Viscous Evolution of a Quark Gluon Plasma

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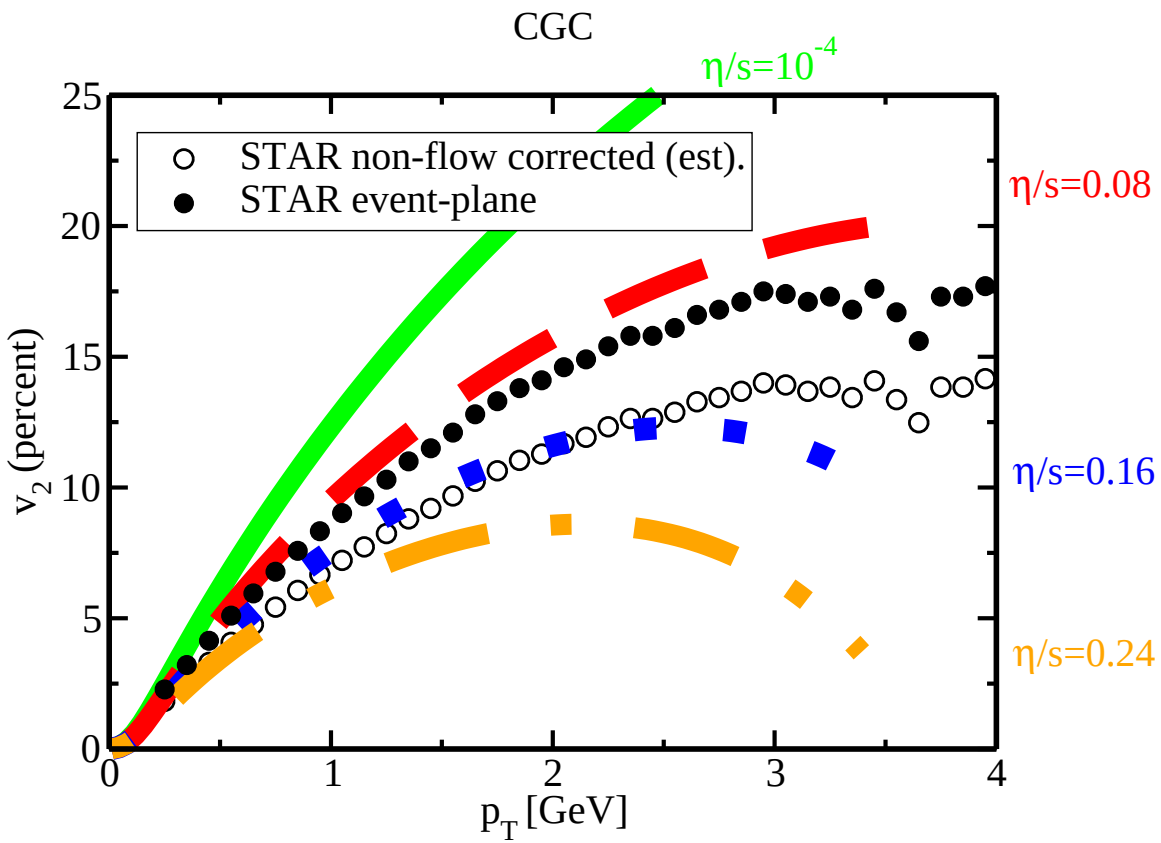
An important figure (Romatschke Luzum)



What are the uncertainties?

How does this become energy loss ? $v_2(p_T)$ or $R_{AA}(\phi)$?

Viscous Corrections

1. Viscous corrections to the equation of motion

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{with} \qquad T^{\mu\nu} = \underbrace{(e+p)u^\mu u^\nu + p g^{\mu\nu}}_{\text{ideal}} - \underbrace{\eta \left\langle \partial^\mu u^\nu \right\rangle}_{\text{viscous}}$$

2. Viscous corrections to the distribution function

$$f \to f_o + \delta f$$

   - Must be proportional to strains – must be a scalar
   - General form in rest frame and ansatz

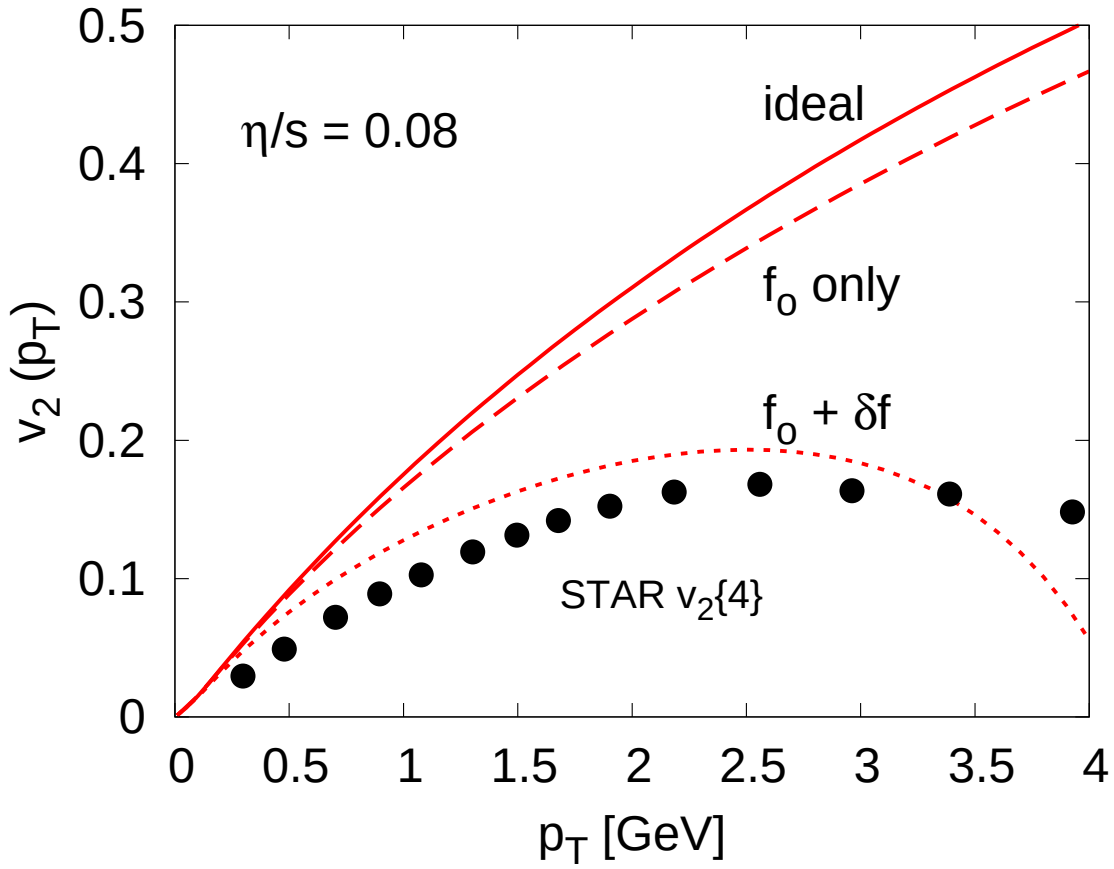
$$\delta f = \chi(p) \times f_o(p)\, \hat{p}^i \hat{p}^j \left\langle \partial_i u_j \right\rangle$$

   - The Quadratic Ansatz $\chi(p) \propto p^2$

$$\delta f = \frac{\eta}{2sT^3} \times f_o(p)\, p^i p^j \left\langle \partial_i u_j \right\rangle$$

All simulations have used the quadratic ansatz!

The role of δf



We should understand δf and the Quadratic Ansatz!

Relaxation Time Approximation

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_x f = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

- Substitute $f = f_o + \delta f$ with $f_o \propto e^{P \cdot u/T}$:

$$\partial_t f_o + v_{\mathbf{p}} \cdot \partial_x f_o = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

- With a bit of algebra:

$$f_o(p)\, \frac{p^i p^j}{2E_{\mathbf{p}} T} \langle \partial_i u_j \rangle = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

- Find:

$$\delta f \propto \frac{\tau_R(E_{\mathbf{p}})}{E_{\mathbf{p}}}\, f_o(p)\, p^i p^j \langle \partial_i u_j \rangle$$

Quadratic ansatz corresponds to $\tau_R \propto E_{\mathbf{p}}$.

What about $\tau_R \propto (E_{\mathbf{p}})^\beta$?

Generalization of Quadratic Ansatz

$$T^{ij} = p\delta^{ij} + \eta \left\langle \partial^i u^j \right\rangle = \int_{\mathbf{p}} \frac{p^i p^j}{E} (f + \delta f)$$

- First moment of δf determines the shear viscosity

$$\delta f = \chi(p)\ f_o(p)\, \hat{p}^i \hat{p}^j \left\langle \partial_i u_j \right\rangle \qquad \text{find} \qquad \eta = \frac{2}{15} \int_{\mathbf{p}} f_o\, \chi(\mathbf{p})\, p$$

- More General Ansatz – massless gas

$$\chi(p) \propto p^{2-\alpha} \qquad \delta f = \underbrace{\frac{\eta}{2sT^3} p^i p^j \left\langle \partial_i u_j \right\rangle}_{\text{old piece}} \underbrace{\left(\frac{p}{T}\right)^{-\alpha} \frac{\Gamma(6)}{\Gamma(6-\alpha)}}_{\text{new piece}}$$

Ansätze only partially constrained by shear viscosity

Two Extreme Limits: Quadratic and Linear Ansatz

1. Relaxation time growing with parton energy

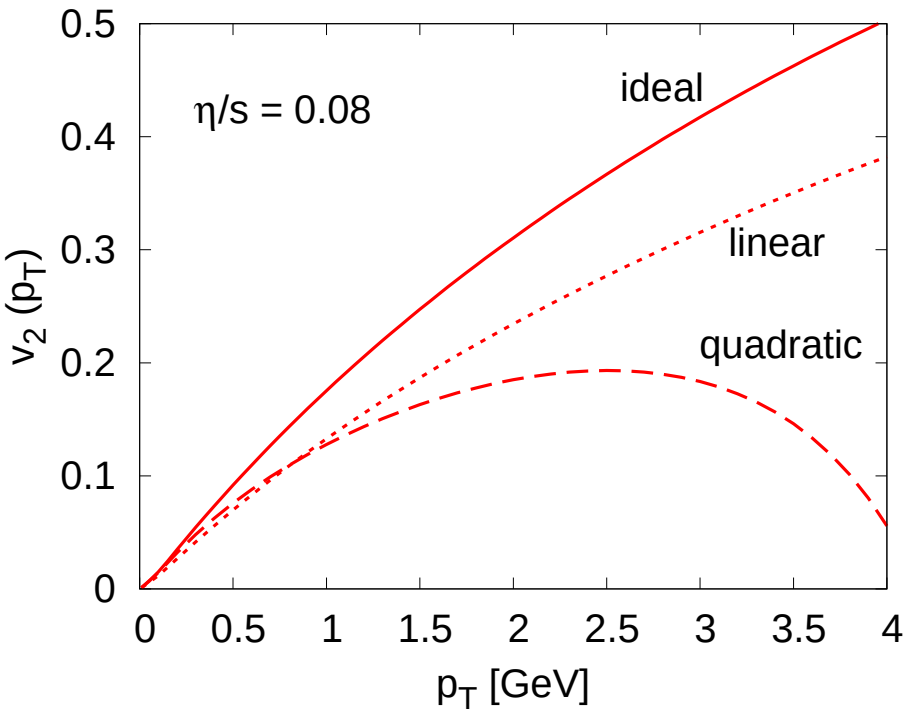
$$\tau_R \propto E_p \qquad \frac{dp}{dt} \propto \text{const} \qquad \chi(p) \propto p^2$$

2. Relaxation time independent of parton energy

$$\tau_R \propto \text{Const} \qquad \frac{dp}{dt} \propto p \qquad \chi(p) \propto p$$

Reality is probably in-between

Two Limits: Quadratic and Linear Ansatz



- The $\overline{v}_2$ independent of δf (arXiv:0905.2433)

$$\delta\overline{v}_2 \propto \int_p f_o\, p^2 \chi(p) \qquad \text{but all Ansätze have} \qquad \eta \propto \int_p f_o\, p\chi(p)$$

Where is reality?

Solving for δf with the Boltzmann Equation:

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_x f = C \circ f$$

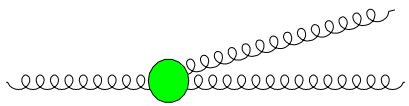
- Substitute $f = f_o + \delta f$

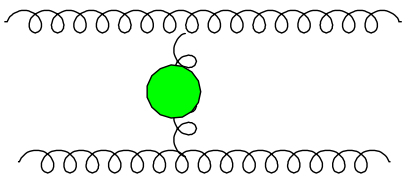
$$\partial_t f_o + v_{\mathbf{p}} \cdot \partial_x f_o = C \circ \delta f$$

- with a bit of algebra

$$f_o(p) \, \frac{p^i p^j}{2E_{\mathbf{p}} T} \, \langle \partial_i u_j \rangle = C \circ \delta f$$

- The collisions and bremsstrahlung is all in C.



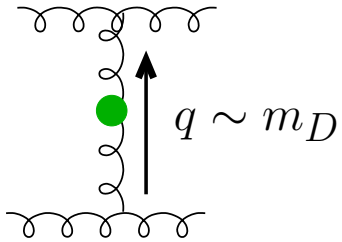


Can go invert this matrix and determine δf

Three Models of Energy Loss

1. Soft Scattering

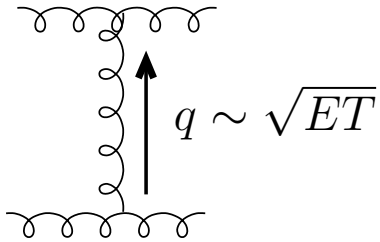
$$\frac{dp}{dt} \propto C_R\, \alpha_s^2 T^2 \log\left(\frac{T}{m_D}\right)$$



find $\chi(p) \propto p^2$

2. Collisional Energy Loss

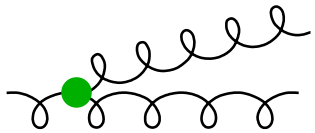
$$\frac{dp}{dt} \sim C_R\, \alpha_s^2 T^2 \log\left(\frac{p}{m_D}\right)$$



find $\chi(p) \propto \frac{p^2}{\log(p\mu)}$

3. Radiative + Collisional Energy Loss in Infinite medium

$$\frac{\Delta p}{\Delta t} \sim \alpha_s \sqrt{\hat{q} E_{\mathbf{p}}}$$

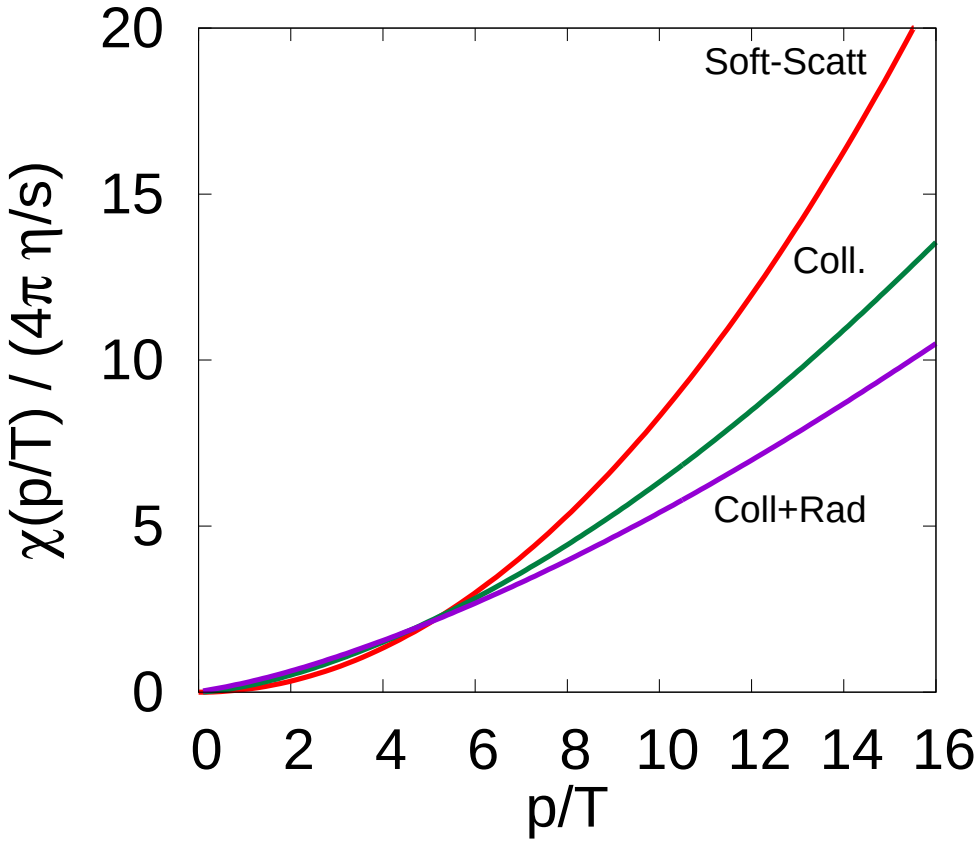


find $\chi \propto p^{3/2}$

These estimates are borne out by our numerical work

Summary – Energy Loss and δf

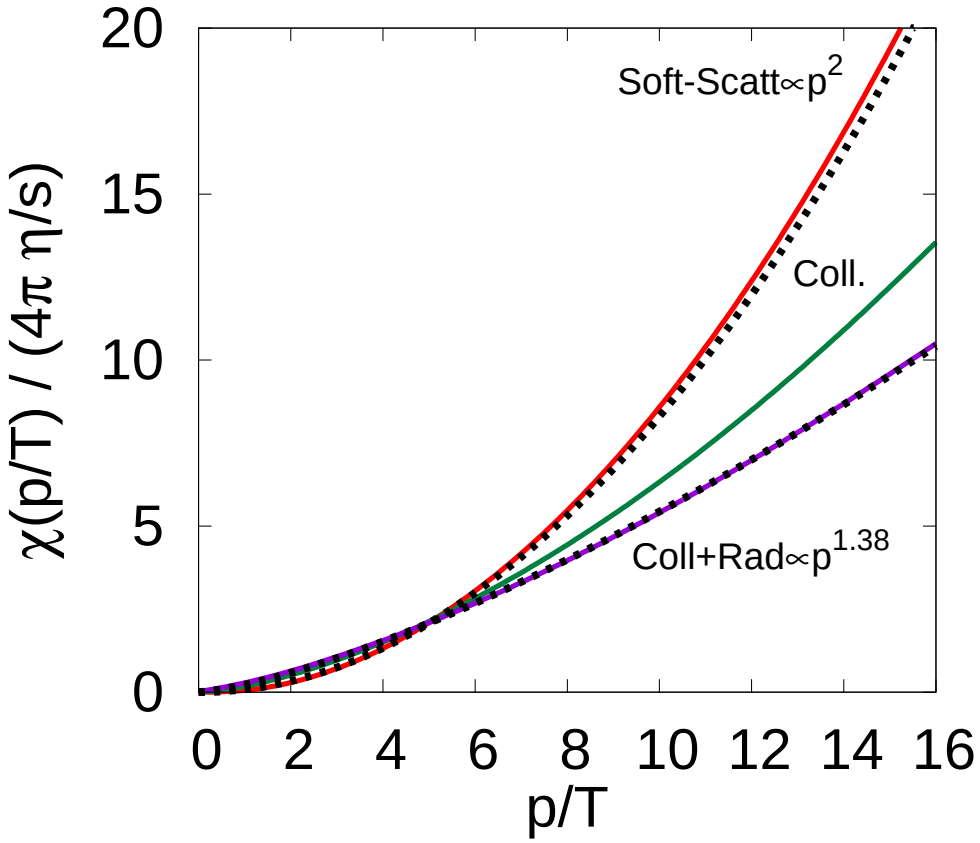
$$\delta f = \chi(p)\ f_o \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$



Energy Loss Determines $\chi(p)$

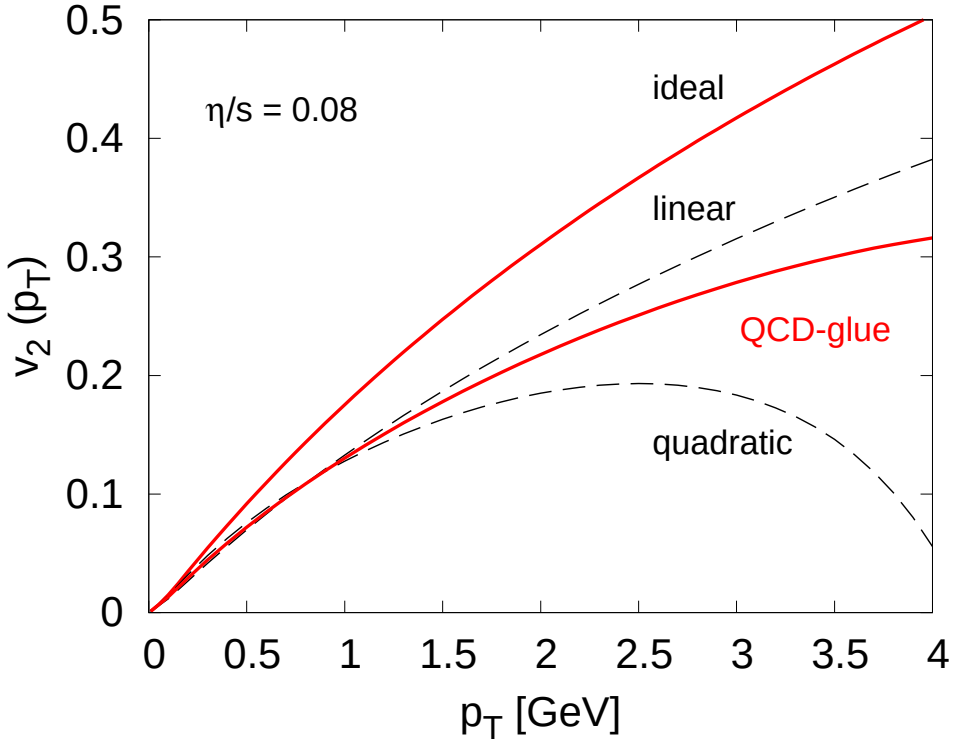
Summary – Energy Loss and δf

$$\delta f = \chi(p)\ f_o \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$



QCD kinetic theory expectation $\chi(p) \propto p^{1.38}$ in relevant range

Phenomenological Summary



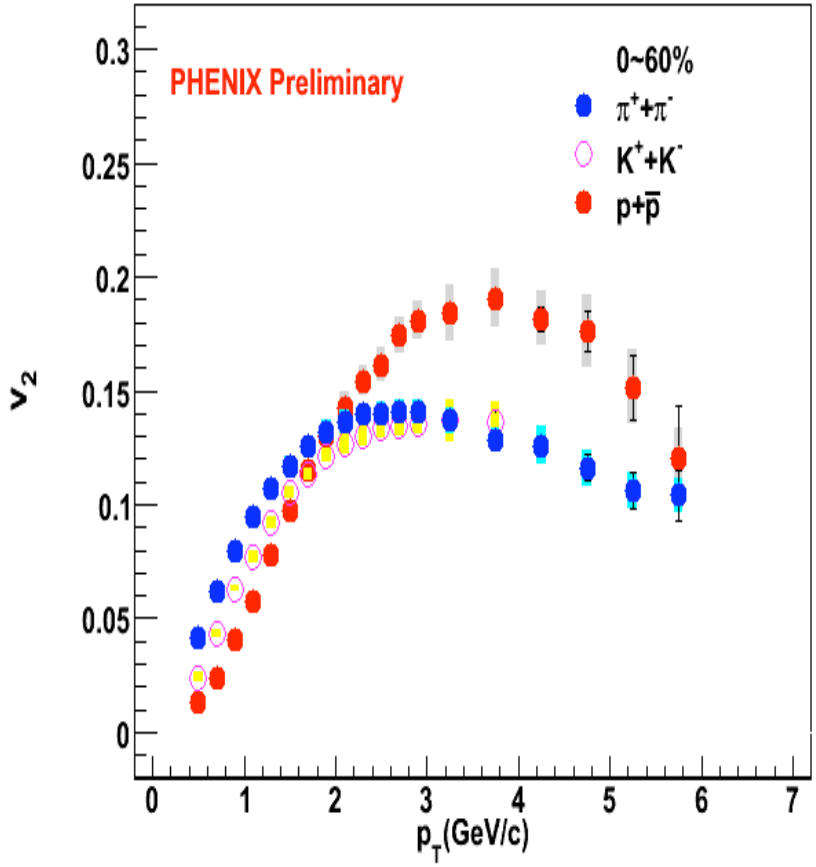
QCD is closer to a linear ($\tau_R =$ const) rather than a quadratic ansatz

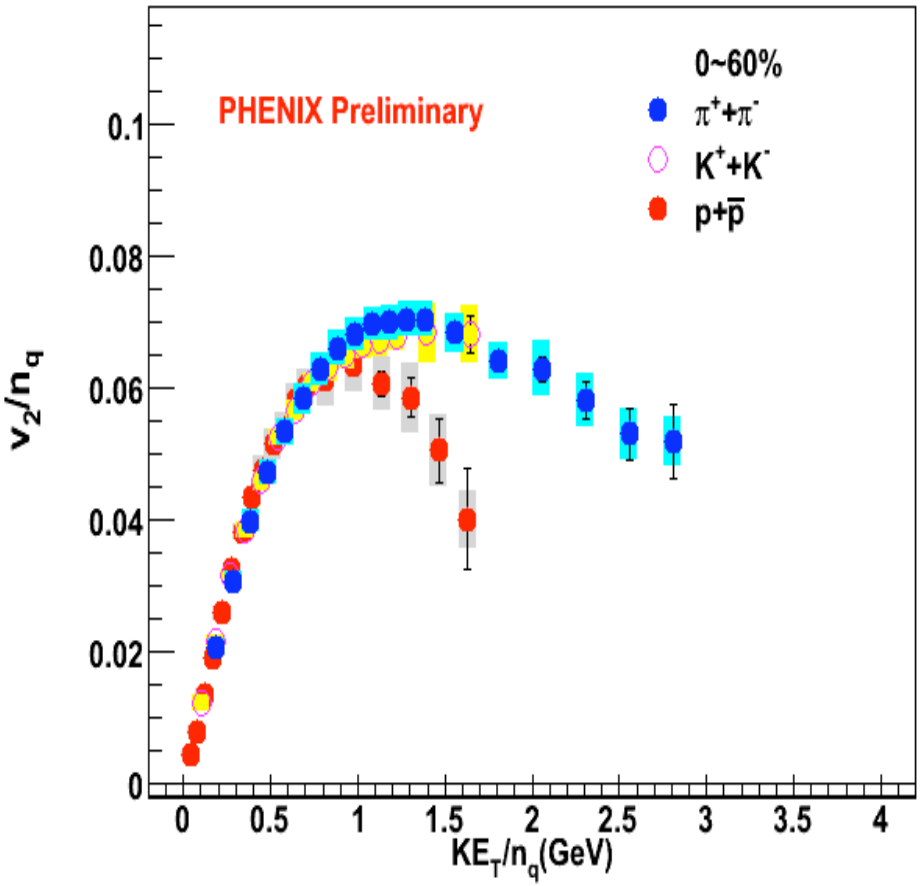
Energy loss sets the tail $p^{3/2}$ for hydro

So far only gluons but ...

QGP = Quarks and Gluons

Two Components observed





Two components in the quark gluon plasma independent of mass

Perhaps due to differences in quarks and gluons

Viscous Casimir Scaling – Quadratic Ansatz

- Quarks and Gluons have different mean free path and δfj

$$\begin{aligned} \delta f^Q &\propto \ell^Q_{\rm mfp} \times f_o(p) p^i p^j \langle \partial_i u_j \rangle \\ \delta f^G &\propto \ell^G_{\rm mfp} \times f_o(p) p^i p^j \langle \partial_i u_j \rangle \end{aligned}$$

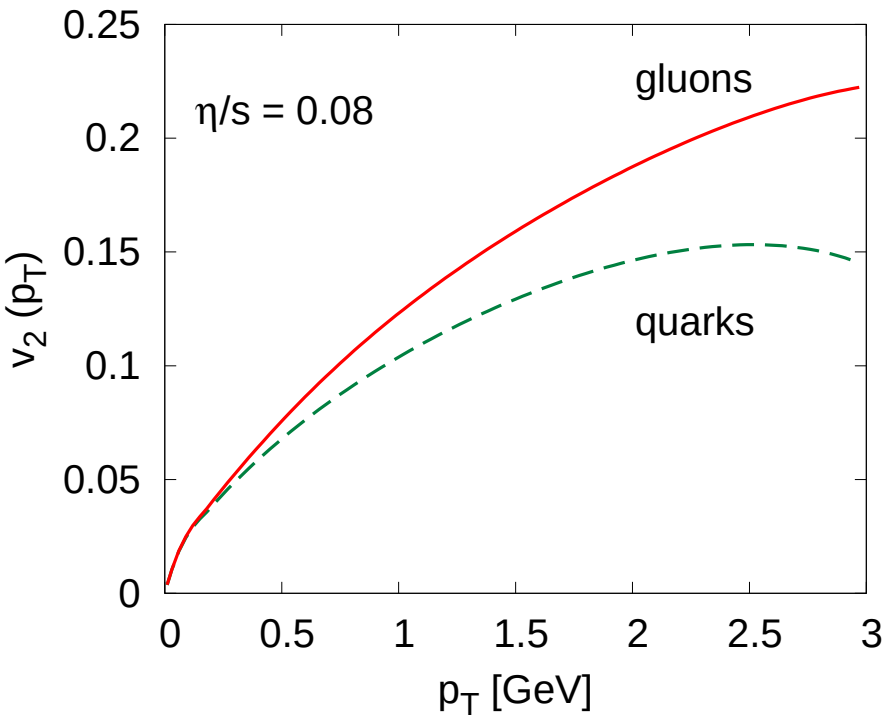
- Casimir Scaling

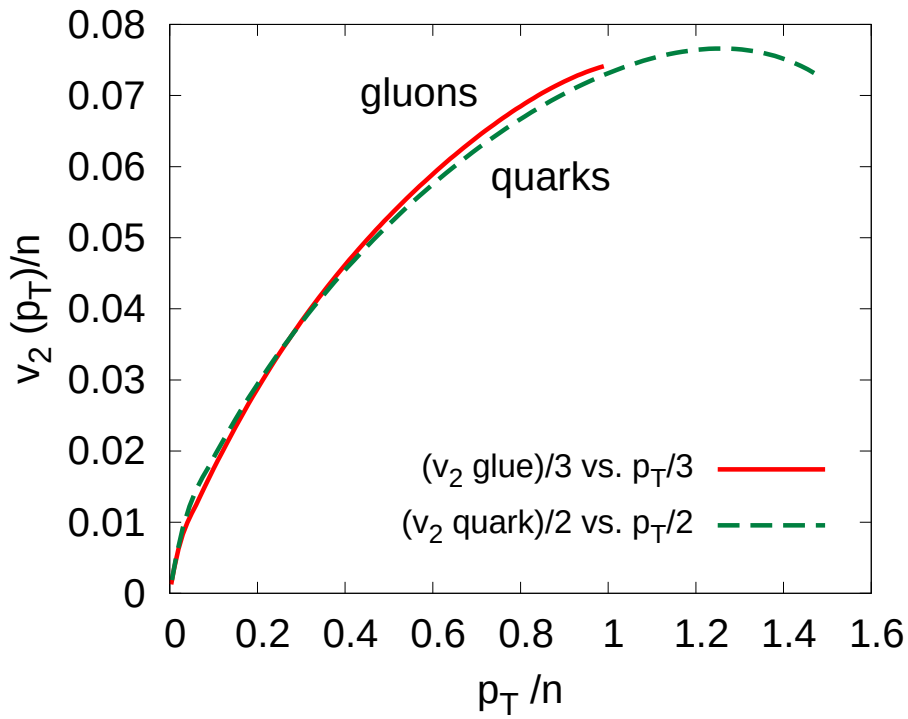
$$\ell_{\rm mfp} = \frac{1}{n\sigma} \qquad \frac{\ell^Q_{\rm mfp}}{\ell^G_{\rm mfp}} = \frac{C_A}{C_F} = \frac{9}{4}$$

- So expect hydro valid while $\delta f / f_o \ll 1$

$$p_{T,\rm max} \propto \sqrt{\frac{R}{\ell_{\rm mfp}}} \qquad so \qquad \frac{p^G_{T\,\rm max}}{p^Q_{T\,\rm max}} = \left(\frac{\ell^Q_{\rm mfp}}{\ell^G_{\rm mfp}}\right)^{1/2} = \left(\frac{9}{4}\right)^{1/2} = 1.5$$

Simple Viscous Casimir Scaling – Quadratic Ansatz

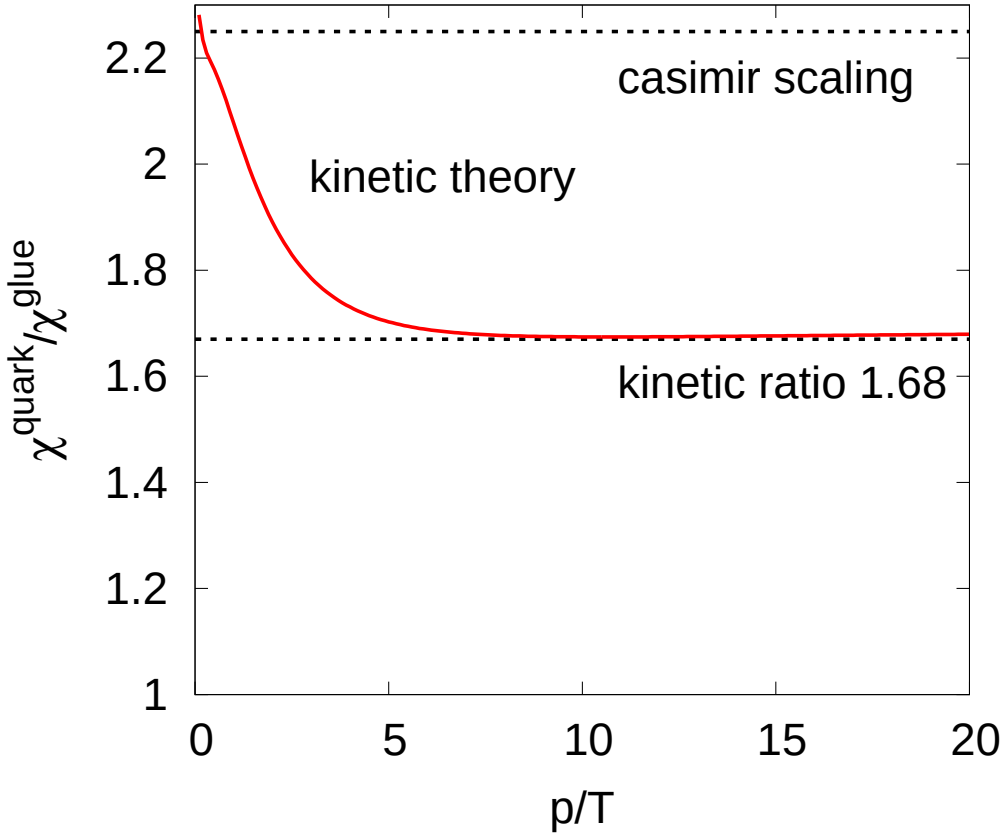




Issues

- Quadratic Ansatz not valid
- Casimir scaling doesn't hold in general

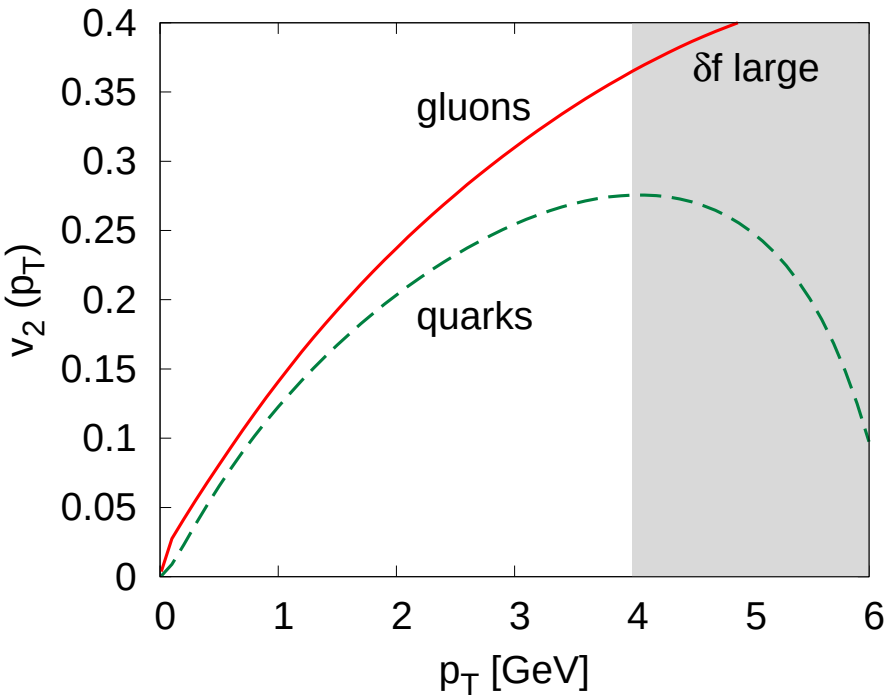
Casimir scaling is not general

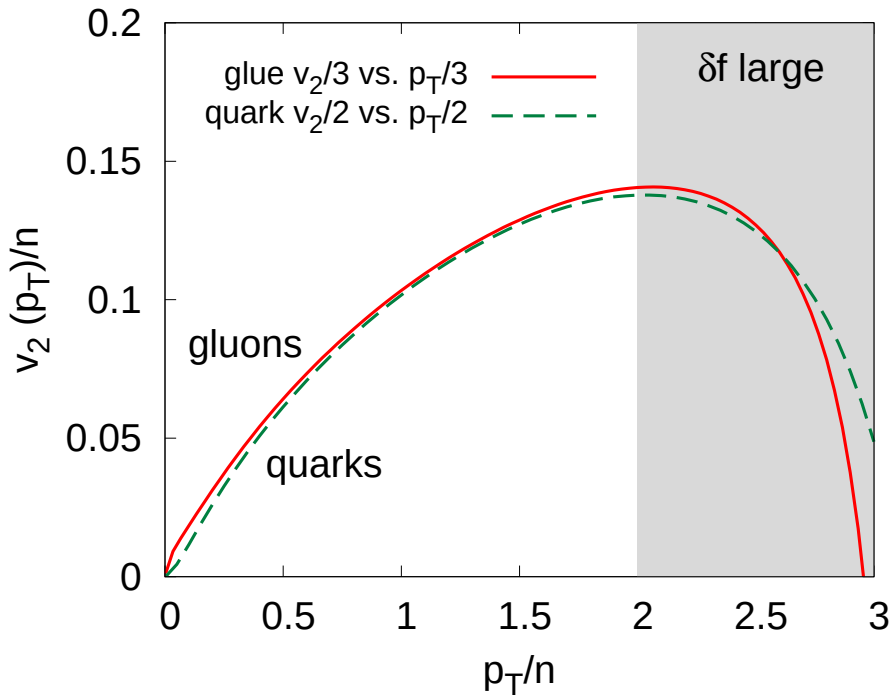


Deviation from Casimir scaling:

1. Compton process – not just soft t-channel exchange
2. Spin dependence of the splitting function $\Leftarrow$ dominant effect

Viscous Casimir Scaling – Real Calculation





$$\frac{p^G_{T\,\max}}{p^Q_{T\,\max}} = \left(\frac{\ell^Q_{\rm mfp}}{\ell^G_{\rm mfp}}\right)^{1/1.38} = (1.68)^{1/1.38} = 1.45$$

Is Quark Number Scaling (QNS) observed really Viscous Casimir Scaling (VCS)?

Conclusions

- Studied the kinetics of Quarks and Gluons and the imprints on elliptic flow
- Radiative energy loss increases the elliptic flow in a certain range of $p_T \simeq 1.5 - 2.5\,\mathrm{GeV}$
- Makes precise the connection between energy loss and viscosity.
- Observed *Viscous Casimir Scaling (VCS)* in the quark and gluon elliptic flow
  - Too early to tell if the quark and gluon explanation holds together.
  - A model which has constituent quark scaling w/out constituent quarks