',s'|\, \bar{q}(-\tfrac{z}{2})\, \gamma^+\, q(\tfrac{z}{2})\, |p,s\rangle_{\substack{z^+=0\\ z_T=0}}$$

$$= \frac{1}{2P^+}\, \bar{u}(p',s')\, \gamma^+ u(p,s)\; H^q(x,\xi,t)$$

$$+ \frac{1}{2P^+}\, \bar{u}(p',s')\, \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m}\, u(p,s)\; E^q(x,\xi,t)$$

$$\tilde{F}^q = (\;\dots\; \gamma^+ \to \gamma^+\gamma_5\;)$$

$$= (\dots\, \gamma^+ \to \gamma^+\gamma_5\,)\; \tilde{H}^q(x,\xi,t) + \frac{1}{2P^+}\, \bar{u}(p',s')\, \frac{(p'-p)^+}{2m}\, u(p,s) \times \tilde{E}(x,\xi,t)$$

AT $p = p'$ $\quad(\xi < 0,\; t = 0)$ "FORWARD LIMIT"

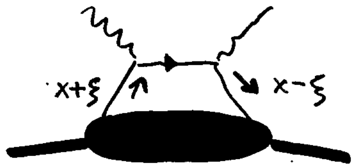
$$H^q(x,0,0) = \begin{cases} q(x) & x>0 \\ -\bar{q}(-x) & x<0 \end{cases}$$

$$\tilde{H}^q(x,0,0) = \begin{cases} \Delta q(x) & x>0 \\ \Delta q(-x) & x<0 \end{cases}$$

E^q AND $\tilde{E}^q$ DECOUPLE

2. PARTON INTERPRETATION

ONE COVARIANT DIAGRAM : $\int_{-\infty}^{\infty} dx$

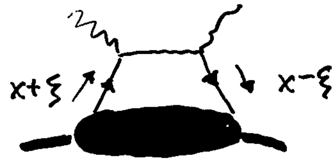


↓

THREE REGIONS

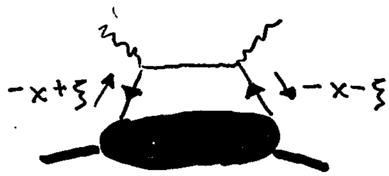
PARTON PLUS-MOMENTUM ≥ 0

$\xi < x < 1$



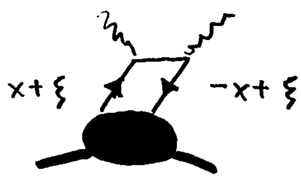
q out
q back in

$-1 < x < -\xi$



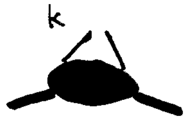
$\bar{q}$ out
$\bar{q}$ back in

$-\xi < x < \xi$



$q\bar{q}$ out

FOR $|x| > 1$

$\int dk^- d^2k_T$  $= 0$

EVOLUTION

- GPDs DEPEND ON FACTORIZATION SCALE μ

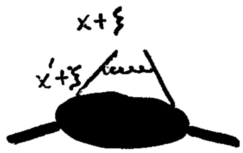
  SCHEMATICALLY, IN THEIR DEFINITION $\int dk^- \int^{\mu^2} d^2k_T$

$$\mu^2 \frac{d}{d\mu^2} \, GPD(x, \xi, t, \mu^2) = \int \frac{dx'}{\xi} \, GPD(x', \xi, t \;\mu^2) \times K\left(\frac{x'}{\xi}, \frac{x}{\xi}\right)$$

- 2 REGIONS :

DGLAP

$\xi < x < 1$

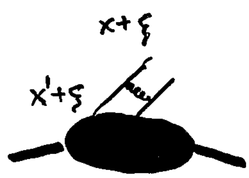


SAME FOR $-1 < x < -\xi$

- SIMILAR TO USUAL DGLAP EVOLUTION

ERBL

$-\xi < x < \xi$



- LIKE MESON DISTRIBUTION AMPLITUDE

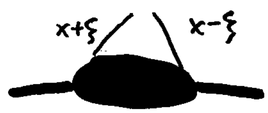
WAVE FUNCTION REPRESENTATION

DECOMPOSE HADRON ON "PARTON STATES"

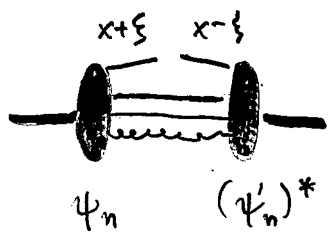
$$|p\rangle = \sum_{\substack{\text{QUANTUM}\\ \#\text{S}}} \int_{\text{MOMENTA}} \Big[\psi_3 \, |qqq\rangle + \psi_4 \, |qqq\,g\rangle + \psi_5 \, |qqqq\bar{q}\rangle + \dots \Big]$$

LIGHT-CONE WAVE FUNCTIONS

$\Rightarrow$ GPD IN DGLAP REGION :

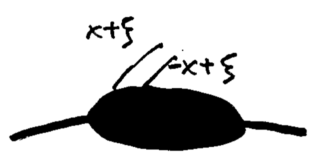


$$= \sum_{n} \sum_{Q.\#s} \int_{\text{MOM.}}$$

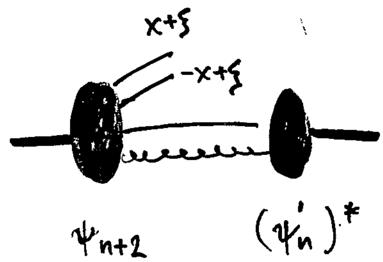


[ANALOGOUS FOR $-1 < x < -\xi$]

IN ERBL REGION :



$$= \sum_{n} \sum_{Q.\#s} \int_{\text{MOM}}$$



WAVE FUNCTION REPRESENTATION

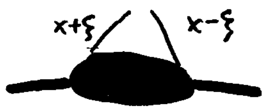
DECOMPOSE HADRON ON " PARTON STATES "

$$|p\rangle = \sum_{\text{QUANTUM \#S}} \int_{\text{MOMENTA}} \Big[\psi_3\, |qqq\rangle + \psi_4\, |qqqg\rangle + \psi_5\, |qqqq\bar{q}\rangle + \dots \Big]$$

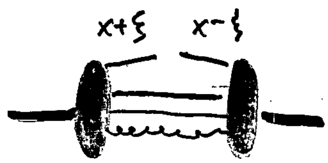
LIGHT - CONE

WAVE FUNCTIONS

$\Rightarrow$ GPD IN DGLAP REGION :

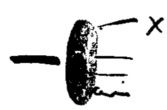


$$= \sum_n \sum_{\text{Q.\#S}} \int_{\text{MOM.}}$$



ψ_n $\quad (\psi_n')^*$

IN FORWARD LIMIT : $= \sum \sum \int \Big|$



$\Big|^2$

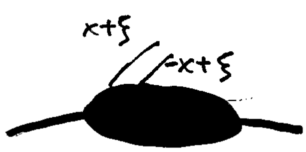
[ANALOGOUS FOR $-1 < x < -\xi$]

$\longrightarrow$ SQUARED AMPLITUDE

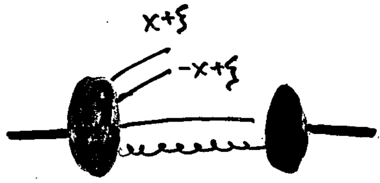
= PROBABILITY

ELSE : INTERFERENCE BETWEEN DIFFERENT CONFIGURATIONS

IN ERBL REGION :



$$= \sum_n \sum_{\text{Q.\#S}} \int_{\text{MOM}}$$



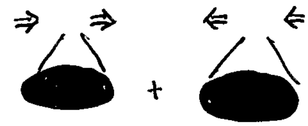
ψ_{n+2} $\quad (\psi_n')^*$

3. HELICITY STRUCTURE

- PARTON SIDE

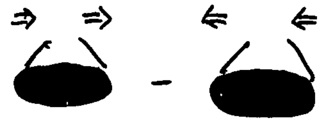
H^q, E^q OPERATOR $\bar{q}\gamma^+ q$

"UNPOLARIZED"



$\tilde{H}^q$, $\tilde{E}^q$ $\bar{q}\gamma^+\gamma_5 q$

"POLARIZED"



ALSO : QUARK HELICITY FLIP

GENERALIZES TRANSVERSITY DISTRIBUTIONS

VERY DIFFICULT TO MEASURE

ALL THIS ALSO FOR GLUONS

H^g, E^g UNPOL.

$\tilde{H}^g$, $\tilde{E}^g$ POL.

AND g HELICITY FLIP (QUITE DIFFICULT TO MEASURE)

- HADRON SIDE

EVALUATE PROTON SPINOR PRODUCTS IN DEF'S p. 5

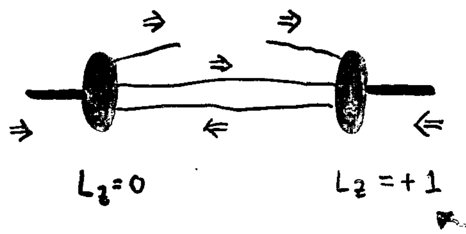
$$\propto \quad H - \frac{\xi^2}{1-\xi^2} E + \left(\tilde{H} - \frac{\xi^2}{1-\xi^2} \tilde{E} \right)$$

$$\propto \left(D^x + i D^y \right) \frac{1}{2m} \left(E + \xi \tilde{E} \right)$$

$$D = \frac{p'}{1-\xi} - \frac{p}{1+\xi}$$

HELICITY NOT CONSERVED $\Rightarrow$ ORBITAL ANGULAR MOMENTUM

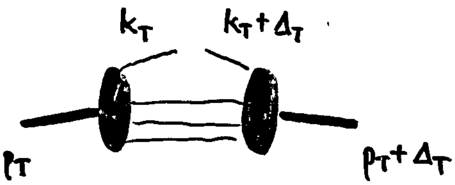
e.g.



4. SPATIAL INTERPRETATION

LOOK AGAIN AT WAVE FCT. REPRESENTATION:

$$H(x,\xi,t) = \sum_{\dots} \int d^2k_T$$



= CONVOLUTION IN PARTON k_T

↓

BECOMES SIMPLER AFTER FOURIER TRANSFORM

- PROTON STATE

$$|p^+, \vec{b}_T\rangle = \frac{1}{(2\pi)^2} \int d^2p_T \; e^{-i\vec{p}_T \vec{b}_T} \; |p^+, \vec{p}_T\rangle$$

IS STATE OF DEFINITE TRANSVERSE POSITION

(= IMPACT PARAMETER) $\vec{b}_T$

- "MIXED REPRESENTATION" : KEEP DEFINTE p^+

⇝ CAN STILL GO TO FRAME WHERE HADRON MOVES FAST

- LOCALIZATION IN 2D IS POSSIBLE

IN 3D IS RESTRICTED WITHIN COMPTON WAVE LENGTH $1/m$

PROTON IS EXTENDED OBJECT.

WHAT MEANS " LOCALIZED AT $\vec{b}_T$ " ?

→ LOOK AT SYMMETRIES

TRANSVERSE **BOOSTS** : PARTICULAR LORENTZ TRF'S

$$k^+ \to k^+$$

$$\vec{k}_T \to \vec{k}_T - k^+ \vec{v}_T \qquad \vec{v}_T \text{ FIXED}$$

$$k^- \to \ldots \qquad (\text{SO THAT } k^2 \to k^2)$$

ANALOG IN NONRELATIVISTIC MECHANICS :

GALILEI TRANSFORMATIONS

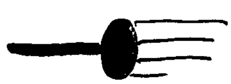
$$m \to m$$

$$\vec{k} \to \vec{k} - m\vec{v}$$

$$\xrightarrow[\text{NOETHER}]{} \quad \text{CENTER OF MASS} \quad \vec{R} = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

⇒ IN RELATIVISTIC CASE :

" CENTER OF MOMENTUM " $\quad \vec{b} = \dfrac{\sum_i \vec{b}_i k_i^+}{\sum_i k_i^+} \qquad (*)$

$\vec{b}$  $\vec{b}_i\,,\ x_i = \dfrac{k_i^+}{\sum_j k_j^+}$

- SEE THIS EXPLICITLY WHEN FOURIER TRF. WAVE FUNCTIONS FROM TRANSVERSE MOMENTUM TO IMPACT PARAMETER

(*) OMIT "T" IN $\vec{b}_T$ FROM NOW

NOW TAKE $\xi = 0$ FOR SIMPLICTY

$$\Rightarrow \quad t = -(\vec{p} - \vec{p}\,')^2$$

$$\langle p^+ \vec{b}\,' | \underbrace{\int \frac{dz^-}{4\pi} e^{i x z^- p^+} \bar{q}(-\tfrac{z}{2})\, \gamma^+ q(\tfrac{z}{2}) \Big|_{\substack{z^+=0 \\ \vec{z}=0}}}_{\mathcal{O}(\vec{z}=\vec{0})} | p^+ \vec{b} \rangle$$

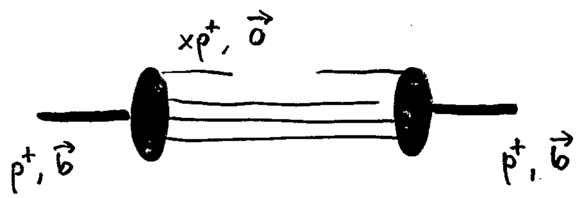
$$= \frac{1}{(2\pi)^4} \int d^2\vec{p}\,' \, d^2\vec{p} \; e^{i \vec{p}\,' \vec{b}\,'} e^{-i \vec{p} \vec{b}} \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$$= \frac{1}{(2\pi)^4} \frac{1}{4} \int d^2(\vec{p}+\vec{p}\,') \, d^2(\vec{p}\,'-\vec{p}) \; e^{i(\vec{p}\,'-\vec{p}) \frac{\vec{b}\,'+\vec{b}}{2} - i(\vec{p}\,'+\vec{p}) \frac{\vec{b}\,'-\vec{b}}{2}}$$

$$\times \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$$= \frac{1}{(2\pi)^2} \delta^{(2)}(\vec{b}\,' - \vec{b}) \int d^2(\vec{p}\,'-\vec{p}) \; e^{i \vec{b} (\vec{p}\,'-\vec{p})} \langle p^+ \vec{p}\,' | \mathcal{O}(\vec{z}=\vec{0}) | p^+ \vec{p} \rangle$$

$\propto$ FOURIER TRF. OF $\mathrm{GPD}(x, \xi=0, t = -(\vec{p}\,'-\vec{p})^2)$



DIAGONAL IN PLUS-MOMENTUM AND IMPACT PARAMETER

$\rightarrow$ DENSITY OF PARTONS IN $(x, \vec{b})$ SPACE

$\int d^2\vec{b} \; \ldots \; = \mathrm{GPD}(x, \xi=0, t=0) \; = $ USUAL PARTON DENSITY

- CLOSER LOOK AT PROTON SPIN :

H : PROTON HELICITY DIAGONAL $\Rightarrow$ DENSITY

E : FLIP

$\Rightarrow$ TRICK : CHANGE BASIS TO "TRANSVERSITY" STATES

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left\{ |+\rangle_z + |-\rangle_z \right\}$$

$${}_x\langle p^+, \vec{0}, + | \mathcal{O}(\vec{z} = \vec{b}) | p^+, \vec{0}, + \rangle_x$$

$$= \frac{1}{2}\left\{ {}_z\langle p^+, \vec{0}, + | \mathcal{O} | p^+, \vec{0}, + \rangle_z + {}_z\langle p^+, \vec{0}, - | \mathcal{O} | p^+, \vec{0}, - \rangle_z \right\}$$

$$+ \frac{1}{2}\left\{ {}_z\langle p^+, \vec{0}, + | \mathcal{O} | p^+, \vec{0}, - \rangle_z + {}_z\langle p^+, \vec{0}, - | \mathcal{O} | p^+, \vec{0}, + \rangle_z \right\}$$

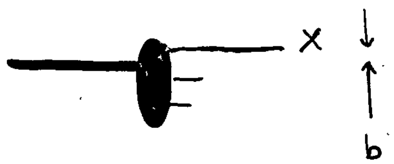
$$\propto \int d^2\Delta \; e^{-i\vec{\Delta}\vec{b}} \; H(x, 0, t = -\vec{\Delta}^2)$$

$$+ \int d^2\Delta \; \frac{i\Delta^y}{2m} \; E(x, 0, t = -\vec{\Delta}^2) \; e^{-i\vec{\Delta}\vec{b}}$$

$$= \int d^2\Delta \; e^{-i\vec{\Delta}\vec{b}} \; H \; \underbrace{- \frac{b^y}{m} \frac{\partial}{\partial \vec{b}^2} \int d^2\Delta \; e^{-i\vec{\Delta}\vec{b}} \; E}_{\text{DENSITY OF QUARKS SHIFTED ALONG } y \text{ AXIS IN STATE } |+\rangle_x}$$

$\Delta = p' - p$

- AS $x \to 1$

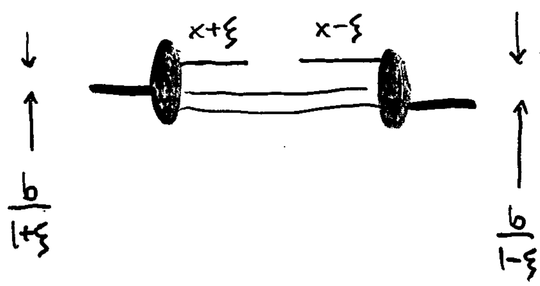


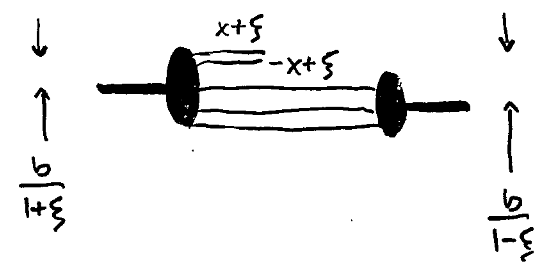
PARTON DOMINATES CENTER OF MOMENTUM

$\Rightarrow$ DISTRIBUTION IN b BECOMES PEAKED AT 0

$\Rightarrow$ t - DEPENCE OF $GPD(x \to 1, \xi, t)$ BECOMES FLAT

- FOR $\xi \neq 0$





- OVERALL SHIFT OF PROTON POSITION
- NO DENSITY ANY LONGER

5. SUM RULES

INTEGRATE DEF. OF H, E OVER x :

$$\int dx \; \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i x z^- P^+} \langle p' |\, \bar{q}(-\tfrac{z}{2})\, \gamma^+\, q(\tfrac{z}{2})\, | p \rangle \Big|_{\substack{z^+=0 \\ \vec{z}_\perp = 0}}$$

$$= \frac{1}{2P^+} \langle p' |\, \bar{q}(0)\, \gamma^+\, q(0)\, | p \rangle$$

LOCAL OPERATOR

$$= \frac{1}{2P^+}\, \bar{u}(p')\gamma^+ u(p) \underbrace{\int_{-1}^{1} dx\; H^q(x,\xi,t)}_{F_1^q(t)} + \frac{1}{2P^+}\, \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2m} u(p) \times \underbrace{\int_{-1}^{1} dx\; E^q(x,\xi,t)}_{F_2^q(t)}$$

DIRAC F.F.   PAULI F.F.

- ξ DEPENDENCE GONE AFTER $\int_{-1}^{1} dx$

LORENTZ INVARIANCE !

- HIGHER MOMENTS IN x

$$\frac{1}{2P^+}\,\bar{u}\gamma^+u\,\boxed{\int_{-1}^{1}dx\;x\,H^q}+\frac{1}{2P^+}\,\bar{u}\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\,u\,\boxed{\int_{-1}^{1}dx\;x\,E^q}$$

$$=\int dx\;x\;\frac{1}{2}\int\frac{dz^-}{2\pi}\,e^{ixz^-P^+}\,\langle p'|\,\bar{q}(-\tfrac{z}{2})\,\gamma^+q(-\tfrac{z}{2})\,|p\rangle_{\substack{z^+=0\\ \vec{z}=0}}$$

$$=\left(\frac{1}{2P^+}\right)^2\langle p'|\,\underbrace{\frac{i}{2}\,\bar{q}(0)\left(\overleftarrow{\partial}^+-\overrightarrow{\partial}^+\right)\gamma^+q(0)}_{T_q^{++}}\,|p\rangle$$

T_q^{++} QUARK CONTRIB'N TO ENERGY – MOMENTUM TENSOR

$$=\left(\frac{1}{2P^+}\right)^2\Big[\;A^q(t)\;\bar{u}\gamma^+u\;P^+ + B^q(t)\;\bar{u}\,\frac{\sigma^{+\alpha}\Delta_\alpha}{2m}\,u\;P^+$$

$$+\;C^q(t)\;\bar{u}u\;\frac{\Delta^+\Delta^+}{m}\;\Big]$$

GORDON IDENTITY: $\quad P^\mu\,\frac{\bar{u}u}{m}=\bar{u}\gamma^\mu u-\bar{u}\,\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m}\,u$

$$\Rightarrow\quad\int_{-1}^{1}dx\;x\;H^q(x,\xi,t)=A^q(t)+4\xi^2\,C^q(t)$$

$$\int_{-1}^{1}dx\;x\;E^q(x,\xi,t)=B^q(t)-4\xi^2\,C^q(t)$$

POLYNOMIAL IN ξ

- JI'S SUM RULE: $\quad J^q=\frac{1}{2}\left[A^q(0)+B^q(0)\right]$

$$=\frac{1}{2}\lim_{t\to 0}\int_{-1}^{1}dx\;x\left(H^q+E^q\right)$$

TOTAL ANGULAR MOM. ALONG z- AXIS

- HIGHER MOMENTS IN x

$$\frac{1}{2P^+}\,\bar{u}\gamma^+u \underbrace{\int_{-1}^{1}dx\; x\, H^q} + \frac{1}{2P^+}\,\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u \underbrace{\int_{-1}^{1}dx\; x\, E^q}$$

$$= \int dx\; x\; \frac{1}{2}\int\frac{dz^-}{2\pi}\, e^{ixz^-P^+}\, \langle p'|\,\bar{q}(-\tfrac{z}{2})\,\gamma^+ q(-\tfrac{z}{2})\,|p\rangle_{\substack{z^+=0\\ \vec{z}_\perp=0}}$$

$$= \left(\frac{1}{2P^+}\right)^2 \langle p'|\underbrace{\frac{i}{2}\,\bar{q}(0)\left(\overleftarrow{\partial}^+ - \overrightarrow{\partial}^+\right)\gamma^+ q(0)}_{T_q^{++}}|p\rangle$$

T_q^{++} QUARK CONTRIB'N TO ENERGY - MOMENTUM TENSOR

$$= \left(\frac{1}{2P^+}\right)^2 \Big[\; A^q(t)\;\bar{u}\gamma^+u\;P^+ + B^q(t)\;\bar{u}\frac{\sigma^{+\alpha}\Delta_\alpha}{2m}u\;P^+ + C^q(t)\;\bar{u}u\;\frac{\Delta^+\Delta^+}{m}\;\Big]$$

GORDON IDENTITY: $\quad P^\mu\,\dfrac{\bar{u}u}{m} = \bar{u}\gamma^\mu u - \bar{u}\,\dfrac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m}\,u$

$$\Rightarrow \int_{-1}^{1}dx\; x\; H^q(x,\xi,t) = A^q(t) + 4\xi^2\, C^q(t)$$

$$\int_{-1}^{1}dx\; x\; E^q(x,\xi,t) = B^q(t) - 4\xi^2\, C^q(t)$$

POLYNOMIAL IN ξ

- JI'S SUM RULE: $\quad J^q = \frac{1}{2}\left[A^q(0) + B^q(0)\right]$

$$= \frac{1}{2}\lim_{t\to 0}\int_{-1}^{1}dx\; x\,\left(H^q + E^q\right)$$

TOTAL ANGULAR MOM. ALONG z- AXIS

LATTICE QCD STUDIES

LOCAL MATRIX ELEMENTS $\langle p', s' | \bar{q}(0) \dots q(0) | p, s \rangle$

$\rightsquigarrow$ MOMENTS OF GPDs

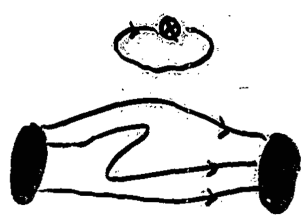
$\hat{=}$ FORM FACTORS

- PREVIOUS STUDIES OF $J^3_q = \frac{1}{2}\left[A_q(0) + B_q(0) \right]$
  MATUR ET AL '99
  GADYAK, JI, JUNG '01

- RECENT : QCDSF
  LPHC, SESAM $A_q(t)$, $B_q(t)$, $C_q(t)$

ONLY "CONNECTED" DIAGRAMS

NOT YET " DISCONNECTED "



FIND IN PARTICULAR :

$B^{u-d}(t)$ BIG $B^{u+d}(t)$ SMALL

AS IN $F_2^{u-d}(t)$ $F_2^{u+d}(t)$

CORRESPONDS TO LARGE N_c PREDICTIONS FOR GOEKE ET AL '01

E^{u-d} E^{u+d}

LATTICE QCD STUDIES

LOCAL MATRIX ELEMENTS $\langle p', s' | \bar{q}(0) \ldots q(0) | p, s \rangle$

$\rightsquigarrow$ MOMENTS OF GPDS

$\equiv$ FORM FACTORS

- PREVIOUS STUDIES OF $J^3_q = \frac{1}{2}\left[A_q(0) + B_q(0) \right]$

  MATUR ET AL '99

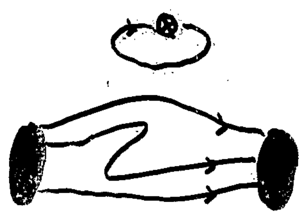
  GADIYAK, JI, JUNG '01

- RECENT : QCDSF

  LPHC, SESAM $\quad A_q(t),\ B_q(t),\ C_q(t)$

ONLY "CONNECTED" DIAGRAMS

NOT YET "DISCONNECTED"



FIND IN PARTICULAR :

$B^{u-d}(t)$ BIG $\qquad B^{u+d}(t)$ SMALL

AS IN $F_2^{u-d}(t) \qquad F_2^{u+d}(t)$

CORRESPONDS TO LARGE N_c PREDICTIONS FOR $\qquad$ GOEKE ET AL '01

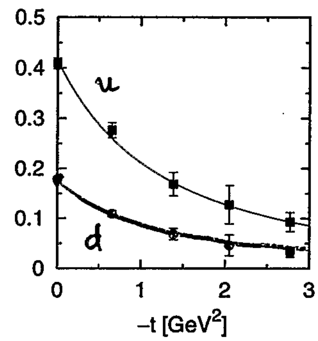
$E^{u-d} \sim N_c^3 \qquad E^{u+d} \sim N_c^2$

$H^{u-d} \sim N_c \qquad H^{u+d} \sim N_c^2 \quad \to u \pm d$

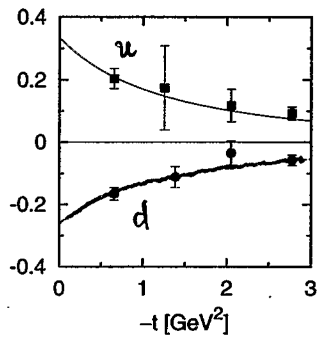
$\tilde{H}^{u-d} \sim N_c^2 \qquad \tilde{H}^{u+d} \sim N_c \quad \to \Delta u \pm \Delta d$

FOR $x, \xi \sim 1/N_c$ AND $-t \ll m^2$

A^q



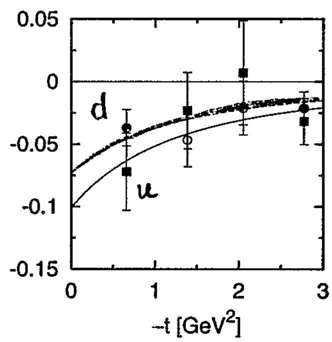
B^q



m_π = 870 MeV

μ = 2 GeV

C^q



M Göckeler *et al*, QCDSF Collaboration, hep-ph/0304249

- FORM COMBINATIONS

$$\underbrace{\frac{1}{2}\left[A^q(0) + B^q(0)\right]}_{J^q} - \frac{1}{2}\Sigma^q = L^q$$

$$\Sigma^q \to \int_0^1 dx\left[\Delta q(x) + \Delta\bar{q}(x)\right]$$

BOTH RECENT STUDIES(*) FIND

L^{u+d} COMPATIBLE WITH ZERO

L^{u-d} ??

(*) QCDSF hep-ph/0304249
LHPC, SESAM hep-lat/0304018

PROTON MATRIX ELEMENTS AT $\mu^2 = 4\,\mathrm{GeV}^2$ ($\overline{MS}$ SCHEME)

FROM QCDSF hep-ph/0304249

$$J_u = 0.37(6) \qquad J_d = -0.04(4)$$

$$\tfrac{1}{2}\Sigma_u = 0.42(1) \qquad \tfrac{1}{2}\Sigma_d = -0.12(1)$$

$$L_u = -0.05(6) \qquad L_d = 0.08(4)$$

EXTRAPOLATED TO $m_\pi \approx 135$ Me

FROM LPHC, SESAM hep-lat/0304018 AND J. NEGELE, PRIVATE COMM.

$$J_{u+d} = 0.338(4) \qquad J_{u-d} = 0.435(25)$$

$$\tfrac{1}{2}\Sigma_{u+d} = 0.341(9) \qquad \tfrac{1}{2}\Sigma_{u-d} = 0.595(9)$$

$$L_{u+d} = -0.003(10) \qquad L_{u-d} = -0.16(3)$$

FOR $m_\pi \simeq 900$ MeV

NB: SOME OF THE ERRORS ARE MINE !

PROTON MATRIX ELEMENTS AT $\mu^2 = 4\,GeV^2$ ($\overline{MS}$ SCHEME)

FROM QCDSF hep-ph/0304249

$J_u = 0.37(6)$ $\qquad$ $J_d = -0.04(4)$

$\frac{1}{2}\Sigma_u = 0.42(1)$ $\qquad$ $\frac{1}{2}\Sigma_d = -0.12\ (1)$

$L_u = -0.05(6)$ $\qquad$ $L_d = 0.08\ (4)$

EXTRAPOLATED TO $m_\pi \approx 135$ M.

FROM LPHC, SESAM hep-lat/0304018 AND J. NEGELE, PRIVATE COMM.

$J_{u+d} = 0.338(4)$ $\qquad$ $J_{u-d} = 0.435\ (25)$

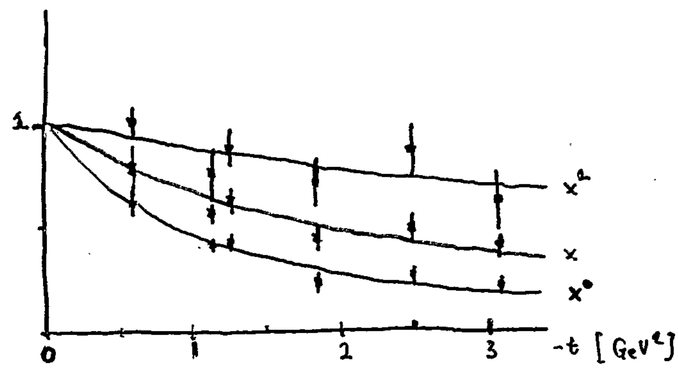
$\frac{1}{2}\Sigma_{u+d} = 0.341\ (9)$ $\qquad$ $\frac{1}{2}\Sigma_{u-d} = 0.595\ (9)$

$L_{u+d} = -0.003\ (10)$ $\qquad$ $L_{u-d} = -0.16\ (3)$

FOR $m_\pi \simeq 900$ MeV

NB: SOME OF THE ERRORS ARE MINE !

EARLIER STUDIES	GADIYAK et al '01	MATHUR et al '99
$J_{u+d}^{\text{connected}}$	0.47 (7)	0.44 (7)
$J_{u+d}^{\text{disconnected}}$	-0.12 (6)	0.09 (2)
	$m_\pi \simeq 210$ MeV	$m_\pi = 135$ MeV



J.NEGELE et al.
hep-lat/0309060

m_π = 870 MeV

$A^{u-d}_{n0}(t) \,/\, A^{u-d}_{n0}(0)$

n = 3
n = 2
n = 1

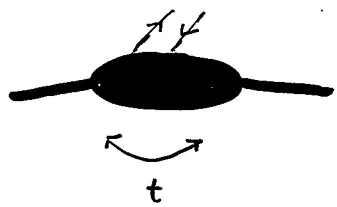
$$A^{u-d}_{n0}(t) = \int_{-1}^{1} dx\, x^{n-1} \; H^{u-d}(x, \xi = 0, t)$$

$\Rightarrow$ t- DEPENDENCE FLATTENS AS x INCREASES

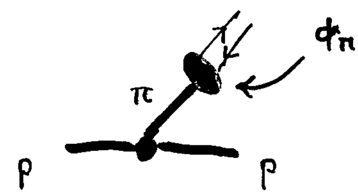
6. CHIRAL DYNAMICS

t-CHANNEL EXCHANGES

- IN $\tilde{E}^u - \tilde{E}^d$ ERBL REGION



=



+ OTHER CONTRIB'S

$\langle p | \bar{q} \gamma^+ \gamma_5 q | p \rangle$

$$\propto \frac{g_{\pi NN}}{t - m_\pi^2} \, \phi_\pi$$

$\langle \pi | \bar{q} \gamma^+ \gamma_5 q | 0 \rangle$
PION DISTRIBUTION AMPLITUDE

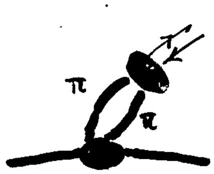
"π POLE CONTRIBUTION"

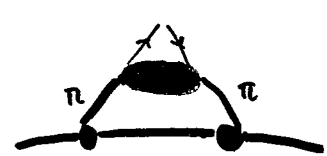
DOMINATES FOR SMALL $|t|$

- 2π EXCHANGE IN H AND E

ERBL REGION

DGLAP (ALSO FOR $\xi = 0$)





NEED SMALL $|t|$ FOR π's TO BE WEAKLY OFF-SHELL

$\tilde{E}^u - \tilde{E}^d$

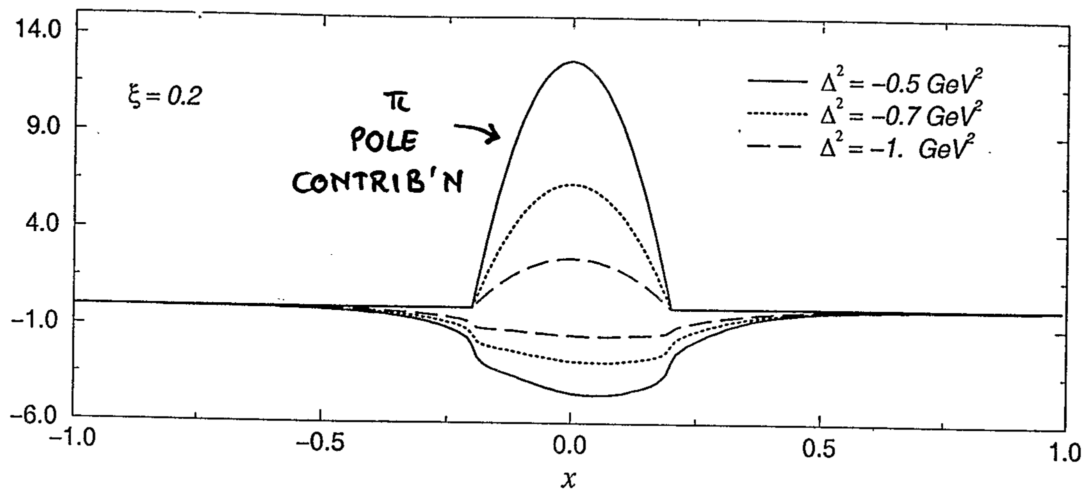
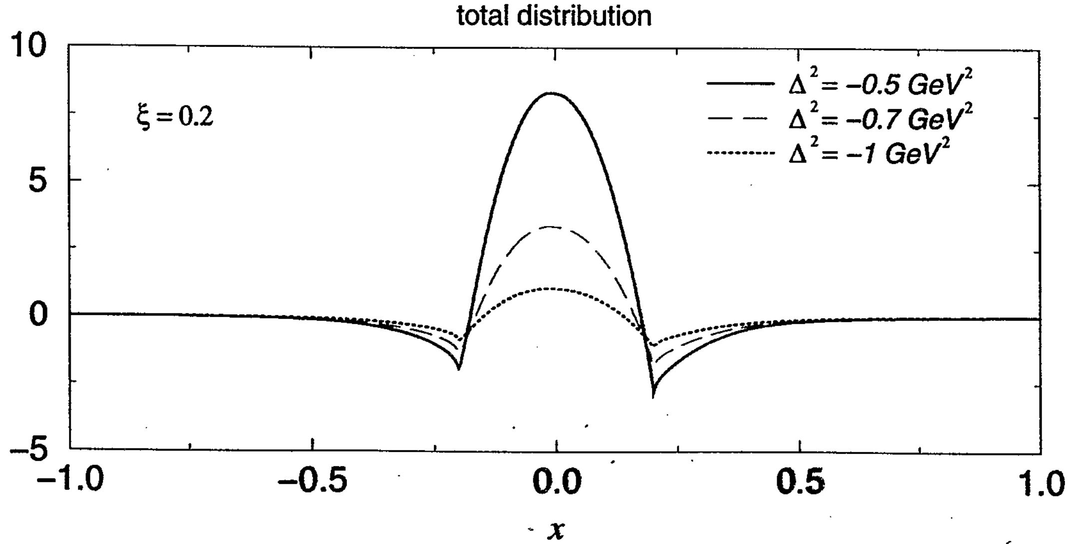


Figure 4: Comparison of pion pole contribution and non-pole part of isovector $\widetilde{E}$ at various values of Δ^2 and ξ. The positive curves correspond to pion pole contributions.

FROM CHIRAL QUARK-SOLITON MODEL

$\widetilde{E}^{(3)}(x, \xi, \Delta^2) \;=\; \tilde{E}^u - \tilde{E}^d$

total distribution



M. PENTTINEN et al.
hep-ph/9909489

7. MODEL BUILDING

(VERY BRIEF)

2 PHYSICS ASPECTS

- INTERPLAY OF t WITH LONGITUDINAL MOMENTUM FRACTION x, ξ

  - SUPER-SIMPLE ANSATZ

    $$H^q(x,\xi,t) = F_1^q(t)\; h^q(x,\xi) \qquad \text{etc.}$$

    $\rightarrow$ CORRECT FIRST x-MOMENT BY CONSTRUCTION

  MORE REALISTIC FORMS BEING EXPLORED

- INTERPLAY BETWEEN x AND ξ

  CHALLENGE :

  $$\int_{-1}^{1} dx\; x^n\, H = \text{POLYNOMIAL}(\xi) \quad \text{OF ORDER} \quad n+1$$

  $$\int_{-1}^{1} dx\; x^n\, E = \quad '' \qquad\qquad '' \qquad n+1$$

  $$\int_{-1}^{1} dx\; x^n\, (H+E) = \quad '' \qquad\qquad '' \qquad n$$

  POLYNOMIALITY

  SIMILAR FOR $\tilde{H}, \tilde{E}$

  - SUPER-SIMPLE ANSATZ "FORWARD MODEL"

    $$H^q(x,\xi,t) = F_1^q(t)\quad q(x) \qquad \text{for } x > 0 \qquad \text{etc.}$$

→ REPRESENT GPDs THROUGH DOUBLE DISTRIBUTIONS

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} d\alpha\, d\beta \;\; \delta(x-\xi\alpha-\beta) \;\; f(\beta,\alpha,t)$$

$$E(x,\xi,t) = \int d\alpha\, d\beta \;\; \delta(x-\xi\alpha-\beta) \;\; k(\beta,\alpha,t)$$

GUARANTEES POLYNOMIALITY (EXERCISE)

→ MODEL f, k

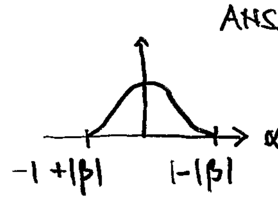
FORWARD LIMIT $\quad q(x) = \int_{|\alpha|\leq 1-x} d\alpha \;\; f(x,\alpha,0) \quad ; x>0$

$\bar{q}(x) = \ldots\ldots$ ANALOGOUS

ANSATZ $\quad f^q(\beta,\alpha,t) = F_1^q(t) \;\; q(\beta) \;\; h(\beta,\alpha)$

WITH $\int_{|\alpha|\leq 1-|\beta|} d\alpha \;\; h(\beta,\alpha) = 1$

↓ SIMPLE ANSATZ



→ REPRESENT GPDs THROUGH DOUBLE DISTRIBUTIONS

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\le 1} d\alpha\, d\beta \;\; \delta(x-\xi\alpha-\beta)\; f(\beta,\alpha,t) + D\left(\tfrac{x}{\xi}, t\right)$$

$$E(x,\xi,t) = \int d\alpha\, d\beta \;\; \delta(x-\xi\alpha-\beta)\; k(\beta,\alpha,t) - D\left(\tfrac{x}{\xi}, t\right)$$

GUARANTEES POLYNOMIALITY (EXERCISE)

$\int dx\, x^n H = \text{POLY}(\xi)$ ORDER n

→ "D - term"

→ MODEL f, k

ONLY ERBL: $D(z,t) \neq 0$ FOR $|z| \le 1$

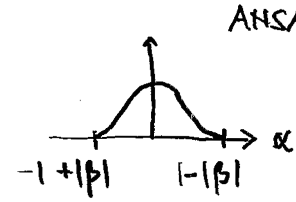
FORWARD LIMIT $\quad q(x) = \int_{|\alpha|\le 1-x} d\alpha \;\; f(x,\alpha,0) \quad ; x>0$

$\bar{q}(x) = \ldots\ldots$ ANALOGOUS

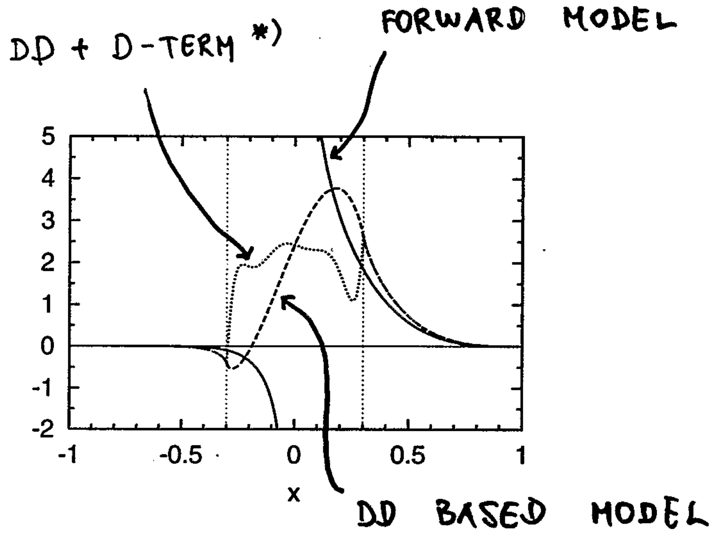
ANSATZ $\quad f^q(\beta,\alpha,t) = F_1^q(t)\;\; q(\beta)\;\; h(\beta,\alpha)$

WITH $\int_{|\alpha|\le 1-|\beta|} d\alpha \;\; h(\beta,\alpha) = 1$

↓ SIMPLE ANSATZ



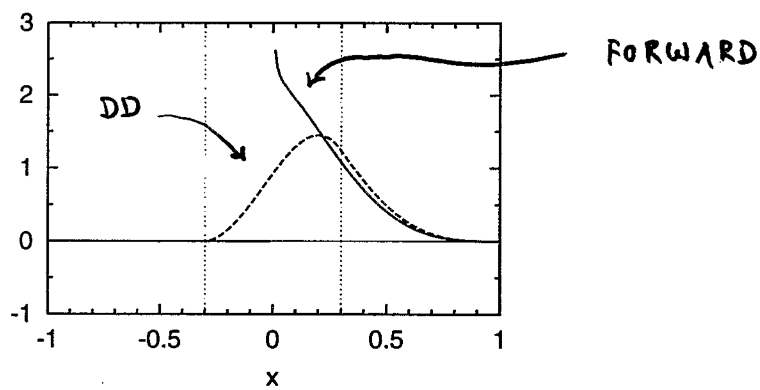
H^{u+d}



$\xi = 0.3$

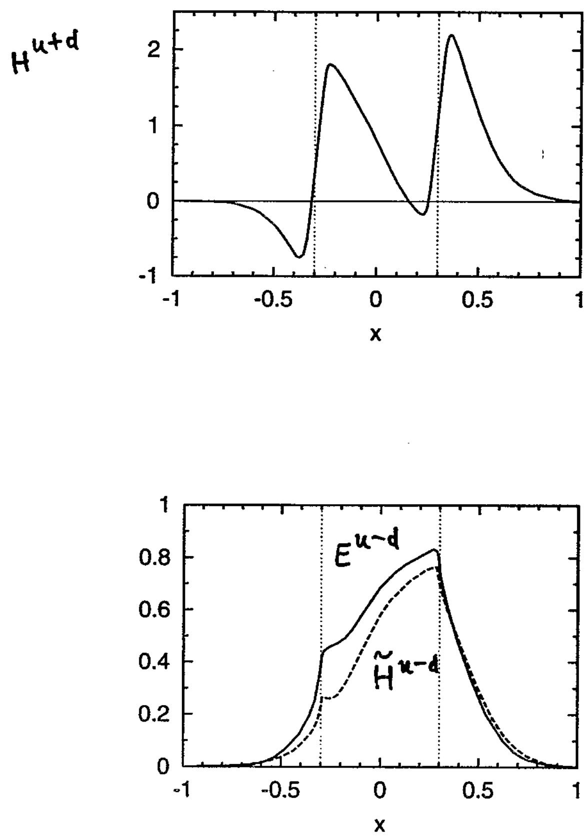
$t = -0.35\ GeV^2$

$\tilde{H}^{u-d}$



*) FROM CHIRAL QUARK-SOLITON MODEL.

CHIRAL QUARK - SOLITON MODEL



$\xi = 0.3$

$t = -0.35\ \mathrm{GeV}^2$

from: K Goeke, M V Polyakov and M Vanderhaeghen, hep-ph/0106012

SUMMARY (FIRST PART)

- GPDs = MATRIX ELEMENTS LIKE USUAL PARTON DESITIES BUT WITH $\langle p', s' | \ldots | p, s \rangle$
- HELICITY DEPENDENCE FOR PARTONS + HADRONS
  ORBITAL ANGULAR MOMENTUM
- $k_T \rightarrow b_T$ WHERE ARE $q, \bar{q}, g$ IN THE PROTON WHEN THEY ARE FAST, SLOW, POLARIZED, ...
- $\int dx \ldots$ GIVES FORM FACTORS
- "HOW MIGHT GPDs LOOK LIKE ?"
  - → DYNAMICAL CALCULATIONS / MODELS
  - → THE ART OF ANSATZ
  - → MEASUREMENTS

8. FACTORIZATION

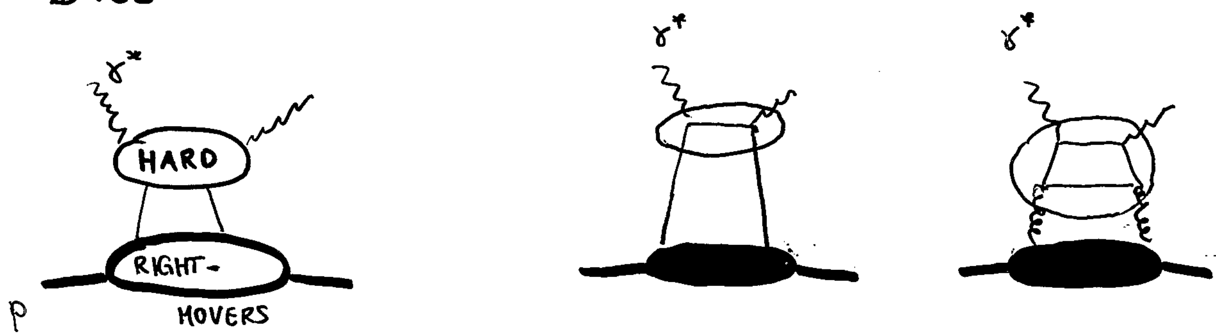
IN BJ LIMIT $\quad Q^2 \to \infty \qquad W^2/Q^2$ FIXED

t FIXED

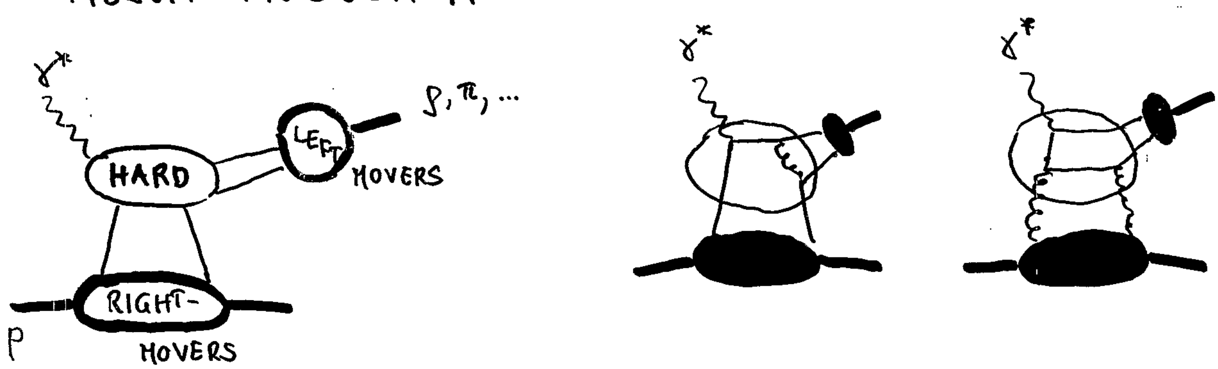
$(\text{i.e. } Q^2 \sim W^2 \gg |t|, \Lambda^2_{QCD})$

UP TO POWER SUPPRESSED $(\sim 1/Q^n)$ TERMS

DVCS



MESON PRODUCTION



- EXTRA INTERNAL PARTON AT LO (α_s)

→ EXTERNAL HARD MOMENTUM $\sim Q$ GETS "DILUTED"

FLAVOR SEPARATION (on p target)

DVCS : $\frac{4}{9}u + \frac{1}{9}d + \frac{1}{9}s \quad + \mathcal{O}(\alpha_s)\, g$

ρ : $\frac{1}{\sqrt{2}}\left(\frac{2}{3}u + \frac{1}{3}d + \frac{3}{8}g \right)$

ω : $\frac{1}{\sqrt{2}}\left(\frac{2}{3}u - \frac{1}{3}d + \frac{1}{8}g \right)$

ϕ : $\frac{1}{3}s + \frac{1}{8}g$

SPIN "FILTER"

DVCS : $H, E, \tilde{H}, \tilde{E}$

Vector Mesons : H, E

Pseudoscalar Mesons : $\tilde{H}, \tilde{E}$

} FROM PARITY INVARIANCE

SCALING :

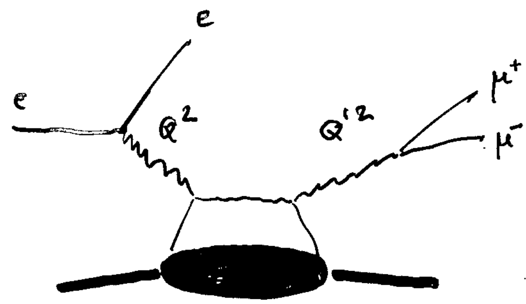
$$\mathcal{A}_{DVCS} \sim \int_{-1}^{1} dx \, \frac{1}{\xi - x - i\epsilon} \, GPD(x, \xi, t; \mu^2 = Q^2) \pm (x \to -x) + \mathcal{O}(\alpha_s)$$

$$= \mathcal{A}_{DVCS}(\xi, t, \ln Q^2) \quad \text{BJ SCALING}$$

$$\mathcal{A}_{MESON} \sim \frac{\alpha_s}{Q} \int_{-1}^{1} \ldots \text{(the same)}$$

$$= \frac{1}{Q} \tilde{\mathcal{A}}(\xi, t, \ln Q^2)$$

DOUBLE DVCS



VARIABLES : DVCS $\rightarrow$ DOUBLE DVCS

$$Q^2, \xi, t \qquad Q^2, Q'^2, \xi, t$$

$$\downarrow$$

$$Q^2 + Q'^2, \rho, \xi, t$$

$$\xi = \frac{Q^2 + Q'^2}{2W^2 + Q^2 - Q'^2}$$

$$\rho = \frac{Q^2 - Q'^2}{2W^2 + Q^2 - Q'^2}$$

$$|\rho| \leq \xi$$

$A(\gamma^* p \rightarrow \gamma^* p)$

$$\sim \int dx \; GPD(x, \xi, t) \frac{1}{\rho - x - i\epsilon} \pm (x \rightarrow -x)$$

$$= i\pi \, GPD(\rho, \xi, t) \pm (\rho \rightarrow -\rho) + \text{real part}$$

- SCAN GPDs IN ERBL REGION

HELICITY SELECTION RULES

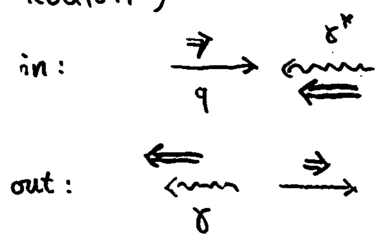
- HARD SCATTERING EVALUATED IN COLLINEAR APPROX.

  $\Rightarrow$ ZERO DEGREE SCATTERING

- NEGLECT LIGHT QUARK MASSES

  $\Rightarrow$ CHIRALITY CONSERVED

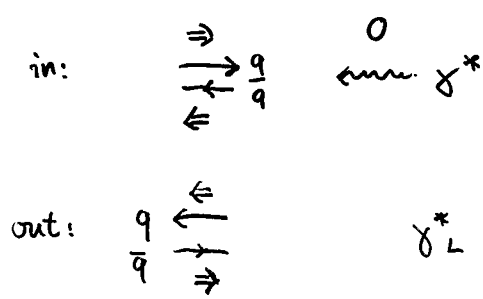
DVCS : (DGLAP REGION)



$\gamma^*_T \to \gamma_T$

$\gamma^*_L \to \gamma_T$ IS $1/Q$ SUPPRESSED

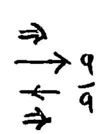
MESONS : (ERBL REGION)



$\gamma^*_L \to \text{Meson}_L$

LEADING

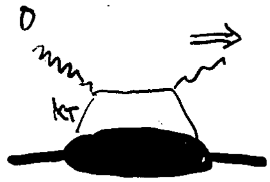
NB :

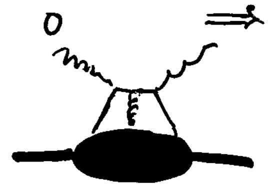


DOES NOT WORK

POWER - SUPPRESSED TRANSITIONS

DVCS





CAN STILL WRITE AS(*) HARD SCATTERING $\otimes$ GPDs

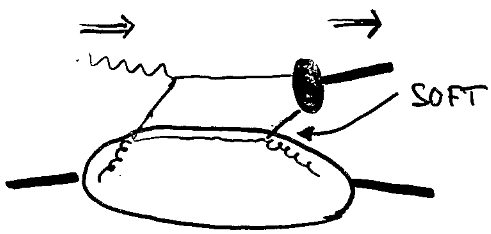
$\sim 1/Q$

" TWIST 3 "

$\sim$ LIKE g_2 IN DIS

(*) EMPIRICAL FACT UP TO $O(\alpha_s)$

MESONS

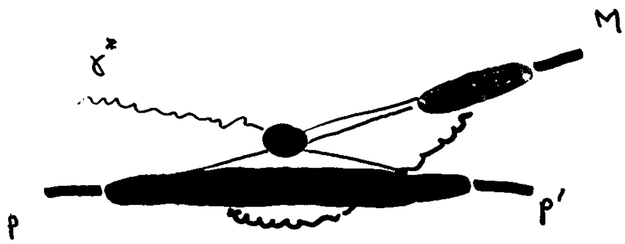


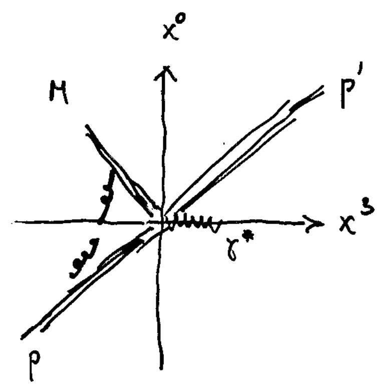
FACTORIZATION IS BROKEN

$\sim 1/Q^2$

IF NEVERTHELESS CALCULATE IN COLLINEAR APPROX. , GET LOGARITHMICALLY DIVERGENT INTEGRALS

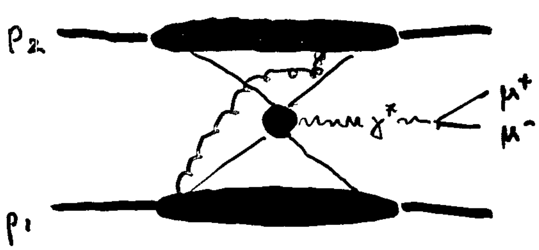
FACTORIZATION BREAKING SOFT EXCHANGES

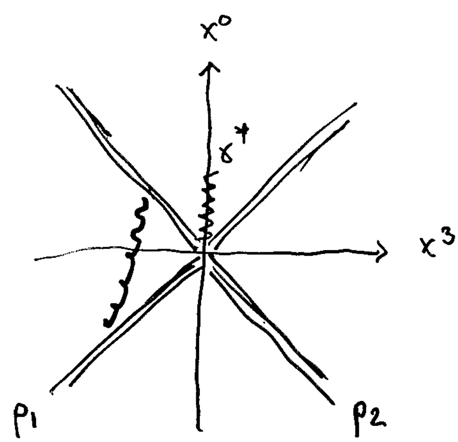




ALSO HAPPENS IN EXCLUSIVE HADRON-HADRON PROCESSES

e.g. $pp \to pp + \mu^+\mu^-$





FOR FACTORIZATION IN pp NEED SUM OVER FINAL STATES

9. MESON PRODUCTION

WITH γ^* AND MESON LONGITUDINAL, HAVE ONLY 2 OBSERVABLES AT LE$_{A}$DING POWER

- $\frac{d\sigma}{dt}(\gamma^* p \to M p) \sim \frac{1}{Q^6}$ (logs Q^2) AT FIXED x_B, t

$$\updownarrow$$

$$\xi = \frac{x_B}{2-x_B}$$

$$\underset{\text{MESON}}{\overset{\text{VECTOR}}{\sim}} \ldots |\underset{\int dx \ldots H}{\underset{\uparrow}{\mathcal{H}}}|^2 + \ldots |\underset{\int dx \ldots E}{\underset{\uparrow}{\mathcal{E}}}|^2 + \ldots \operatorname{Re}(\mathcal{H}^* \mathcal{E})$$

FOR PSEUSCALAR: ADD A ~ , DIFFERENT KINEMATICAL FACTORS

- SSA FOR TRANSVERSELY POLARIZED TARGET

$$\frac{d\sigma}{dt}(p^\downarrow) - \frac{d\sigma}{dt}(p^\uparrow) \underset{\text{MESONS}}{\underset{\text{VECTOR}}{\sim}} \ldots \operatorname{Im}(\mathcal{H}^* \mathcal{E})$$

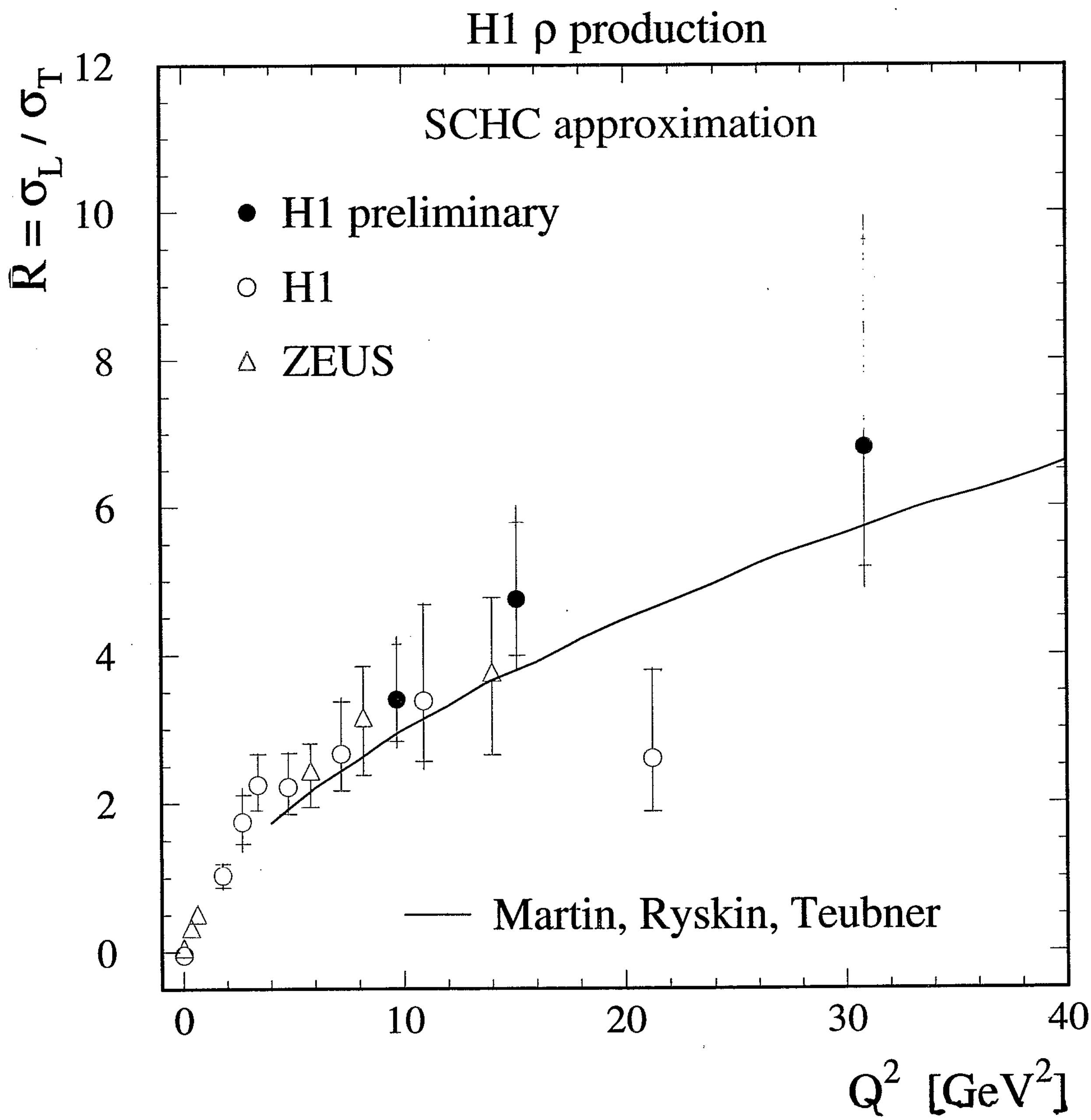
X. JANSSEN, H1 COLL.

DIS 2002

H1 - prelim. - 02-015

ρ HELICITY FROM ANGULAR DISTRIBUTION OF $\rho \to \pi\pi$

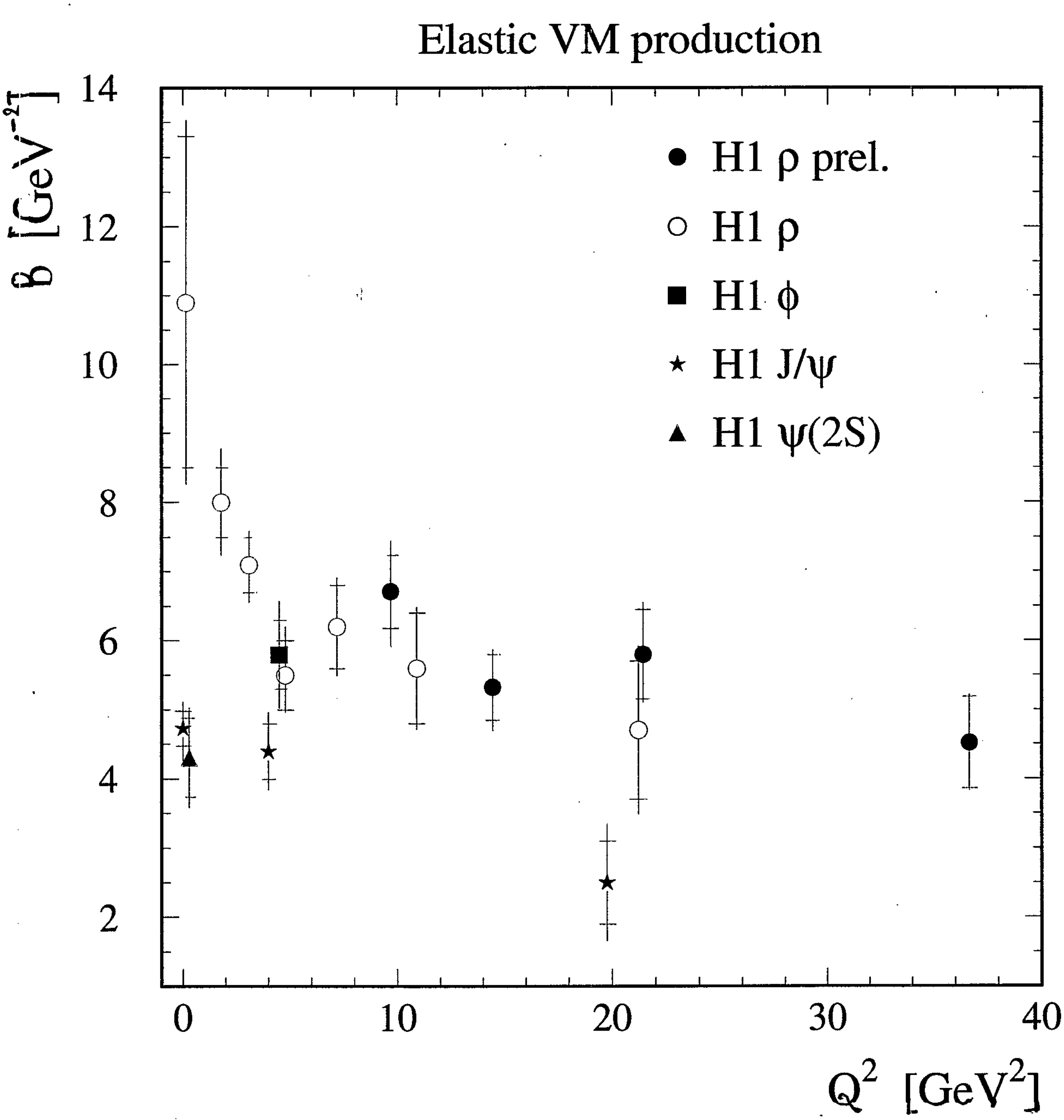
$\xrightarrow[\text{SCHC}]{}$ γ^* HELICITY



X. JANSSEN , H1 COLL. , DIS 2002

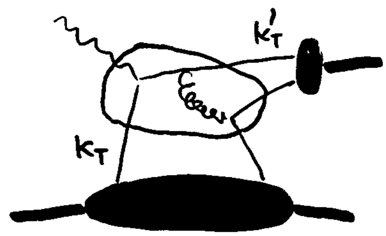
H1 - prelim - 02 - 015

FROM $\frac{d\sigma}{dt} \propto e^{Bt}$ AT SMALL $|t|$



ONE SOURCE OF POWER - SUPPRESSED EFFECTS :

"INTRINSIC k_T"



$$\int dx\, dz \quad GPD(x, \dots)\ T_H(x, z, \xi)\ \phi(z)$$

$$\Big\downarrow$$

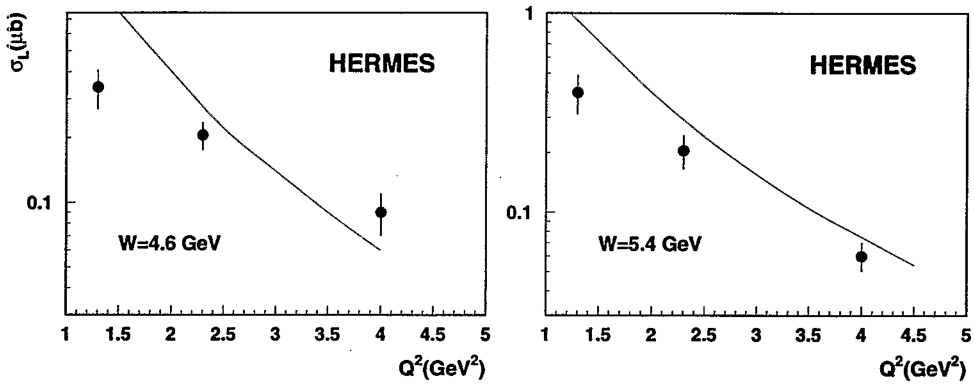
$$\int dx\, dz\ d^2k_T\, d^2k_T'\ GPD'(x, k_T, \dots)\ T_H'(x, z, \xi, k_T, k_T')\ \psi(z, k_T')$$

MODEL CALCULATIONS $\rightarrow$

SUBSTANTIAL EFFECTS EVEN

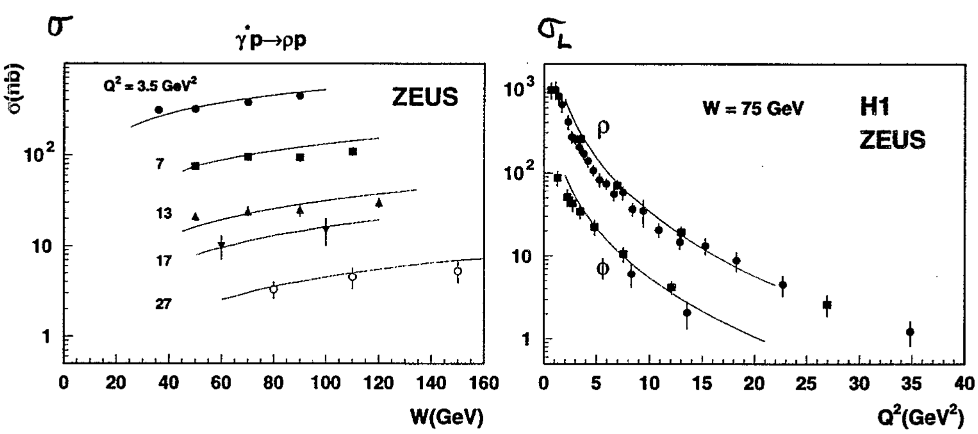
FOR $Q^2 \sim$ few GeV^2

$\gamma^* p \to \rho^0 p$



data: A Airapetian *et al*, HERMES Collaboration, hep-ex/0004023

theory curves: M Guidal WITH INTRINSIC k_T IN MESON AND PROTON



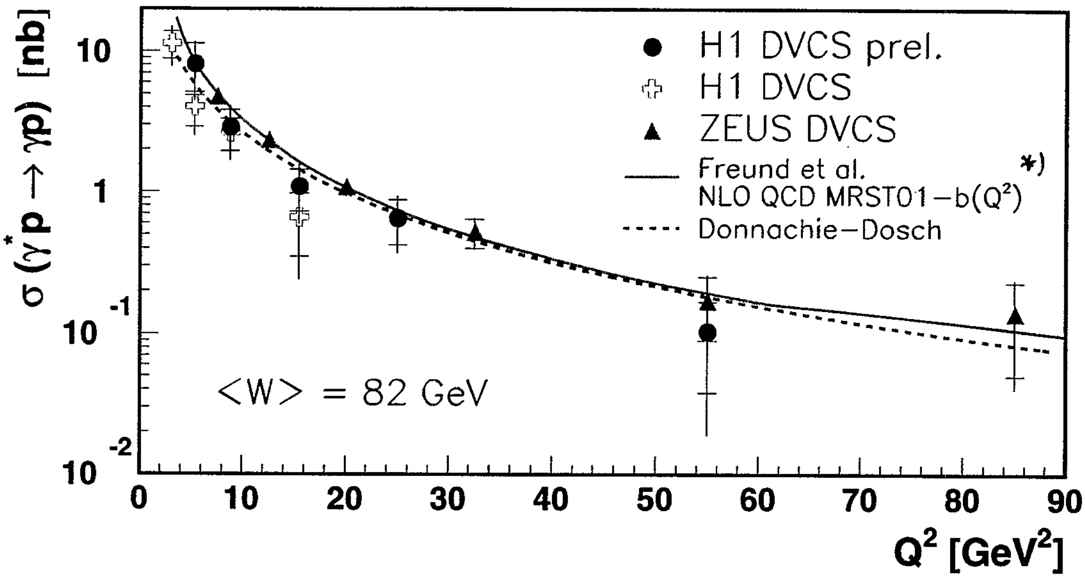
data from: hep-ex/9507001, hep-ex/9808020, hep-ph/9908519

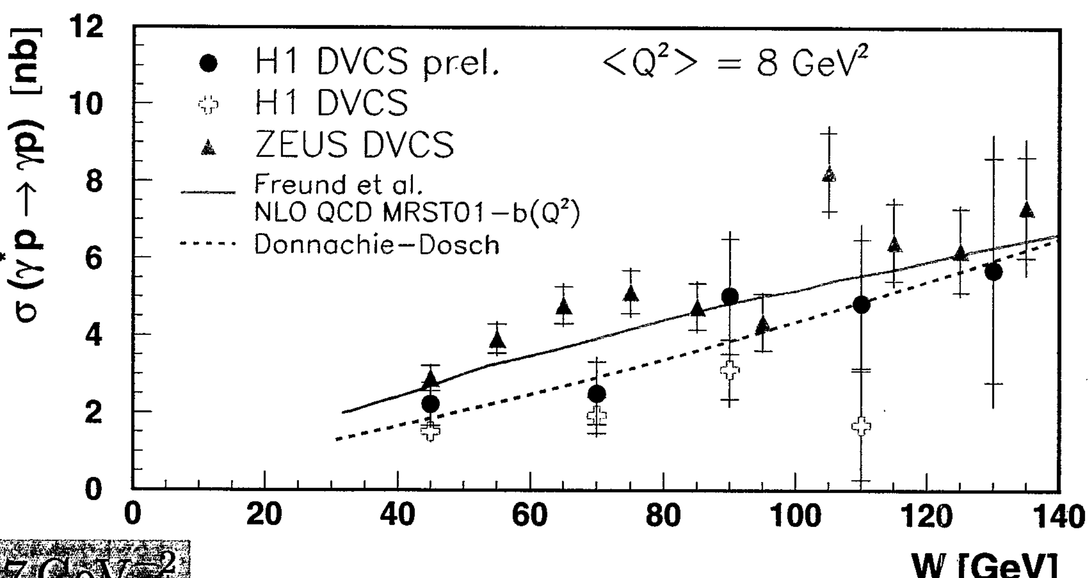
theory curves: S V Goloskokov, P Kroll and B Postler, hep-ph/0308140

WITH INTRINSIC k_T IN MESON

L. FAVART EPS 03 Aachen

All H1 and ZEUS Results





$b = 7\ \mathrm{GeV}^{-2}$

$\Longrightarrow$ Good agreement between H1 results

$\Longrightarrow$ Fair agreement between H1-prel and ZEUS results except for $W \sim 70$ GeV: H1 lower by 2σ

*) hep-ph/0306012

10. DVCS $\quad ep \to ep\gamma$

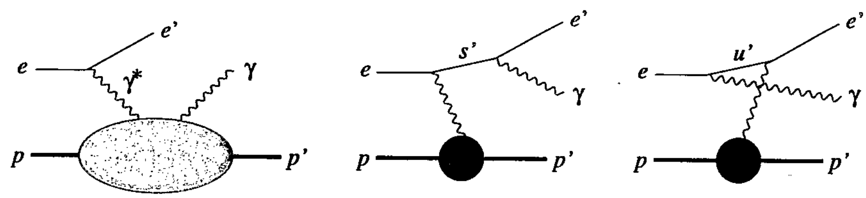
VARIABLES : $Q^2, t, \quad x_B$ (OR ξ)

$$y = \frac{q \cdot p}{k \cdot p} = \frac{Q^2}{x_B\,(s_{ep} - m^2)}$$

BEAM MOMENTUM

ϕ

COMPTON $\qquad$ BETHE - HEITLER



(a) $\qquad$ (b)

PARAMETRIZED BY

$F_1(t)$, $F_2(t)$

10. DVCS $ep \to ep\gamma$

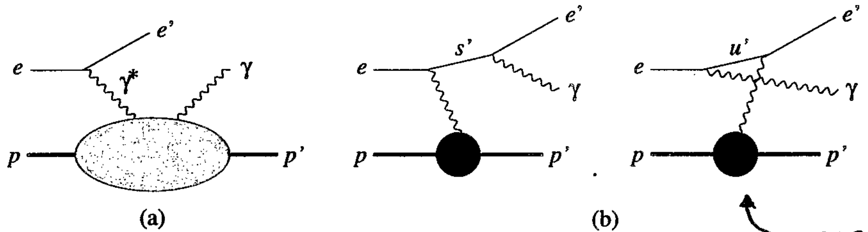
VARIABLES : $Q^2,\ t,\quad x_B$ (OR ξ)

$$y = \frac{q\cdot p}{k\cdot p} = \frac{Q^2}{x_B(s_{ep} - m^2)}$$

BEAM MOMENTUM

ϕ

COMPTON BETHE - HEITLER



(a) (b)

PARAMETRIZED BY $F_1(t)$, $F_2(t)$

$d\sigma_{COMP} \sim \frac{1}{Q^2}\ \frac{1}{y^2}$ FROM $e \to \gamma^* e$ VERTEX

FROM γ^* PROPAGATORS

$d\sigma_{BH} \sim \frac{1}{t}$

$\Rightarrow \quad d\sigma_{COMP} \ll d\sigma_{BH}$ UNLESS y SMALL (s_{ep} LARGE)

$d\sigma_{INT}$ INTERFERENCE $\rightarrow$ PHASE INFORMATION ON $A(\gamma^* p \to \gamma p)$

FILTERED OUT IN $d\sigma(e^+) - d\sigma(e^-)$

10. DVCS $\quad ep \to ep\gamma$

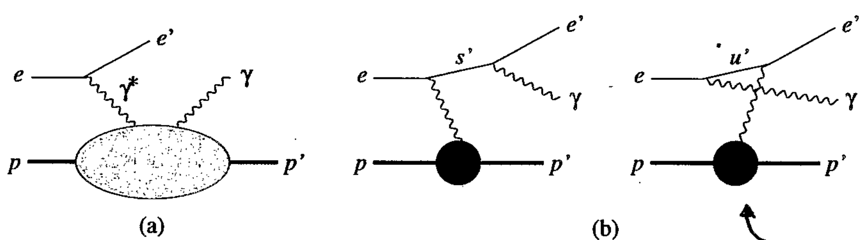
VARIABLES : $Q^2,\ t,\ \ x_B$ (OR ξ)

$$y = \frac{q\cdot p}{k\cdot p} = \frac{Q^2}{x_B\,(s_{ep} - m^2)}$$

BEAM MOMENTUM

ϕ

COMPTON $\qquad$ BETHE - HEITLER



PARAMETRIZED BY

$F_1(t)\ ,\ F_2(t)$

MODEL - INDEPENDENT ANALYSIS :

PARAMETRIZE $\gamma^* p \to \gamma p$ BY ITS HELICITY AMPLITUDES

OR FORM FACTORS

FUNCTIONS OF Q^2, t, x_B

HOW CAN GET INFORMATION ON

γ^* HELICITY ??

10. DVCS $\quad ep \to ep\gamma$

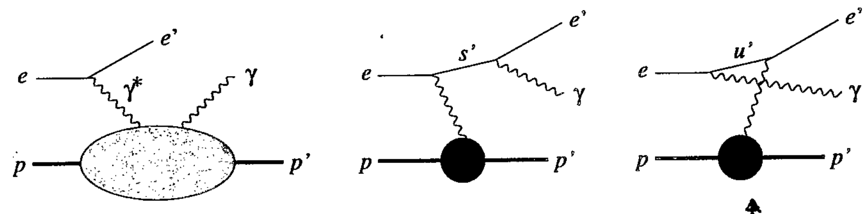
VARIABLES : $Q^2,\ t,\ x_B$ (OR ξ)

$$y = \frac{q \cdot p}{k \cdot p} = \frac{Q^2}{x_B\,(s_{ep} - m^2)}$$

k ← BEAM MOMENTUM

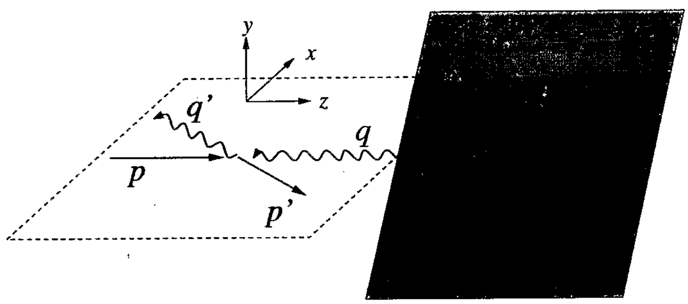
ϕ ROTATION ANGLE AROUND γ^* AXIS

COMPTON $\qquad$ BETHE - HEITLER



(a) $\qquad$ (b)

PARAMETRIZED BY $F_1(t)$, $F_2(t)$



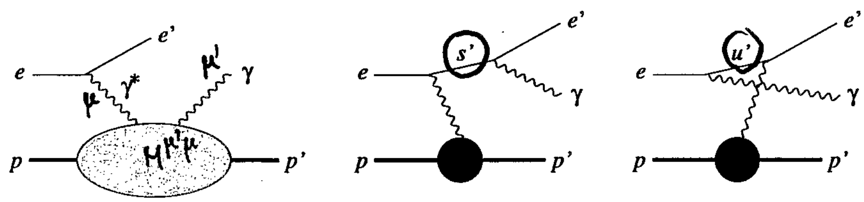
OPERATOR OF ROTATION AROUND z :

$$e^{i\phi J_z}$$

- $d\sigma_{BH}$ *) DROPS OUT IN SINGLE SPIN ASY'S (BEAM OR SPIN $\frac{1}{2}$ TARGET)

(*) BUT NOT $d\sigma_{COMP}$

→ ALSO ACCESS TO $d\sigma_{INT}$



(a) (b)

$$\frac{d\sigma_{INT}}{d\phi\, dx_B\, dQ^2\, dt} = e_\ell \Big[\ldots \cos\phi \quad \mathrm{Re}\, \hat{M}^{++} + \ldots \cos 2\phi \quad \mathrm{Re}\, \hat{M}^{+0}$$

$$+ \ldots \cos 3\phi \quad \mathrm{Re}\, \hat{M}^{+-} \quad + O(1/Q) \Big]$$

$$+ \quad e_\ell P_\ell \Big[\ldots \quad \sin\phi \quad \mathrm{Im}\, \hat{M}^{++} + \ldots \sin 2\phi \quad \mathrm{Im}\, \hat{M}^{+0}$$

$$+ O(1/Q) \Big]$$

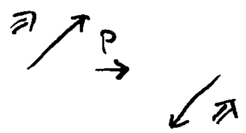
$\hat{M}^{\mu'\mu}$ = SUPERPOSITION OF DVCS HELICITY AMPLITUDES

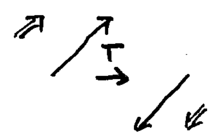
ϕ - DEPENDENCE : ANALYZER OF γ^* HELICITY

HAVE EXTRA KNOWN ϕ -DEP'CE FROM PROPAGATORS $1/s'u'$

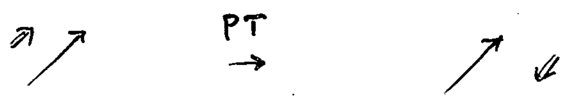
- WHY DOES BH DROP OUT IN SINGLE-SPIN ASYMM'IES ?

- IF PARITY IS CONSERVED, ANY SSA FOR SPIN $\frac{1}{2}$ REQUIRES DYNAMICAL PHASE (="Re AND Im PART" IN AMPLITUDE)





SO THAT



T ALSO CHANGES INITIAL ↔ FINAL STATE

$$d\sigma(\vec{s}) - d\sigma(-\vec{s}) \sim |\langle f|T|i\rangle|^2 - |\langle i|T|f\rangle|^2$$

$$= |\langle f|T|i\rangle|^2 - |\langle f|T^{+}|i\rangle|^2$$

$$= \tfrac{1}{2}\left(\langle f|T|i\rangle + \langle f|T^{+}|i\rangle\right)^{*} \cdot \left(\langle f|T|i\rangle - \langle f|T^{+}|i\rangle\right) + \text{c.c.}$$

NB: $\langle f|T^{+}|i\rangle = \langle f|T|i\rangle^{*}$ ONLY IN SIMPLE CASES

- HOW TO GET $\langle f | T | i \rangle \neq \langle f | T^+ | i \rangle$?

→ UNITARITY

TRANSITION FROM $|i\rangle$ TO $|f\rangle$ $\sim$ $\langle f | S | i \rangle$

$$\sum_f | \langle f | S | i \rangle |^2 = \sum_f \langle i | S^+ | f \rangle \langle f | S | i \rangle$$

$$= \langle i | S^+ S | i \rangle \overset{!}{=} 1 = \langle i | i \rangle$$

$\Rightarrow$ $S^+ S = \mathbb{1}$ UNITARY

TRANSITION MATRIX $S = \mathbb{1} + i T$

$$\mathbb{1} = S^+ S = (\mathbb{1} - i T^+)(\mathbb{1} + i T) = \mathbb{1} + T^+ T + i (T - T^+)$$

$$\Rightarrow \quad T^+ T = \frac{1}{i} (T - T^+)$$

$$\sum_z \langle Y | T^+ | z \rangle \langle z | T | X \rangle = \sum_z \langle z | T | Y \rangle^* \langle z | T | X \rangle$$

$$\frac{1}{i} \left(\langle Y | T | X \rangle - \langle Y | T^+ | X \rangle \right)$$

for $X = Y$ get

$$\sum_z | \langle z | T | X \rangle |^2 = 2 \, \mathrm{Im} \, \langle X | T | X \rangle$$

OPTICAL THEOREM

WHICH GPDs SENSITIVE TO ?

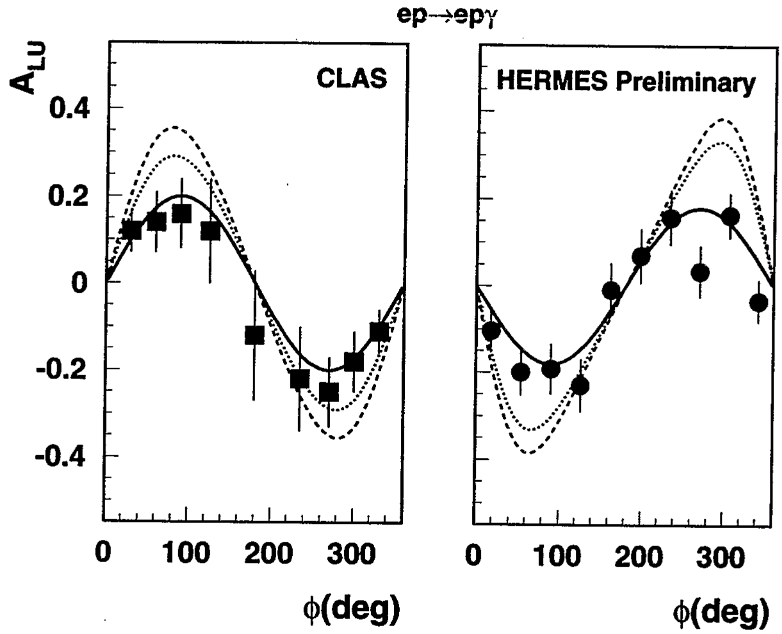
UNPOLARIZED TARGET :

INTEGRAL $\int dx \ldots H$

$$\hat{M}^{++} = \sqrt{1-\xi^2}\ \frac{\sqrt{t_0-t}}{2m} \left[F_1\, \mathcal{H} + \xi (F_1+F_2)\, \tilde{\mathcal{H}} - \frac{t}{4m^2} F_2\, \mathcal{E} \right]$$

SENSITIVITY TO E WITH TRANSVERSE TARGET

$$\rightarrow \quad -\frac{t}{4m^2} F_2\, \mathcal{H} + \frac{t}{4m^2} F_1\, \mathcal{E} + \text{SUPPRESSED TERMS}$$



BEAM SPIN ASY

$$A_{LU}(\phi) = \frac{d\sigma(e\uparrow) - d\sigma(e\downarrow)}{\text{''} + \text{''}}$$

S Stepanyan *et al*, CLAS Collaboration, hep-ex/0107043

F Ellinghaus *et al*, HERMES Collaboration, hep-ex/0212019

	$\langle Q^2 \rangle$ [GeV^2]	$\langle x_B \rangle$	$\langle -t \rangle$ [GeV^2]	$\langle y \rangle$
CLAS	1.25	0.19	0.19	0.82
HERMES	2.5	0.12	0.18	0.4

$$A_c(\phi) = \frac{d\sigma(e^+)/d\phi \;-\; d\sigma(e^-)/d\phi}{'' \;+\; ''}$$

$\langle Q^2 \rangle = 2.8\ GeV^2$

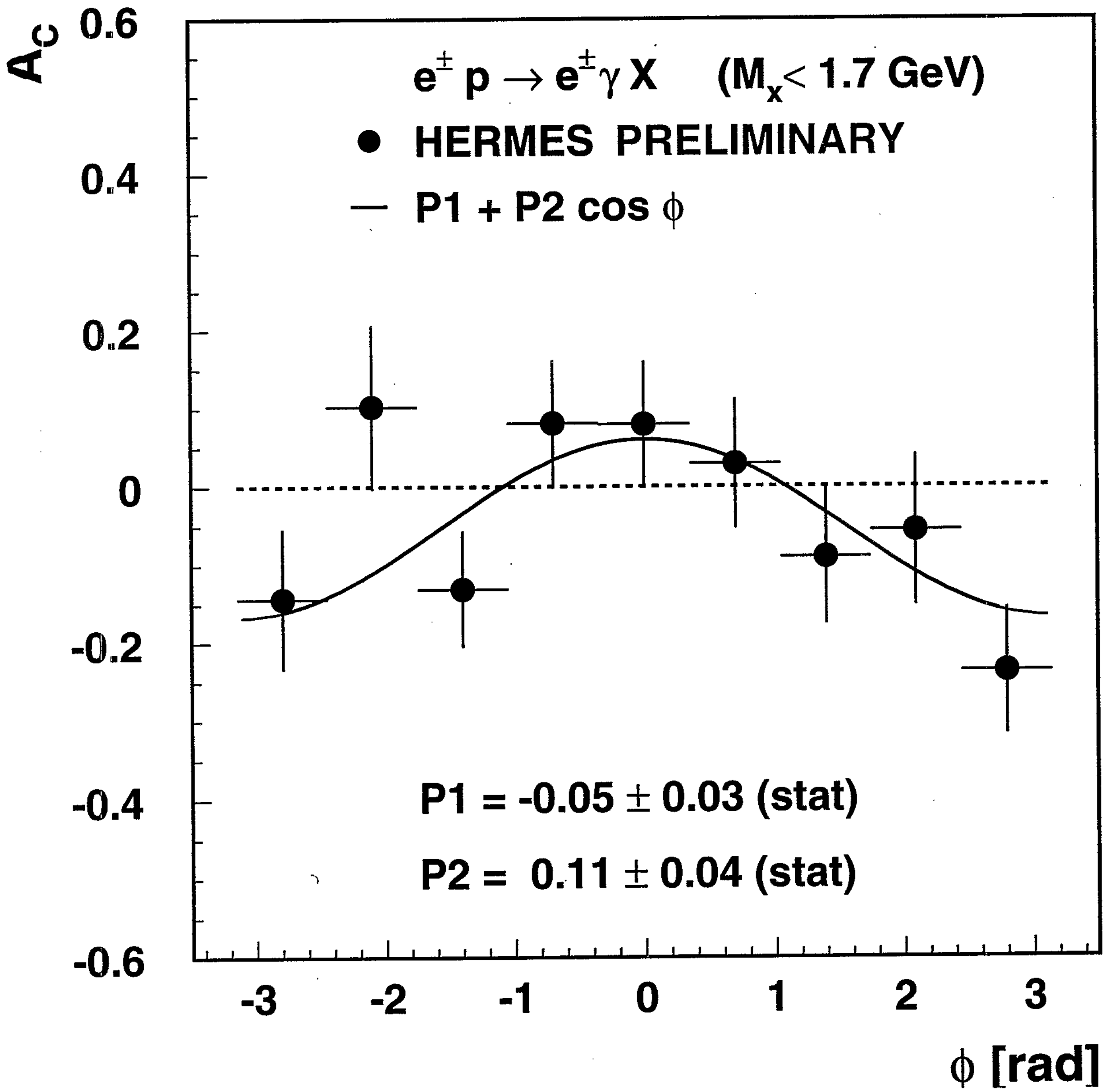
$\langle x_B \rangle = 0.12$

$\langle -t \rangle = 0.27\ GeV^2$

$\langle y \rangle = 0.45$

F. ELLINGHAUS, HERMES COLL.

HERMES 02-022



SUMMARY (PART 2)

- FACTORIZATION → SCALING BEHAVIOR
  HELICITY SELECTION RULES

  NOT EVERYTHING FACTORIZES

- DVCS : MOST DETAILED ACCESS TO GPDS

- MESONS : MANY CHANNELS
  → HANDLE ON q VS. g, FLAVOR, SPIN

  POWER CORRECTIONS MORE SERIOUS

SUMMARY

- GPDs : * FRAMEWORK TO CONNECT VARIOUS ASPECTS OF q/g STRUCTURE IN HADRONS

  * "PUT PARTON PICTURE INTO 3D"
    → O.A.M.

- SPIN : * ASPECT OF HADRON STRUCTURE

  * TOOL TO CONTROL HARD SUBPROC.