Understanding the spin structure of the nucleon

C. Weiss (Regensburg U.), June 8-9, 2004

- Quark / Antiquark / Gluon distributions are long-distance characteristics of the nucleon, but refer explicitly to QCD degrees of freedom!

- Basic "philosophy"

Interpolation

Spontaneous breaking of chiral symmetry		Asymptotic freedom
π as Goldstone	"Constituent" quarks	Quarks / Gluons
Chiral Lagrangian	Nucleon as soliton ($N_c \to \infty$ limit)	pert. QCD
1 fm		0.1 fm

Language: Field theory!

Outline

- Quark/Antiquark distributions in the nucleon
  - Factorization
  - Partonic sum rules

- Spontaneous breaking of chiral symmetry
  - Analogy with Ferromagnetism
  - Chiral Lagrangian for soft pions
  - Nucleon as chiral soliton ("Skyrmion")

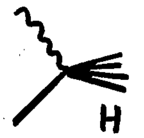
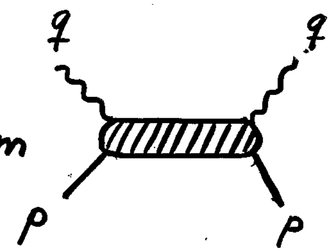
- "Interpolating" between chiral dynamics and pert. QCD
  - Constituent quarks
  - Chiral quark-soliton model of the nucleon

- Results for quark/antiquark distributions
  - unpolarized
  - polarized, $\Delta\bar{u} - \Delta\bar{d}$ asymmetry

- Transversity
  - How different are $\delta q(x)$ and $\Delta q(x)$

    $\delta\bar{q}$ and $\Delta\bar{q}$

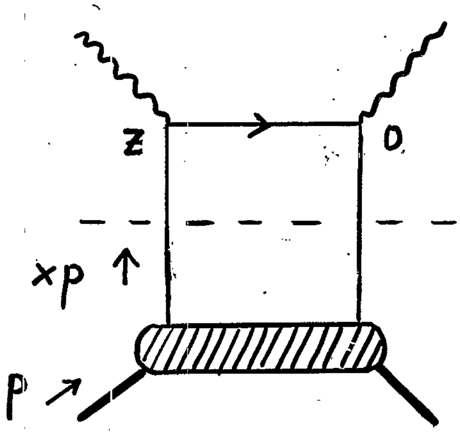
Quark/Antiquark distributions in the nucleon

$$\sigma_{tot}^{\gamma^* N} = \sum_H \Big|$$  $$\Big|^2 = \mathrm{Im}$$ 

virtual Compton amp. (forward scattering)

Q^2, W^2 large!

Bjorken Limit



$\sim Q^2$

"Factorization"

$\sim$ hadronic

$$\langle p | \bar{\psi}(0)\, \gamma_\mu\, \psi(z) | p \rangle = \bar{u} \gamma_\mu u \int_0^1 dx \left[e^{-ix p\cdot z}\; q(x) - e^{ix p \cdot z}\; \bar{q}(x) \right]$$

$z^2 = 0$
light cone

quark/antiquark distributions

> Factorization: Separation of momenta (virtualities)
> $Q^2 \longleftrightarrow$ hadronic

$$\langle p|\,\bar{\psi}(0)\,\gamma_\mu \overset{\gamma_5}{\downarrow}\,\psi(z)\,|p\rangle \;=\; \bar{u}\gamma_\mu \overset{\gamma_5}{\downarrow} u \int_0^1 dx \left[e^{-ixp\cdot z}\,\Delta q(x) + e^{ixp\cdot z}\,\Delta\bar{q}(x) \right]$$

$z^2 = 0$
light cone

polarized quark/antiquark distributions

Factorization: Separation of momenta (virtualities)
$Q^2 \longleftrightarrow$ hadronic

Partonic sum rules

$z_\mu \to 0$ $\quad \bar{\psi}(0)\,\gamma_\mu\,\psi(0)$ $\quad$ local vector current

$\gamma_\mu\gamma_5$ $\quad$ axial vector

... conserved (scale-independent)!

... Matrix elements known from low-energy em/weak interactions!

$$\to \int_0^1 dx\,[u(x) - \bar{u}(x)] = 2$$

$d \quad \bar{d}$

number of valence quarks

$$\int_0^1 dx\,[\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}] = g_A^{(3)}$$

isovector axial coupling

Bjorken sum rule

Isoscalar axial current not conserved.
[U(1) anomaly of QCD]

«spin crisis»

Spontaneous breaking of chiral symmetry

- Ground state of many-particle system / field theory can have "less symmetry" than dynamics ($\equiv$ Hamiltonian).

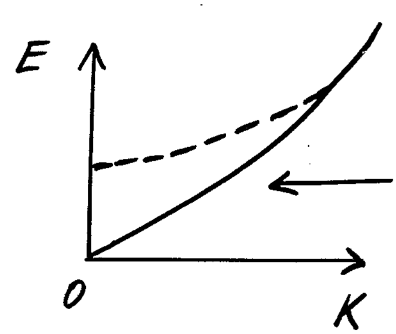
- Example: Ferromagnetism

$$H = \sum_{i,j} c_{ij}\, \vec{S}_i \cdot \vec{S}_j \qquad \text{Spins on a lattice}$$

... invariant under $O(3)$ rotations $\vec{S}_i \to \mathcal{O}\,\vec{S}_i$

Ground state	$\nearrow \searrow \nearrow$ $\leftarrow \nearrow \rightarrow$ $\downarrow \searrow \uparrow$	$\uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow$ $\vec{M} \equiv \frac{1}{V}\left\langle \sum_i \vec{S}_i \right\rangle \neq 0$ "order parameter"
...invariant under	$O(3)$	$O(2)$ subgroup only

Excitation spectrum

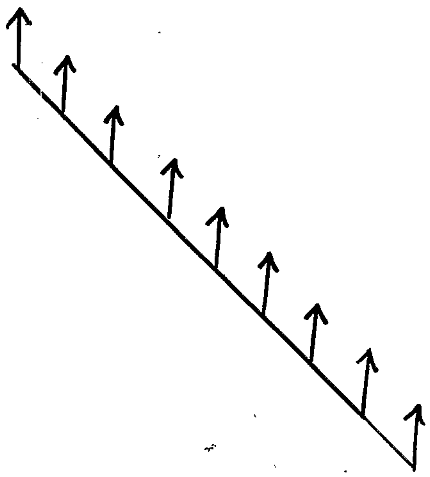


spin waves (massless)

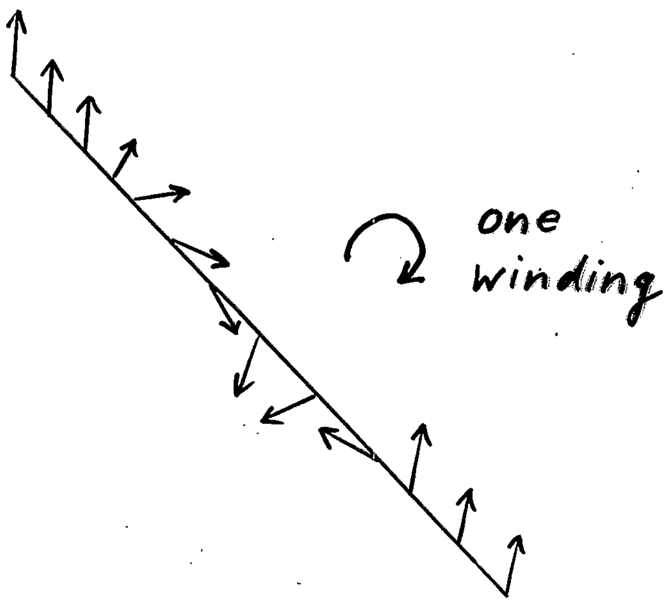
"Goldstone boson"

New type of "large" excitation: Topological soliton

... here: 1D system



Ground state



Soliton
("particle")

- Chiral symmetry of QCD

$\gamma_5^2 = 1 \qquad \psi_{L,R} \equiv \underbrace{\frac{1 \pm \gamma_5}{2}}_{\text{projector}} \psi$

left/right-handed components of quark field

$\gamma_5 \psi_{L,R} = \pm \psi_{L,R}$

$$\mathcal{L}_{QCD} = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R + \cancel{m} (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) + \text{gluonic part}$$

... invariant under independent flavor rotations of L and R components

$$\left.\begin{aligned} \psi_L &\to e^{i\vec{\alpha}_L \vec{\tau}} \psi_L \\ \psi_R &\to e^{i\vec{\alpha}_R \vec{\tau}} \psi_R \end{aligned}\right\} \quad SU(2)_L \times SU(2)_R \text{ group}$$

- Alt. representation:

$\psi \to e^{i\vec{\alpha}\cdot\vec{\tau}} \psi$ isospin rot'ns, $SU(2)$ subgroup

$\psi \to e^{i\gamma_5 \vec{\beta}\cdot\vec{\tau}} \psi$

$\rightsquigarrow$ conserved currents

$\bar{\psi} \gamma_\mu \tau^a \psi$ vector

$\bar{\psi} \gamma_\mu \gamma_5 \tau^a \psi$ axial vector

- QCD ground state ("vacuum")

$$\langle 0|\bar\psi\psi|0\rangle = \langle 0|\bar\psi_L\psi_R + \bar\psi_R\psi_L|0\rangle \neq 0$$ chiral condensate

... invariant under isospin rotations only!

$$SU(2)_L \times SU(2)_R \xrightarrow{\text{spont. broken}} SU(2)_{isospin}$$

- Goldstone bosons: Pions

$$\underbrace{\langle 0|\bar\psi(x)\gamma^\mu\gamma_5\tau^a\psi(x)|\pi^b(p)\rangle}_{\text{axial current}} = i F_\pi\, p^\mu \delta^{ab} e^{-ip\cdot x}$$

F_π: pion decay constant ($\pi^+ \to e^+\bar\nu$), 93 MeV

$$\partial_\mu(\dots) = 0 \;\rightarrow\; p^2 = M_\pi^2 = 0 \quad \text{... massless!}$$

[If quark mass non-zero $m \neq 0$

$$\partial_\mu(\bar\psi\gamma^\mu\gamma_5\tau^a\psi) \propto m \;\rightarrow\; M_\pi^2 \propto m \;]$$

- Interactions of pions: Chiral Lagrangian

$$U(x) \equiv e^{i\tau^a \pi^a(x)/F_\pi} \qquad \text{unitary matrix field}$$

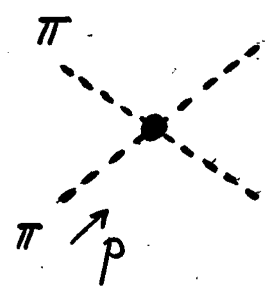
... transforms as $\psi_L \bar{\psi}_R$ — "Phase" of chiral condensate

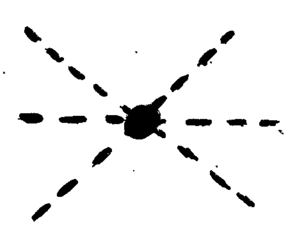
Construct effective Lagrangian from invariants
[Weinberg 66; Callan, Coleman, Wess, Zumino 66; Gasser, Leutwyler 80's]

$$\mathcal{L}_{chiral} = \frac{F_\pi^2}{4} tr[\partial_\mu U^\dagger \partial^\mu U] + \text{terms } (\partial U)^4 + \dots$$

$\Longrightarrow$ # of derivatives

... describes all possible scattering processes of pions in zero-momentum limit $p_\mu \to 0$



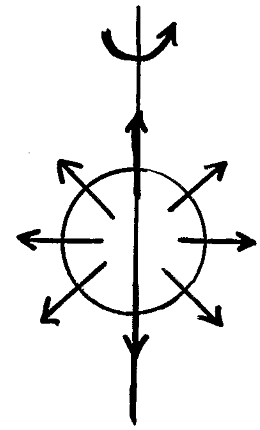


... increasing number of coupling constants!

- Intriguing suggestion: Nucleon as topological soliton [Skyrme 60's]

$$U(\vec{x}) = e^{i\tau^a \hat{r}^a P(r)}$$

static field



$\pi^a \propto \hat{r}^a$

"hedgehog"

Spin/Isospin states: Quantization of collective rotations

$$J = I = \tfrac{1}{2} \quad N$$
$$\tfrac{3}{2} \quad \Delta$$

$$M = M_{static} + \frac{J(J+1)}{2\Theta}$$

moment of inertia

Winding number of chiral phase (topolog. invariant) $\longleftrightarrow$ Baryon number in QCD [Witten 79, 83]

$\rightarrow$ magnetic moments, g_A
form factors,
⋮

[Simple model: Skyrme's $(\partial U)^4$ term]

Questions:

- How to estimate coupling constants in chiral Lagrangian?

  ... "Economics"

- How to turn Skyrme's picture into a quantitative dynamical model?

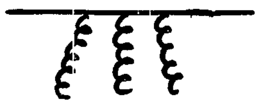
  $(\partial U)^4, (\partial U)^6, \ldots$ is not enough

- How to compute quark/antiquark distributions in nucleon?

spontaneous breaking of chiral symmetry $\longleftrightarrow$ QCD degrees of freedom

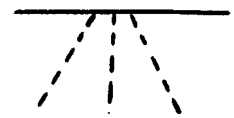
?

Interpolating between chiral dynamics and pert. QCD



non-pert. gluon field

$\xrightarrow{\text{spont. br. chiral sym.}}$



pion field

$M \approx 350$ MeV dynamical mass

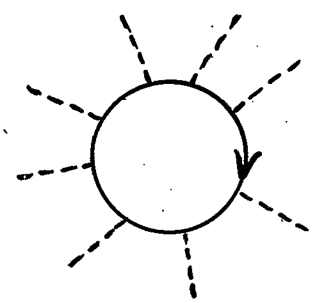
$g_{\pi qq} = \frac{M}{F_\pi} \gg 1$!

$$\rightarrow \quad \mathcal{L}_{eff} = \bar{\psi}\left(i\partial\!\!\!/ - M e^{i\gamma_5 \tau^a \pi^a / F_\pi}\right)\psi$$

[Diakonov, Petrov 86]

... valid for momenta $-p^2 < \Lambda^2 \approx (600\,\text{MeV})^2$ Ultraviolet cutoff

- Integrate over constituent quark fields



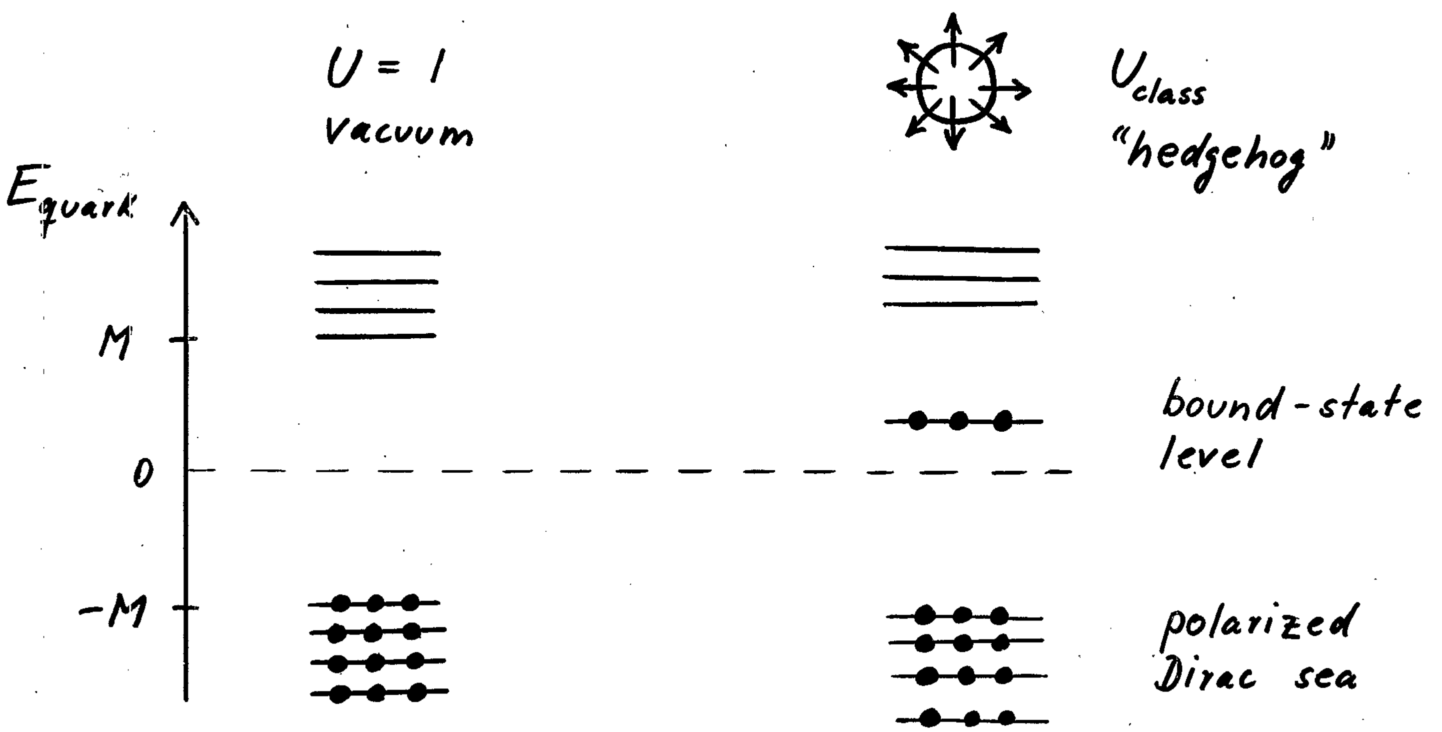
$$S_{eff}[\pi] = \text{Det}\left[i\partial\!\!\!/ - M e^{i\gamma_5 \tau^a \pi^a}\right]$$

$$= \frac{F_\pi^2}{4}\,\text{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \text{terms } (\partial U)^4 + \ldots$$

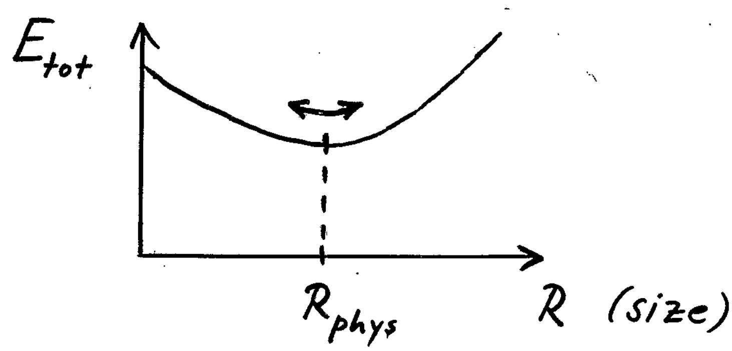
with $F_\pi^2, \ldots = \text{function}(M, \Lambda)$

$$\boxed{\text{Field theory of massive “constituent” quarks coupled to pion field interpolates between chiral Lagrangian and free quarks}}$$

- Nucleon ($N_c \to \infty$): Constituent quarks/antiquarks moving in classical pion field [Diakonov, Petrov, Pobylitsa 88]



$$E_{tot} = N_c E_{level} + N_c \sum_{sea} (E_n - E_n^{vac})$$



... Contains both skyrmion and non-rel. quark model as limiting cases!

... Fully field-theoretical description of nucleon!

- Calculate quark / antiquark distributions
  [Diakonov, Petrov, Pobylitsa, Polyakov, CW 96/97]

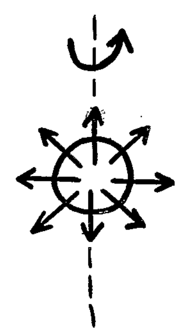
$$\underbrace{\langle N | \, \bar{\psi}(0) \ldots \psi(z) \Big|_{\mu^2} | N \rangle}$$

Identify QCD operator at scale $\mu^2 = \Lambda^2 = (600 \text{ MeV})$
with constituent quark operator

calculate
matrix element

$$\sum_{\substack{\text{level} \\ + \text{sea}}} \langle n | \ldots | n \rangle$$

quark states
in background of
classical pion field



$\Omega \sim \frac{1}{N_c}$

slowly
rotating

$$\left.\begin{array}{ll} u+d, & \bar{u}+\bar{d} \\ \Delta u - \Delta d, & \Delta \bar{u} - \Delta \bar{d} \end{array}\right\} \sim \Omega^0$$

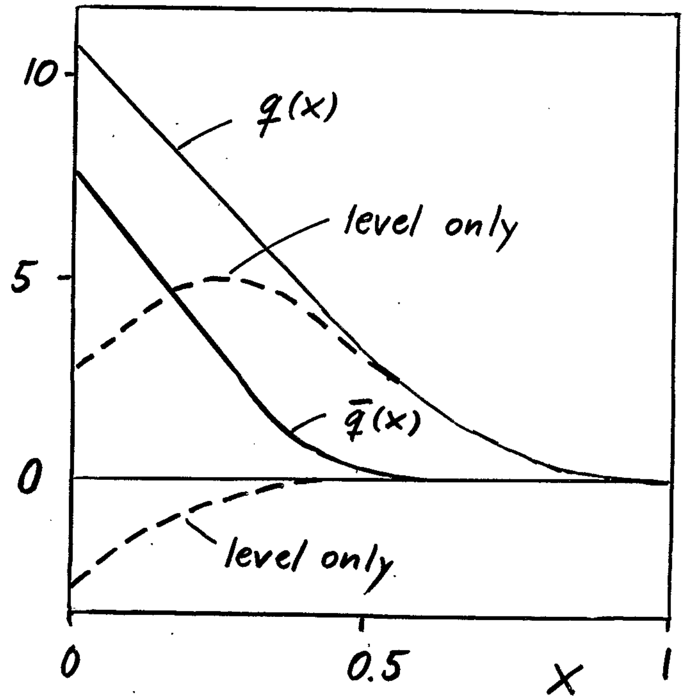
... leading in $1/N_c$

$$\left.\begin{array}{ll} u-d, & \bar{u}-\bar{d} \\ \Delta u + \Delta d, & \Delta \bar{u} + \Delta \bar{d} \end{array}\right\} \sim \Omega^1$$

... suppressed

Isovector polarized distributions leading in $1/N_c$ expansion

- Unpolarized distributions — isoscalar



$q \equiv u + d$

[Diakonov, Petrov, Pobylitsa, Polyakov, CW 96/97]

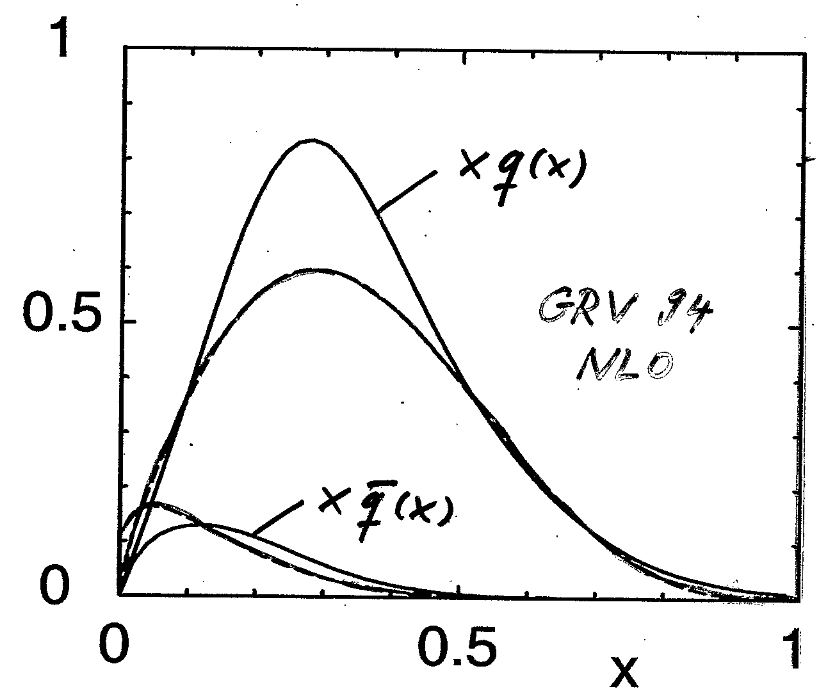
$$\bar{q}(x) = \underbrace{\bar{q}(x)_{level}}_{<0} + \bar{q}(x)_{sea} > 0$$ as should be!

↯ bag model

$$\int dx \left[q(x) - \bar{q}(x) \right] = 3$$ quark number sum rule

$\boxed{\text{Positivity and sum rules satisfied thanks to field-theoretical description of nucleon ... "Completeness"}}$

Comparison with parametrizations
(from fits to DIS & other data)



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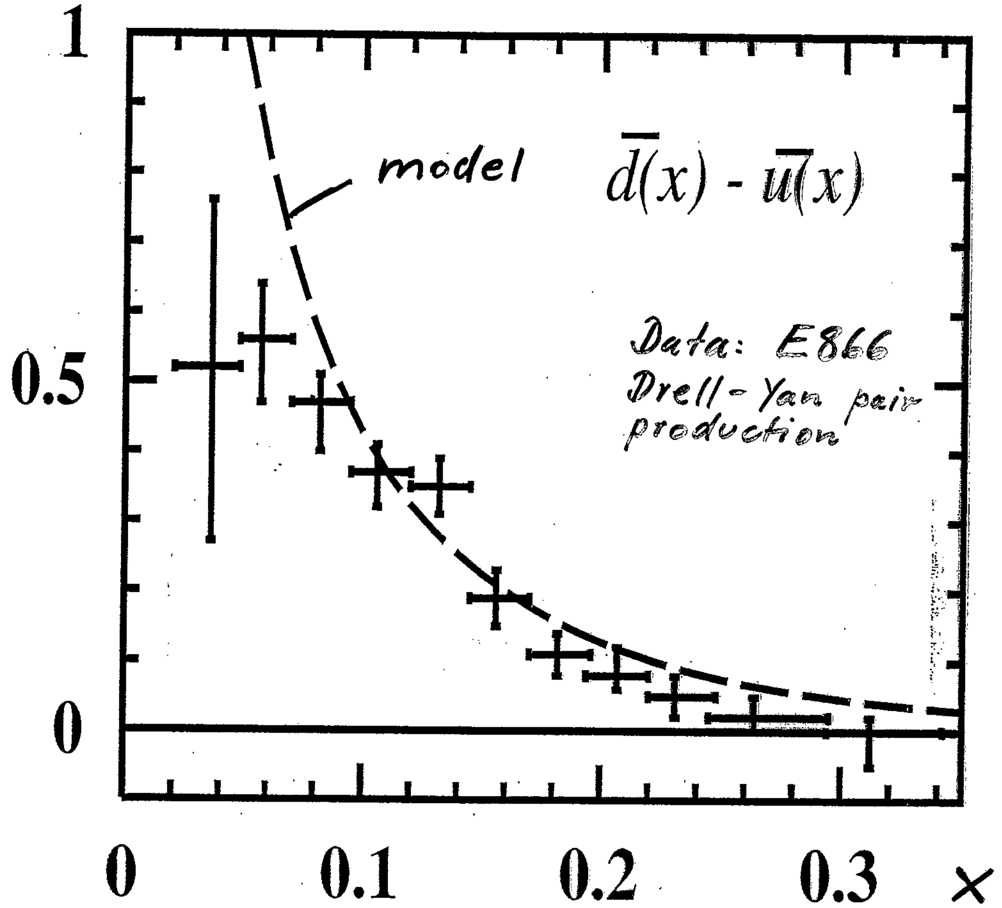
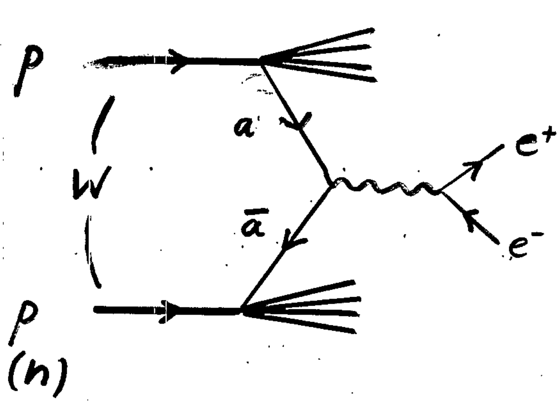
- *Unpolarized distribution — isovector*

[Pobylitsa, Polyakov, Goeke, Watabe, CW 98]





$M^2, p_{\parallel}$

$$\sigma(pp \to e^+e^- + X)$$

$$\propto \sum_a e_a^2\, q_a(x_1)\, q_{\bar{a}}(x_2)$$

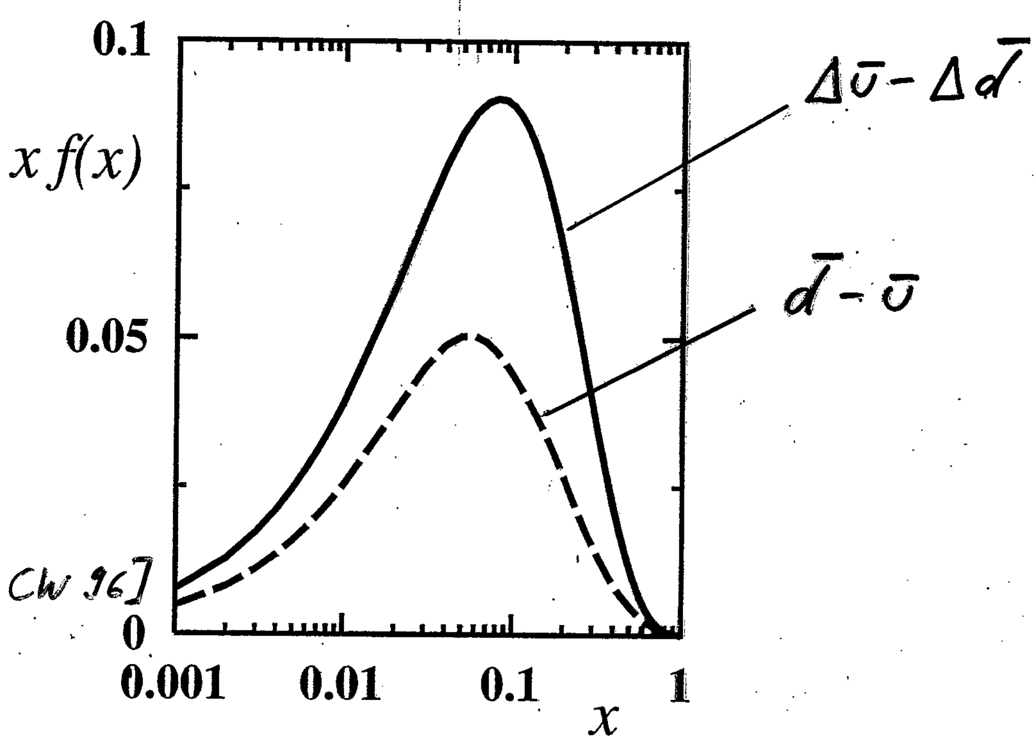
$$x_{1,2} = \text{function}\,(W, p_{\parallel})$$

● Polarized distributions – isovector

$1/N_c$ - expansion suggests: $|\Delta\bar{u} - \Delta\bar{d}| \gg |\bar{u} - \bar{d}|$

leading — suppressed

...parameter-free prediction!



[Diakonov, Petrov, Pobylitsa, Polyakov, CW 96]

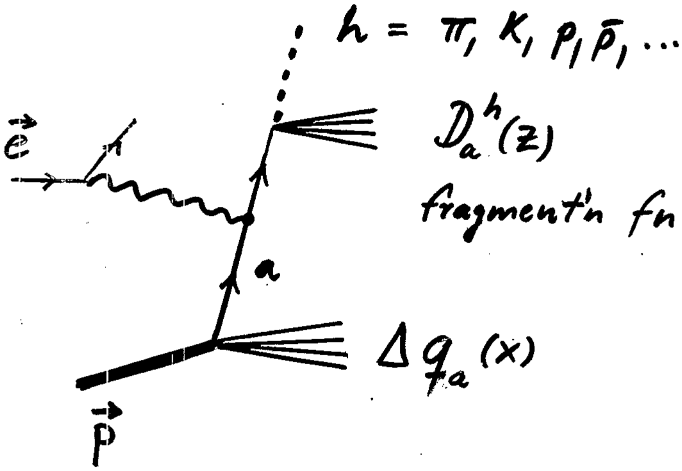
... Large polarized antiquark flavor asymmetry!

[Alt. explanations:
- Pauli blocking in quark model
- Meson cloud models with "π-σ interference]

How to measure?

● Semi-inclusive DIS $\gamma^* p \to h + X$

[SMC, HERMES 98-03, COMPASS, JLAB 9 11 GeV]



$$A_1^h \equiv \frac{\sigma_+^h - \sigma_-^h}{\sigma_+^h + \sigma_-^h} = \frac{\sum_a e_a^2 \, \Delta q_a(x) \, D_a^h(z)}{\sum_a e_a^2 \, q_a(x) \, D_a^h(z)}$$

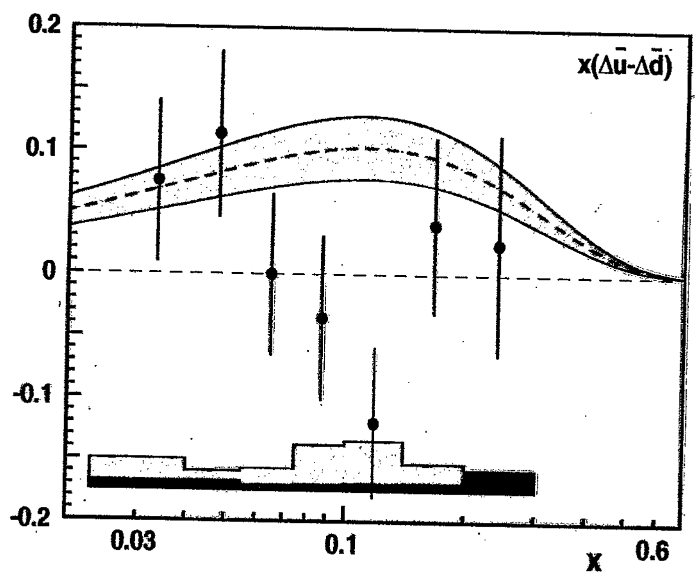
spin asymmetry

$a = u, \bar{u}, d, \bar{d}, \ldots$

HERMES 03

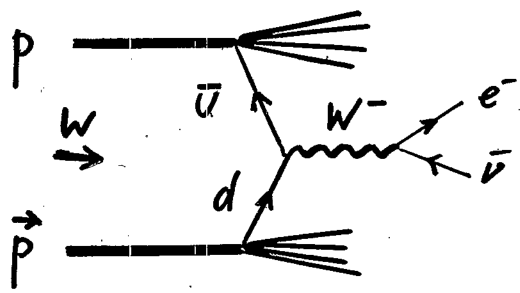
semi-inclusive

$\gamma^* + \vec{N} \to \pi^\pm + X$
$K^\pm$
$p\bar{p}$



... Problem: Low sensitivity of A_1^h to $\Delta\bar{u} - \Delta\bar{d}$!

- $W^\pm$ production in $\vec{p}p$ at RHIC [Leader, Sridhar; Bourrely, Soffer]



$p_{\parallel}$

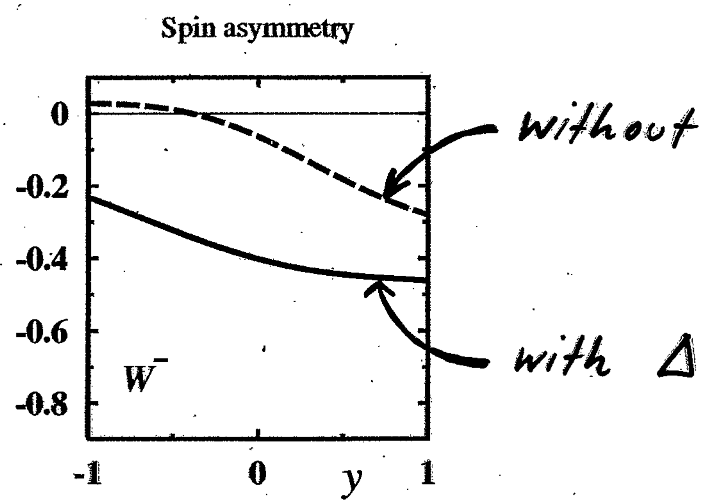
$$A_L \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

$$= \frac{-\bar{u}(x_1)\,\Delta d(x_2) + d(x_1)\,\Delta\bar{u}(x_2)}{\bar{u}(x_1)\,d(x_2) + d(x_1)\,\bar{u}(x_2)}$$

Scale: M_W^2

$$x_{1,2} = \text{function}(W, p_{\parallel})$$

single spin asymmetry

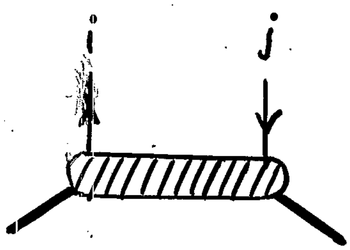


[Goeke, Polyakov, Schweitzer, Strikman, CW 00]

... high sensitivity to $\Delta\bar{u} - \Delta\bar{d}$!

Transversity

[Ralston, Soper; Artru, Mekhfi; Jaffe, Ji]



Quark density matrix
(Dirac spinor)

$$\langle N | \bar{\psi}_j(0) \ldots \psi_i(z) | N \rangle \quad \propto \quad (\gamma_\mu)_{ij} \qquad (\gamma_\mu \gamma_5)_{ij} \qquad (\sigma_{\mu\nu})_{ij}\, z^\nu$$

$q(x)$	$\Delta q(x)$	$\delta q(x)$
unpol.	pol.	transversity

... chirally odd!
$L \to R$

Sum rule

$$\int_0^1 dx \left[\delta q(x) - \delta \bar{q}(x) \right] = g_T$$

$$\langle N | \underbrace{\bar{\psi}(0)\, \sigma_{\mu\nu} \gamma_5\, \psi(0)}_{\text{not conserved!}} | N \rangle = \bar{u}\, \sigma_{\mu\nu} \gamma_5\, u \; g_T$$

"tensor charge"

cf. $\ldots \gamma_\mu \gamma_5 \ldots$ $\qquad \ldots \gamma_\mu \gamma_5 \ldots \; g_A$ axial charge

- How different is $\delta q(x)$ from $\Delta q(x)$?
  $\delta\bar{q}$ $\Delta\bar{q}$

– Non-rel. system: $\delta q(x) = \Delta q(x)$ no spin-orbi[t] interaction

– Chiral dynamics: Coupling to pion field

$$\left.\begin{array}{lll} \bar{\psi}\gamma_\mu\gamma_5\tau^a\psi & \longrightarrow & \partial_\mu\pi^a \\ \bar{\psi}\sigma_{\mu\nu}\gamma_5\psi & \longrightarrow & \varepsilon^{abc}\varepsilon_{\mu\nu\rho\sigma}\partial_\rho\pi^b\partial_\sigma\pi^c \end{array}\right\}\text{very different}$$

↷ Expect: Quark distributions similar
Antiquark " very diffcrent

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- Chiral quark-soliton model: $\delta q(x)$ vs. $\Delta q(x)$
  $\delta\bar{q}$ $\Delta\bar{q}$

[Schweitzer et al. 01,
Pobylitsa, Polyakov 96]

$g_T^{(3)} = 1.06$

$g_T^{(0)} = 0.63$

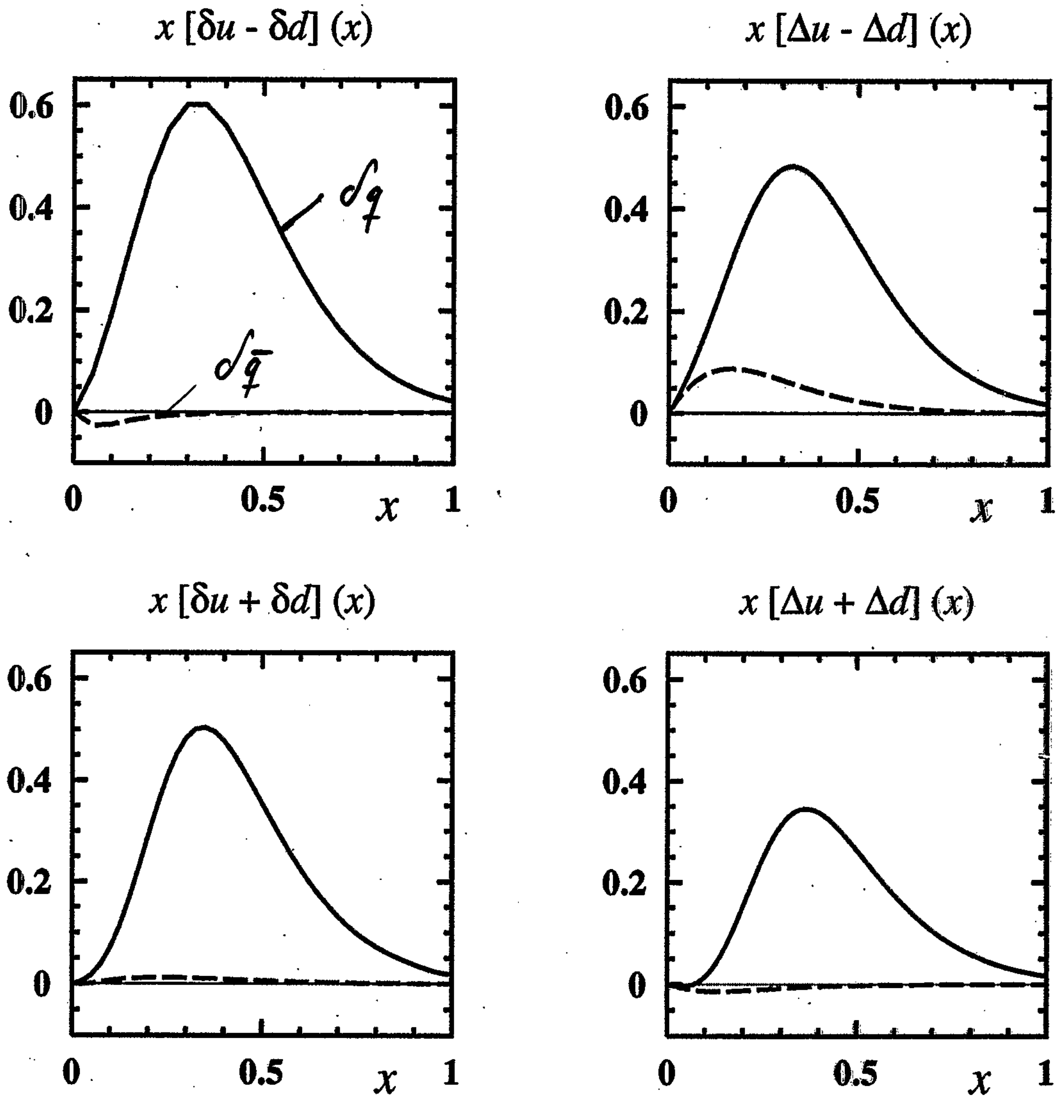


Figure 2: *The total isovector (top row) and isoscalar (bottom row) transversity and longitudinally polarized quark– and antiquark distributions, multiplied by x. Shown are the total results (sum of level and continuum contributions), corresponding to the solid lines in Fig. 1.)* <u>*Solid lines:*</u> *Quark distributions.* <u>*Dashed lines:*</u> *Antiquark distributions.*

• Drell-Yan pair production with transversely polarized protons – spin asymmetry

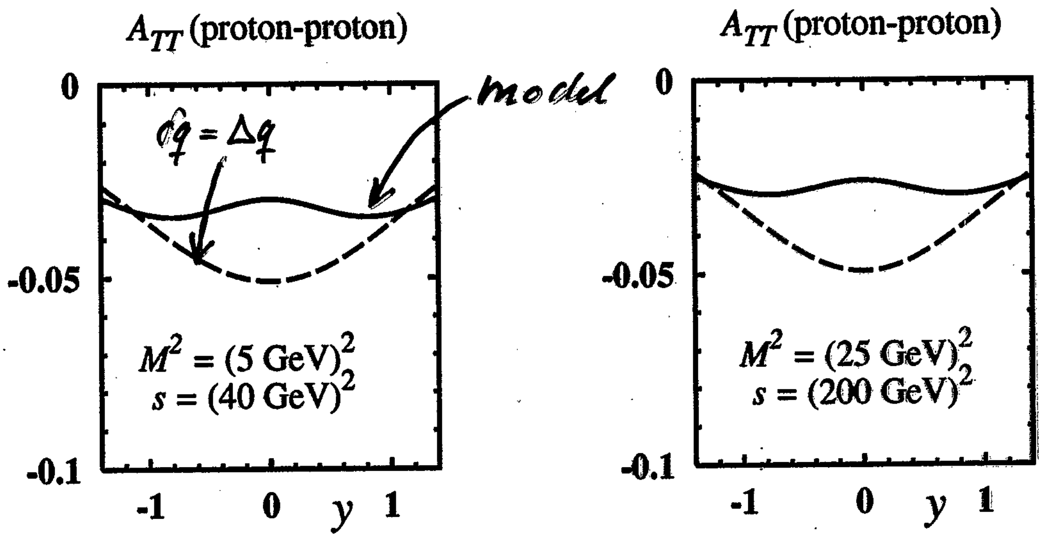
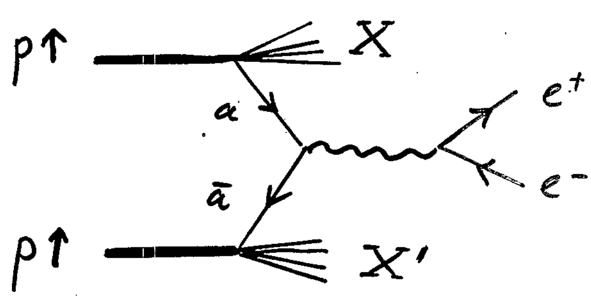


Figure 6: *The transverse spin asymmetry, A_{TT}, Eq.(5.1), in collisions of transversely polarized protons, in two different kinematical regions: $s = (40\,\mathrm{GeV})^2, M^2 = (5\,\mathrm{GeV})^2$ (left), and $s = (500\,\mathrm{GeV})^2, M^2 = (25\,\mathrm{GeV})^2$ (right). Solid lines: Asymmetries calculated with the quark- and antiquark distributions computed in the chiral quark–soliton model, cf. Fig.2. For the quark distributions we used the calculated ratios of transversity to longitudinally polarized distributions, Eqs.(4.6) and (4.7), together with the GRSV95 parameterizations* [3] *for $[\Delta u - \Delta d](x)$ and $[\Delta u + \Delta d](x)$. Dashed lines: Asymmetries obtained assuming that $\delta q(x) \equiv \Delta q(x)$ and $\delta\bar{q}(x) \equiv \Delta\bar{q}(x)$ $(q = u, d)$, using the GRSV95 parameterizations* [3] *for $\Delta q(x)$ and $\Delta\bar{q}(x)$.*



Summary

- Model of constituent quarks coupled to pion field interpolates between chiral Lagrangian and asymptotic freedom

- Nucleon at $N_c \to \infty$: Constituent quarks/antiquarks moving in self-consistent pion field ("soliton")

  → fully field-theoretical description ("completeness")
  → antiquarks, positivity, sum rules

- Interesting qualitative predictions

$$|\Delta\bar{u} - \Delta\bar{d}| \gg \bar{d} - \bar{u}$$

$$\delta q \approx \Delta q, \qquad \delta\bar{q} \neq \Delta\bar{q}$$

We actually understand A LOT about the spin structure of the nucleon ...