Single Spin Asymmetries

Lecture Two

Jianwei Qiu
Iowa State University

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Outline of Lecture Two

- Review: Single spin asymmetry within the collinear factorization
- Semi-inclusive deeply inelastic scattering
- K_T- factorization
- Sivers effect and Sivers functions
- Collins effect and Collins functions
- Open questions and discussion

Perturbative QCD factorization (II)

- **K_T–factorization** $\Leftrightarrow$ $Q \sim xp\cdot n \approx k_T \sim \sqrt{k^2}$

$$\sigma^h_{phys} = \hat{\sigma}^i_2(x,k_T)\otimes T_2^{i/h}(x,k_T)$$

Next lecture

No all-order proof of k_T – factorization for an arbitrary k_T

- **Collinear factorization:**

$Q \sim xp\cdot n \sim k_T, \sqrt{k^2}$

$$\sigma^h_{phys} = \hat{\sigma}^i_2 \otimes [1+\alpha_s+\alpha_s^2+\ldots]\otimes T_2^{i/h}(x) + \frac{\hat{\sigma}^i_3}{Q}\otimes[1+\alpha_s+\alpha_s^2+\ldots]\otimes T_3^{i/h}(x) + \frac{\hat{\sigma}^i_4}{Q^2}\otimes[1+\alpha_s+\alpha_s^2+\ldots]\otimes T_4^{i/h}(x) + \ldots$$

Leading Twist

perturbative

Power corrections

Provides systematic ways to quantify high order corrections

Factorization fails beyond the next-to-leading powers in hadronic collisions

Single spin asymmetry within the collinear factorization

❖ **Generic twist-3 factorized contributions**

$$\Delta\sigma_{AB\to h}(\vec{s}_T) = \sum_{abc} T^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes f_{b/B}(x') \otimes \hat{\sigma}_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z) \quad (1)$$

Provides hadron spin dependence

$$+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T)$$

transversity

Even γ's !

$$\otimes \left\{ f_{b/B}(x') \otimes \hat{\sigma}'_{ab\to c}(\vec{s}_T) \otimes D^{(3)}_{c\to h}(z_1, z_2) \right\} \quad (2)$$

Even γ's !

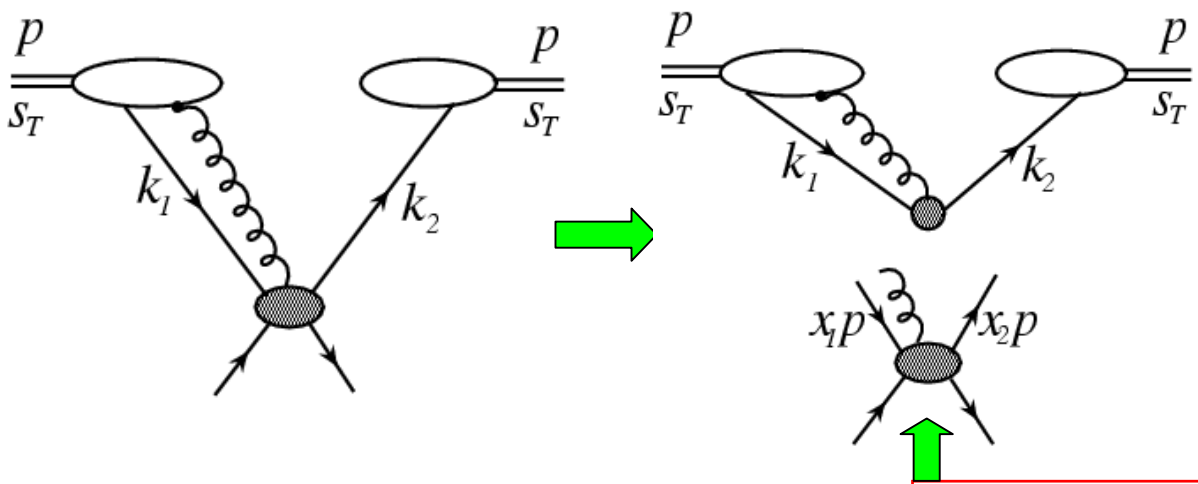
$$(3) \left\{ + f^{(3)}_{b/B}(x'_1, x'_2) \otimes \hat{\sigma}''_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z) \right\}$$

(1) Calculated by Qiu and Sterman, Phys. Rev. D, 1999
(2) Not yet be calculated
(3) Calculated by Kanazawa and Koike, Phys. Lett. B, 2000

(3) << (1)

Dynamical twist-3 contribution to A_N

❖ Factorization:



$$T_F(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

$$T_D(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp \psi \rangle$$

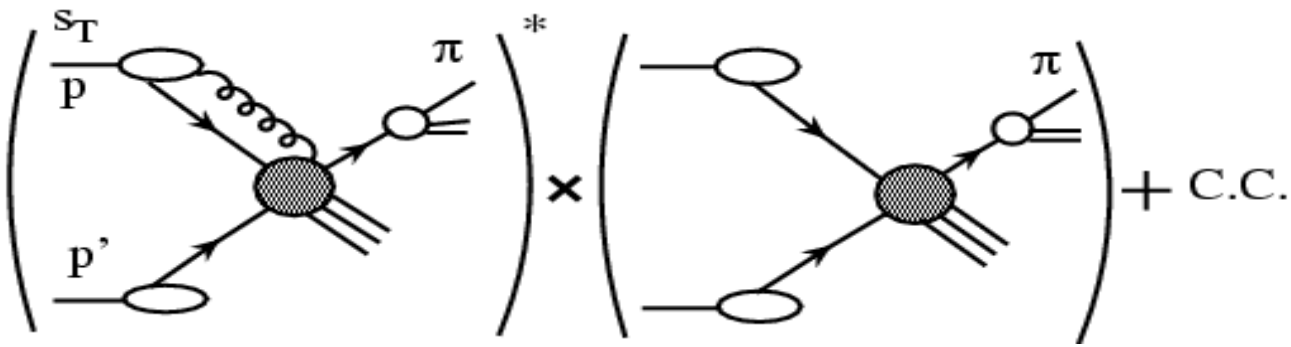
Normal twist-2 distributions

❖ Twist-3 correlation functions:

- $T_F(x_1,x_2)$ and $T_D(x_1,x_2)$ **have different properties under the P and T transformation**
- $T_D(x_1,x_2)$ **does not contribute to the A_N**
- $T_F(x_1,x_2)$ **is universal, x_1=x_2 for A_N due to the pole**

Leading twist-3 contribution to A_N

❖ **Minimal approach (within the collinear factorization):**



❖ Leading $(\partial/\partial x)T_F(x,x)$ contribution to the asymmetries

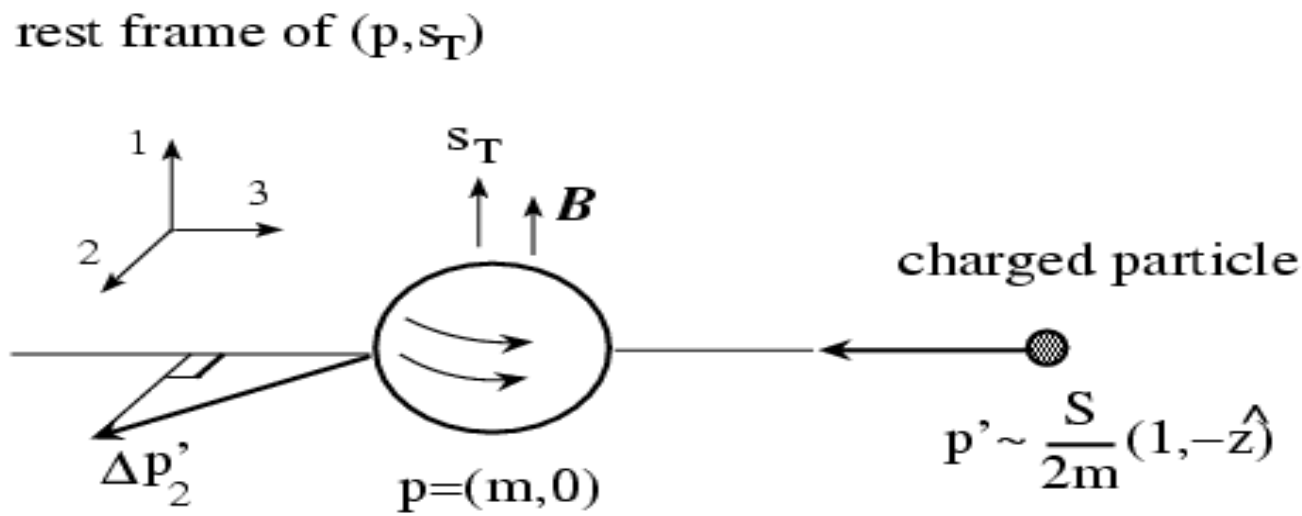
$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}} D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right]$$

$$\otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

❖ $$A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if } T_F(x,x)\propto q(x)\propto(1-x)^n$$

What $T_F(x,x)$ tells us – (I)

❖ **Consider a classical (Abelian) situation:**



– change of transverse momentum

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

– in the c.m. frame

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}$$

– total change: $\boxed{\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y^-)}$

What $T_F(x,x)$ tells us – (II)

$$T_F(x,x) \propto \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- $T_F(x,x)$ tells us something about quark's transverse motion in a transversely polarized hadron

- **Note:** Color field strength $F^{+\sigma}$ alone is not gauge invariant

- T_F Represents a fundamental quantum correlation Between quark and gluon inside a hadron

Model for quark-gluon correlation $T_F(x,x)$

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-}$$

$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^+(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

Model for $T_F(x,x)$ – (II)

- Model for $T_F(x,x)$ of quark flavor a:

$$T_{F_a}(x,x) \equiv \kappa_a \, \lambda \, q_a(x)$$

- with $\kappa_u = +1$ and $k_d = -1$ for proton

Fitting parameter $\lambda \sim O(\Lambda_{\rm QCD})$

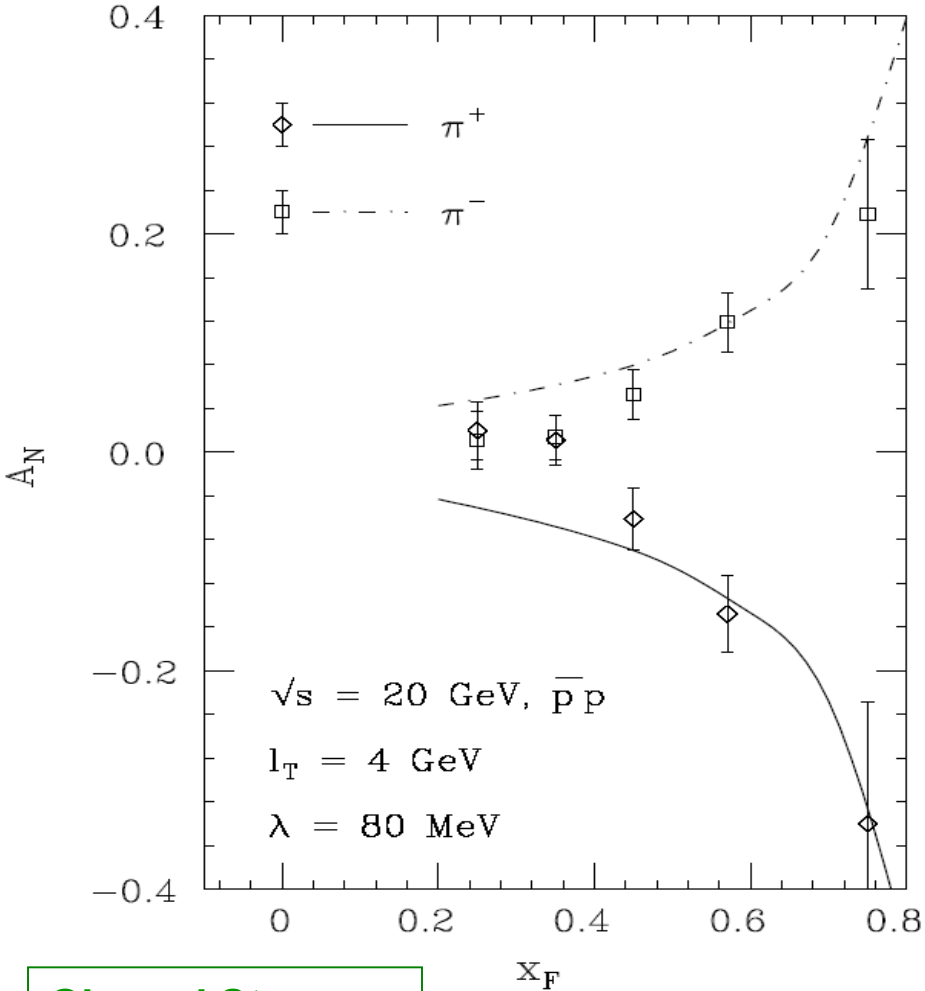
One parameter and one sign!

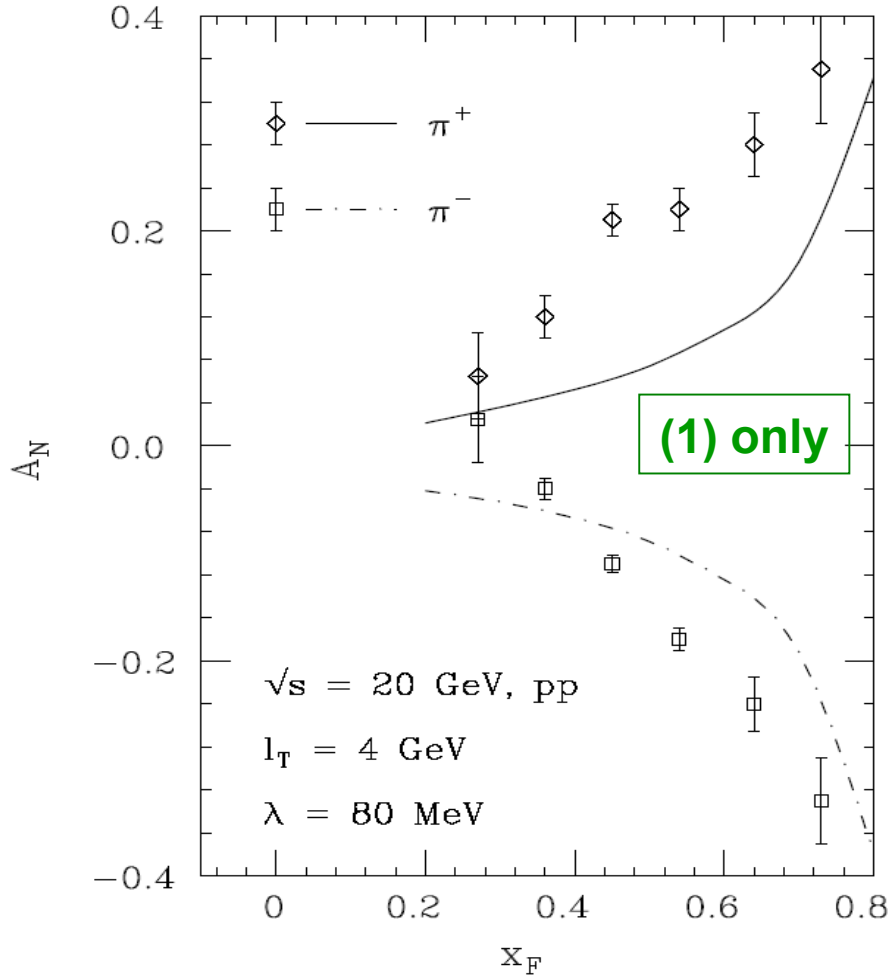
Predictive power of the factorization approach

- extract $T_F(x,x)$ from one observable, say π^+ or π^-
- use it to predict other observable, say π^0
- $(\partial/\partial x)T_F(x,x)$ leads to enhancement of the asymmetries in forward region
- same partonic parts can be used for calculating the asymmetries in production of other types of single hadron, say in k, or p production

Numerical results – (I)

(compare apples with oranges)



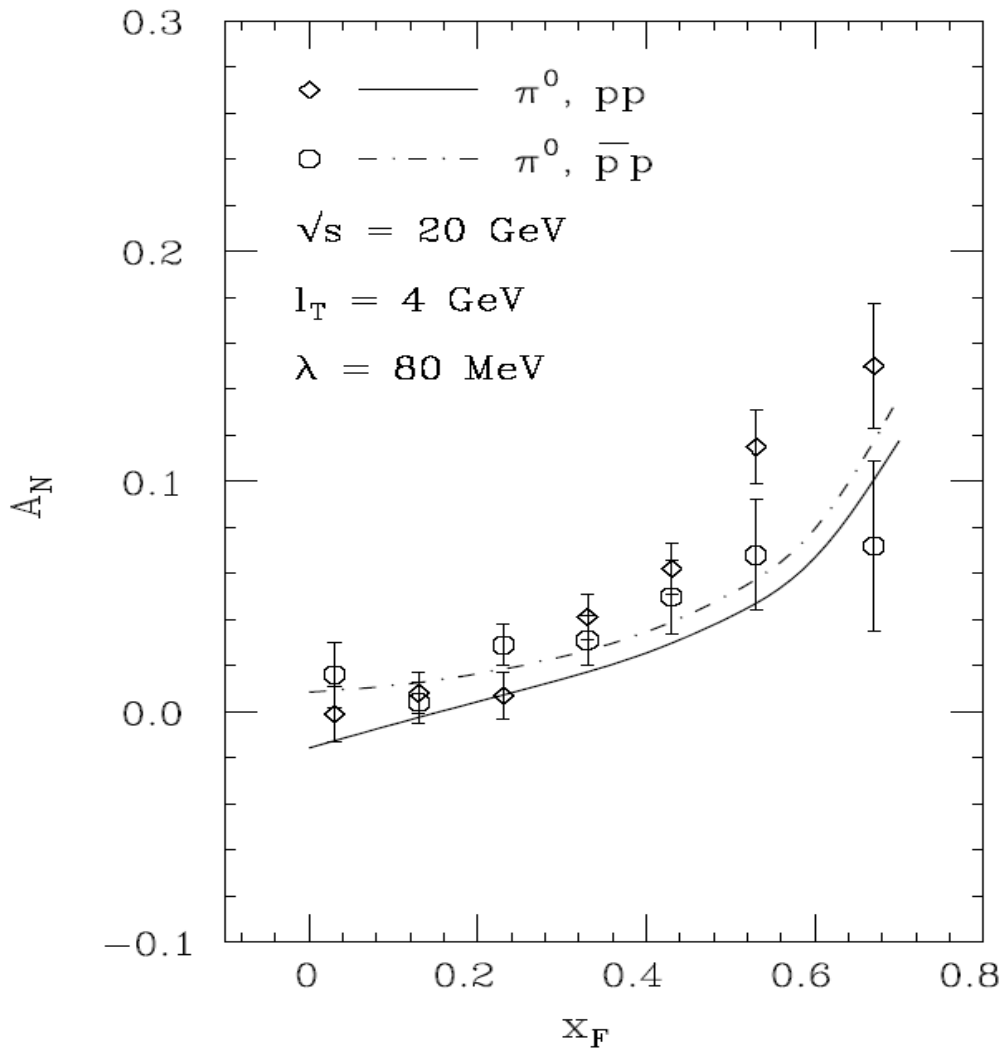


Qiu and Sterman
Phy. Rev. D, 1999

Fermilab data with ℓ_T up to 1.5 GeV

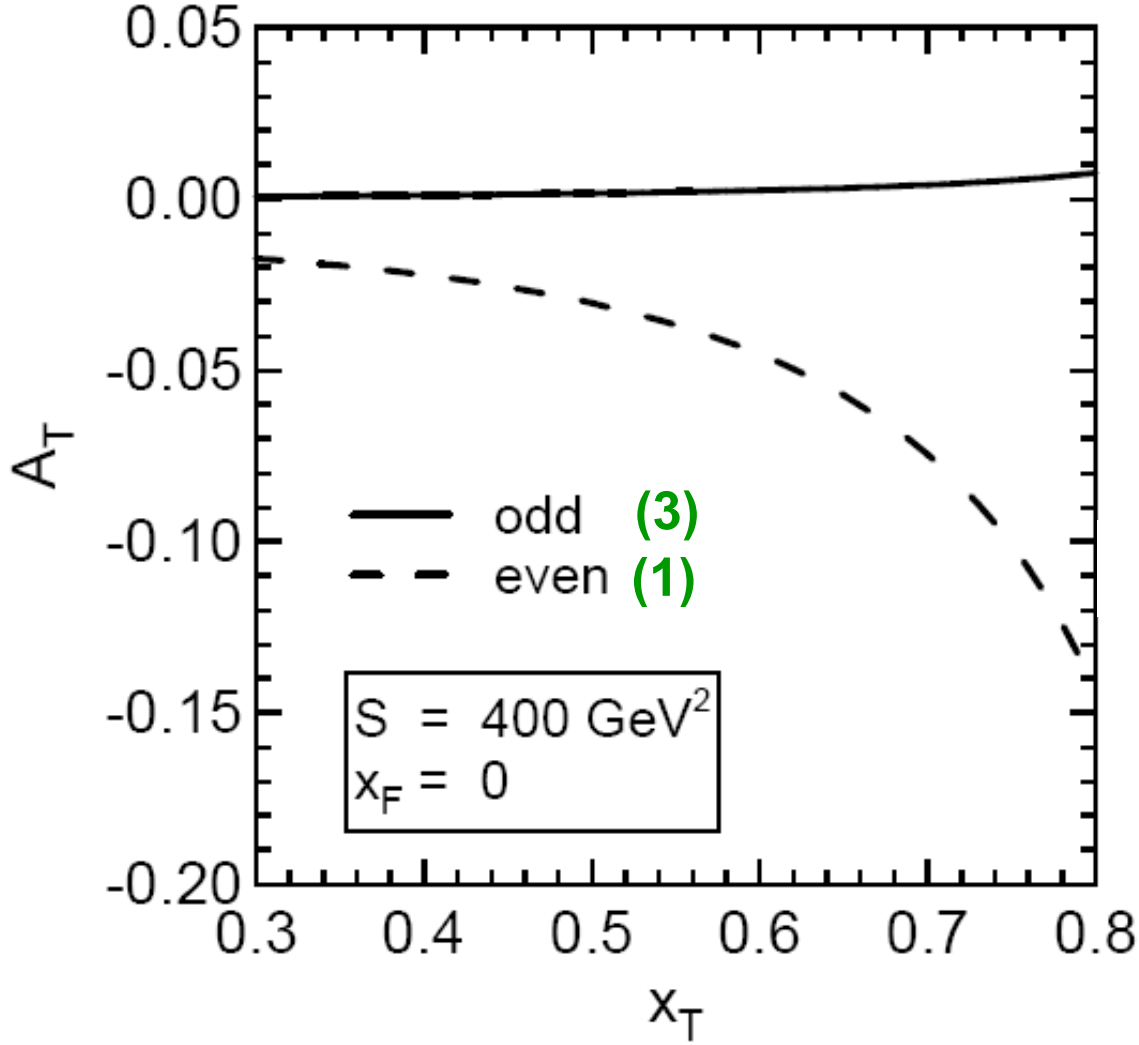
Numerical results – (II)

(compare apples with oranges)



Qiu and Sterman
Phy. Rev. D, 1999

Numerical results – (III)

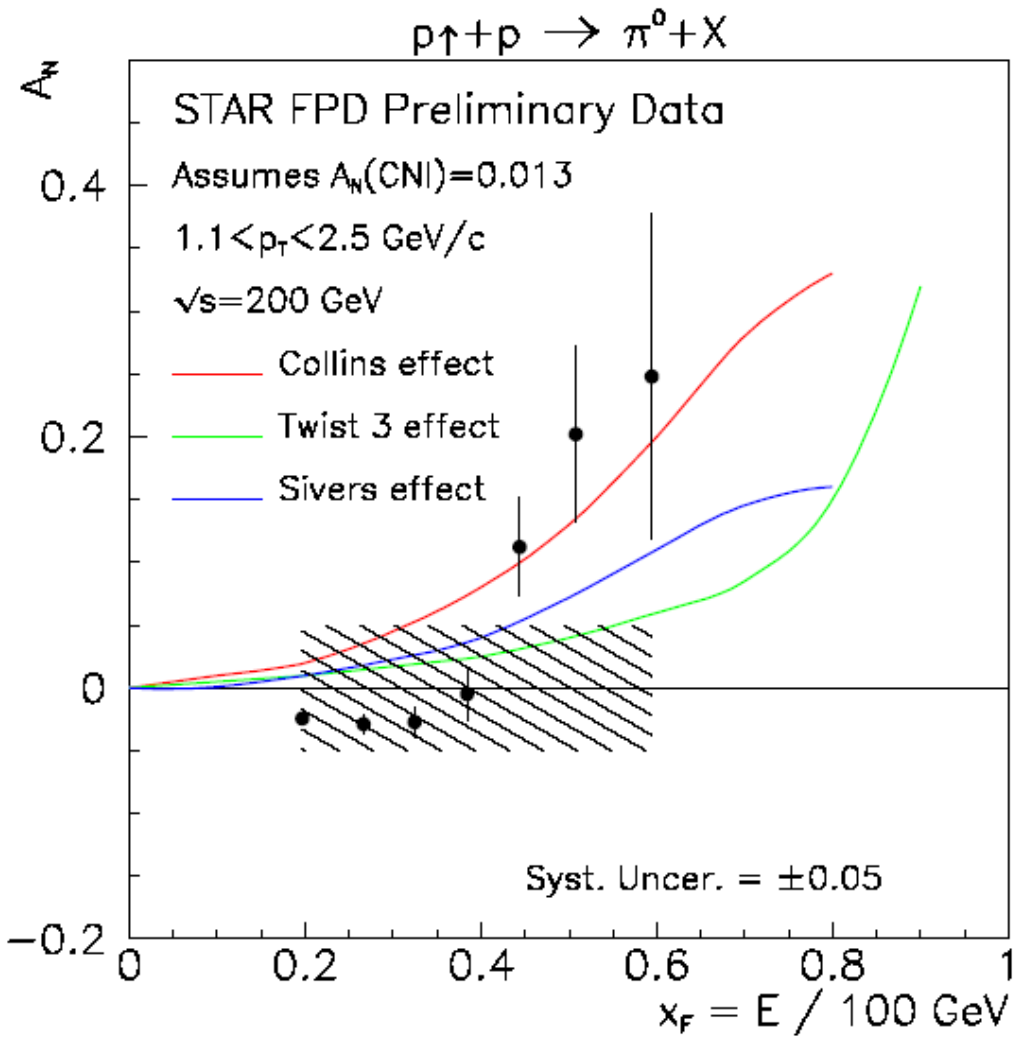


Kanazawa and Koike
Phys. Lett. B, 2000

Numerical results – (IV)
(compare apples with oranges)

STAR

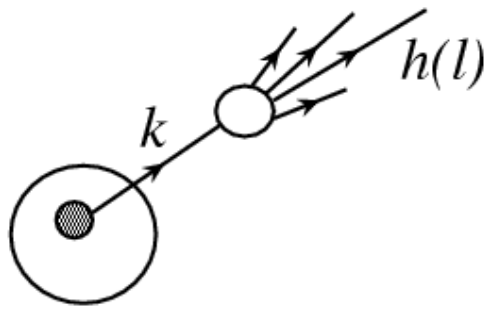
Rakness



When does the factorization lose its predictive power?

At the time when the nonperturbative functions lose their universality

❖ **For final-state fragmentation:**



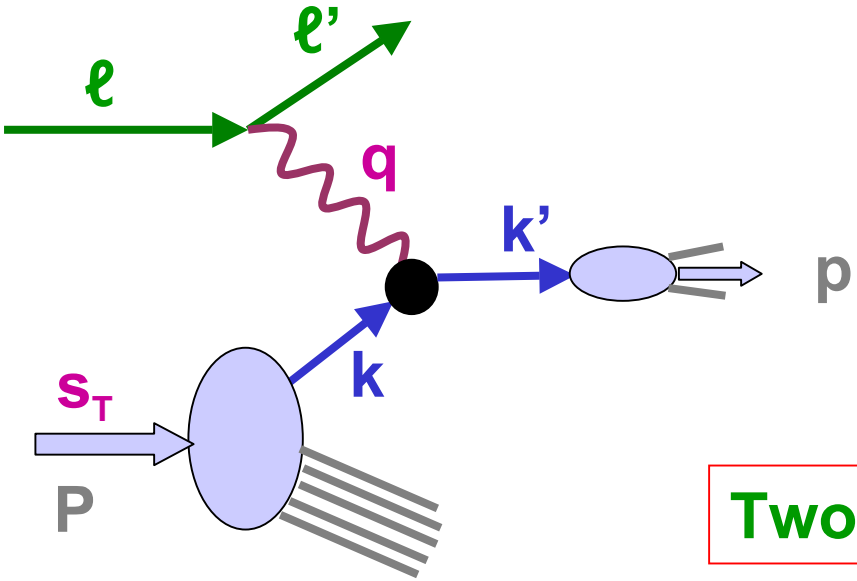
Factorization breaks if the fragmentation took place inside the hadronic medium

⟺ **If the lifetime of the parton state of momentun *k* is shorter than the medium size**

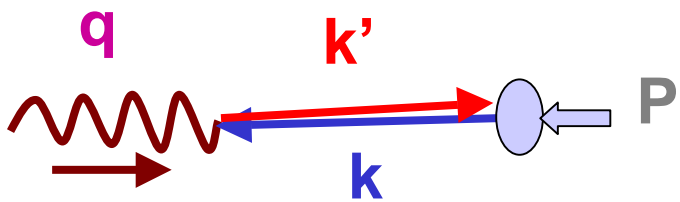
Lifetime: $\Delta y_\perp \sim \Delta t \sim \frac{1}{\Delta E} \sim \frac{2\ell_\perp}{m_{jet}^2} \sim L_{medium} \sim 2r_0 \sim 2\ \text{fm}$

$$\ell_\perp \sim 5\ \text{GeV}\left(\frac{m_{jet}^2}{\text{GeV}^2}\right)$$

Semi-inclusive deeply inelastic scattering



Breit frame:

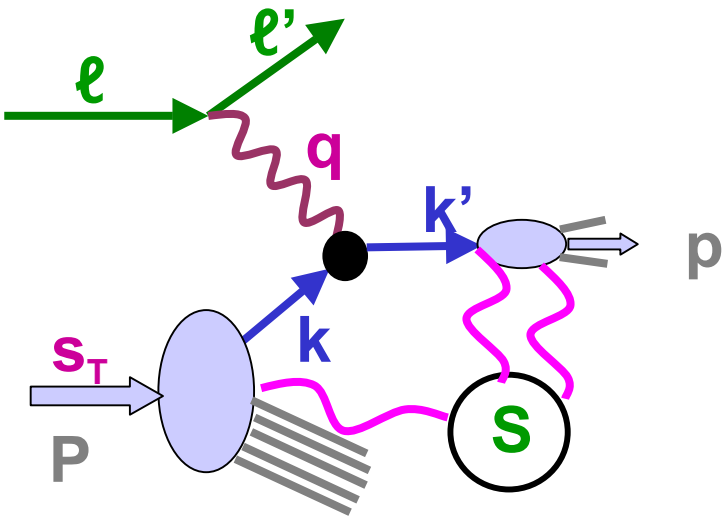


Two scale problem: $Q^2=-q^2$, p_T

- **Fixed order pQCD:** $Q \gg p_T \gg \Lambda_{\rm QCD}$
- **Sudakov resummation:** $Q \gg p_T$ ← **Always need Nonperturbative Input!**
- **Single spin asym:** $A_N \propto \vec{s}_T \cdot (\vec{P} \times \vec{p}) \Rightarrow 0$

If P is anti-parallel to p

K_T - Factorization



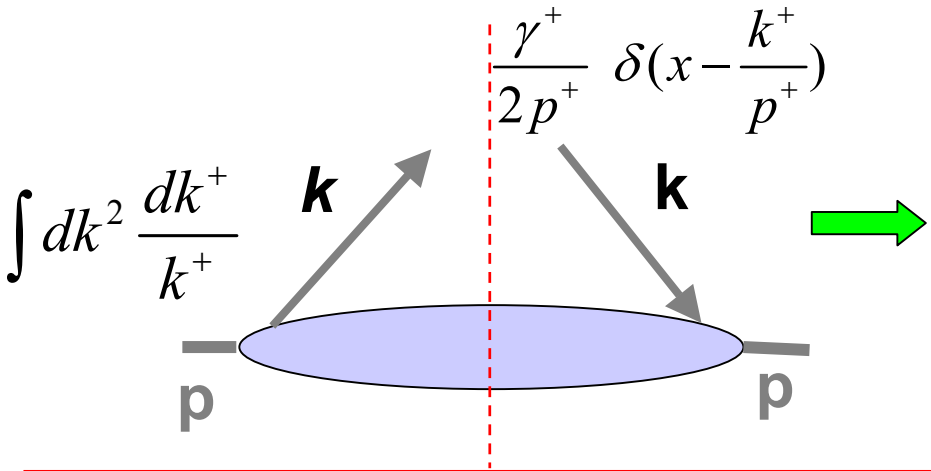
In q-P frame, if $k_T \ll p_T \ll Q$

- ❖ we can neglect k^2 in partonic part
- ❖ But, we cannot neglect k_T in partonic part

$$k^\mu = xP^\mu + k_\perp^\mu + \frac{k_T^2}{2k\cdot n} n^\mu$$

- ❑ One can define k_T-dependent and gauge invariant parton distributions $\neq$ factorization
- ❑ Soft interaction between the hadrons can spoil factorization
- ❑ Sudakov resummation done in b-space, and need nonperturbative information for coming back to momentum space

K_T – dependent parton distributions



$$\mathcal{Q}(x, k_\perp, \mu, x\zeta)$$

$$\Psi_v(\xi) = \mathcal{L}_v(\infty;\xi)\psi(\xi)$$

$$\mathcal{L}_v(\infty;\xi) = \exp\left(-ig\int_0^\infty d\lambda v\cdot A(\lambda v+\xi)\right)$$

$$\mathcal{Q}(x, k_\perp, \mu, x\zeta) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2 b_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \left\langle P \left| \overline{\Psi}_v(\xi^-, 0, \vec{b}_\perp)\gamma^+\Psi_v(0) \right| P \right\rangle$$

Proposed factorized formula:

Ji et al. hep-ph/0404183

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$

$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$

$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) ,$$

K_T- PDF

p_T-frag

soft

Sivers' functions

- **Sivers' functions:** $f_{1T}^{\perp}(x)$

$$q(x,k_\perp,\vec{s}_\perp)-q(x,k_\perp,-\vec{s}_\perp)=f_{1T}^{\perp}(x)\ \varepsilon_{\mu\nu\rho\sigma}\frac{\gamma^\mu n^\nu k_\perp^\rho s_\perp^\sigma}{M}$$

Sivers' functions are connected to a polarized hadron beam

Sivers' functions are T- odd, but do not need another T- odd function to produce nonvanish asymmetries – the extracted proportional factor is T- odd, proportional to A_N

- **Polarized SIDIS cross section:**

$$\Delta\sigma(x,Q,p_\perp)\propto[q(x,k_\perp,\vec{s}_\perp)-q(x,k_\perp,-\vec{s}_\perp)]\otimes D(z)$$

$$\propto\left(\varepsilon_{\mu\nu\rho\sigma}P^\mu n^\nu k_\perp^\rho s_\perp^\sigma\right)f_{1T}^{\perp}(x)\otimes D(z)$$

Collins' functions

❖ **Collins' functions:** $H_1^{\perp}(x)$

$$D(z,k_\perp,\vec{s}_\perp)+D(x,k_\perp,-\vec{s}_\perp)=H_1^{\perp}(z)\ \sigma_{\mu\nu}\frac{k_\perp^{\mu}\overline{n}^{\nu}}{M}$$

Collins' functions are connected to the unpolarized Fragmentation contributions to a hadron

Collins' functions are T- odd, need another T- odd function to produce nonvanish asymmetries

❖ **Polarized SIDIS cross section:**

$$\Delta\sigma(x,Q,p_\perp)\propto\delta q(x,\vec{s}_\perp)\otimes[D(z,k_\perp,\vec{s}_\perp)+D(z,k_\perp,-\vec{s}_\perp)]$$

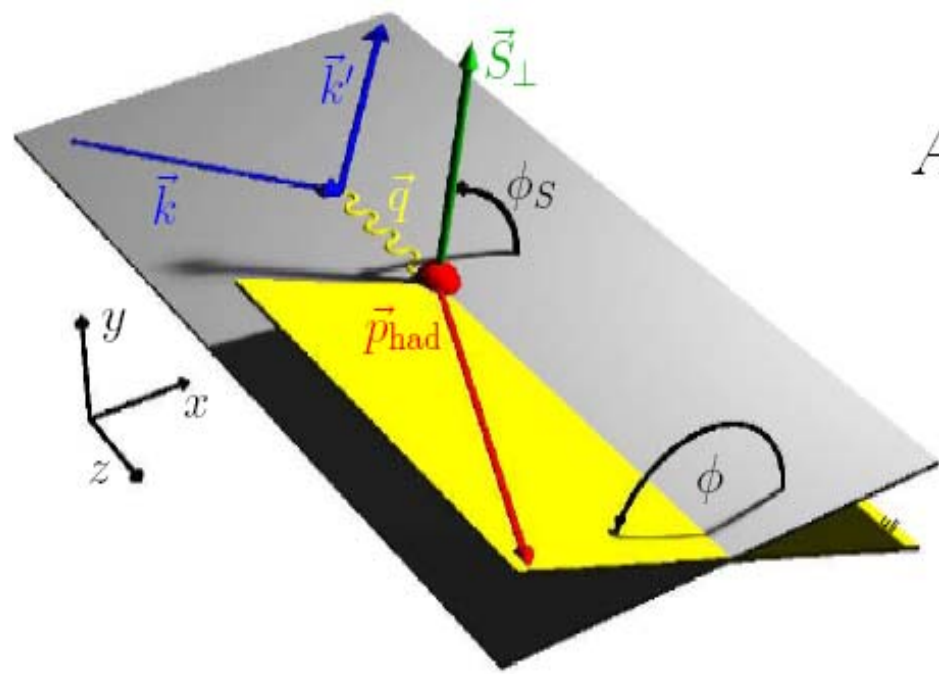
$$\propto\left(\sigma_{\mu\nu}k_\perp^{\mu}\overline{n}^{\nu}\right)\ \delta q(x,\vec{s}_\perp)\otimes H_1^{\perp}(z)$$

Measure Sivers' and Collins' functions

❖ **SIDIS:** $ep^{\uparrow} \longrightarrow e'\pi X$

study azimuthal distribution of π's:

with transversely polarized target:
(unpolarized beam)



❖ **Collins functions:**

$$A_{UT}^{\rm Sin\Phi} = \sum_q \frac{e_q^2 \delta q(x) H_1^{\perp}(z)}{e_q^2 q(x) D(z)}$$

$\Phi = \phi + \phi_S$ Collins angle

Seperate Sivers' and Collins' functions

Transversely polarized target

Sivers	Collins
$\langle \sin(\phi - \phi_s) \rangle$ moment	$\langle \sin(\phi + \phi_s) \rangle$ moment
$\downarrow$	$\downarrow$
$f_{1T}^{\perp}(x)$	$h_1(x)$, $H_1^{\perp}(z)$

Φ_s : Angle between $\vec{S}_{\perp}$ and L.P.

ϕ : Angle between L.P. and H.P.

Open questions and discussion

❖ Dynamics for single transverse spin asymmetries:

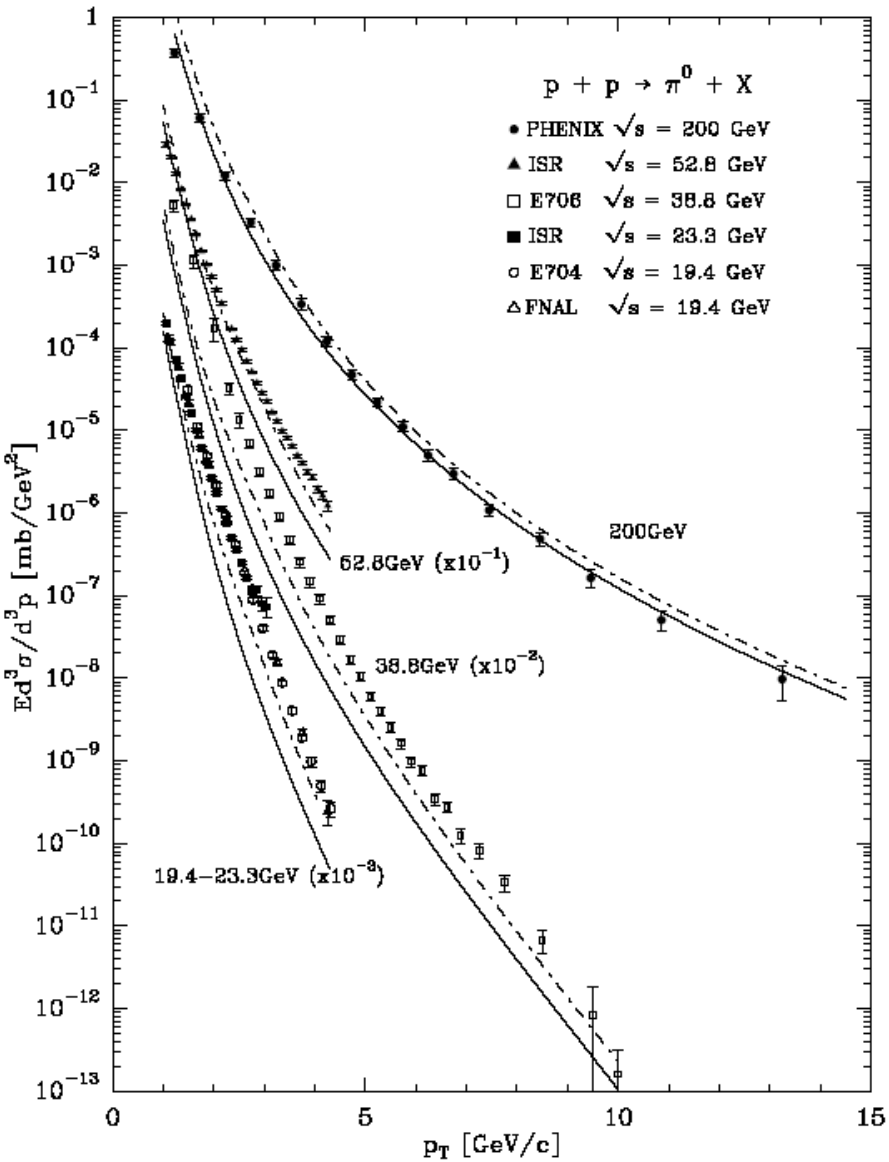
- ❑ Connection between the twist-3 and finite k_T approach?
- ❑ Transition between the k_T approach at low p_T and twist-3 mechanism at high p_T?

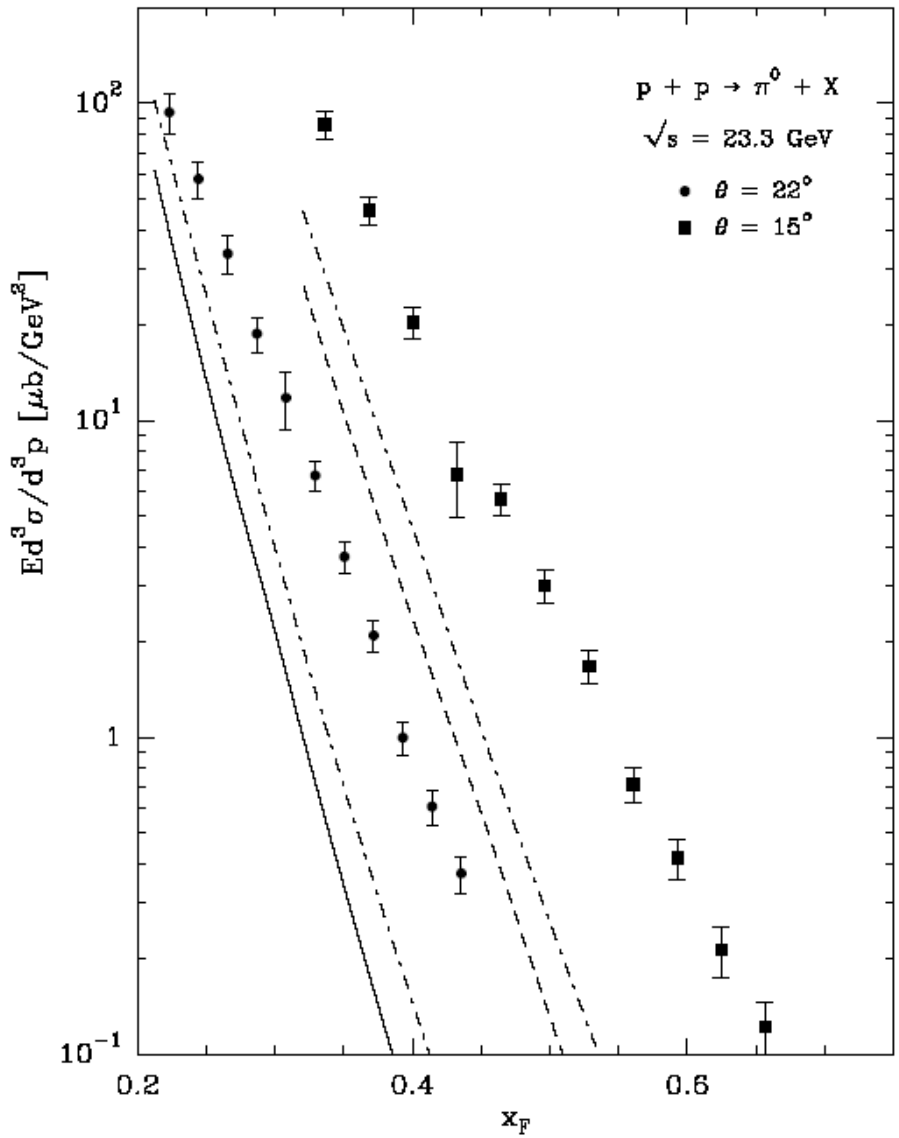
	k_T approach	Twist-3
Sivers:	$f_{1T}^{\perp}(x)$	$T_F(x,x)$
Collins:	$H_1^{\perp}(z)$	$D^{(3)}(z,z)$

❖ Asymmetries vs absolute cross sections:

PQCD NLO formalism does not fit low energy π data!

Bourrely and Soffer, EPJC (in press)





- If there is no K_T – factorization, how universal are Sivers and Collins functions?
  - KT – factorization in SIDIS and Drell-Yan might be a reasonable approximation due to a large scale Q and $P_T \sim K_T << Q$
  - It is very unlikely to have the K_T – factorization in hadronic collisions when $P_T \sim K_T$ due to a lack of perturbative scale
- What these nonperturbative functions try to tell us?
- K_T – dependent distributions do include information on power corrections in a twist expansion
  How much are not included in the K_T – distributions?
- … …