THE IMPORTANCE OF POSITIVITY IN SPIN PHYSICS

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BNL JUNE 12, 2004)

WE WILL SHOW WITH SEVERAL SPECIFIC EXAMPLES THE RELEVANCE OF POSITIVITY TO PUT MODEL-INDEPENDANT CONSTRAINTS ON SPIN OBSERVABLES

WE WILL USE SIMPLE MATHEMATICAL TOOLS (CAUCHY-SWARZ INEQUALITY, POSITIVITY OF HERMITIAN MATRIX, ...)

- THE TRANSVERSITY DISTRIBUTION h_1^q
  HOW TO USE? HOW TO MEASURE IT?
  $A_{LL} \gg A_{TT}$

- QUADRATIC RELATIONS IN TWO-BODY REACTIONS
  $pp \to pp$
  $\bar{p}p \to \bar{\Lambda}\Lambda$

- HADRONIC INCLUSIVE REACTIONS
  $A_1 + A_2 \to X$
  $A_1 + A_2 \to B + X$

- INCLUVISE DEEP INELASTIC SCATTERING
  $\vec{e}\vec{p} \to eX$
  $\nu p \to \vec{\tau} X$

THE $h_1^{q,\bar{q}}(x, Q^2)$ DISTRIBUTIONS AND THE NUCLEON TENSOR CHARGES

1- DEFINITIONS AND BASIC PROPERTIES

IN DEEP-INELASTIC SCATT. ONE HAS THE USUAL DISTRIBUTIONS

$$F_2^{p,n} \to q(x, Q^2) = q_+ + q_- \qquad (q_\pm \text{ HEL. DIST.})$$
$$q = u, d, s, \ldots \qquad (\text{TWIST-2})$$

$$F_3^{\nu,\bar{\nu}} \to \bar{q}(x, Q^2) = \bar{q}_+ + \bar{q}_-$$

$$g_1^{p,n} \to \Delta q(x, Q^2) = q_+ - q_- \qquad (\text{TWIST-2})$$
$$q = u, d, s$$

q IS RELATED TO THE MATRIX ELEMENT OF THE VECTOR CURRENT

$$\langle p, s | \bar{q} \gamma_\mu q | p, s \rangle$$

Δq IS RELATED TO THE MATRIX ELEMENT OF THE AXIAL CURRENT

$$\langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle$$

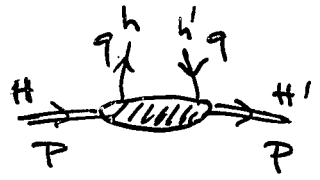
HOWEVER THERE IS ONE MORE DISTRIBUTION RELATED TO

$$\langle p, s | \bar{q} \underbrace{\sigma_{\mu\nu} i \gamma_5}_{\text{SPIN OPERATOR}} q | p, s \rangle \qquad (\text{TWIST-2})$$

CALLED $h_1^q(x, Q^2)$ (OR $\Delta q_T(x, Q^2)$) WHICH DESCRIBES THE QUARK DISTRIBUTION INSIDE A TRANSVERSELY POLARIZED PROTON

$$h_1^q(x, Q^2) = q_{m\uparrow} - q_{m\downarrow}$$

POSITIVITY CONSTRAINTS FOR FORWARD PARTON DISTRIBUTIONS



LET'S CONSIDER THE THREE FORWARD QUARK DISTRIBUTIONS

$$q(x)\ ,\quad \Delta q(x)\ ,\quad h_1^q(x)$$

WHICH CAN BE RELATED TO THE FORWARD QUARK-NUCLEON ELASTIC AMPLITUDE

$$q_{h'}(q) + N_H(P) \to q_h(q) + N_{H'}(P)$$

h_1^q CORRESPONDS TO $H = h' = +1/2$, $H' = h = -1/2$

LET'S USE THE MATRIX ELEMENTS

$$a_X^{H,h}(P,q) = \langle N_H(P)\,|\,O\,|\,q_h(q), X\rangle$$

ONE CAN SHOW THAT

$$q_+(x) \equiv \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

$$= \frac{1}{2}\sum_X \left(|a_X^{+,+}|^2 + |a_X^{-,-}|^2\right)$$

$$h_1^q(x) = \frac{1}{2}\sum_X \left[(a_X^{+,+})^*(a_X^{-,-}) + (a_X^{+,+})(a_X^{-,-})^*\right]$$

SCHWARZ INEQUALITY LEADS IMMEDIATELY TO

$$2\,|h_1^q(x)| \leq q(x) + \Delta q(x)$$

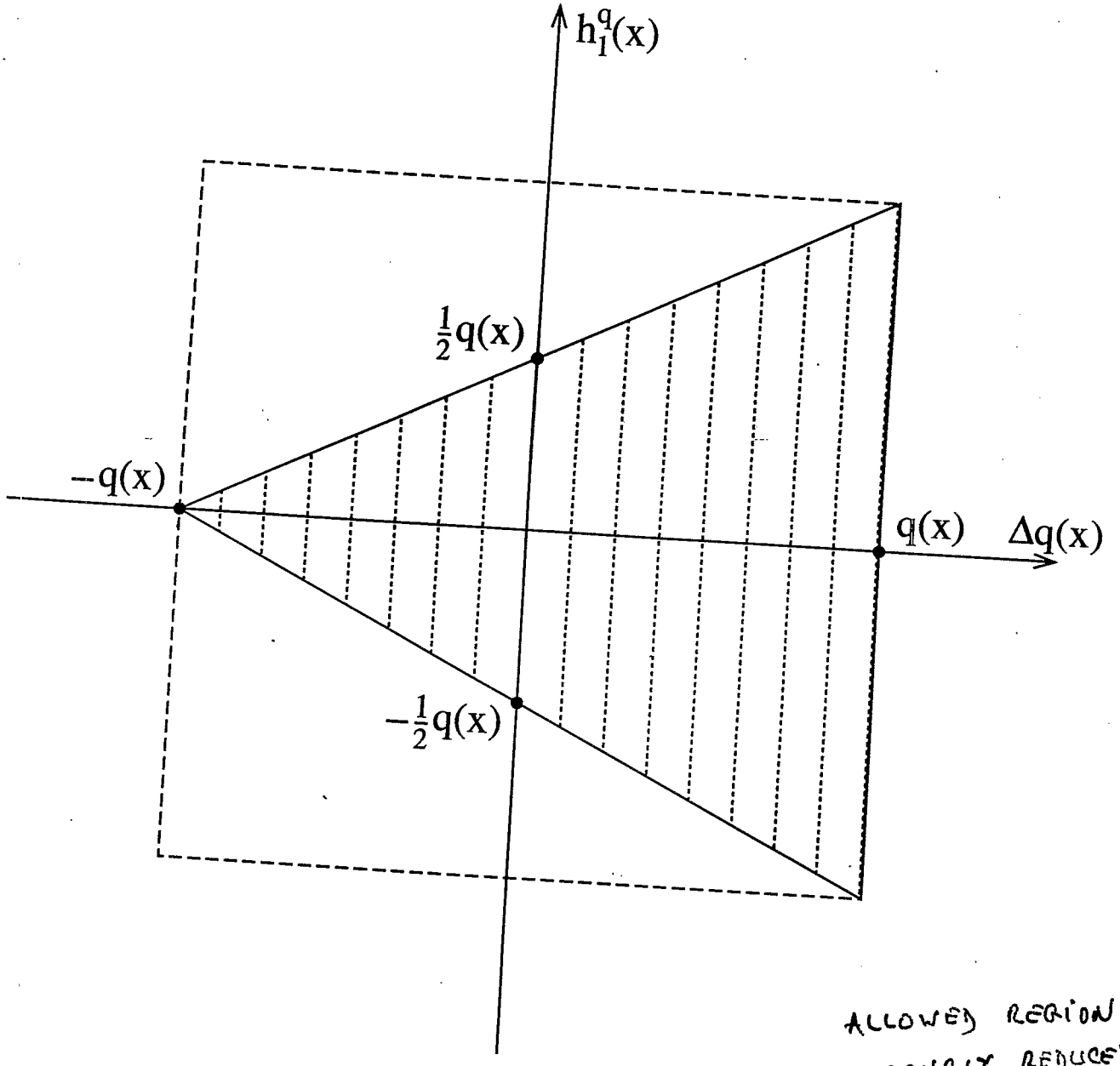
(J.S. PHYS. REV. LETT. 74 (1995) 1292)

STRONGER THAN THE TRIVIAL ONE $|h_1^q(x)| \leq q(x)$

THIS SIMPLE RESULT IS <u>NOT</u> AFFECTED BY QCD CORRECTIONS UP TO NLO

(W. VOGELSANG, PHYS. REV. D57 (1998) 1886 ;
C. BOURRELY, J.S., O. TERYAEV, PHYS. LETT. B420 (1998) 375)

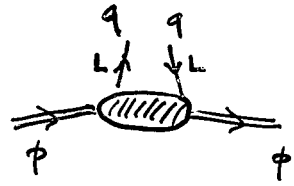
$$\underset{Pq\to Pq}{M} = q(x)\,\mathbb{1}\otimes\mathbb{1} + \Delta q(x)\,\vec{\sigma}\cdot\vec{e}_3\otimes\vec{\sigma}\cdot\vec{e}_3 + h_1^q(x)\sum_{i=1,2}\vec{\sigma}\cdot\vec{e}_i\otimes\vec{\sigma}\cdot\vec{e}_i$$

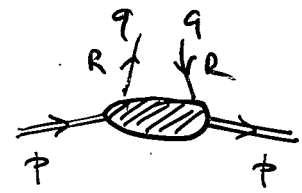


ALLOWED REGION
STRONGLY REDUCED

Fig.1

IN TERMS OF DIAGRAMS WE HAVE

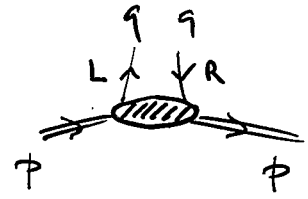
$q =$ 

$+$ 

CHIRAL EVEN

$\Delta q =$ " $-$ "

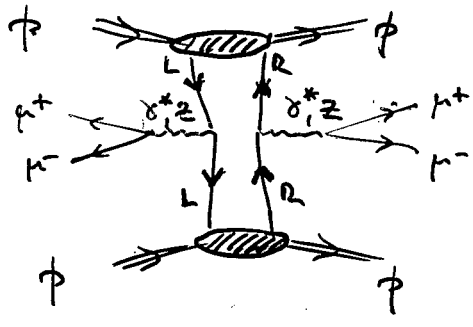
BUT ALSO

$h_1^q =$ 

CHIRAL ODD

(LEFT-RIGHT INT.)

CLEARLY FROM CHIRALITY CONSERVATION h_1^q DECOUPLES FROM DIS BUT IT CAN BE MEASURED IN DRELL-YAN TYPE PROCESSES WITH BOTH INITIAL PROTONS TRANSVERSELY POLARIZED



$$A_{TT}^{\gamma^*} = \hat{a}_{TT} \frac{\sum_q e_q^2 h_1^q(x_a)\, h_1^{\bar{q}}(x_b)}{\sum_q e_q^2 q(x_a)\, \bar{q}(x_b)}$$

ANALOGOUS FOR

$$A_{TT}^{Z} = \frac{\sum_q (a_q^2 - b_q^2)(a_{\bar{q}}^2 - b_{\bar{q}}^2)\, h_1^q h_1^{\bar{q}}}{\sum_q (a_q^2 + b_q^2)(a_{\bar{q}}^2 - b_{\bar{q}}^2)\, q\bar{q}}$$

$A_{TT}^{W^\pm} = 0$ SHOULD VANISH FOR $W^\pm$ PRODUCTION SINCE W IS LEFT-HANDED (NO L-R INT.)

IF $\neq 0$ NEW PHYSICS

SO FAR NO DATA ON $h_1^q(x, Q^2)$ AND HOPE TO GET FIRST DETERMINATION WITH THE POLARIZED pp COLLIDER AT RHIC-BNL (2000?)

AS ONE DEFINES THE AXIAL CHARGES, FIRST MOMENT OF $\Delta q(x)$

$$\Delta q(Q^2) = \int_0^1 dx \left[\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2)\right]$$

ONE DEFINES THE TENSOR CHARGES

(PURE VALENCE) $\delta q(Q^2) = \int_0^1 dx \left[h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2)\right]$

MARTIN, SCHÄFER, STRATMANN, VOGELSANG
PHYS. REV. D57 (1998) 3084

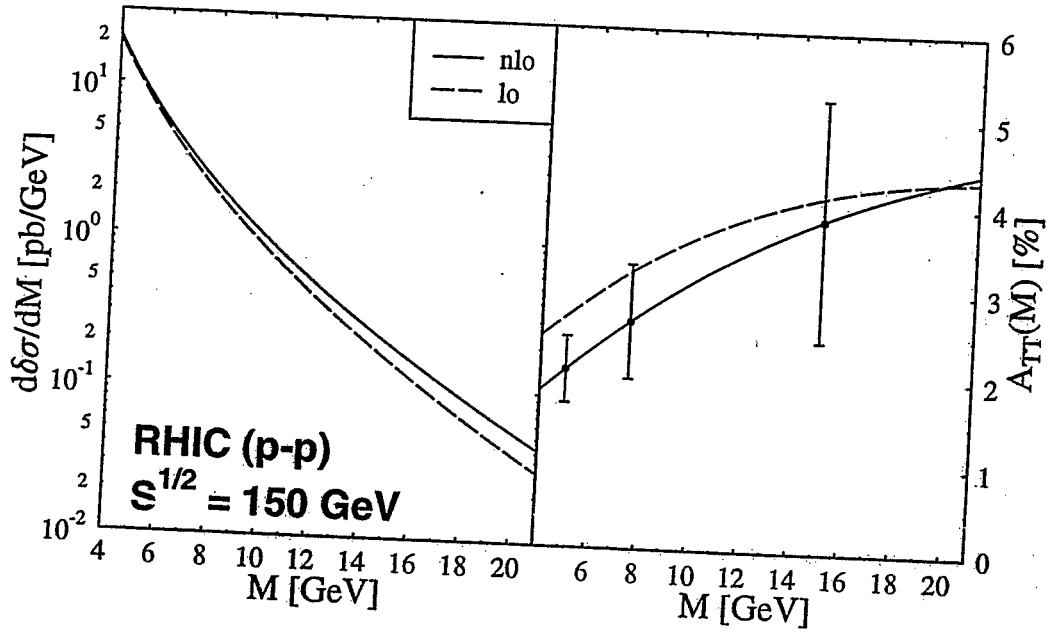


Figure 4: As in Fig. 3, but for RHIC at $\sqrt{S} = 150\,\mathrm{GeV}$ assuming $\mathcal{L} = 240\,\mathrm{pb}^{-1}$, 70% polarisation of each beam and 100% detection efficiency.

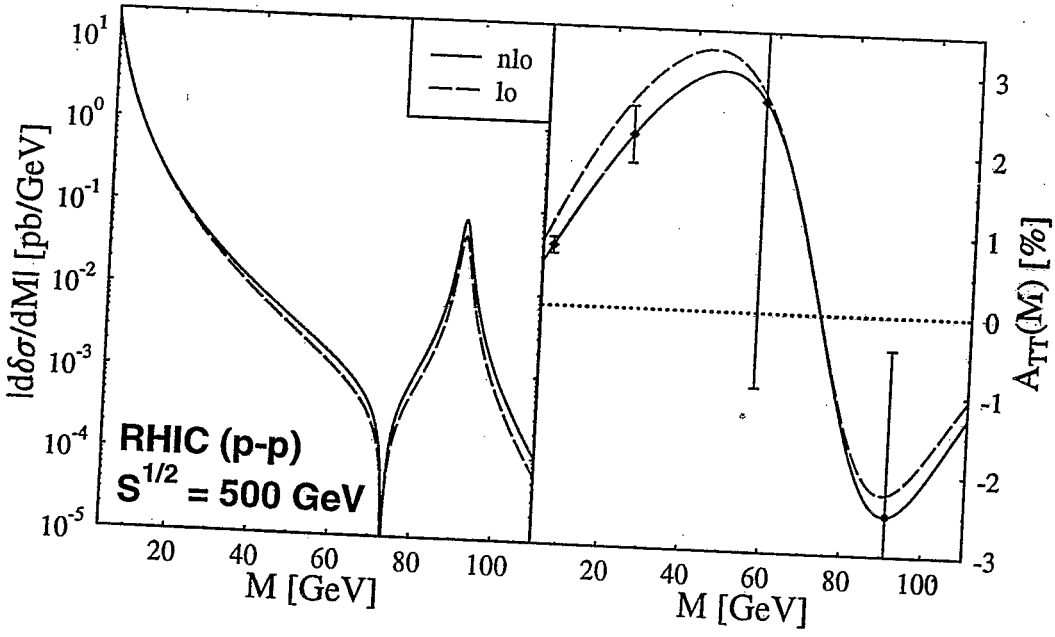
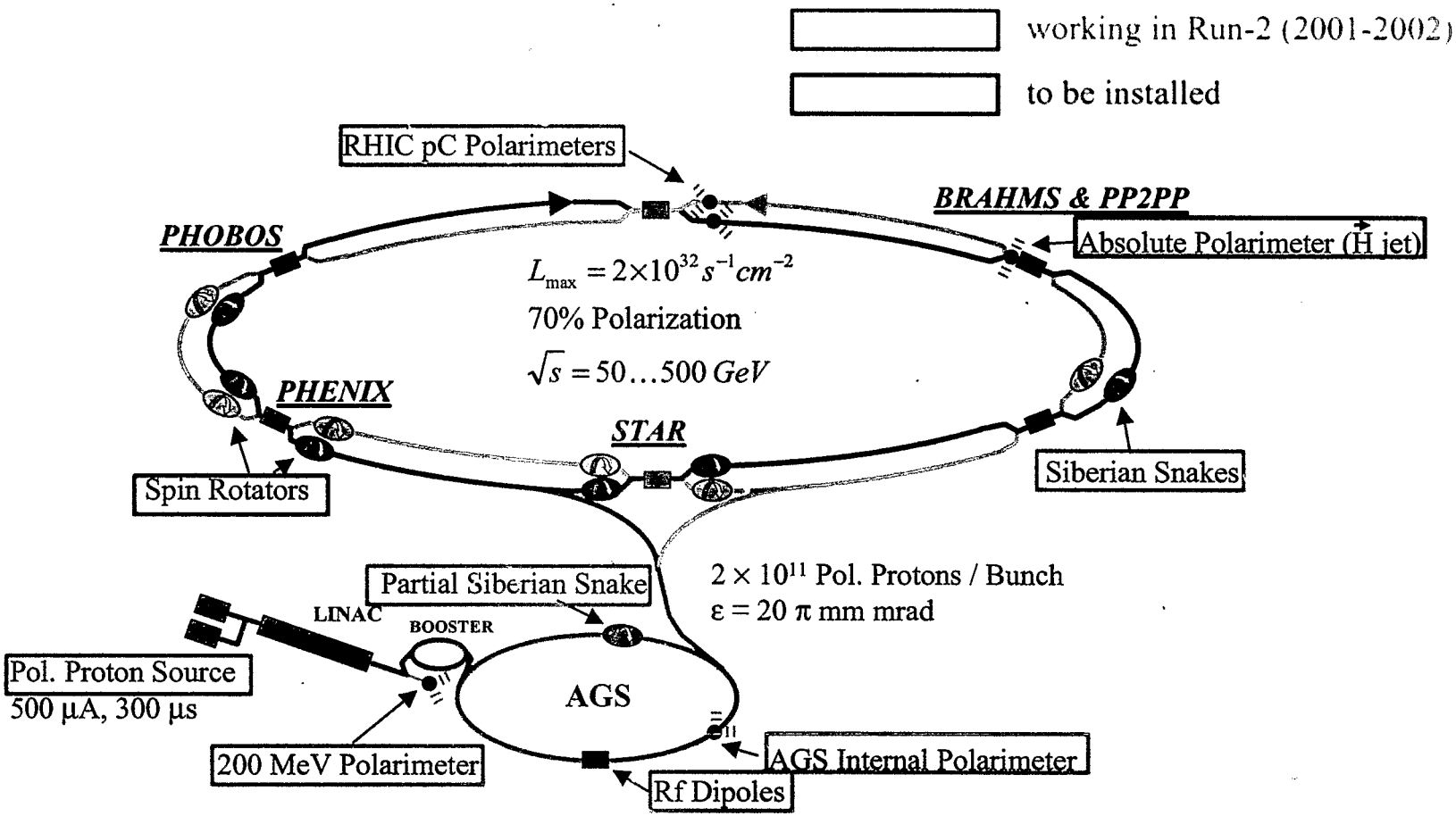


Figure 5: As in Fig. 4, but for $\sqrt{S} = 500\,\mathrm{GeV}$ and $\mathcal{L} = 800\,\mathrm{pb}^{-1}$.

RHIC Polarized Proton Collider



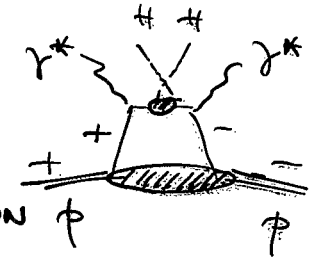
TRANSVERSITY OF A QUARK IN THE NUCLEON

$h_1^q(x, Q^2)$ ($\delta q(x, Q^2)$) A FUNDAMENTAL LEADING TWIST PARTON DISTRIBUTION

NOT ACCESSIBLE IN DIS BECAUSE CHIRAL ODD

BUT CAN BE MEASURED

- IN SI DIS $e p^{\uparrow} \to e\, H(\vec{k}_T)\, X$



SO MEASURE THE PRODUCT OF h_1^q WITH A CHIRAL ODD FRAGMENTATION FUNCTION.

THIS EFFECT HAS BEEN ALREADY UNCOVERED IN THE HERMES AZIMUTHAL ASYMMETRY A_{UL}

SHOULD BE CONFIRMED AND LARGER WITH A TRANSVERSE TARGET

- IN pp COLLISIONS AT RHIC

  A_{TT} IN DIMUON PRODUCTION

  EXPECTED TO BE SMALL

  ALSO ANTICIPATE STRONG DILUTION OF A_{TT} IN GLUON INDUCED PROCESSES LIKE JET PRODUCTION OR PROMPT PHOTON PRODUCTION

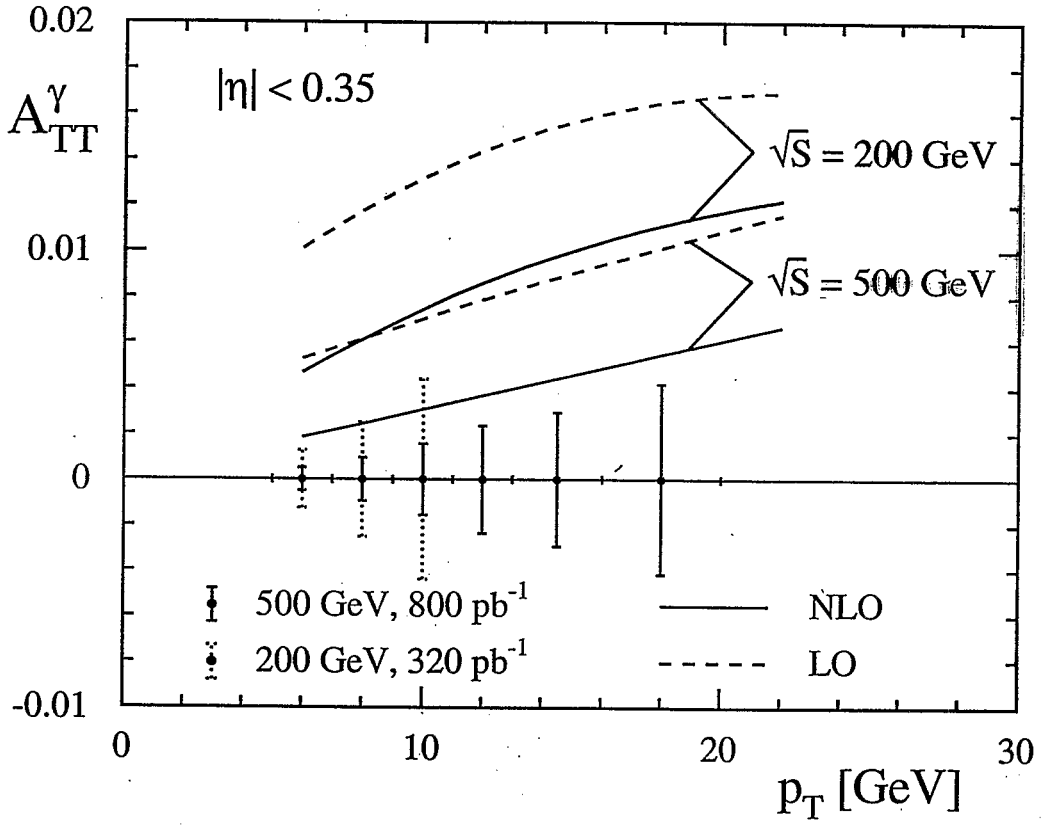
$$|A_{TT}| \ll |A_{LL}|$$

MUST BE CHECKED

(USE POSITIVITY BOUND $|2h_1^q(x, Q^2)| \leq q(x, Q^2) + \Delta q(x, Q^2)$)

(J.S. PRL 74, 1292 (1995))

$$\vec{p} + \vec{p} \to \gamma + X$$



(L.O.) J. S., M. STRATMANN, W. VOGELSANG
PHYS. REV. D65, 114024 (2002)

(NLO) M. STRATMANN, W. VOGELSANG (2003)

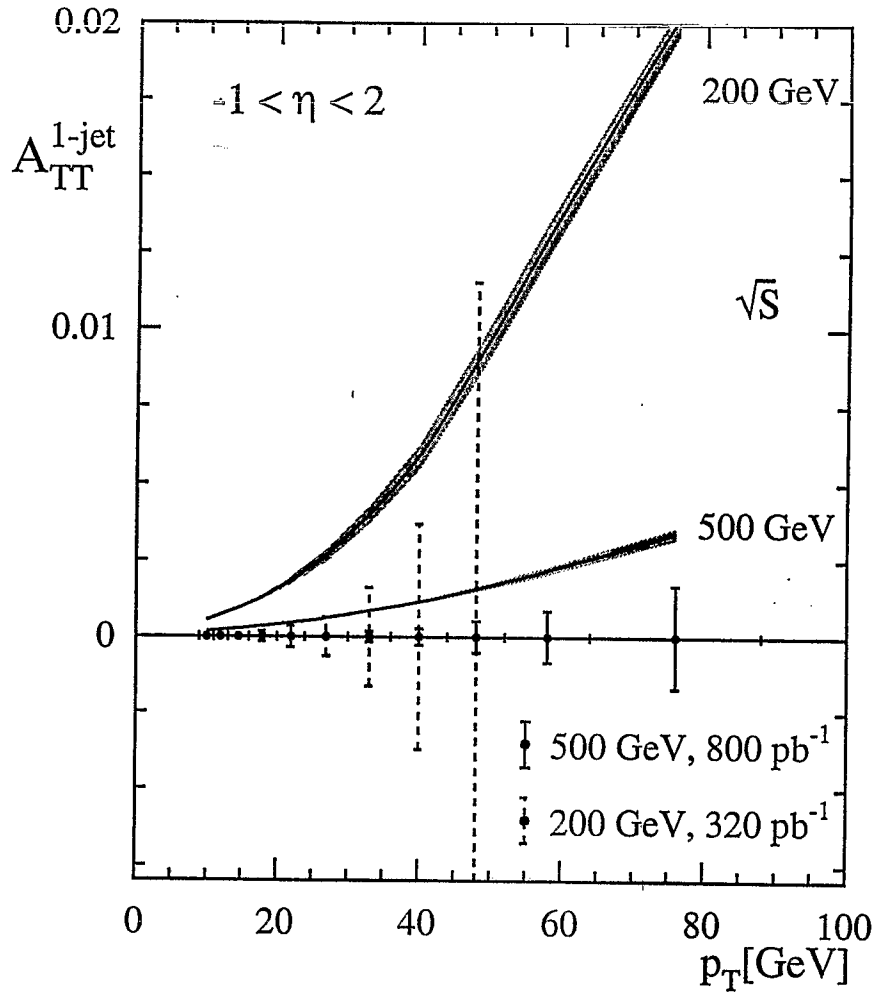


Figure 1: "Maximally possible" A_{TT} for single-inclusive jet production at RHIC c.m.s. energies of 200 GeV and 500 GeV as a function of p_T. Jet rapidities are integrated over the detector acceptance ($-1 \leq \eta \leq 2$). The shaded bands represent the theoretical uncertainty in A_{TT} if μ_F is varied in the range $p_T/2 \leq \mu_F \leq 2p_T$. Also indicated as "error bars" is the expected statistical accuracy for certain bins in p_T.

J.S., M. STRATMANN, W. VOGELSANG, PRD 65, 114024 (2002)

QUADRATIC RELATIONS IN TWO-BODY REACTIONS

1) $0 + 1/2 \rightarrow 0 + 1/2$

(SCATT. AMP.) $M = f + g\, \vec{\sigma} \cdot \vec{n}$ $\quad$ $\vec{n}$ normal to scatt. plane

f AND g ARE TWO COMPLEX AMPLITUDES
SO NEED TO DETERMINE 4 REAL NUMBERS
CANNOT DETERMINE ONE OVERALL PHASE
SO NEED ONLY 3

HOWEVER CAN CONSTRUCT 4 BILINEAR PRODUCTS

$$|f|^2, \; |g|^2, \; \mathrm{Im}(f g^*), \; \mathrm{Re}(f g^*)$$

BECAUSE IN FACT 4 OBSERVABLES

$$\frac{d\sigma}{dt}, \; P, \; R, \; A$$

SO IT MUST BE A RELATION AMONG THEM
INDEED

$$P^2 + R^2 + A^2 = 1 \qquad \text{(QUAD. REL.)}$$

VERY USEFUL TO KNOW BECAUSE FOR EXAMPLE
IF $P \sim 1$, R <u>AND</u> A MUST BE ~ 0

THIS CAN BE EXTENDED TO MORE
COMPLICATED CASES

MORE GENERALLY FOR A REACTION WITH M COMPLEX AMPLITUDES, IT IS FULLY DESCRIBED IN TERMS OF $2M-1$ REAL NUMBERS

THERE ARE M^2 $\left(M + \frac{M(M-1)}{2} \times 2\right)$ OBSERVABLES AND THEREFORE $(M-1)^2$ INDEPENDANT QUADRATIC RELATIONS BETWEEN THE M^2 OBSERVABLES $\left(M^2 - (M-1)^2 = 2M-1\right)$

IN ORDER TO GET THESE RELATIONS ONE DEFINES THE MATRIX OF OBSERVABLES AS

$$\sigma = \varphi \otimes \varphi^+ \qquad \varphi \text{ is a } M\text{-DIM VECTOR}$$

IT IS A $M \times M$ HERMITIAN MATRIX WHICH IS POSITIVE AND IT HAS THE PROPERTY

$$\sigma^2 = \mathrm{Tr}(\sigma)\,\sigma$$

GIVEN THE FORM OF σ THE NECESSARY AND SUFFICIENT CONDITIONS ARE

$$|\sigma_{ij}|^2 = \sigma_{ii}\,\sigma_{jj} \quad (i \neq j)$$

$$\sigma_{ij}\,\sigma_{jk} = \sigma_{ik}\,\sigma_{jj} \quad (i \neq k)$$

FOR $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$

$M = 5$ FOR $pp \to pp$ (15 QUADRATIC REL.)

$M = 6$ FOR $\bar{p}p \to \bar{\Lambda}\Lambda$ (25 " ")

$$A_{LL}^2 + D_{NN}^2 \leq 1$$

FROM THE MEASUREMENT OF $A_{LL} \sim -1$ ONE EXPECTS $D_{NN} \sim 0$

HAVE ALSO SIMPLER LINEAR RELATIONS

$$
\begin{aligned}
&(1+D_{NN}+C_{NN}+K_{NN}\pm 4P)(1-D_{NN}+K_{NN}-C_{NN})=(K_{PP}+K_{KK}\pm 2H_{PNP})^2+(H_{PNK}-H_{KNP}\mp 2K_{KP})^2,\\
&(1+D_{NN}+C_{NN}+K_{NN}\pm 4P)(1+D_{NN}-K_{NN}-C_{NN})=(D_{PP}+D_{KK}\pm 2H_{NPP})^2+(H_{NPK}-H_{NKP}\mp 2D_{KP})^2,\\
&(1+D_{NN}+C_{NN}+K_{NN}\pm 4P)(1-D_{NN}-K_{NN}+C_{NN})=(C_{PP}-C_{KK}\pm 2H_{PPN})^2+(H_{PKN}+H_{KPN}\pm 2C_{KP})^2,\\
&(1-C_{NN})^2-(D_{NN}-K_{NN})^2=(C_{PP}+C_{KK})^2+(H_{PKN}-H_{KPN})^2,\\
&(1-D_{NN})^2-(K_{NN}-C_{NN})^2=(D_{PP}-D_{KK})^2+(H_{NPK}+H_{NKP})^2,\\
&(1-K_{NN})^2-(D_{NN}-C_{NN})^2=(K_{PP}-K_{KK})^2+(H_{PNK}+H_{KNP})^2,\\
&-(1+D_{NN}+C_{NN}+K_{NN})(G_{KKKK}-1)=2\left[P^2+(G_{KKKK}-1)^2+G^2_{KKKP}\right].
\end{aligned}
\tag{6}
$$

The first nine equations enable us to calculate the third-rank spin tensor H_{ijk} (nine quantities) if we know all one-spin and two-spin measurements. Only the last equation involves the four-spin measurements which are not yet accessible to experiment at 6 GeV/c.[2]

It follows from the general case that six independent relations result from Eqs. (3). They are quadratic in the observables and will determine the phases of the off-diagonal elements O_{ij} in terms

C. BOURRELY, J. S.
PR D12 (1975) 2932

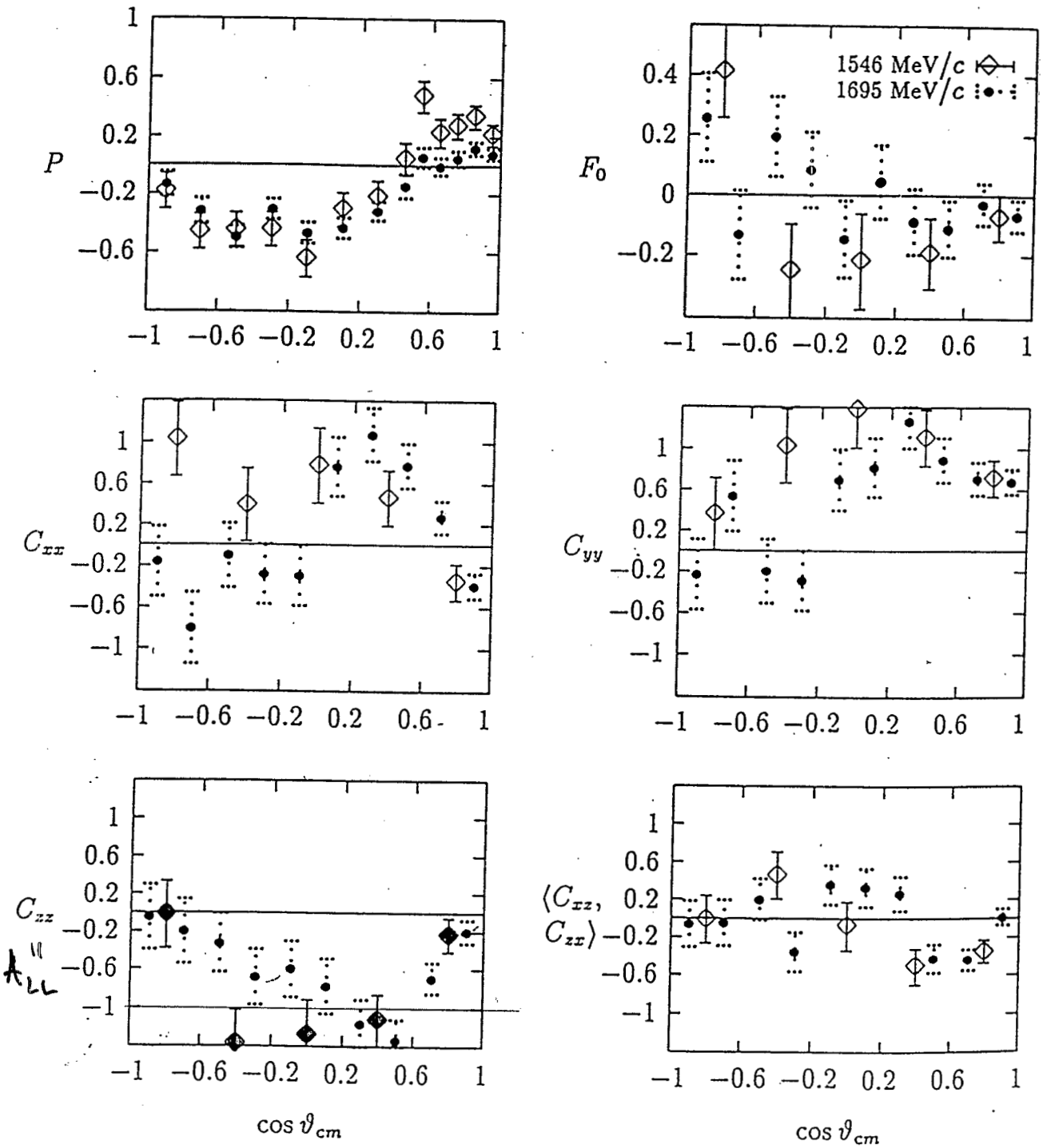
SEE ALSO M. ELCHIKH and J. M. RICHARD PHYS. REV. C61, 035205 (2000)

$$1 + A_{NN} \geq \pm (D_{NN} + K_{NN})$$

IF $D_{NN} \sim 0$

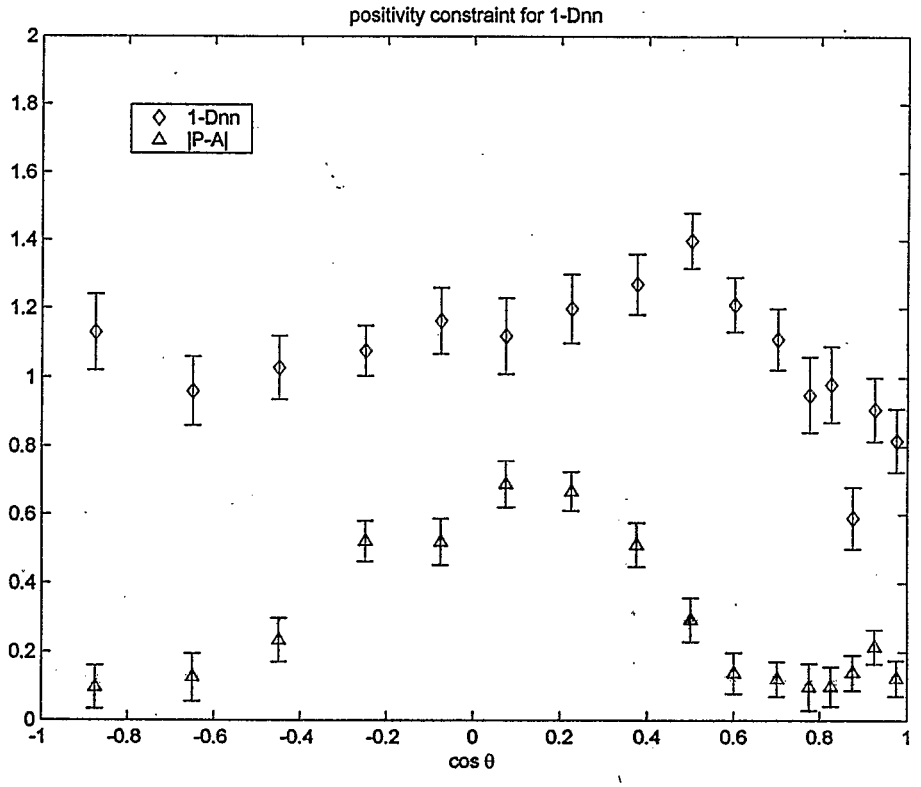
$$1 + A_{NN} \geq \pm K_{NN}$$

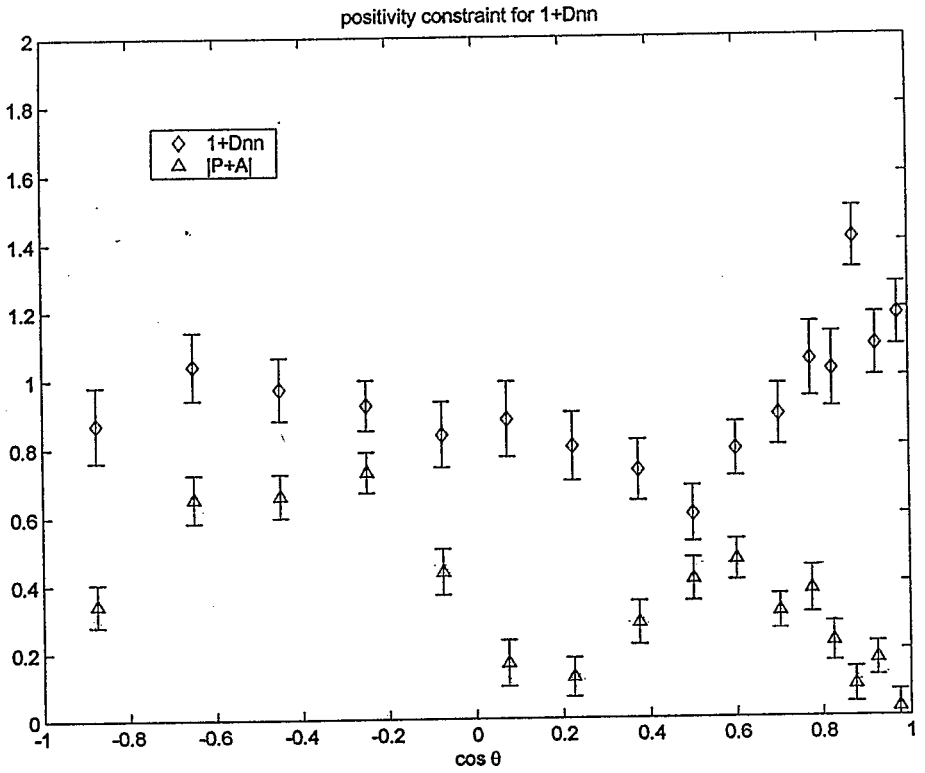
J.-M. Richard / Physics Letters B 369 (1996) 358–361



1. Polarisation P, spin-singlet fraction F_0, and spin-correlation coefficients C_{ij} for the reaction $\bar{p}p \to \bar{\Lambda}\Lambda$ at 1446 (open squ
5 MeV/c (black dots). Data are from Ref. [2].

$1+D_{NN} \geq |P_\Lambda + A_N|$ $\quad$ $|1-D_{NN}| \geq |P_\Lambda - A_N|$





PS185 at LEAR $\bar{p}p \to \bar{\Lambda}\Lambda$

(FROM K. PASCHKE Thesis 2001)

NN POLARIZED TOTAL CROSS SECTION

$$a(1/2) + b(1/2) \rightarrow X$$

GENERALIZED OPTICAL THEOREM

$$\sigma_{tot}(\vec{P}_a, \vec{P}_b) = Tr(M\rho)$$

$$\rho = 1/4\,(1 + \vec{P}_a\vec{\sigma}) \otimes (1 + \vec{P}_b\vec{\sigma})$$

M is THE FORWARD SCATTERING MATRIX

$$a + b \rightarrow a + b \quad (\theta = 0)$$

IT is A 4x4 MATRIX IN THE SPACE OF THE 4 VECTORS

$$|+1/2, +1/2\rangle, \; |+1/2, -1/2\rangle, \; |-1/2, +1/2\rangle, \; |-1/2, -1/2\rangle$$

IN FACT

$$\sigma_{tot}(\vec{P}_a, \vec{P}_b) = \sigma_{tot} + P_{az}P_{bz}\,\Delta\sigma_L + \Delta\sigma_T(P_{ax}P_{bx} + P_{ay}P_{by})$$

WHERE

$$\sigma_{tot} = 1/2\,(\sigma_{tot}(++) + \sigma_{tot}(+-))$$

$$= 1/2\,(\sigma_{tot}(\uparrow\uparrow) + \sigma_{tot}(\uparrow\downarrow))$$

$$\Delta\sigma_L = \sigma_{tot}(+-) - \sigma_{tot}(++)$$

$$\Delta\sigma_T = \sigma_{tot}(\uparrow\downarrow) - \sigma_{tot}(\uparrow\uparrow)$$

M IS THE FOLLOWING HERMITIAN MATRIX (SYMMETRIC)

$$M = \begin{pmatrix} \sigma_{tot} + \Delta\sigma_L/2 & 0 & 0 & \Delta\sigma_T \\ 0 & \sigma_{tot} - \Delta\sigma_L/2 & 0 & 0 \\ 0 & 0 & \sigma_{tot} - \Delta\sigma_L/2 & 0 \\ \Delta\sigma_T & 0 & 0 & \sigma_{tot} + \frac{\Delta\sigma_L}{2} \end{pmatrix}$$

WHICH MUST BE POSITIVE

i.e. ALL DIAGONAL ELEMENTS AND PRINCIPAL MINORS (i.e. WITH DIAGONAL ELEMENTS) OF M MUST BE POSITIVE

$\Rightarrow$ $|\Delta\sigma_L|/2 \leq \sigma_{tot}$ TRIVIAL

$|\Delta\sigma_T| \leq \sigma_{tot} + \Delta\sigma_L/2$ NOT TRIVIAL

IF THESE CROSS SECTIONS ARE MEASURED AT RHIC ENERGIES THEY SHOULD FULFILL THIS CONSTRAINT.

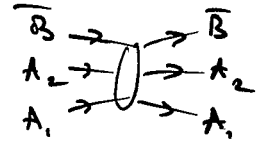
POSITIVITY CONSTRAINTS ON INITIAL SPIN OBSERVABLES IN INCLUSIVE REACTIONS

$$A_1(1/2) + A_2(1/2) \to B + X$$

DON'T OBSERVE SPINS IN FINAL STATE

OBSERVABLES ARE SPIN-DEPENDENT DIFF. C.S. WITH RESPECT TO p_B, i.e. DISCONTINUITIES (W.R.T. M_X) OF THE FORWARD 3-3 AMPLITUDES

$$A_1 + A_2 + \bar{B} \to A_1 + A_2 + \bar{B}$$



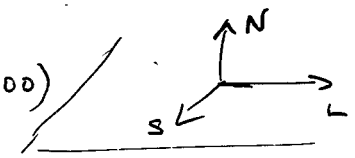
EIGHT OBSERVABLES $\quad (A_1 A_2 | BX) = f(\sqrt{s}, p_T, y)$ (GENERALIZED OPTICAL TH.)

$\sigma_0 = (00|00) \qquad A_{1N} = (N0|00)$

$A_{2N} = (0N|00)$

$A_{LL} = (LL|00)$, $A_{SS} = (SS|00)$, $A_{NN} = (NN|00)$

$A_{LS} = (LS|00)$, $A_{SL} = (SL|00)$



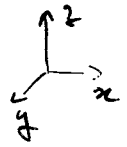
2×2 DENSITY MATRIX OF SPIN-½ PARTICLE

$$\rho_i = \frac{1}{2}\left(\mathbb{1}_2 + \vec{e}_i \vec{\sigma}\right) \qquad i = 1, 2$$

STATE OF POL. OF INCOMING SYSTEM DESCRIBED BY

$$\rho = \rho_1 \otimes \rho_2$$

$$\Rightarrow \qquad \sigma(\vec{e}_1, \vec{e}_2) = \mathrm{Tr}(M\rho)$$



WITH

$$M = \sigma_0 \Big[\mathbb{1}_4 + A_{1N}\, \sigma_{1z} \otimes \mathbb{1}_2 + A_{2N}\, \mathbb{1}_2 \otimes \sigma_{2z} + A_{NN}\, \sigma_{1z} \otimes \sigma_{2z}$$
$$+ A_{LL}\, \sigma_{1x} \otimes \sigma_{2x} + A_{SS}\, \sigma_{1z} \otimes \sigma_{2y} + A_{LS}\, \sigma_{1x} \otimes \sigma_{2y} + A_{SL}\, \sigma_{1y} \otimes \sigma_{2x} \Big]$$

THIS PARAMETRIZATION IS JUSTIFIED SINCE WE HAVE

$$\sigma(\vec{e}_1, \vec{e}_2) = \sigma_0 \Big[1 + A_{1N} e_{1z} + A_{2N} e_{2z} + A_{NN} e_{1z} e_{2z}$$

$$+ A_{LL} e_{1x} e_{2x} + A_{SS} e_{1y} e_{2y} + A_{LS} e_{1x} e_{2y} + A_{SL} e_{1y} e_{2x} \Big]$$

M IS A 4x4 (CROSS SECTION) MATRIX WHICH IS HERMITIAN AND POSITIVE

TO DERIVE THE POSITIVITY CONDITIONS SHOULD WRITE THE EXPLICIT EXPRESSION OF M.

AFTER PERMUTING TWO ROWS AND TWO COLUMNS IT REDUCES TO

$$\left(\begin{array}{c|c} M_+ & 0 \\ \hline 0 & M_- \end{array} \right)$$

$$M_\pm = \begin{pmatrix} (1 \pm A_{NN}) \pm (A_{1N} \pm A_{2N}) & (A_{LL} \pm A_{SS}) \mp i (A_{LS} \pm A_{SL}) \\ (A_{LL} \pm A_{SS}) \pm i (A_{LS} \pm A_{SL}) & (1 \pm A_{NN}) \mp (A_{1N} \pm A_{2N}) \end{pmatrix}$$

$\Rightarrow$ TWO STRONGEST CONDITIONS (NECESSARY AND SUFFICIENT)

$$(1 \pm A_{NN})^2 \geq (A_{1N} \pm A_{2N})^2 + (A_{LL} \pm A_{SS})^2 + (A_{LS} \pm A_{SL})^2$$

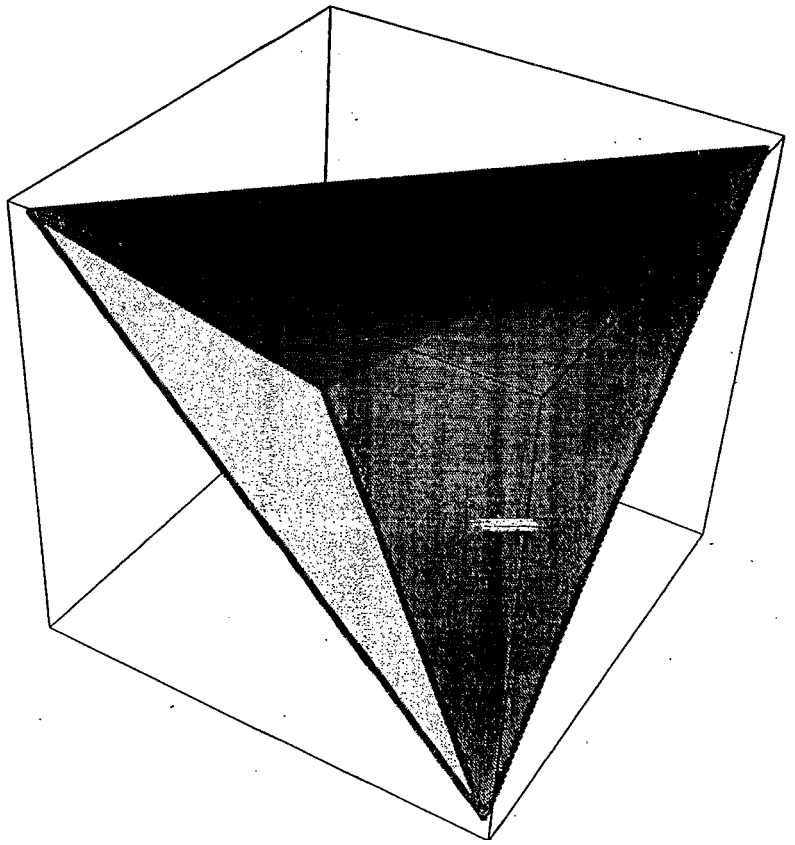
[IF IN A PROCESS $A_{NN} = \pm 1 \Rightarrow A_{1N} = \pm A_{2N}$, $A_{LL} = \pm A_{SS}$, $A_{LS} = \pm A_{SL}$ AND $1 \geq A_N^2 + A_{LL}^2 + A_{LS}^2$]

SIX WEAKER CONDITIONS

$$1 \pm A_{NN} \geq |A_{1N} \pm A_{2N}|$$
$$1 \pm A_{NN} \geq |A_{LL} \pm A_{SS}|$$
$$1 \pm A_{NN} \geq |A_{LS} \pm A_{SL}|$$

(SHOW GRAPH)

VERY GENERAL, MUST HOLD IN ANY KINEMATICAL REGION $(\sqrt{s}, p_T, y)$ ep, e^+e^-, qq', $q\bar{q}$, ETC....



VOLUME ALLOWED $2^3 \longrightarrow 8/3$

SPECIFIC CASE OF pp SCATTERING

FOR IDENTICAL INITIAL PARTICLES

$$A_{IN}(y) = A_{2N}(-y)$$

$$A_{LS}(y) = A_{SL}(-y)$$

TWO CONSTRAINTS AMONG FIVE SPIN OBSERVABLES

$$[1 \pm A_{NN}(y)]^2 \geq [A_{IN}(y) \pm A_{IN}(-y)]^2 + [A_{LL}(y) \pm A_{SS}(y)]^2 + [A_{LS}(y) \pm A_{SL}(-y)]^2$$

SO FOR $y = 0$

$$1 + A_{NN}(0) \geq 2|A_N(0)|$$

$$1 + A_{NN}(0) \geq 2|A_{LS}(0)|$$

IF A_{NN} LARGE AND NEGATIVE $\Rightarrow$ A_N, $A_{LS} \sim 0$

IF A_{NN} LARGE AND POSITIVE USELESS CONDITIONS

HOWEVER ACCORDING TO QCD CALCULATIONS (BNL-RHIC)

FOR $pp \to \gamma X$ AND $pp \to \begin{matrix} jet + X \\ \pi^0 + X \end{matrix}$ $A_{NN} \sim 0$

(ABSENCE OF h_1 FOR GLUONS IN DOMINANT PROCESS $Gq \to Gq$, $Gq \to Gq$)

SO ONE GETS

$$|A_N| \lesssim 1/2 \quad , \quad |A_{LS}| \lesssim 1/2$$

AND ALSO $|A_{LL} + A_{SS}| \lesssim 1$

SO THESE RESULTS ARE STRONG CONSTRAINTS FOR FUTURE DATA AND MODEL BUILDERS

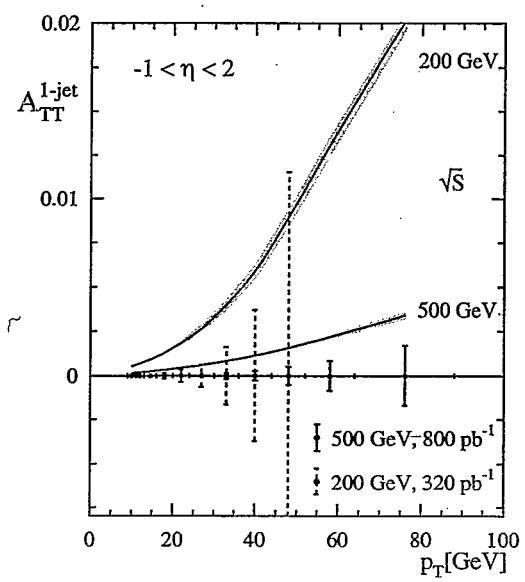
LO results assuming $\delta q(x,\mu_0) = \frac{1}{2}\,[q(x,\mu_0) + \Delta q(x,\mu_0)]$

Soffer, MS, Vogelsang

PRD65, 114024 (2002)

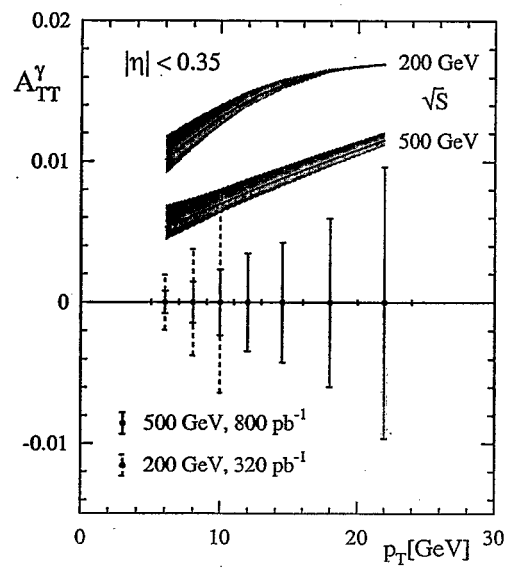
$p^\uparrow p^\uparrow \to \text{jet}\, X$

STAR



$p^\uparrow p^\uparrow \to \gamma X$

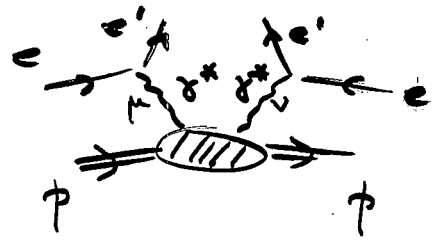
PHENIX



A_{NN} IS SMALL DUE TO THE ABSENCE OF h_1 FOR GLUON ($Gq \to \gamma q$ DOMINATES FOR $pp \to \gamma X$, $GG \to GG$ FOR $pp \to \text{jet}\, X$ AT SMALL $p_\perp$)

DEEP INELASTIC SCATTERING

$$ep \to e'X$$



$$\frac{d^2\sigma}{dE'd\Omega_e} = \frac{4\alpha^2}{Q^4}\frac{E'}{E} L^{(e)}_{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu} = \underset{\substack{\uparrow \\ W_1\ W_2}}{W^{(S)}_{\mu\nu}} + \underset{\substack{\uparrow \\ G_1\ G_2}}{W^{(A)}_{\mu\nu}}$$

THESE FOUR STRUCTURE FUNCTIONS CORRESPOND TO THE FOUR SCATT. AMPLITUDES FOR $\gamma^* p \to \gamma^* p$ WHICH SURVIVE IN THE FORWARD DIRECTION

THEY ARE ALSO RELATED TO THE FOUR TOTAL C.S.

$$\sigma^T_{3/2}\ ,\ \sigma^T_{1/2}\ ,\ \sigma^L_{1/2}\ ,\ \sigma^{TL}_{1/2}$$ ← CAN BE NEGATIVE

UNPOL. DIS LEAD TO $\sigma^T_{3/2} + \sigma^T_{1/2}$ AND $\sigma^L_{1/2}$

POL. DIS TO $\sigma^T_{3/2} - \sigma^T_{1/2}$ AND $\sigma^{TL}_{1/2}$

IN FACT ONE MEASURES

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} \qquad A_2 = \frac{2\sigma^{TL}_{1/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}}$$

OLD POSITIVITY CONSTRAINTS

$$|A_1| \le 1 \qquad |A_2| \le \sqrt{R} \qquad (R = \sigma_L/\sigma_T)$$

TRIVIAL ↑ RECENTLY MEASURED

$R = \sigma_L / \sigma_T$

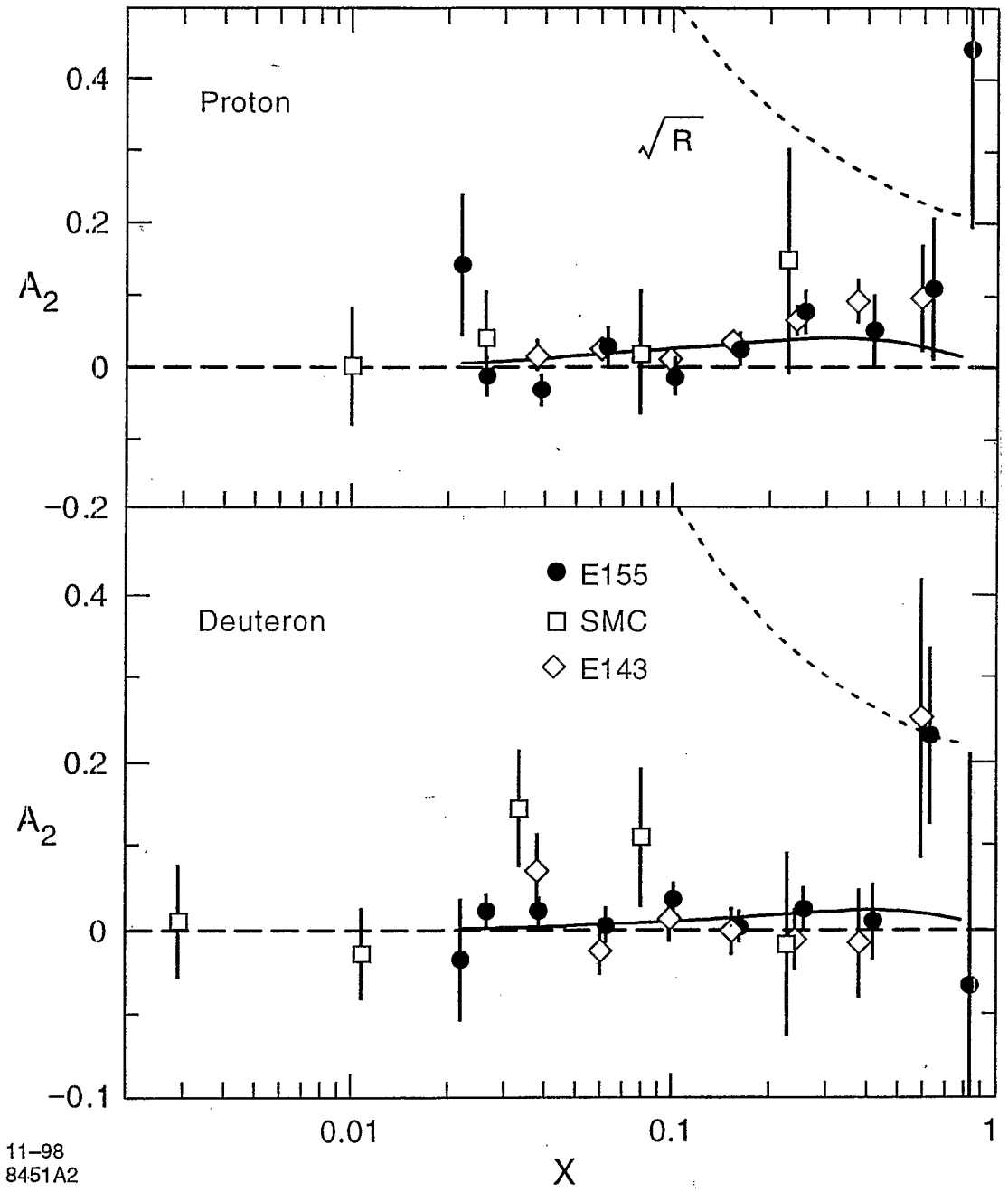


Figure 2: The asymmetries A_2 for proton and deuteron for this experiment (E155) with data from all spectrometers averaged (Table 1). The errors are statistical; the systematic errors are negligible. Also shown are the data from SLAC E143 [7] and SMC[6]. Our A_2^{WW} calculation is shown as the solid line and the $\sqrt{R}$ positivity limit is shown as the dotted curve, evaluated at the average Q^2 for this experiment at each x.

$$(\sigma^T_{3/2,1/2})\ \sigma_T^{\pm} = \sum_x |\langle +1/2, \pm 1 | x \rangle|^2$$

$$(\sigma^L_{1/2})\ \sigma_L = \sum_x |\langle +1/2, 0 | x \rangle|^2 = \sum_x |\langle -1/2, 0 | x \rangle|^2$$

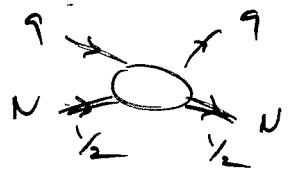
$$(\sigma^{LT}_{1/2})\ \sigma_{LT} = \mathrm{Re} \sum_x \langle +1/2, +1 | x \rangle \langle -1/2, 0 | x \rangle$$

C-S inequality $\sum_x |\langle +1/2, +1 | x \rangle \pm a \langle -1/2, 0 | x \rangle|^2 \geq 0$

a real positive

$$\Rightarrow \quad |\sigma_{LT}| \leq \sqrt{\sigma_L \sigma_T^+}$$

$$|A_2| \leq \sqrt{R\left(\frac{1+A_1}{2}\right)}$$



VERY SIMILAR TO THE CASE WHICH LED TO ANOTHER WELL KNOWN POSITIVITY BOUND

$$2\,|h_1^q(x)| \leq \underbrace{q(x) + \Delta q(x)}_{2q_+(x)}$$

POSITIVITY CONSTRAINTS FOR LEPTON POLARIZATION IN ν-N INTERACTIONS

MOTIVATION

POLARIZATION OF LEPTONS IN νN AND $\bar{\nu} N$ COLLISIONS IS IMPORTANT TO STUDY NEUTRINO OSCILLATIONS

SUPER-KAMIOKANDE MEASURING THE ATMOSPHERIC NEUTRINO FLUX SUGGEST THAT ν_μ OSCILLATES INTO ν_τ WITH NEARLY MAXIMAL MIXING.

TO DEMONSTRATE THIS IT IS IMPORTANT TO DETECT EFFECTS OF ν_τ APPEARANCE AND THEREFORE

$$\nu_\tau (\bar{\nu}_\tau) + N \to \tau^- (\tau^+) + X$$

IS A CRUCIAL REACTION (QE, RES, DIS). ($E_\nu \gtrsim 3.5$ GeV)

THE STUDY OF THE τ POLARIZATION IS NEEDED TO IDENTIFY THE τ PRODUCTION SIGNAL (ALSO TO ESTIMATE THE BACKGROUND) DECAY DIST. DEPENDS ON τ POL.

(WARNING: SINCE $M_\tau > M_p$ CANNOT BE NEGLECTED THE KINEMATICS IS NOT THE USUAL DIS)

$$\frac{d\sigma^{\pm}}{dE_\tau d\cos\theta} = \frac{G_F}{2\pi}\frac{M_W p_\tau}{(Q^2+M_W^2)^2}R_\pm , \qquad \left(L^{\pm}_{\mu\nu} W_{\mu\nu}\right) \tag{3}$$

where G_F is the Fermi constant and M_W is the W-boson mass. Here

$$\begin{aligned} R_\pm &= \frac{1}{M}\left\{\left(2W_1 + \frac{m_\tau^2}{M^2}W_4\right)(E_\tau - p_\tau\cos\theta) + W_2(E_\tau + p_\tau\cos\theta)\right. \\ &\left. \pm\frac{W_3}{M}\left(E_\nu E_\tau + p_\tau^2 - (E_\nu + E_\tau)p_\tau\cos\theta\right) - \frac{m_\tau^2}{M}W_5\right\}, \end{aligned} \tag{4}$$

where the $\pm$ signs correspond to $\tau^\mp$ productions.

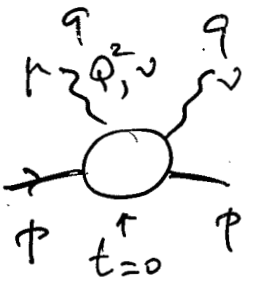
Because of time reversal invariance, the polarization vector $\vec{P}$ of the τ in its rest frame, lies in the scattering plane defined by the momenta of the incident neutrino and the produced τ. It has a component P_L along the direction of $\vec{p_\tau}$ and a component P_P perpendicular to $\vec{p_\tau}$, whose expressions are, in the laboratory frame, [8, 9, 12]

$$P_P = \mp\frac{m_\tau\sin\theta}{MR_\pm}\left(2W_1 - W_2 \pm \frac{E_\nu}{M}W_3 - \frac{m_\tau^2}{M^2}W_4 + \frac{E_\tau}{M}W_5\right), \tag{5}$$

$$\begin{aligned} P_L = \mp\frac{1}{MR_\pm}&\left\{\left(2W_1 - \frac{m_\tau^2}{M^2}W_4\right)(p_\tau - E_\tau\cos\theta) + W_2(p_\tau + E_\tau\cos\theta)\right. \\ &\left. \pm\frac{W_3}{M}\left((E_\nu + E_\tau)p_\tau - (E_\nu E_\tau + p_\tau^2)\cos\theta\right) - \frac{m_\tau^2}{M}W_5\cos\theta\right\}. \end{aligned} \tag{6}$$

In addition, it is convenient to introduce also the degree of polarization defined as $P = \sqrt{P_P^2 + P_L^2}$. As previously the $\pm$ signs correspond to $\tau^\mp$ productions and it is clear that if $W_3 = 0$, one has $R_+ = R_-$ and τ^+ and τ^- have opposite polarizations. We also note that if one can neglect the mass of the produced lepton ($m_\tau = 0$), $P_P = 0$, so such a lepton is purely left-handed, if negatively charged, or purely right-handed, if positive.

$$\begin{aligned} W_{\mu\nu} = &-g_{\mu\nu}W_1(\nu,Q^2) + \frac{p_\mu p_\nu}{M^2}W_2(\nu,Q^2) \\ &- i\varepsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2M^2}W_3(\nu,Q^2) + \frac{q_\mu q_\nu}{M^2}W_4(\nu,Q^2) \\ &+ \frac{p_\mu q_\nu + p_\nu q_\mu}{2M^2}W_5(\nu,Q^2) \left[\cancel{i\frac{(p_\mu q_\nu - p_\nu q_\mu)}{2M^2}W_6(\nu,Q^2)}\right] \end{aligned}$$



$Q^2 = -q^2$

$\nu = \frac{pq}{2M}$

3 Positivity constraints and numerical results

From Eq. (2) clearly the hadronic tensor $W_{\mu\nu}(p,q)$ is Hermitian

$$W_{\mu\nu}(p,q) = W^*_{\nu\mu}(p,q) \ , \tag{7}$$

and semi-positive. This last property implies that

$$a^*_\mu W_{\mu\nu}(p,q) a_\nu \geq 0 \ , \tag{8}$$

for *any* complex 4-vector a_μ. The 4x4 matrix representation of $W_{\mu\nu}(p,q)$ in the laboratory frame where $p = (M,0,0,0)$ and $q = (\nu, \sqrt{\nu^2+Q^2}, 0, 0)$ reads $\begin{pmatrix} M_1 & 0 \\ 0 & M_0 \end{pmatrix}$ where M_1 and M_0 are the following 2x2 Hermitian matrices

$$M_1 = \begin{pmatrix} -W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5 & \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M} W_4 + \frac{1}{2} W_5) \\ \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M} W_4 + \frac{1}{2} W_5) & W_1 + \frac{\nu^2+Q^2}{M^2} W_4 \end{pmatrix} \ , \tag{9}$$

and

$$M_0 = \begin{pmatrix} W_1 & \frac{-i\sqrt{\nu^2+Q^2}}{2M} W_3 \\ \frac{+i\sqrt{\nu^2+Q^2}}{2M} W_3 & W_1 \end{pmatrix} \ . \tag{10}$$

THE NECESSARY AND SUFFICIENT CONDITIONS ARE

$$W_1 \geq 0 \,, \tag{11}$$

$$-W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5 \geq 0 \,, \tag{12}$$

$$W_1 + \frac{\nu^2 + Q^2}{M^2} W_4 \geq 0 \,, \tag{13}$$

d from the 2x2 determinants of M_0 and M_1 we get two inequalities quadratic the W_i's namely

$$W_1^2 \geq \frac{\nu^2 + Q^2}{4M^2} W_3^2 \,, \tag{14}$$

equivalently

$$W_1 \geq \frac{\sqrt{\nu^2 + Q^2}}{2M} |W_3| \,, \tag{15}$$

d

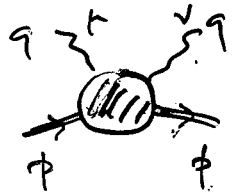
$$\left(-W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5\right)\left(W_1 + \frac{\nu^2 + Q^2}{M^2} W_4\right) \geq \frac{\nu^2 + Q^2}{M^2}\left(\frac{\nu}{M} W_4 + \frac{1}{2} W_5\right)^2 . \tag{16}$$

PHYSICS INTERPRETATION

IF WANT TO CONSIDER THE FORWARD VECTOR-BOSON NUCLEON COMPTON AMPLITUDES

$$M(h, h') = \varepsilon_\mu^*(h')\, W_{\mu\nu}\, \varepsilon_\nu(h)$$

- LINEAR ARE POSITIVITY OF C.S.
- QUADRATIC IS A CAUCHY-SCHWARZ INEQUALITY



WILL NOT USE ANY SPECIFIC MODEL TO TEST USEFULNESS OF POSITIVITY

GENERATE RANDOMLY THE VALUES OF THE W_i's

$$W_1, W_2 \in [0,1]$$

$$W_3, W_4, W_5 \in [-1,+1]$$

NOTE IF IMPOSE $R_\pm > 0$ IT IS TOO WEAK TO IMPLY $|P| < 1$

- SO FIRST IMPOSE THE TRIVIAL POSITIVITY $R_\pm > 0$ $|P| < 1$ $(\Rightarrow |P_L| \leq 1$, $|P_P| \leq 1)$

GET THE GREY DOTS

- THEN IMPOSE THE NON-TRIVIAL ONES

GET THE BLACK DOTS

WE OBSERVE A STRONG REDUCTION OF THE ALLOWED REGIONS

$x \nearrow$ LEAD TO A SITUATION WHERE τ^+ (τ^-) IS ALMOST PURELY RIGHT-HANDED (LEFT HANDED)

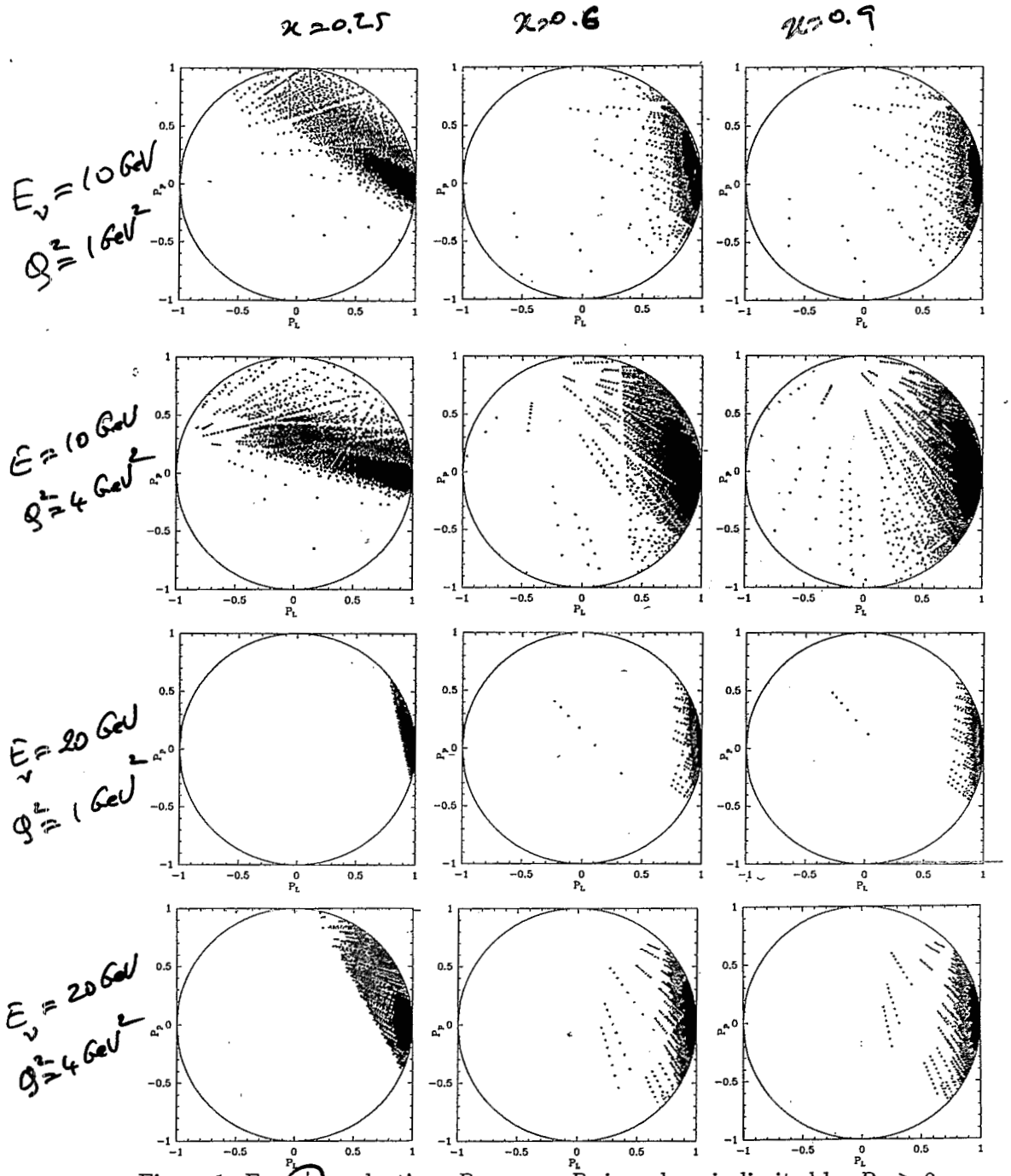


Figure 1: For τ^+ production, P_P versus P_L in a domain limited by $R_+ \geq 0$, $P \leq 1$ (grey area) plus non trivial positivity constraints (black area). From top to bottom and left to right, $E_\nu = 10\text{GeV}$, $Q^2 = 1\text{GeV}^2$, $x = 0.25, 0.6, 0.9$, $E_\nu = 10\text{GeV}$, $Q^2 = 4\text{GeV}^2$, $x = 0.4, 0.6, 0.9$, $E_\nu = 20\text{GeV}$, $Q^2 = 1\text{GeV}^2$, $x = 0.25, 0.6, 0.9$, $E_\nu = 20\text{GeV}$, $Q^2 = 4\text{GeV}^2$, $x = 0.25, 0.6, 0.9$.

FOR τ^- GET SYMMETRIC WITH RESPECT TO THE CENTER

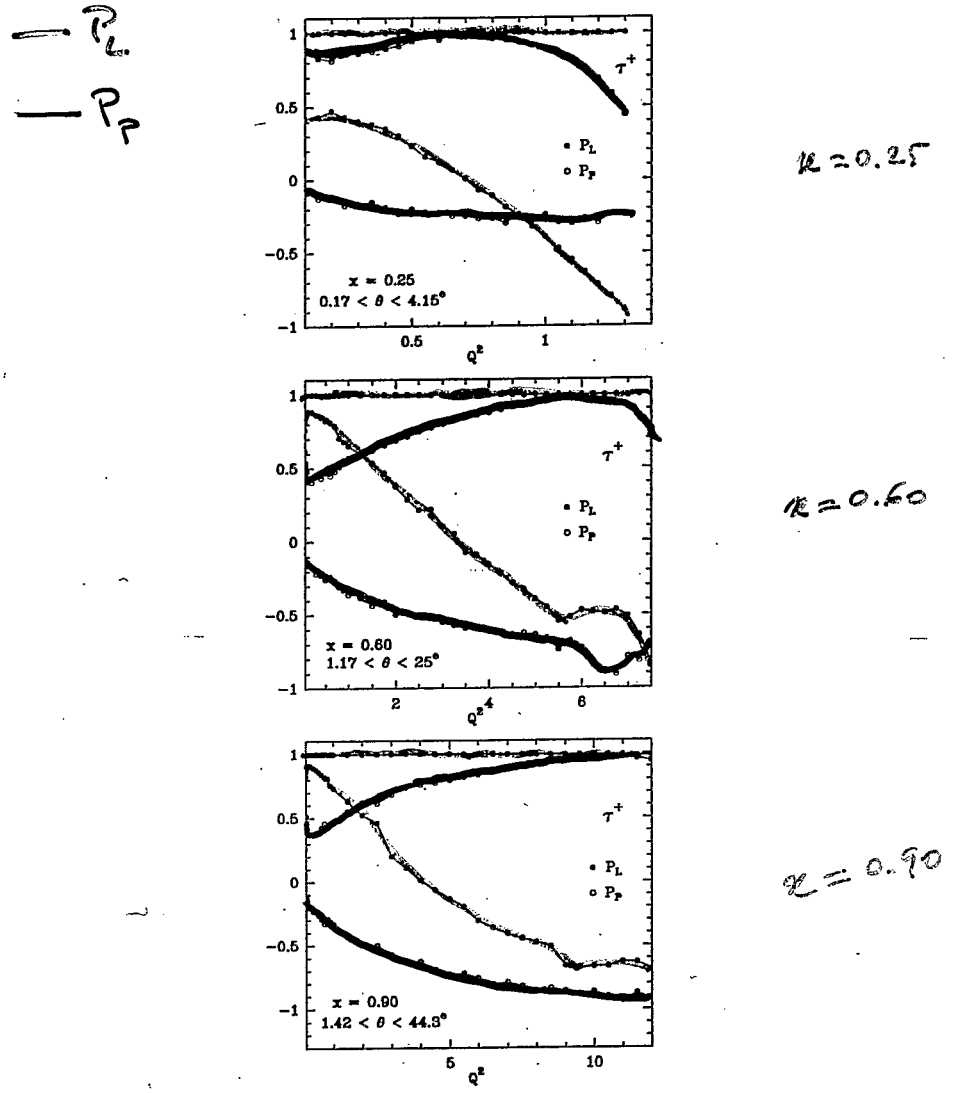
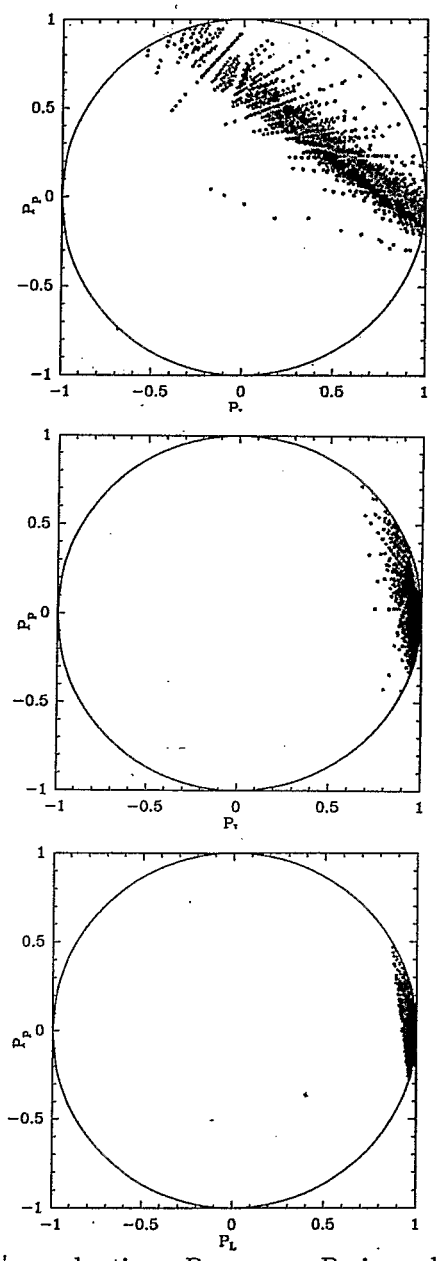


Figure 2: For τ^+ production, upper and lower bounds on P_P (open circles) and P_L (full circles) as a function of Q^2 for $E_\nu = 10$GeV and $x = 0.25, 0.6, 0.9$.



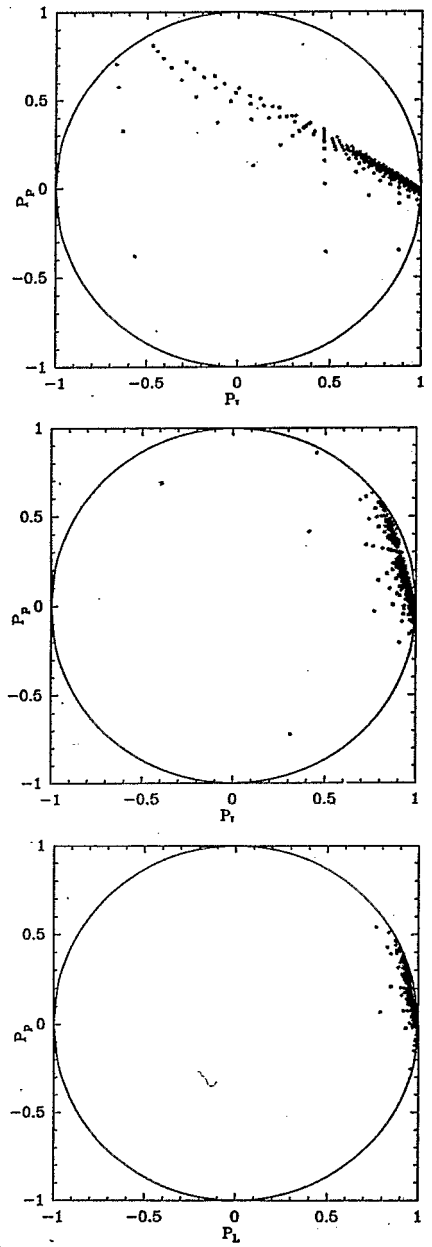
$x = 0.25$

$x = 0.60$

$x = 0.90$

Figure 3: For τ^+ production, P_P versus P_L in a domain limited by $R_+ \geq 0$, $P \leq 1$ assuming the Callan-Gross relation (grey area) plus non trivial positivity constraints (black area). $E_\nu = 10\text{GeV}$, $Q^2 = 1\text{GeV}^2$, from top to bottom, $x = 0.25, 0.6, 0.9$.

$Q^2 W_1 = \nu^2 W_2$



$x = 0.25$

$x = 0.60$

$x = 0.90$

Figure 4: For τ^+ production, P_P versus P_L in a domain limited by $R_+^2 \geq 0$, $P \leq 1$ assuming the Albright-Jarlskog relations (grey area) plus non trivial positivity constraints (black area). $E_\nu = 10\text{GeV}$, $Q^2 = 1\text{GeV}^2$, from top to bottom, $x = 0.25, 0.6, 0.9$.

$MW_1 = \nu W_5$

$W_4 = 0$