QCD Resummations for Hadronic Collisions

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RBRC and BNL Nuclear Theory

RHIC/AGS Users' Meeting 2005

Outline:

- Introduction: why resum?

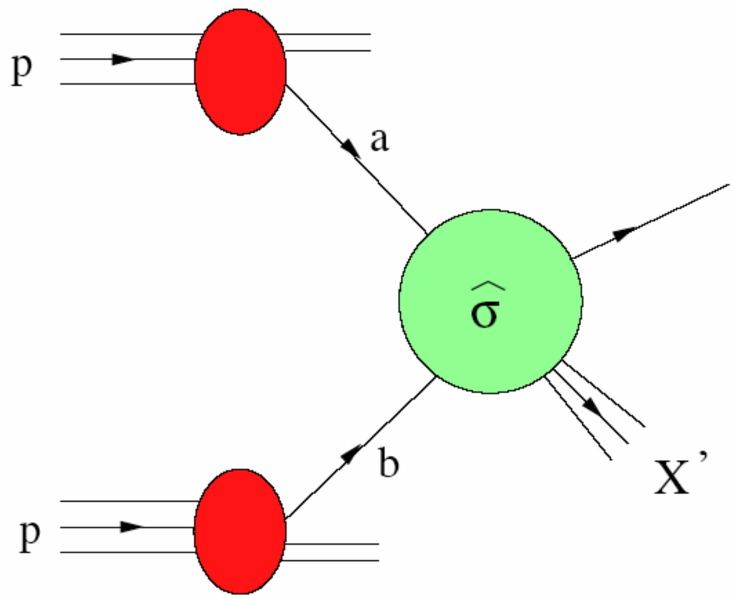
  q_T resummations and threshold resummations
- How is resummation done?
- Phenomenology
- Conclusions

Collab. with   G. Sterman, A. Kulesza; D. de Florian

I. Introduction

Hard scattering in hadron collisions

Characterized by large scale $Q \sim p_T,\ M_{\mu\mu},\ M_W, \ldots$



(for pions, additional fragmentation functions)

$$\mathbf{Q^2\, d\sigma} = \int \mathbf{dx_a} \int \mathbf{dx_b}\ \mathbf{f_a(x_a}, \mu)\, \mathbf{f_b(x_b}, \mu)\ \mathbf{Q^2 d}\hat{\sigma}\left(\frac{\mathbf{Q}}{\mu}, \alpha_\mathbf{s}(\mu)\right)_{\substack{p_a = x_a P_a \\ p_b = x_b P_b}}$$

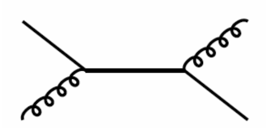
"collinear fact."

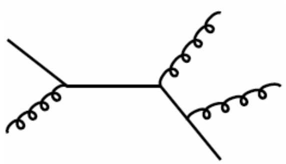
$$\mathbf{Q^2\,d\sigma} = \int \mathbf{dx_a} \int \mathbf{dx_b}\ \mathbf{f_a(x_a,\mu)\,f_b(x_b,\mu)}\ \mathbf{Q^2 d}\hat\sigma\left(\frac{\mathbf{Q}}{\mu}, \alpha_\mathbf{s}(\mu)\right) + \mathcal{O}\left(\frac{\lambda}{\mathbf{Q}}\right)^\mathbf{p}$$

- $f_{a,b}$ parton distributions : non-perturbative, but universal
- $\hat\sigma$ partonic cross section : process-dependent, but pQCD
- μ factorization / renormalization scale : μ ~ Q
- corrections to formula : down by inverse powers of Q
- higher order QCD corrections to $\hat\sigma$

$$\hat\sigma = \underbrace{\hat\sigma^0}_{\text{LO}} + \underbrace{\alpha_s\hat\sigma^1}_{\text{NLO}} + \ldots$$

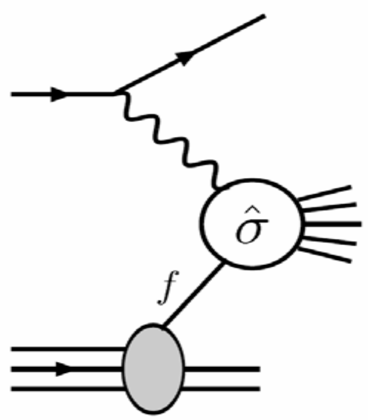
NLO = state of the art



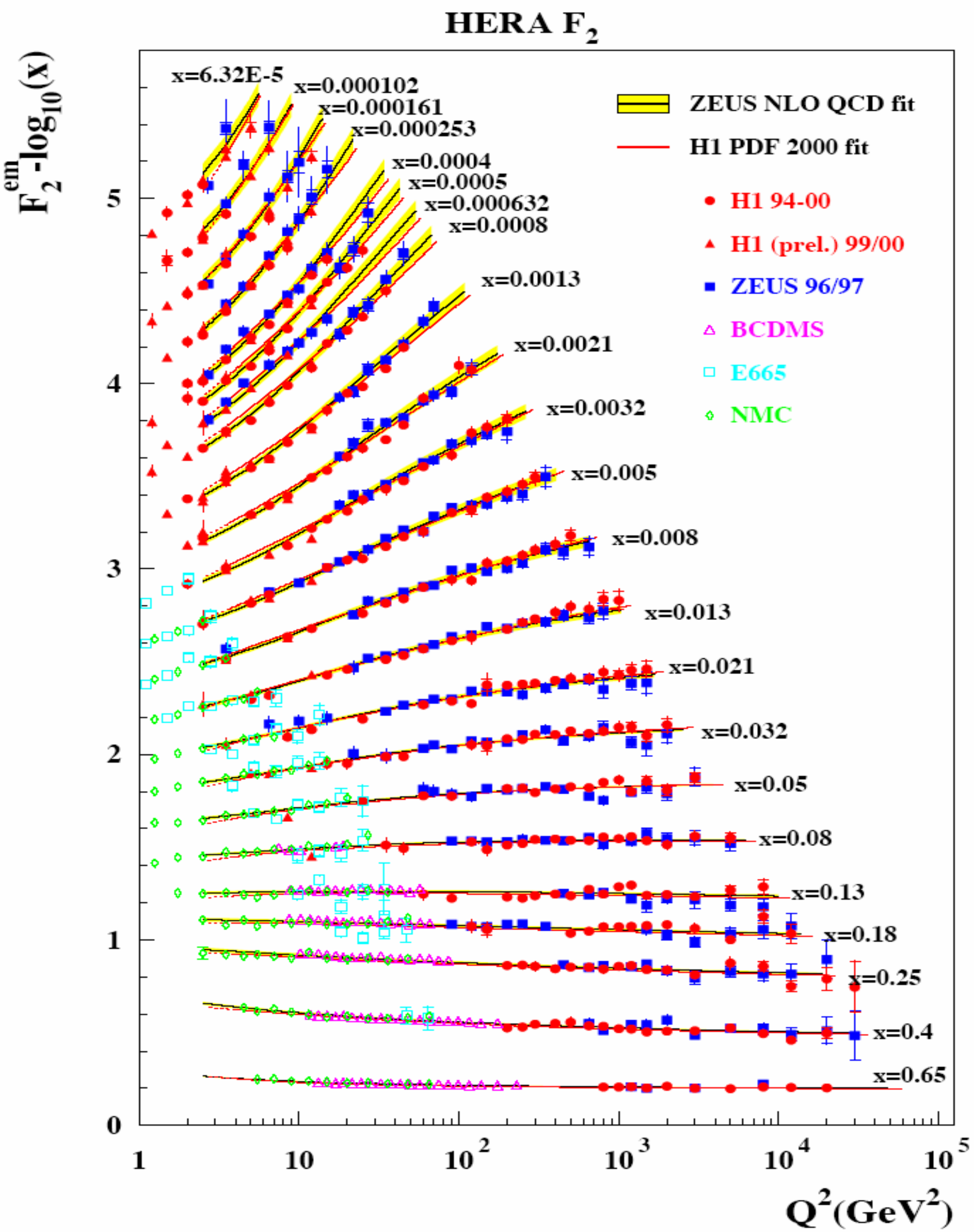


may be sizable & will reduce scale dependence

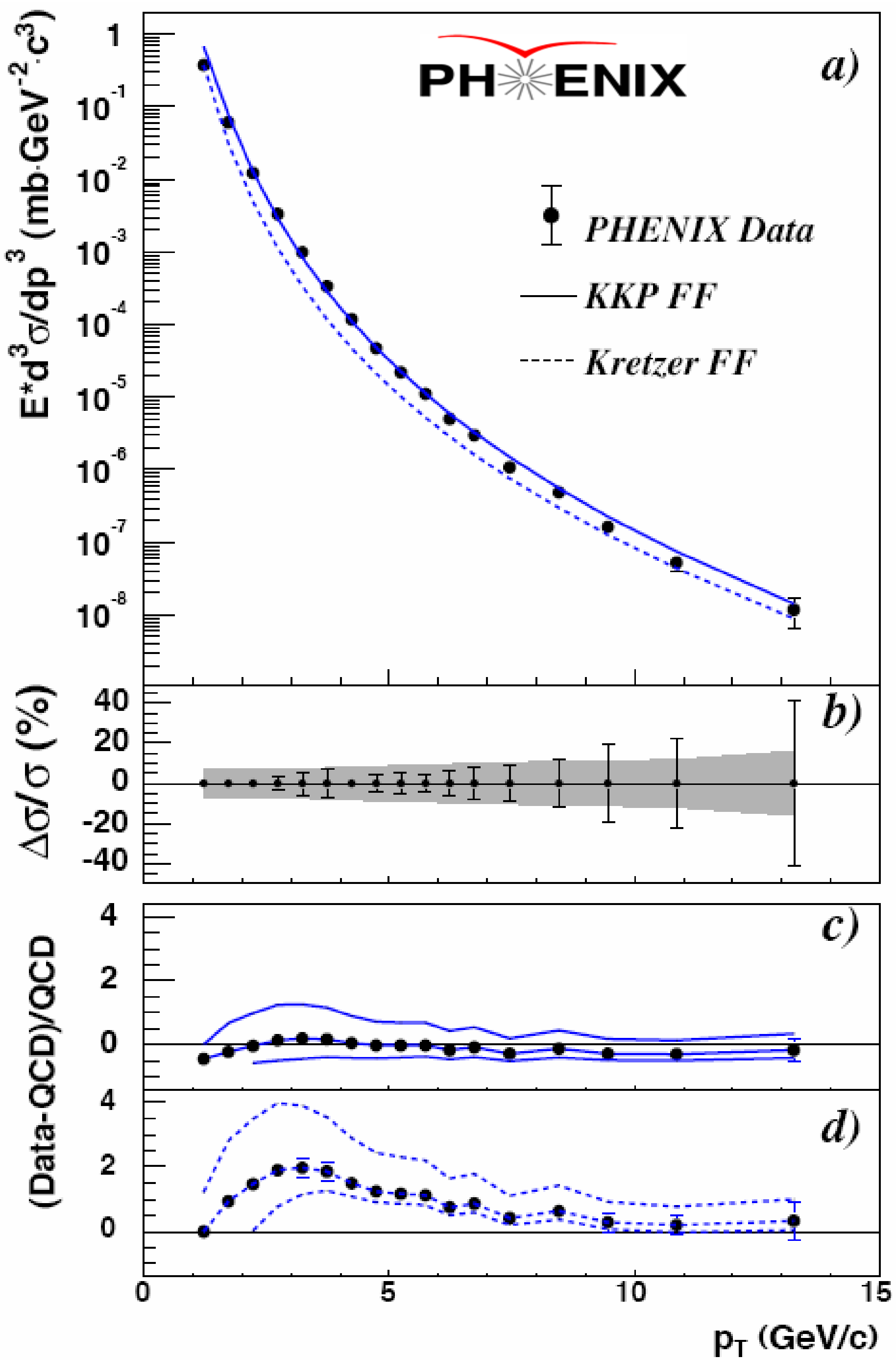
Successes :



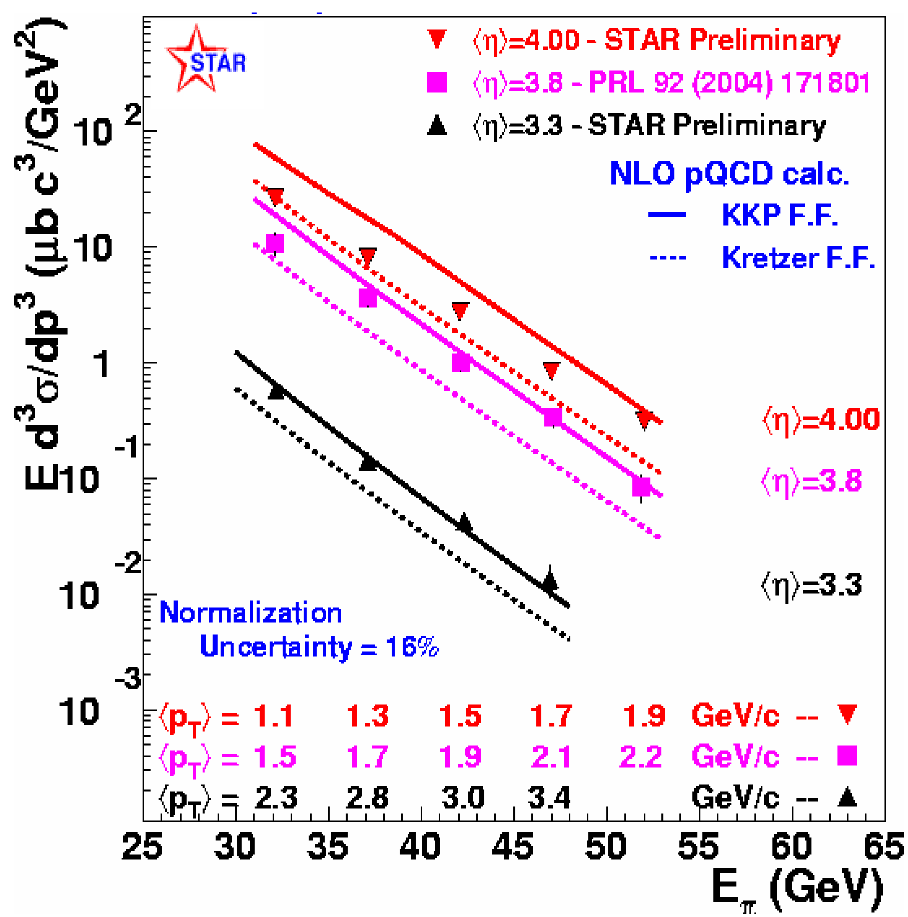
$\hat{\sigma}$
f



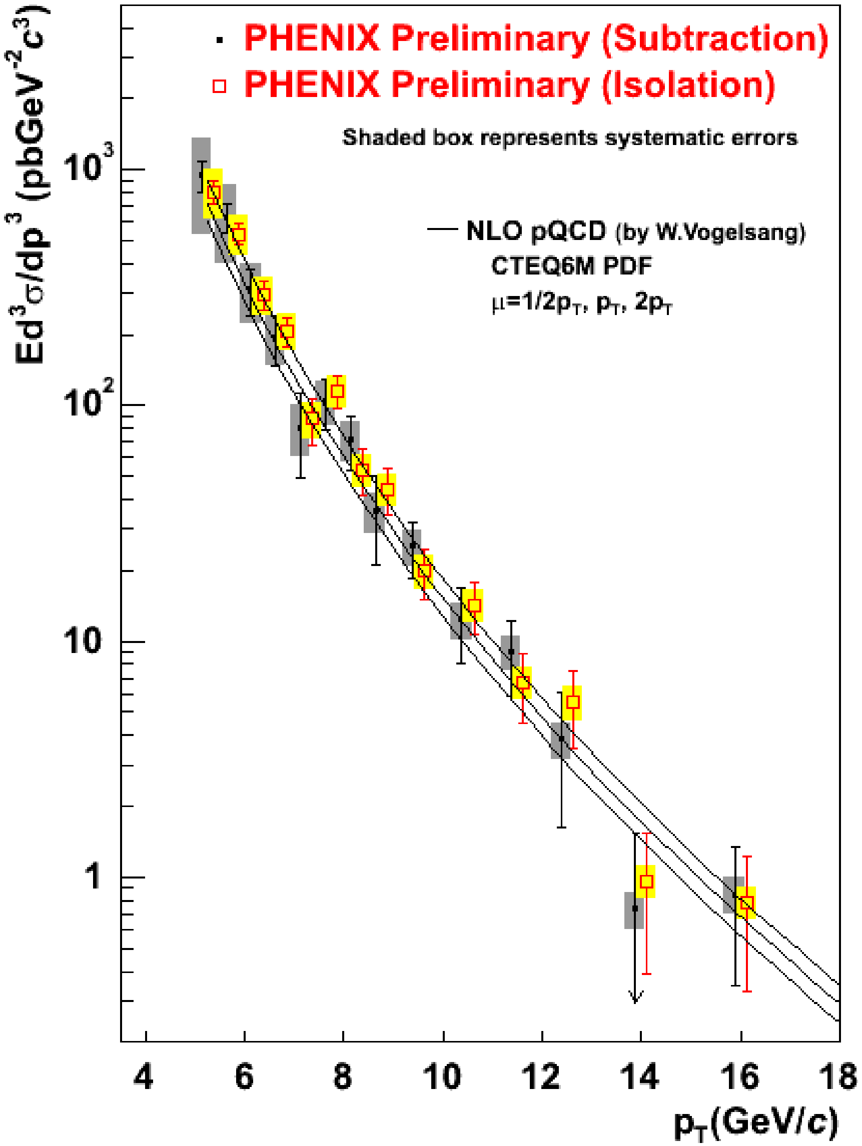
HERA F_2
F_2^{em}-$\log_{10}(x)$
$Q^2(GeV^2)$
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65



$pp \to \pi^0 X$ at RHIC



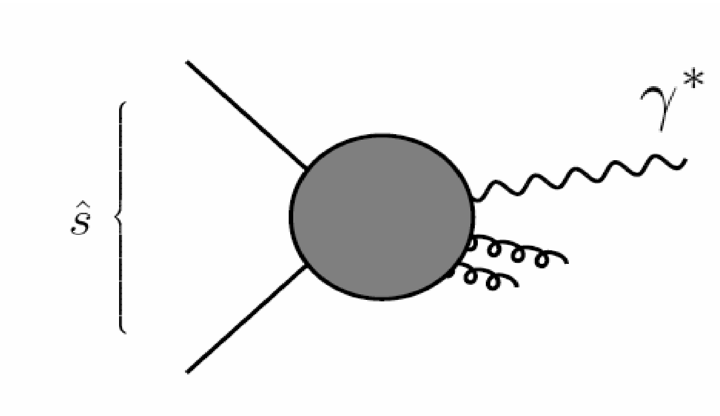
$pp \rightarrow \gamma X$



Sometimes, fixed-order calculation in perturbation theory fails even in the presence of a hard scale

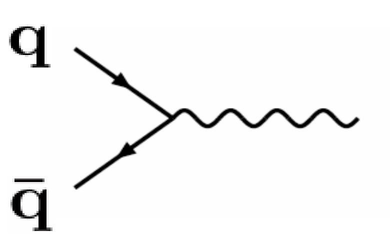
- **when one reaches an "exclusive boundary"**

- **example : Drell-Yan cross section**



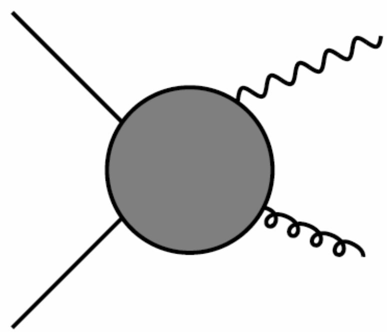
mass Q, transv. momentum q_T

- **LO partonic cross section :**



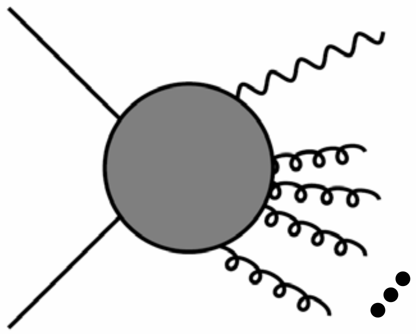
$$\frac{d^2\hat{\sigma}^{(0)}_{q\bar{q}}}{dQ^2 d^2 q_T} \sim \delta(\vec{q}_T)$$

- **first-order correction :**



$$\text{as} \quad q_T \to 0 \qquad \frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}}}{dQ^2 d^2 q_T} \sim \alpha_s \frac{\ln(q_T/Q)}{q_T^2} + \ldots$$

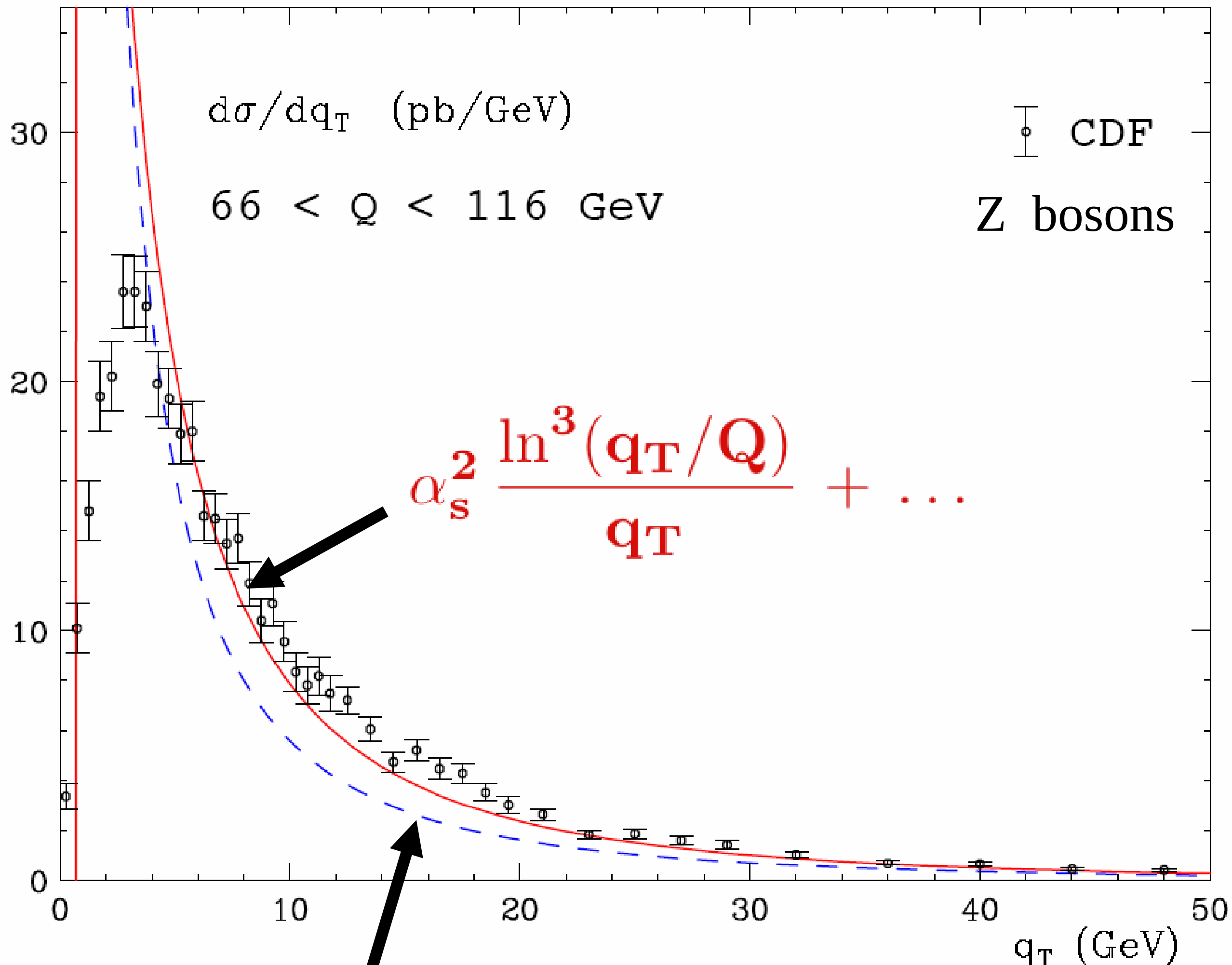
- **higher orders :**



$$\frac{d^2\hat{\sigma}^{(k)}_{q\bar{q}}}{dQ^2 d^2 q_T} \sim \alpha_s^k \frac{\ln^{2k-1}(q_T/Q)}{q_T^2} + \ldots$$

"recoil logarithms"

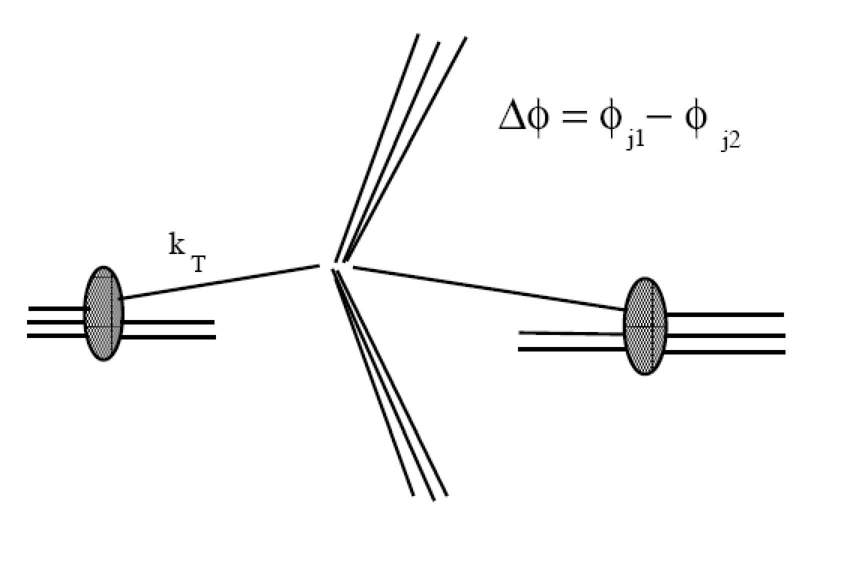
q_T distribution is measurable :



$$\alpha_s^2 \frac{\ln^3(q_T/Q)}{q_T} + \dots$$

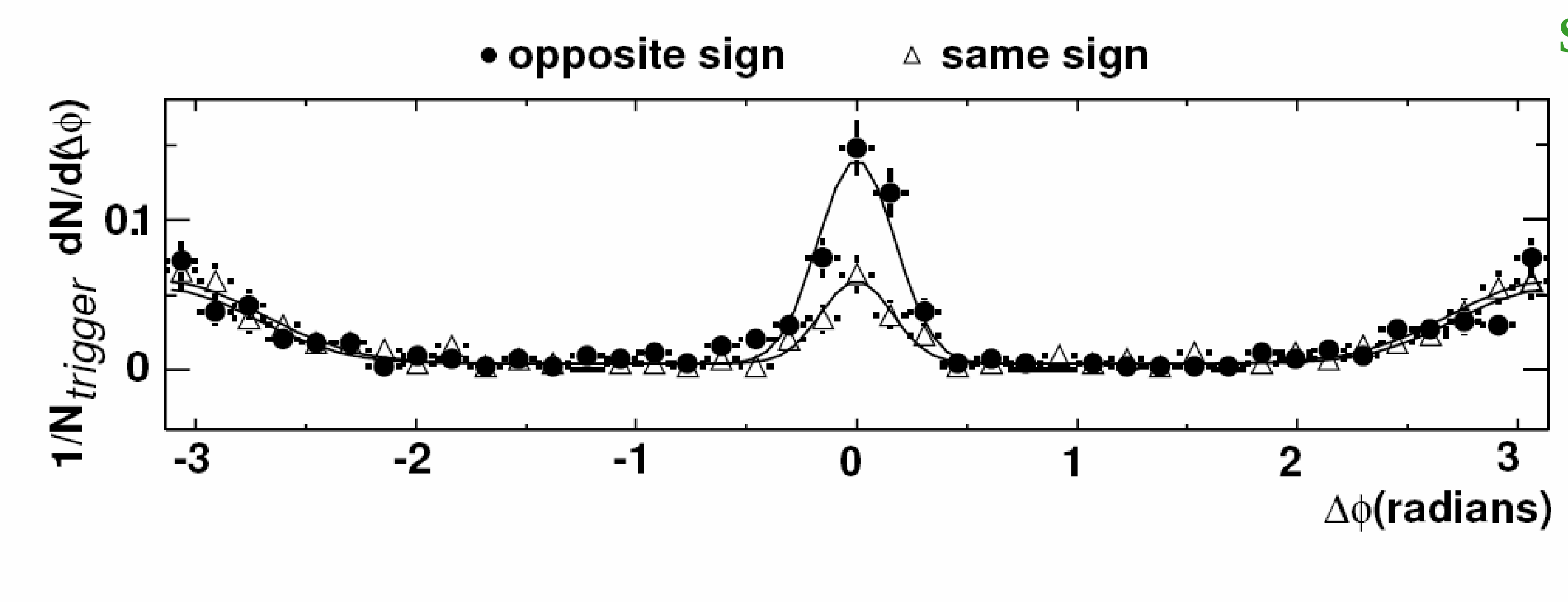
$$\alpha_s \frac{\ln(q_T/Q)}{q_T} + \dots$$

- other example of this type : back-to-back correlatn.

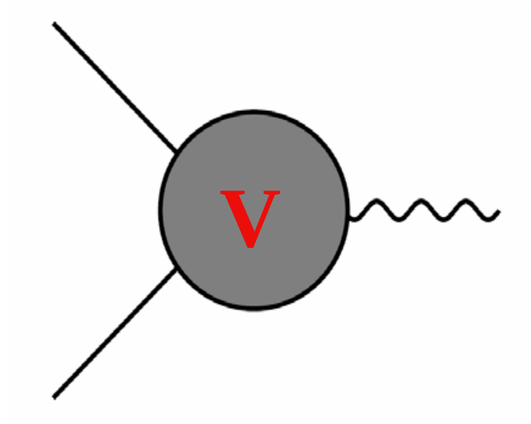


large scale is
jet p_T

STAR

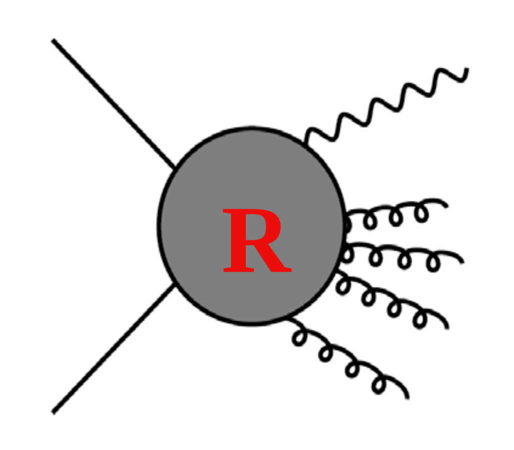


- **perturbation theory appears in distress**
- **phenomenon (and solution) well understood**



virtual corrections

$q_T = 0$

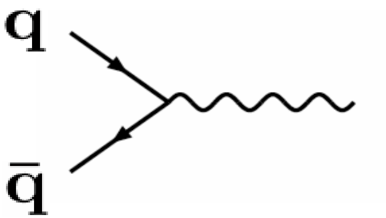


real emission

$q_T \neq 0$

For $q_T \to 0$ real radiation is inhibited, only soft emission is allowed: affects IR cancellations

- the cross sections just considered are in a way special: large scale Q, differential in (small) observed q_T

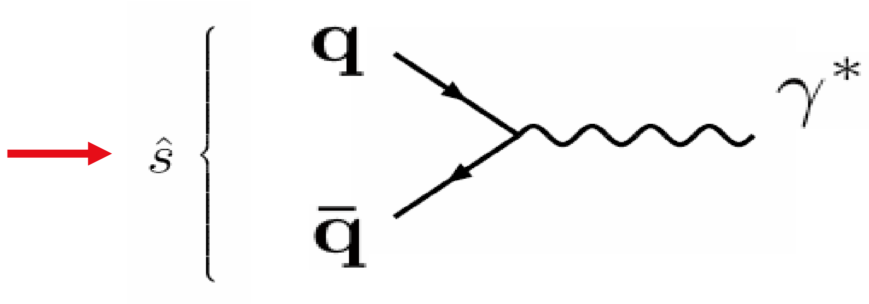


$q_T \neq 0$ from parton k_T

- what about more inclusive (single-scale) cross sections ?

$$\frac{d\sigma}{dQ^2}$$ for Drell-Yan

$$\frac{d\sigma}{dp_T}$$ for high-p_T reactions, $pp \rightarrow \pi X$, …



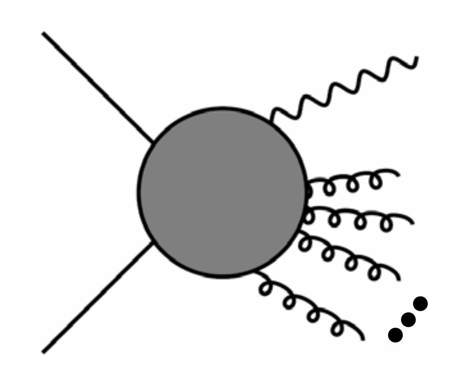
mass Q

$$\hat{\mathbf{s}} = \mathbf{Q^2} \quad , \quad \text{or} \quad \mathbf{z} \equiv \frac{\mathbf{Q^2}}{\hat{\mathbf{s}}} = \mathbf{1}$$

- **partonic cross section :**

$$\frac{d\hat{\sigma}^{(0)}_{q\bar{q}}}{dQ^2} \propto \delta(1-z)$$

- **higher orders :**



$$\frac{d\hat{\sigma}^{(k)}_{q\bar{q}}}{dQ^2} \propto \alpha_s^k \, \frac{\ln^{2k-1}(1-z)}{1-z} + \ldots$$

"threshold logarithms"

- **unlike q_T, one doesn't really measure partonic energy …**

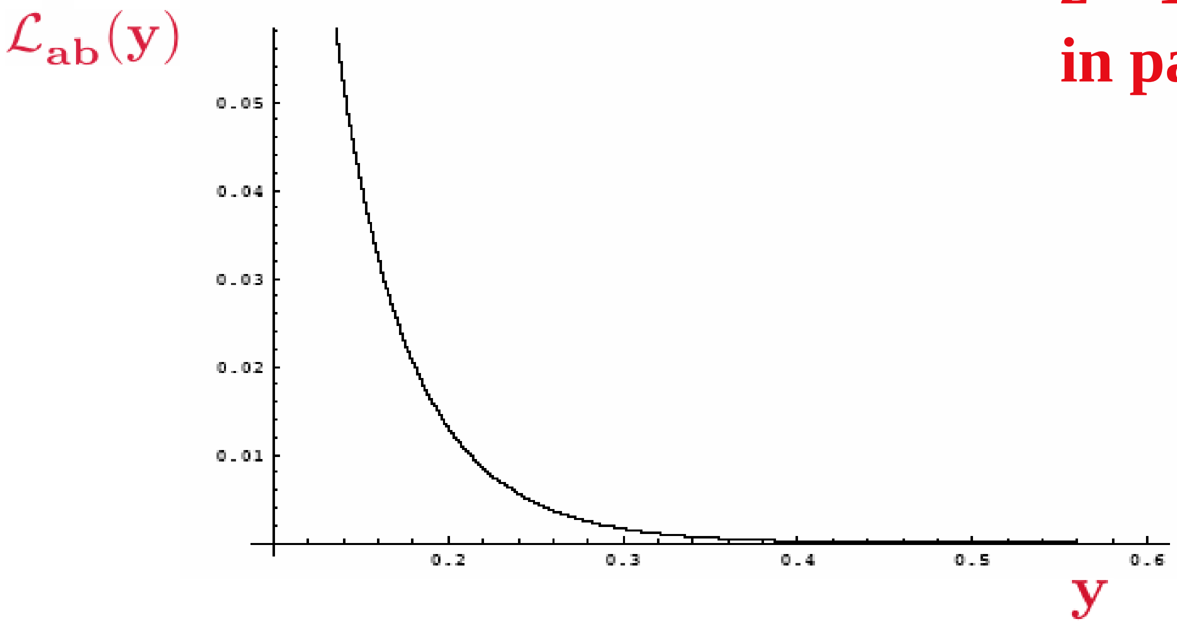
- **nevertheless :**

$$\tau = \mathbf{Q^2/S}$$

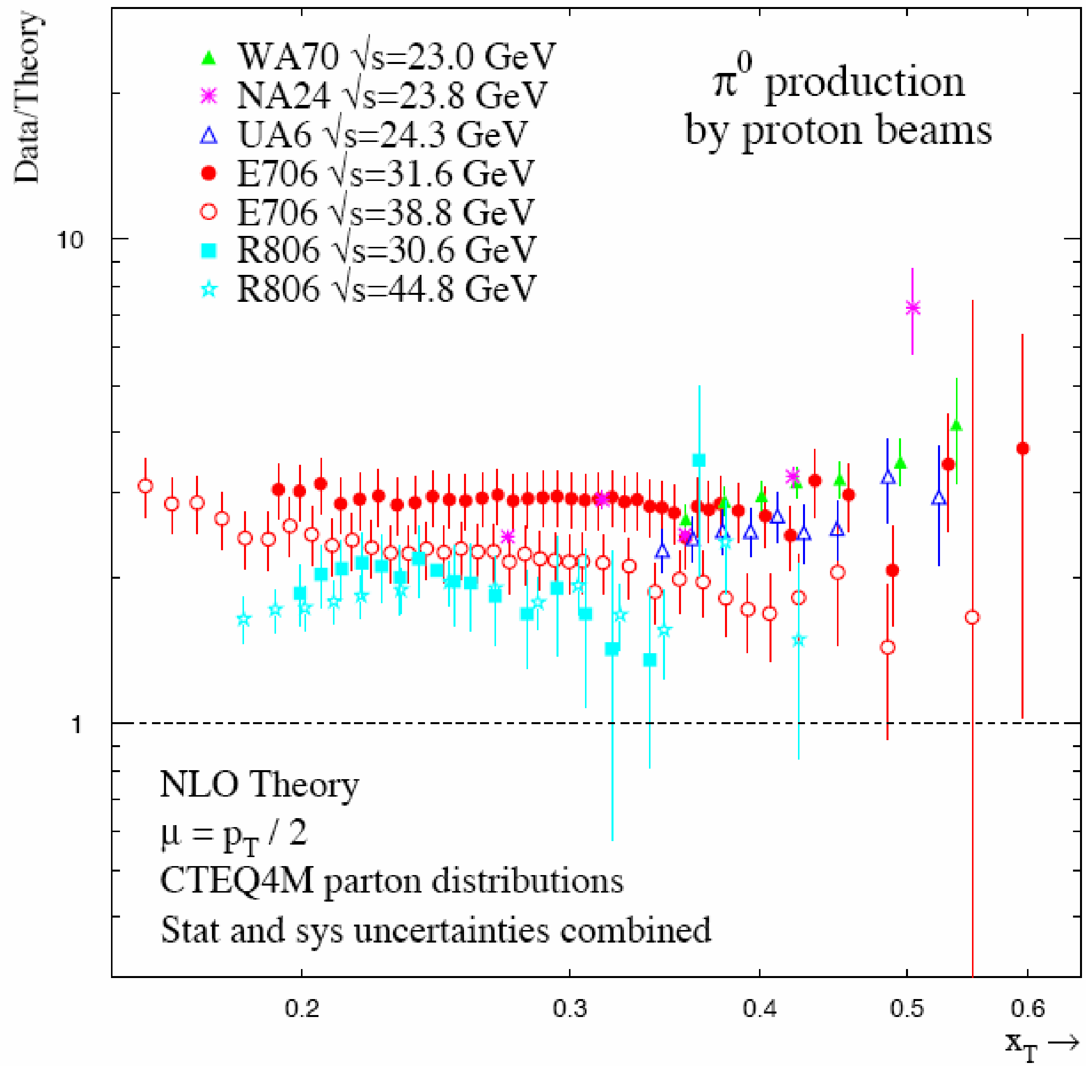
$$\sigma^{\mathrm{DY}} \propto \sum_{\mathbf{a,b}} \int_{\tau}^{1} \frac{\mathbf{dx_a}}{\mathbf{x_a}}\, \mathbf{f_a(x_a)} \int_{\tau/\mathbf{x_a}}^{1} \frac{\mathbf{dx_b}}{\mathbf{x_b}}\, \mathbf{f_b(x_b)}\, \hat{\sigma}_{\mathbf{ab}}(\mathbf{z} = \tau/\mathbf{x_a x_b})$$

$$= \sum_{\mathbf{a,b}} \int_{\tau}^{1} \frac{\mathbf{dz}}{\mathbf{z}}\, \mathcal{L}_{\mathbf{ab}}\left(\frac{\tau}{\mathbf{z}}\right)\, \hat{\sigma}_{\mathbf{ab}}(\mathbf{z})$$

z = 1 emphasized, in particular as $\tau \to 1$



Perhaps involved in explanation of shortfall of NLO for $pp \to \pi^0 X$ at lower energies ?



Apanasevich et al.
(see also Aurenche et al.; Bourrely, Soffer)

- two-scale problems

$$\mathrm{q_T} \ \text{vs.} \ \mathrm{Q} \qquad\qquad \mathrm{Q(1-z)} \ \text{vs.} \ \mathrm{Q}$$

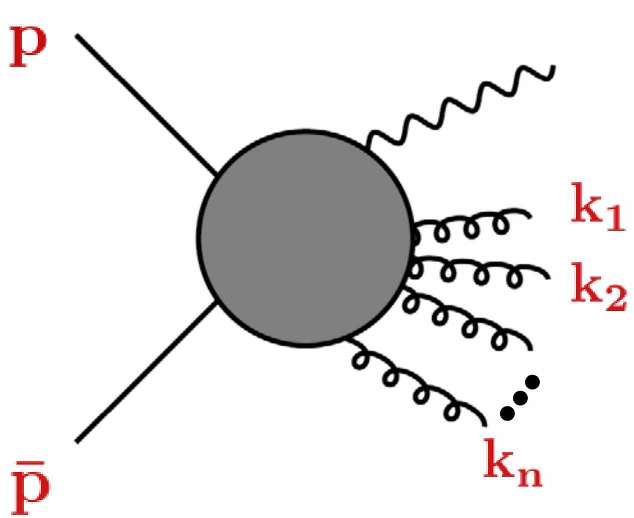
- $\alpha_s^k \, L^{2k-1}$ will spoil perturbative series -
  unless taken into account to all orders

  = Resummation !

- work began in the '80s with Drell-Yan
  * q_T resummation — Dokshitzer et al.; Parisi Petronzio; Collins, Soper, Sterman; …
  * threshold resummation — Sterman; Catani, Trentadue; …
- today : high level of sophistication
  * many other reactions in QCD
  * more general "joint" formalism — Laenen, Sterman, WV

II. How resummation is done

Consider emission of n soft gluons :



$$\sigma_{n\,g} \propto \int |M|^2 (p, \bar{p}; k_1, \ldots, k_n) \times d\Phi_{n\,g} (p, \bar{p}; k_1, \ldots, k_n)$$

matrix element

phase space

(1) Factorization of dynamics:

$$|M|^2 (p, \bar{p}; k_1, \ldots, k_n) \approx \frac{1}{n!} |M|^2 (p, \bar{p}) \prod_{i=1}^{n} \rho(k_i)$$

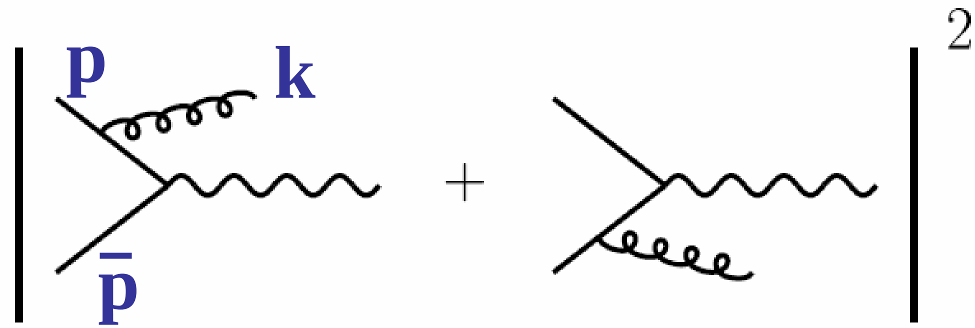
(2) Factorization of phase space:

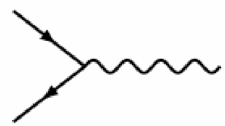
$$\Phi_{n\,g}(p, \bar{p}; k_1, \ldots, k_n) \sim \prod_{i=1}^{n} \Phi_{1\,g}(k_i)$$

If both occur → exponentiation of 1-gluon em.

Concerning (1) :

- first-order correction :



$$|M(p,\bar{p};k)|^2 \;\approx_{k\to 0}\; \left|M\left(\right.\right.$$  $$\left.\left.\right)\right|^2 \cdot\; g^2\, C_F\, \frac{2\,(p\cdot\bar{p})}{(p\cdot k)\;(\bar{p}\cdot k)}$$

- soft emission : simplification of matrix elements. In QED : emission of n soft photons in hard process uncorrelated :

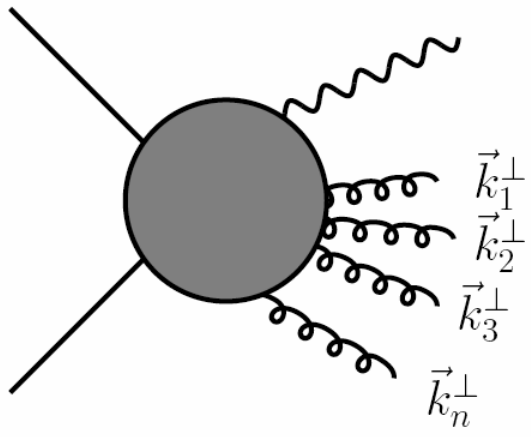
$$|\mathbf{M}|^2\,(\mathbf{p},\bar{\mathbf{p}};\mathbf{k_1},\ldots,\mathbf{k_n}) \;=\; \frac{\mathbf{1}}{\mathbf{n!}}\left[\prod_{\mathbf{i=1}}^{\mathbf{n}} \frac{(\mathbf{p}\cdot\bar{\mathbf{p}})}{(\mathbf{p}\cdot\mathbf{k_i})(\bar{\mathbf{p}}\cdot\mathbf{k_i})}\right]|\mathbf{M}|^2\,(\mathbf{p},\bar{\mathbf{p}})$$

- in QCD : an emitted soft gluon carries color $\Rightarrow$ emission correlated
- still, QED-like factorization in soft limit

  (Ermolaev, Fadin; Bassetto, Ciafaloni, Marchesini; Gatheral; Frenkel, Taylor)

Concerning (2) : **phase-space delta-functions connect gluons**

For Drell-Yan cross section at small q_T :

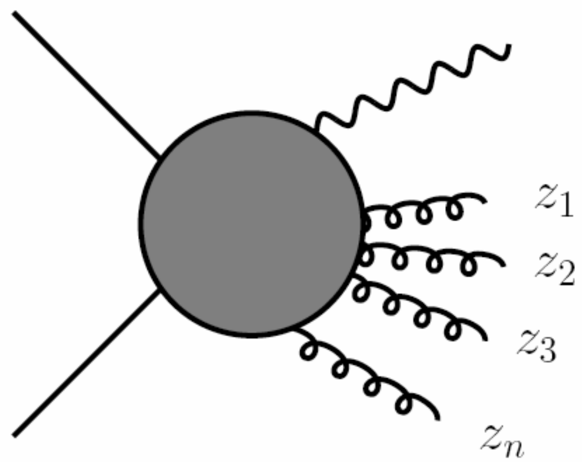


$$\delta^2\left(\vec{\mathbf{q}}_{\mathbf{T}}+\sum_{\mathbf{i}}\vec{\mathbf{k}}^{\mathbf{i}}_{\mathbf{T}}\right) = \frac{\mathbf{1}}{(\mathbf{2}\pi)^{\mathbf{2}}}\int \mathbf{d^2b}\; \mathbf{e}^{-\mathbf{i}\vec{\mathbf{b}}\cdot\left(\vec{\mathbf{q}}_{\mathbf{T}}+\sum_{\mathbf{i}}\vec{\mathbf{k}}^{\mathbf{i}}_{\mathbf{T}}\right)}$$

$$\frac{\ln^{\mathbf{2k-1}}(\mathbf{q_T}/\mathbf{Q})}{\mathbf{q_T}} \leftrightarrow \ln^{\mathbf{2k}}(\mathbf{bQ})$$

Choice of transform depends on observable :

$$\frac{\mathbf{d}\hat{\sigma}}{\mathbf{dQ^2}}$$



$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$$\delta\left(\mathbf{1} - \mathbf{z} - \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{z_i}\right) = \frac{\mathbf{1}}{\mathbf{2\pi i}} \int_{\mathcal{C}} \mathbf{dN}\, \mathbf{e}^{\mathbf{N}\left(\mathbf{1-z-\sum_{i=1}^{n} z_i}\right)}$$

$$\frac{\ln^{\mathbf{2k-1}}(\mathbf{1-z})}{\mathbf{1-z}} \leftrightarrow \ln^{\mathbf{2k}} \mathbf{N}$$

for threshold logs

- **exponentiation gives :**

$*$ **for q_T resummation in DY, find LL**

$$\hat{\sigma} \;\propto\; \exp\left[-\frac{\mathbf{2C_F}}{\pi}\,\alpha_{\mathbf{s}}\ln^{\mathbf{2}}(\mathbf{bQ}) + \ldots + ..\alpha_{\mathbf{s}}^{\mathbf{k}}\ln^{\mathbf{k+1}}(\mathbf{bQ}) + \ldots\right]$$

suppression

$*$ **for threshold resummation in DY**

$$\hat{\sigma} \;\propto\; \exp\left[+\frac{\mathbf{2C_F}}{\pi}\,\alpha_{\mathbf{s}}\ln^{\mathbf{2}}(\mathbf{N}) + \ldots\right]$$

enhancement

- **What is the general structure ?**

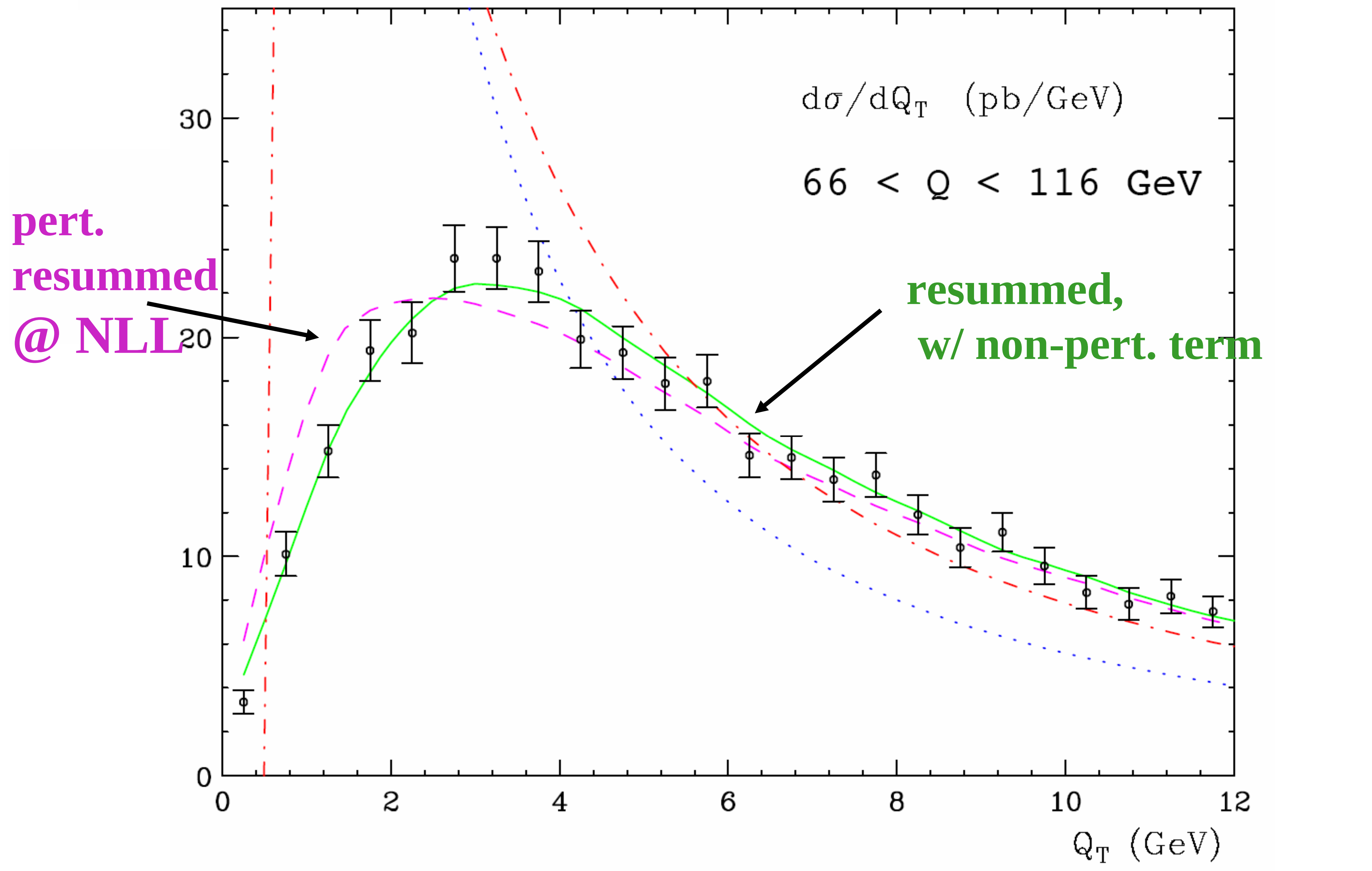
Fixed order

Resummation

LO	1			
NLO	$\alpha_s\,L^2$	$\alpha_s\,L$	α_s	$+\ \dots$
NNLO	$\alpha_s^2\,L^4$	$\alpha_s^2\,L^3$	$\alpha_s^2\,L^2$	$\alpha_s^2\,L\quad+\ \dots$
	$\alpha_s^3\,L^6$	$\alpha_s^3\,L^5$	$\alpha_s^3\,L^4$	$\alpha_s^3\,L^3\quad+\ \dots$
	$\alpha_s^4\,L^8$	$\alpha_s^4\,L^7$	$\alpha_s^4\,L^6$	$\alpha_s^4\,L^5\quad+\ \dots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N^{k}LO	$\alpha_s^k\,L^{2k}$	$\alpha_s^k\,L^{2k-1}$	$\alpha_s^k\,L^{2k-2}$	$\alpha_s^k\,L^{2k-3}\quad+\ \dots$
	LL	**NLL**	**NNLL**	

III. Phenomenology of Resummation

Kulesza, Sterman, WV



Why non-perturbative piece ?

$$\hat{\sigma} \;\propto\; \exp\left[-\frac{\mathbf{2C_F}}{\pi}\,\alpha_{\mathbf{s}}\ln^{\mathbf{2}}(\mathbf{bQ})\;+\;\ldots\right]$$

really comes from:

$$\exp\left[\int_{\mathbf{0}}^{\mathbf{Q^2}}\frac{\mathbf{dk_\perp^2}}{\mathbf{k_\perp^2}}\left(\mathbf{J_0}(\mathbf{bk_\perp})-\mathbf{1}\right)\left\{\frac{\mathbf{2C_F}}{\pi}\,\alpha_{\mathbf{s}}(\mathbf{k_\perp^2})\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k_\perp^2}}\right)\;+\;\ldots\right\}\right]$$

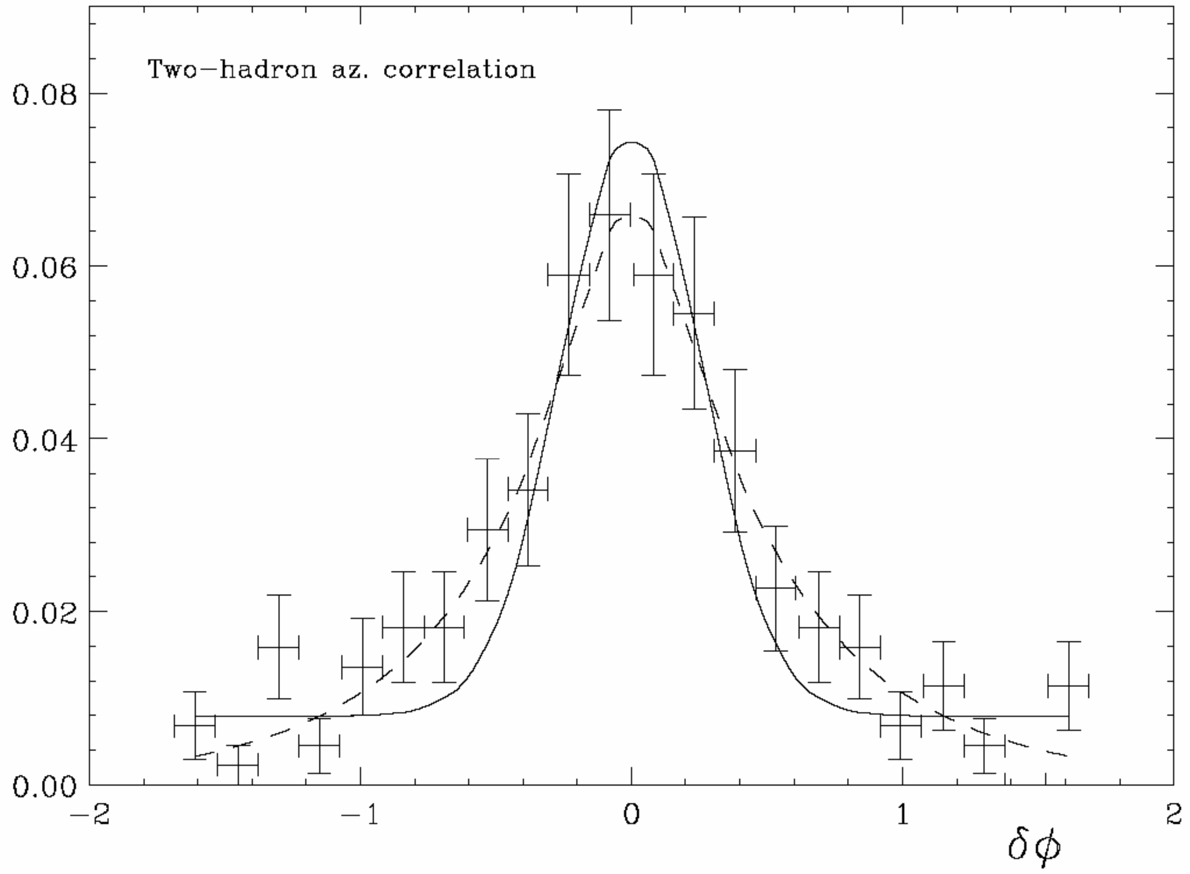
Logarithms are contained in

$$\exp\left[-\int_{\mathbf{1/b^2}}^{\mathbf{Q^2}}\frac{\mathbf{dk_\perp^2}}{\mathbf{k_\perp^2}}\left\{\frac{\mathbf{2C_F}}{\pi}\,\alpha_{\mathbf{s}}(\mathbf{k_\perp^2})\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k_\perp^2}}\right)\;+\;\ldots\right\}\right]$$

Contribution from very low $k_\perp$

$$\exp\left[-\mathbf{b}^2\,\underbrace{\frac{C_F}{\pi}\int d\mathbf{k}_\perp^2\,\alpha_s(\mathbf{k}_\perp^2)\ln\left(\frac{\mathbf{Q}}{\mathbf{k}_\perp}\right)}_{g_1 + g_2\ln(Q^2/Q_0^2)}\right]$$

- suggests Gaussian non-pert. contribution with logarithmic Q dependence
- for Z production at CDF & D0, fit gives coefficient **0.8 GeV^2**
- if one thinks of it as a "parton $k_\perp$" : $\langle \mathbf{k}_\perp \rangle \approx 1\ \mathrm{GeV}$
- more "global" fits see log(Q) dependence Davies, Webber, Stirling; Landry et al., Ladinsky, Yuan; Qiu, Zhang



$\sqrt{\langle k_\perp^2 \rangle} \approx 2$ GeV

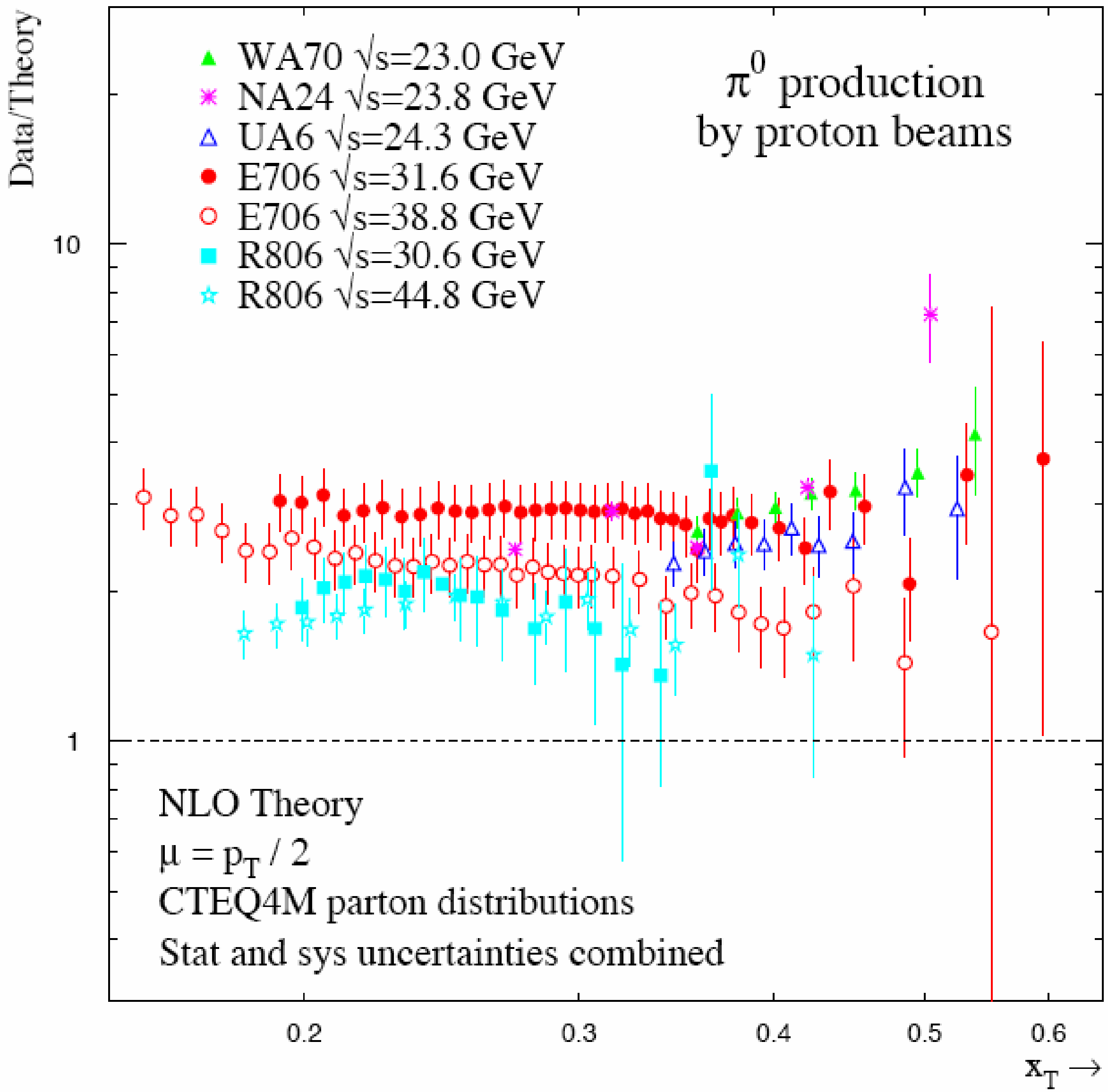
STAR

- dashed line: including Sudakov effects

  $*$ markedly better agreement with data; $\sqrt{\langle k_\perp^2 \rangle} \approx 0.9$ GeV

$pp \to \pi^0 X$

at lower energies



Apanasevich et al.
(see also Aurenche et al.; Bourrely, Soffer)

Why problems in fixed-target regime ?

Why "near perfect" at colliders ?

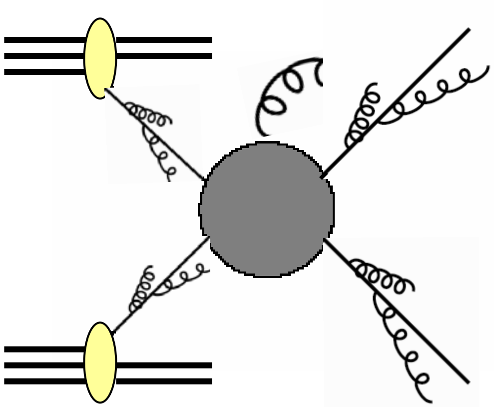
- **higher-order corrections beyond NLO ?**

$$p_T^3 \frac{d\hat{\sigma}_{ab}}{dp_T} = p_T^3 \frac{d\hat{\sigma}_{ab}^{\text{Born}}}{dp_T} \Bigg[1 + \underbrace{\mathcal{A}_1\, \alpha_s\, \ln^2\left(1-\hat{x}_T^2\right) + \mathcal{B}_1\, \alpha_s\, \ln\left(1-\hat{x}_T^2\right)}_{\text{NLO}}$$

$$+ \ldots + \mathcal{A}_k\, \alpha_s^k\, \ln^{2k}\left(1-\hat{x}_T^2\right) + \ldots \Bigg] + \ldots$$

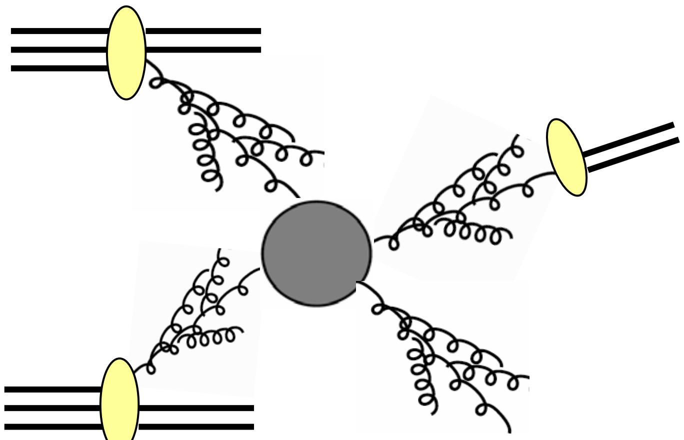
$$\hat{x}_T \equiv \frac{2p_T}{\sqrt{\hat{s}}}$$

"threshold" logarithms



$\hat{x}_T \rightarrow 1$: only soft/collinear gluons allowed

Large terms may be resummed to all orders in α_s



Sterman; Catani, Trentadue;

Laenen, Oderda, Sterman;

Catani, Mangano, Nason, Oleari, WV;

Sterman, WV; Kidonakis, Owens

Kidonakis, Sterman; Bonciani et al.;
de Florian, WV

- soft-gluon effects exponentiate :

  Leading logarithms

Mellin moment in $\hat{x}_T^2$

$gg \to gg$

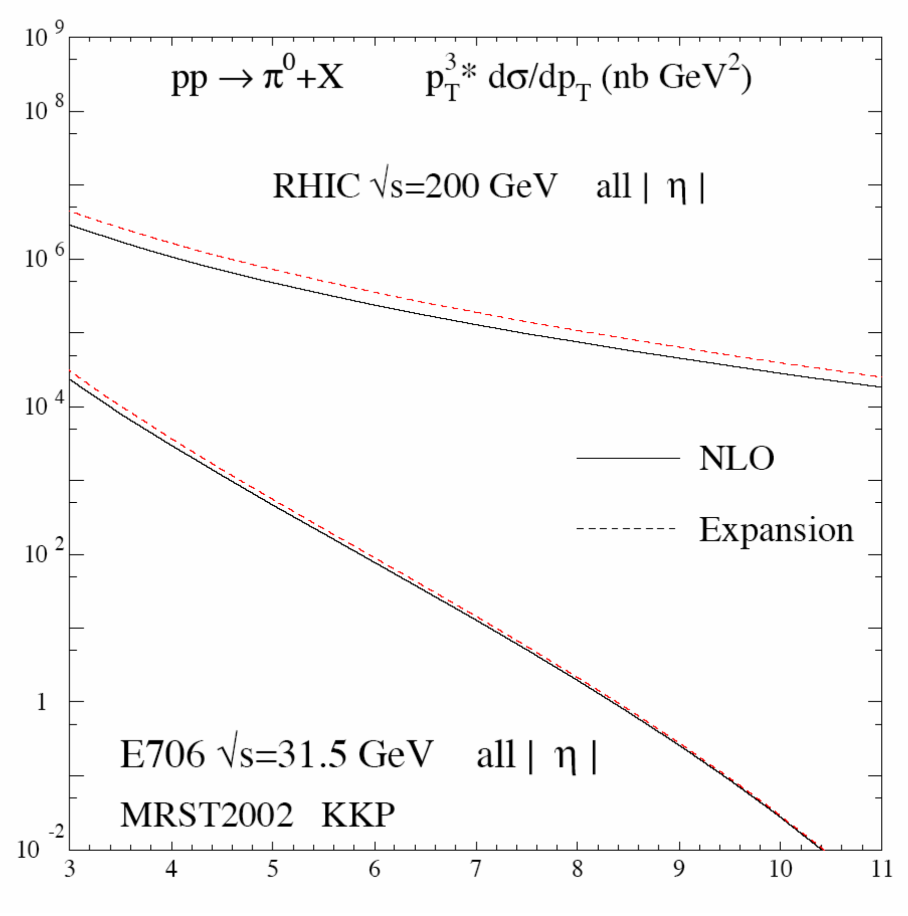
$$\exp\left[\left(C_A + C_A + C_A - \frac{1}{2}C_A\right)\frac{\alpha_s}{\pi}\ln^2(N)\right]$$

- expect large enhancement !

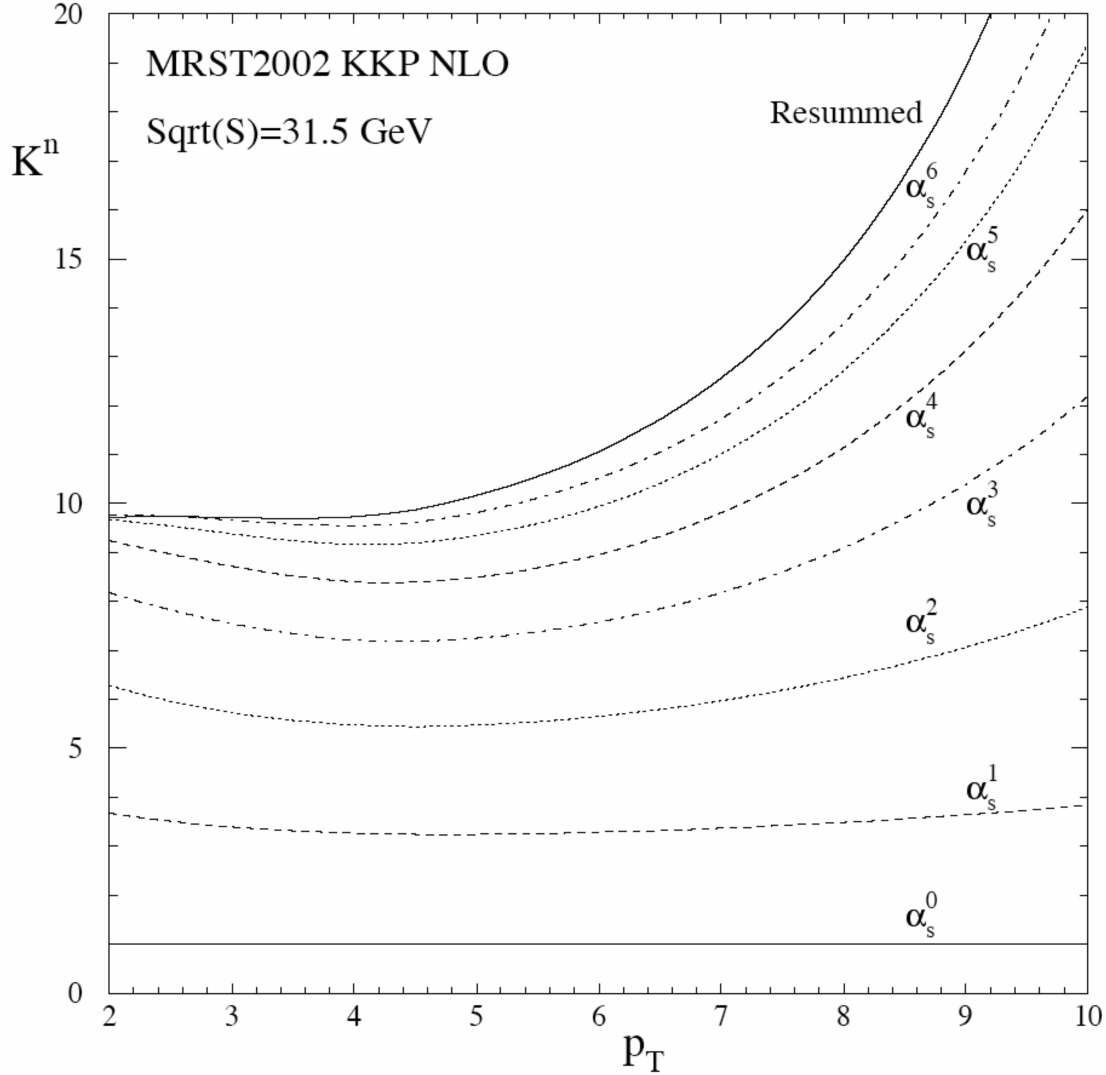
  (NLL far more complicated, known)

de Florian, WV

How dominant are they ?

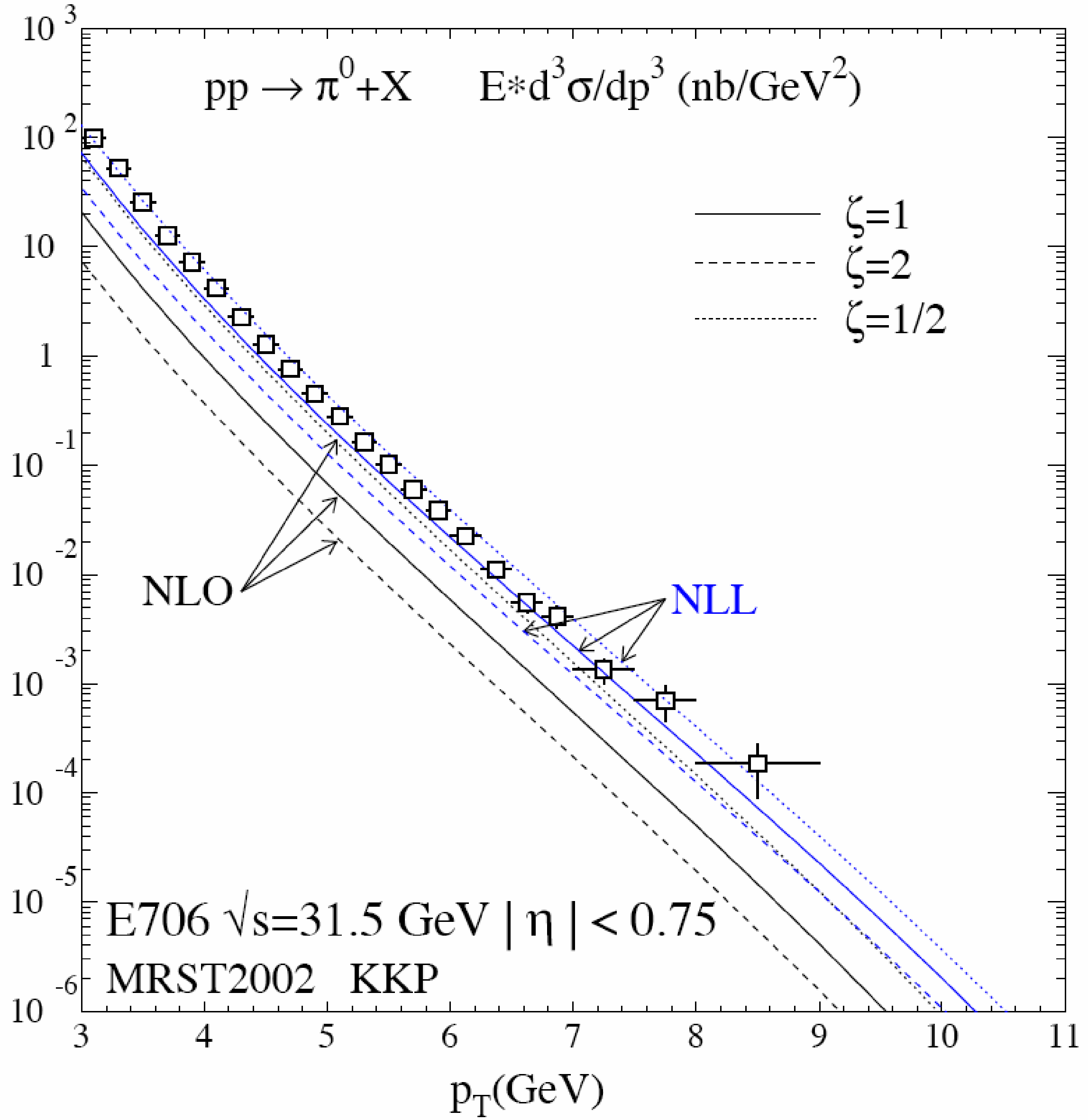


→ subleading terms important at collider energies


MRST2002 KKP NLO
Sqrt(S)=31.5 GeV
Resummed
K^n
p_T
α_s^6
α_s^5
α_s^4
α_s^3
α_s^2
α_s^1
α_s^0
0
5
10
15
20
2
3
4
5
6
7
8
9
10

E706

$pp \rightarrow \pi^0 X$



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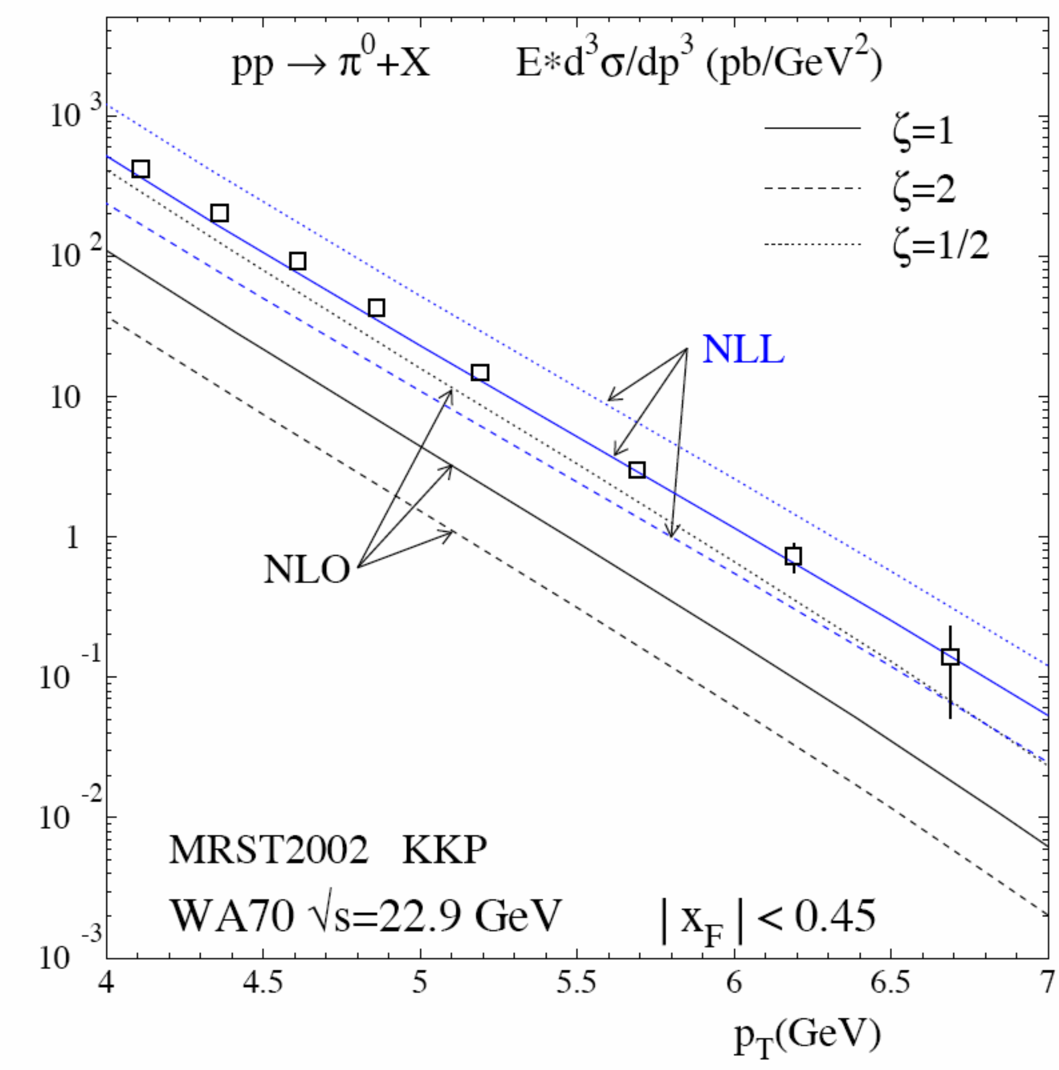
WA70

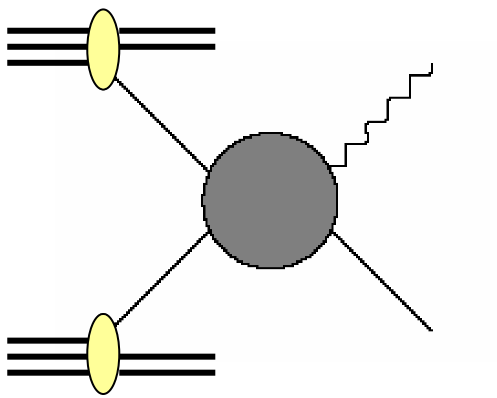


however, effects at colliders are much smaller!

bigger effects at forward rapdities $x_T \cosh(\eta) < 1$

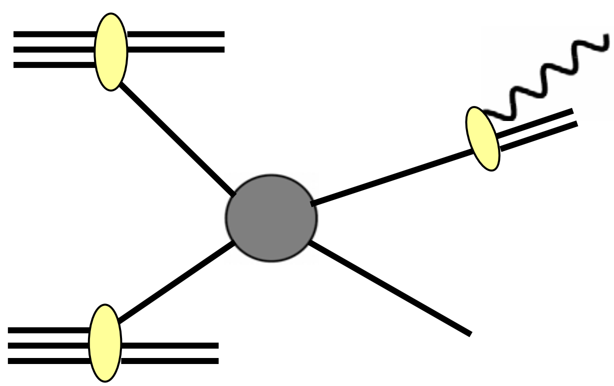
Application to prompt photons: $pp \to \gamma X$

"direct" contributions:



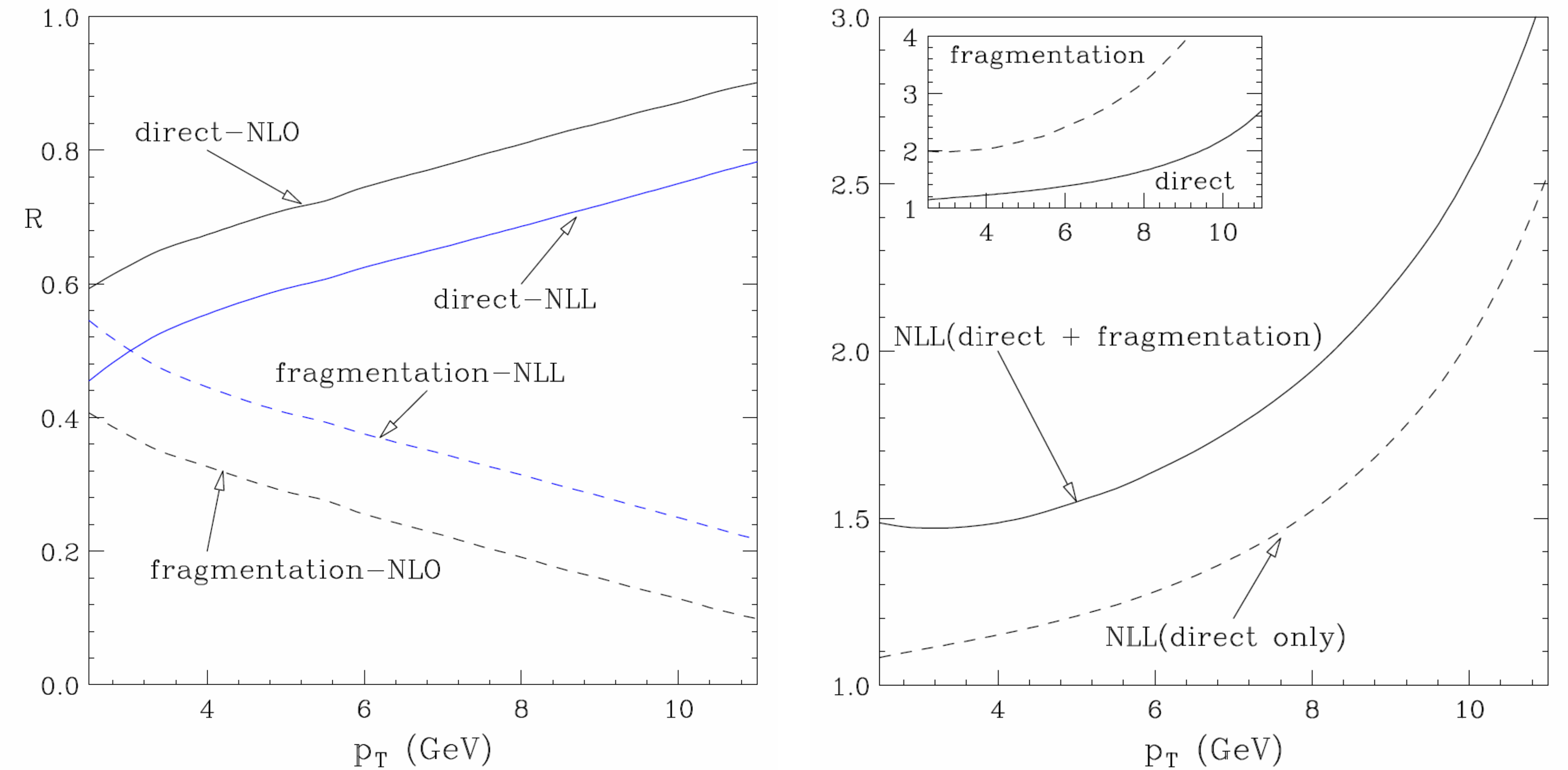
relatively small resum. effects

"fragmentation" contributions:

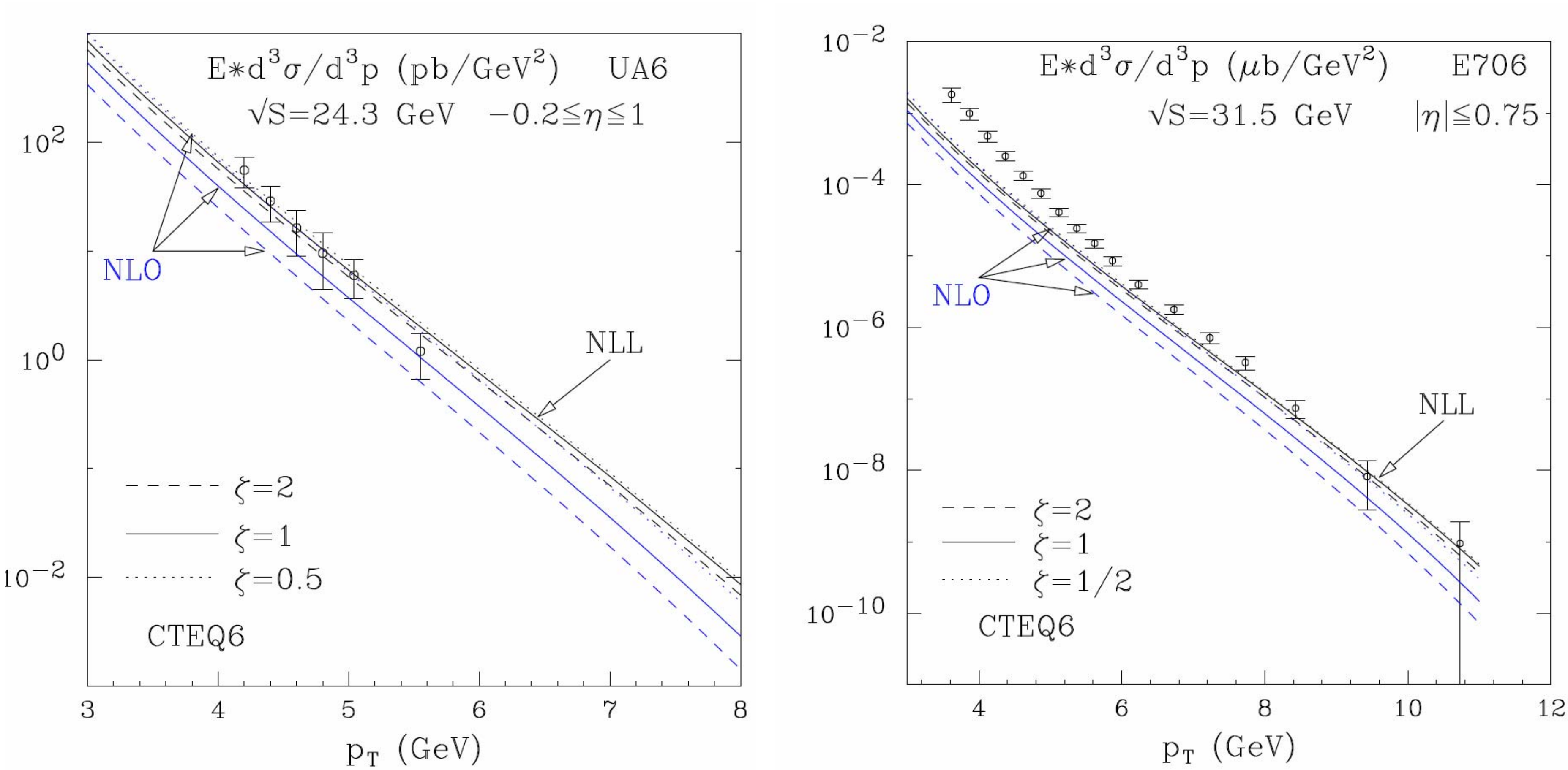


a bit like π^0 production,
but less $gg \to gg$ because
D_g^γ is smaller

de Florian, WV



de Florian, WV



Can one estimate non-pert.
power corrections here ?

DY: $$\exp\left[\frac{2C_F}{\pi}\int_0^1 dy\,\frac{y^N-1}{1-y}\int_{Q^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2)\;+\;\ldots\right]$$

- ill-defined because of strong-coupling regime

$$\exp\left[\frac{2C_F}{\pi}\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2)\;\left\{K_0\left(\frac{2Nk_\perp}{Q}\right)\;+\;\ln\left(\frac{Nk_\perp}{Q}\right)\right\}\;+\;\ldots\right]$$

- regime of very low $k_\perp$:

$$\exp\left[\frac{2C_F}{\pi}\,\frac{N^2}{Q^2}\int_0^{\lambda^2}dk_\perp^2\;\alpha_s(k_\perp^2)\;\ln\left(\frac{Q}{Nk_\perp}\right)\right]\;\sim\;\exp\left[\frac{2C_F}{\pi}\,\frac{N^2}{Q^2}\;\left\{g_1\;+\;g_2\;\ln\left(\frac{Q}{NQ_0}\right)\right\}\right]$$

- overall powers all even, exponentiating !

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- **connects g_1 and g_2 to values from q_T resummation ?**

- **for single-inclusive cross sections** $pp \to \gamma X$

$$Q \leftrightarrow 2p_T$$

$$N \leftrightarrow \frac{1}{\ln x_T^2} \qquad x_T \equiv \frac{2p_T}{\sqrt{s}}$$

- **then,**

$$\frac{N^2}{Q^2} \sim \frac{1}{p_T^2 \ln^2\left(\frac{4p_T^2}{s}\right)}$$

fairly small effect

Sterman, WV

- **qualitatively, for inclusive hadrons, replace** $p_T \to p_T/z$

IV. Conclusions

- resummation can be crucial in hadronic cross sections

- cross sections at "small - q_T" (but otherwise large scale Q) :

  * resummation leads to suppression

  * at small q_T (Gaussian) non-perturbative effects important

- cross sections characterized just by large scale Q :

  * threshold resummation important when "x~1"

  * leads to enhancement

  * great phenomenological relevance in $pp \to \pi X$ in fixed-target regime

  * non-perturbative effects appear less important

  * implications for k_T models …