Viscosity near the critical point

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Longest running lab experiment ?

Pitchdrop Experiment

University of Queensland, Australia

Year	Event
1927	Experiment set up
1930	The stem was cut
1938(Dec)	1st drop fell
1947(Feb)	2nd drop fell
1954(Apr)	3rd drop fell
1962(May)	4th drop fell
1970(Aug)	5th drop fell
1979(Apr)	6th drop fell
1988(Jul)	7th drop fell
2000(Nov)	8th drop fell

Navier-Stokes Hydrodynamics

$$\boxed{\begin{aligned} T^{\alpha\beta} &= (p+e)u^\alpha u^\beta - pg^{\alpha\beta} + \delta T^{\alpha\beta} \\ N^\alpha &= nu^\alpha + \delta N^\alpha \end{aligned}}$$

Local rest frame: $T^{00} = e \qquad u^0 = 1$

$N^0 = n \qquad u^i = 0$

Landau frame: $\delta T^{0i} = 0$

Eckart frame: $\delta N^\alpha = 0$

Shear and bulk viscosity:

$$\delta T^{ij} = -\eta \left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} - \frac{2}{3} \vec{\nabla} \cdot \vec{u}\, \delta_{ij} \right) - \zeta\, \vec{\nabla} \cdot \vec{u}\, \delta_{ij}$$

mean free path

$$\eta = \frac{4}{5} p \, \tau_{\rm coll}$$

Gyulassy, Pang, Zhang, Nucl. Phys. A 626 (1997)
Teaney PRC 68 (034913) 2003

Non-zero interaction range

$$\zeta \propto p \frac{r_{\rm int}^2}{\tau_{\rm coll}} \qquad \eta \propto p \frac{r_{\rm int}^2}{\tau_{\rm coll}}$$

Cheng et al, PRC 65 (024901) 2002

Chemical non-equilibrium

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{1}{\tau_{\mathrm{chem}}}\left(N - N_{\mathrm{eq}}\right)$$

offset from equilibrium: $\delta N = -\tau_{\mathrm{chem}} \frac{\mathrm{d}N_{\mathrm{eq}}}{\mathrm{d}t}$

$$\delta P = \left.\frac{\partial p}{\partial n}\right|_{e=\mathrm{const}} \frac{\delta N}{\Omega}$$

$$\boxed{\zeta = \left.\frac{\partial p}{\partial n}\right|_{e=\mathrm{const}} \frac{\partial n}{\partial s} s\tau_{\mathrm{chem}}}$$

larger when T falls or m rises

Dynamic mean fields

$$\frac{\partial^2\sigma(t)}{\partial t^2} + \Gamma\frac{\partial\sigma(t)}{\partial t} + m_\sigma^2\left(\sigma(t) - \sigma_{\mathrm{eq}}(t)\right) = R(t)$$

$$m\frac{\partial^2 x(t)}{\partial t^2} + \gamma\frac{\partial x(t)}{\partial t} + k\left(x(t) - x_0(t)\right) = R(t)$$

Offset: $\delta x = -\frac{\gamma}{k}v_0$

$$\delta\phi = -\frac{\Gamma}{m_\sigma^2(T)}\frac{\mathrm{d}\sigma_{\mathrm{eq}}}{\mathrm{d}t}$$

$$\zeta = \left.\frac{\partial p}{\partial\sigma}\right|_{e=\mathrm{const}}\frac{\Gamma}{m_\sigma^2(T)}\frac{\partial\sigma_{\mathrm{eq}}}{\partial s}s$$

Example: Linear Sigma Model

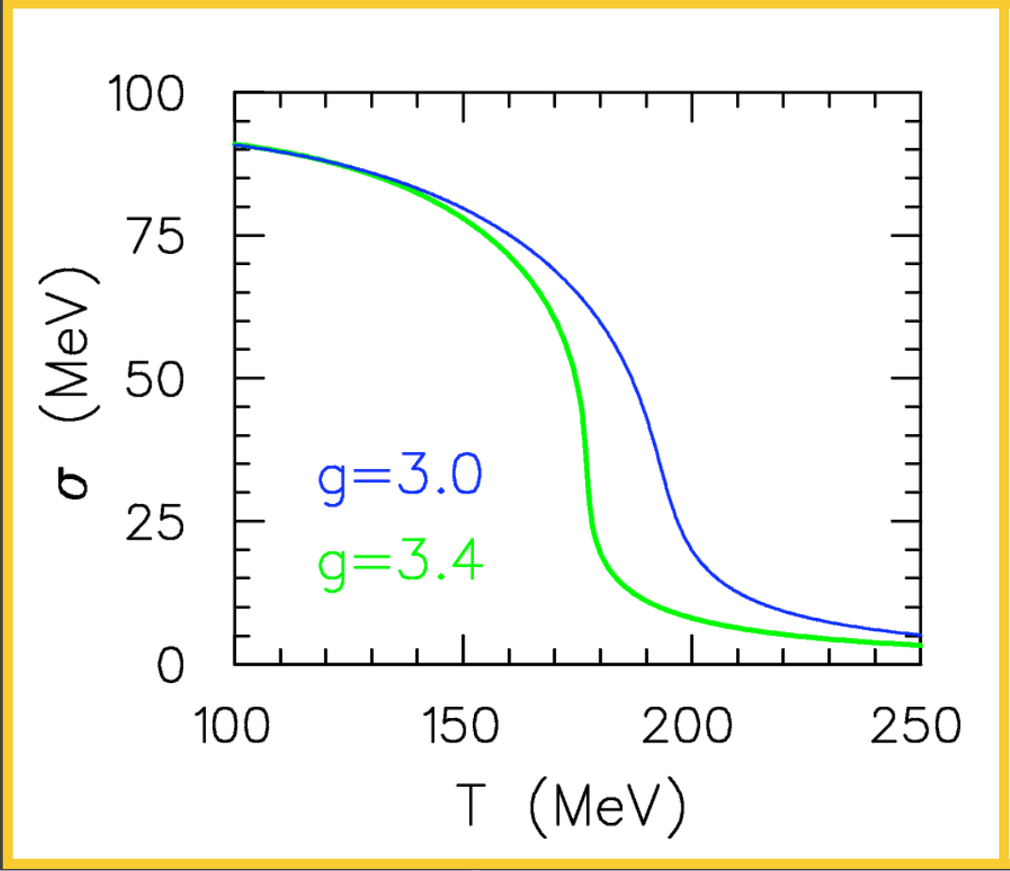
$$H = -\frac{1}{2}\sigma\nabla^2\sigma + \frac{\lambda^2}{4}\left(\sigma^2 - f_\pi + \frac{m_\pi}{\lambda^2}\right)^2 - h_{\mathrm{q}}\sigma + H_{\mathrm{quarks}}(m = g\sigma)$$

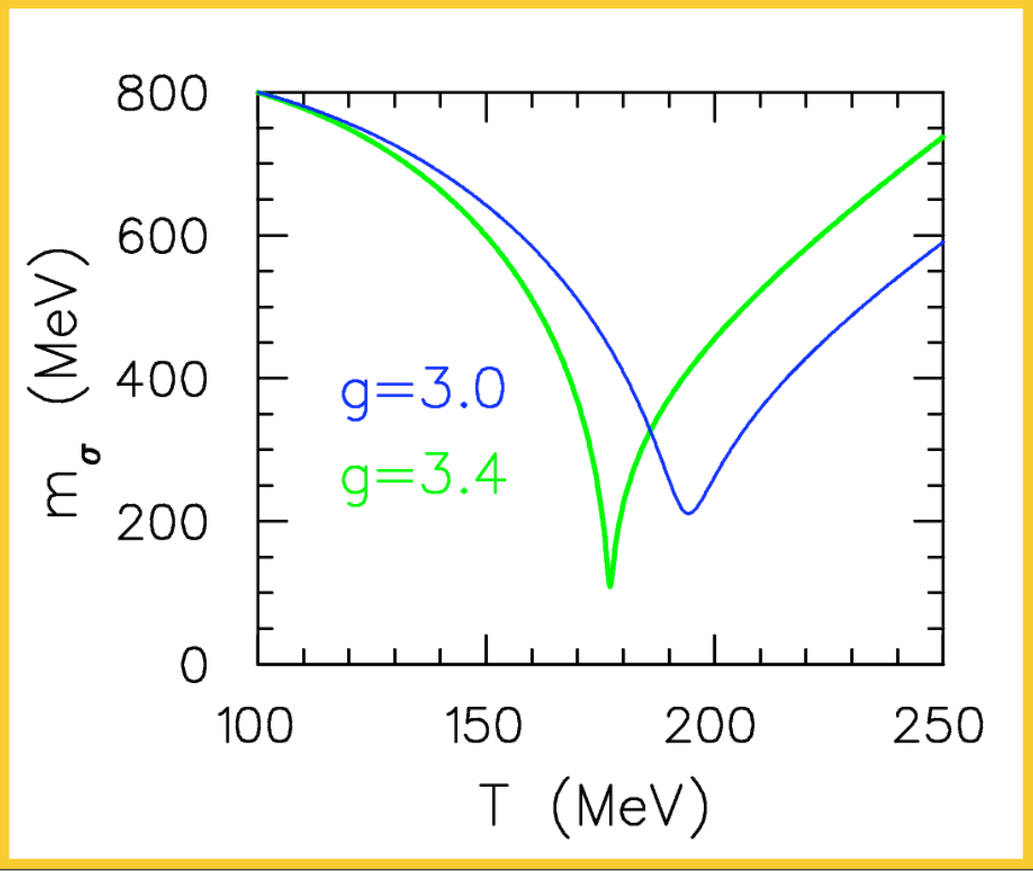
1st order when

g > 3.5549

crossover when

g < 3.5549



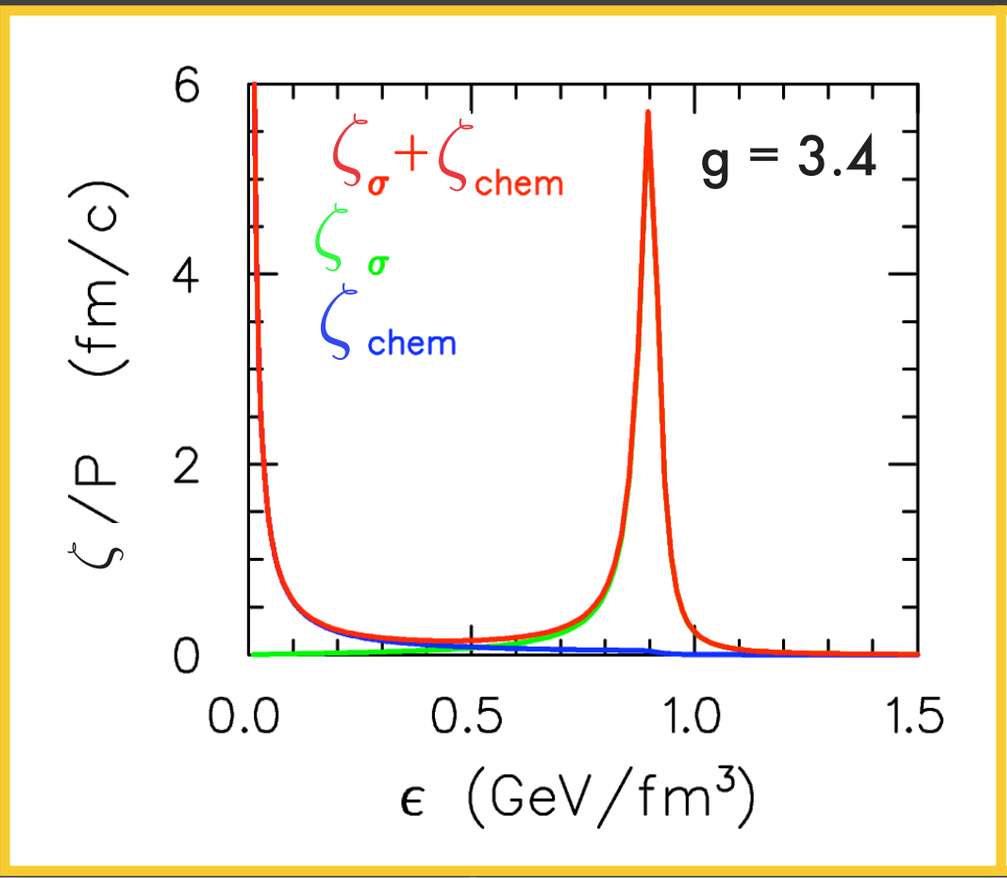


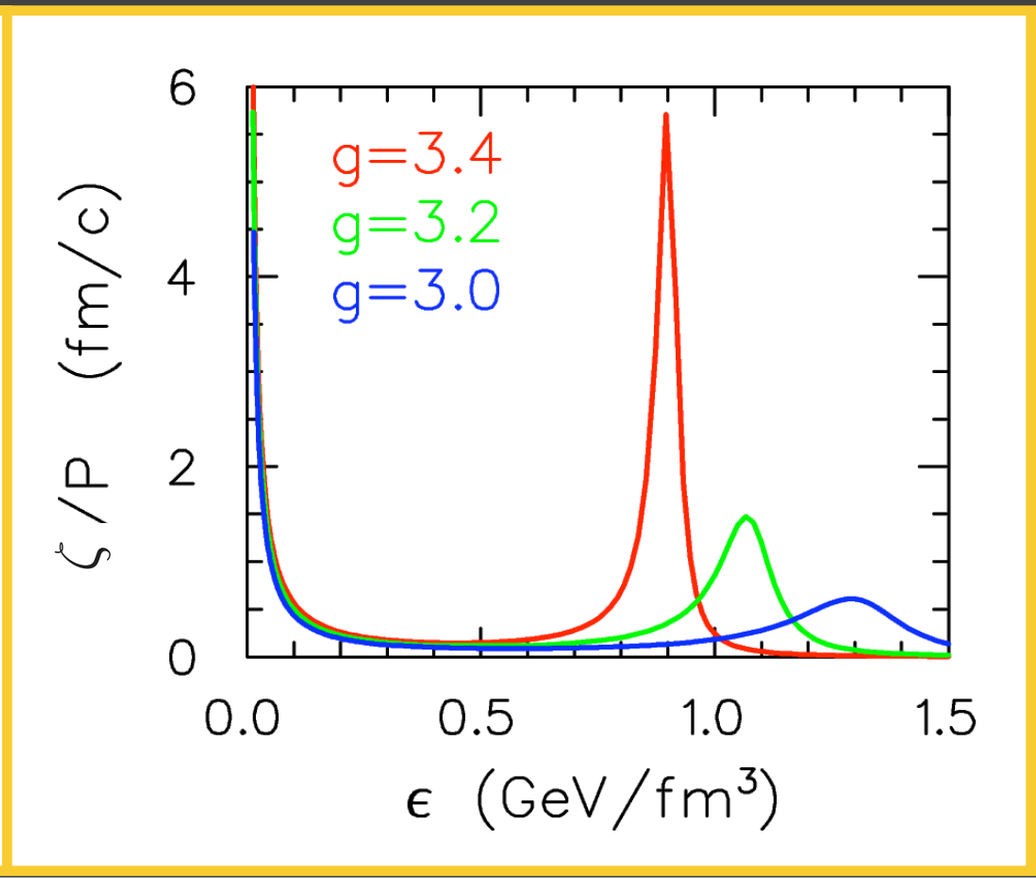
Linear Sigma Model

$$\zeta = \frac{\Gamma}{m_\sigma^2} \left.\frac{\partial p}{\partial n}\right|_{e=\mathrm{const}} \frac{\partial \sigma_{\mathrm{eq}}}{\partial s} s$$

1st order when
g > 3.5549

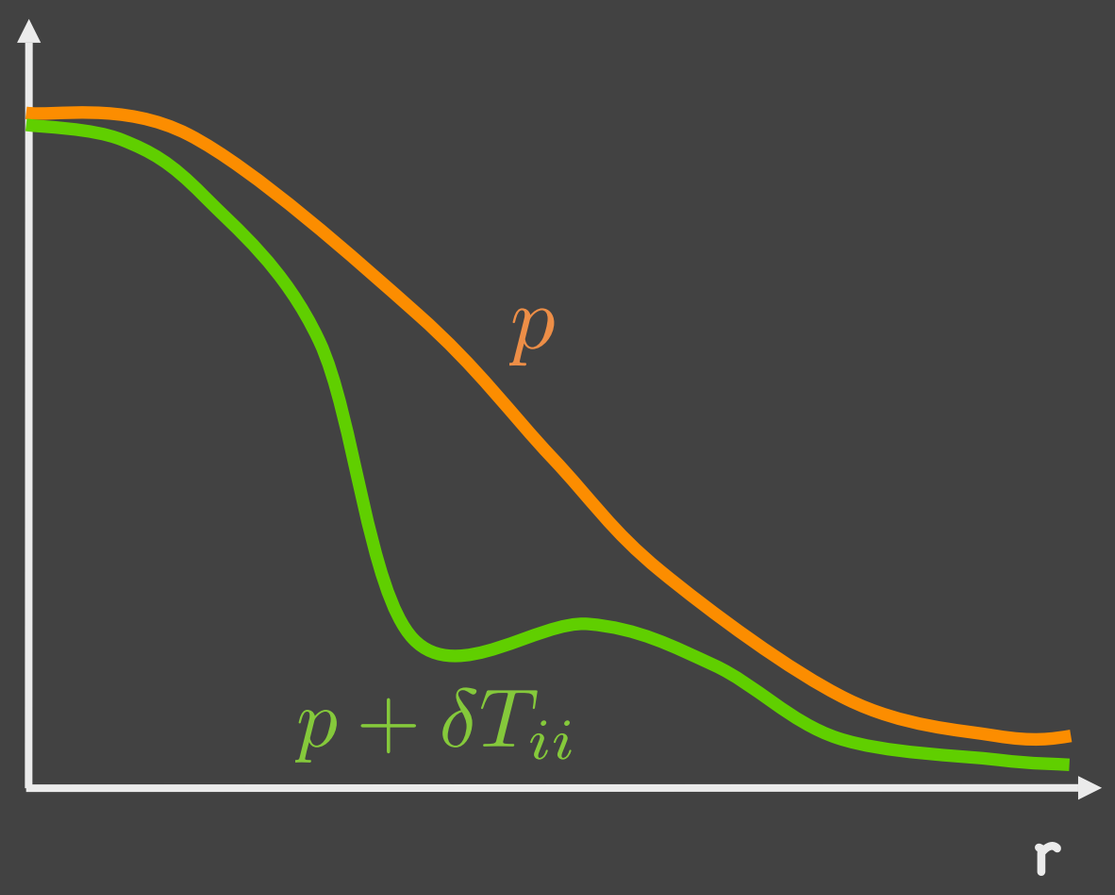
crossover when
g < 3.5549





How might this effect the dynamics?

- "traffic jam"
- flash-like emission



Summary

Numerous sources of viscosity
- finite collision time (shear)
- finite interaction range (shear + bulk)
- chemical nonequilibrium (bulk)
- non-equilibrium fields (bulk)

Shear viscosity important at early times
affects elliptic flow

Bulk viscosity important near T_C, alternatively can be included through:
- explicit chemical evolution
- explicit evolution of fields