# Status of shear viscosity determination from flow at RHIC - theoretical perspective

**Denes Molnar**

**RIKEN/BNL Research Center & Purdue University**

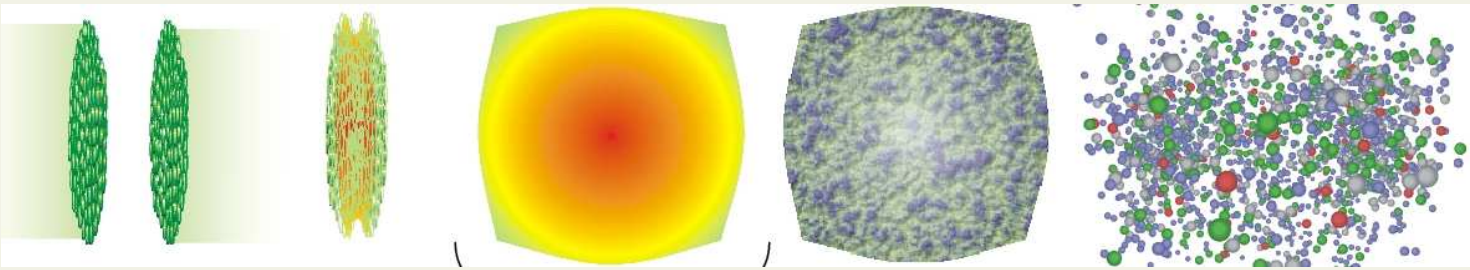
**RHIC AGS Users Meeting**

**June 7-11, 2010, Brookhaven National Laboratory, Upton, NY**

# Heavy ion collisions

**goal: experimentally test QCD at finite T - equation of state, viscosity, ...**

initial nuclei  parton plasma  hadronization  hadron gas



$\sim 10^{-14}$ **m**
**= 10 fermis**

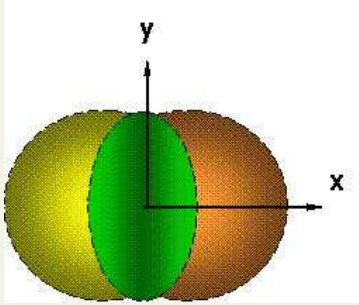
<——— $\sim 10^{-23}$ **sec = 10 yochto(!)-secs** ———>

- **rapid expansion, inhomogeneities (time and length scales comparable)**
- **early and late dynamics out of equilibrium**

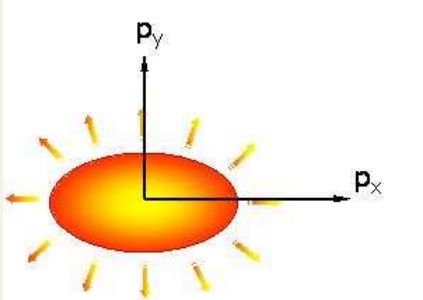
**does it thermalize?**

# Thermalization at RHIC

efficient conversion of spatial eccentricity to **momentum anisotropy**

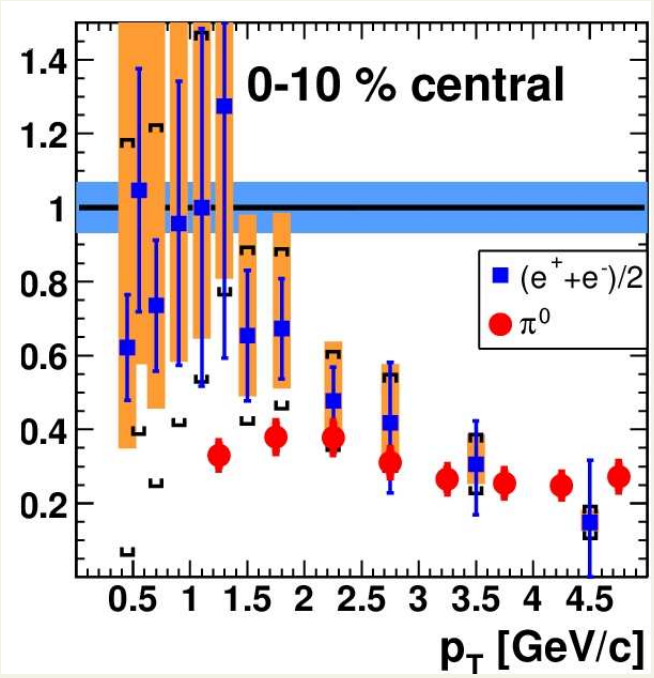



$\longrightarrow$




"elliptic flow"

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \qquad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle$$



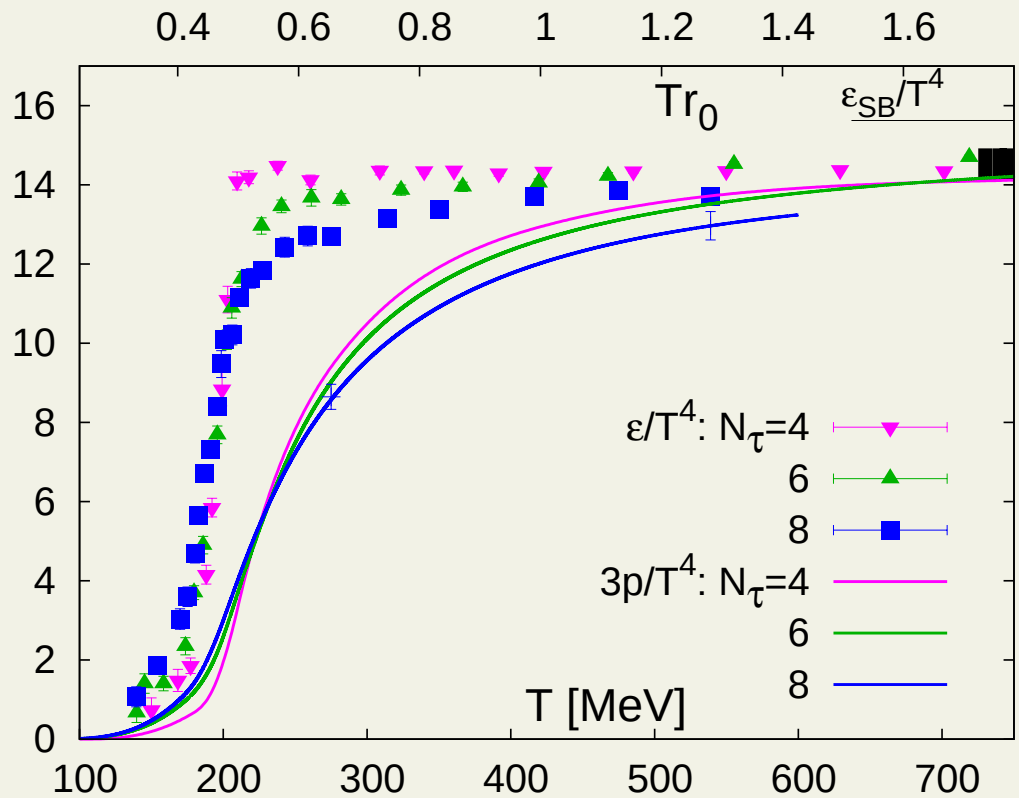

very opaque - **large energy loss, even for heavy quarks**

$$R_{AA} = \frac{measured\ yield}{expected\ yield\ for\ dilute\ system}$$

# QCD equation of state ($\mu_B = 0$)

**lattice QCD** - $Z = \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}A\, e^{-S_E^{QCD}}$

A. Bazavov et al, arXiv:0903.4379



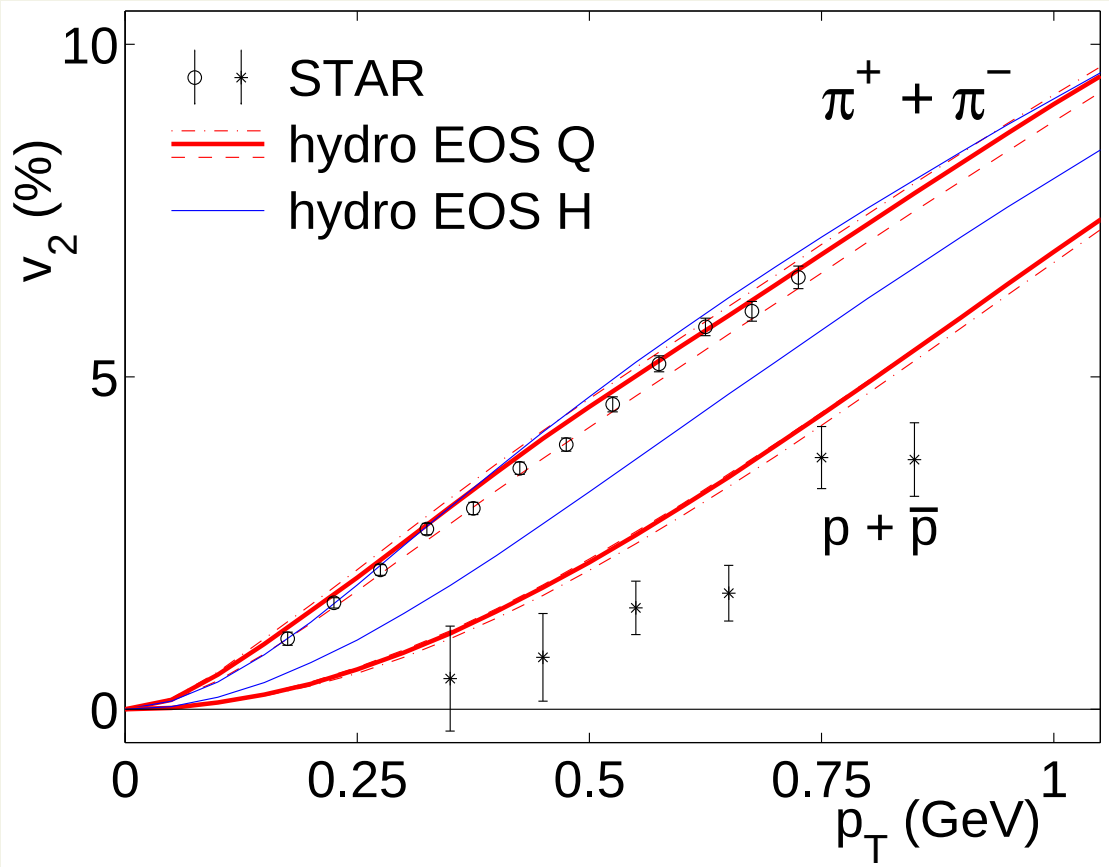

$$\frac{e_{SB}}{T^4} \approx \frac{d.o.f.}{3}$$

**quite robust results, though still evolving** - $T_c \approx 170 - 180 - 200$ **MeV**

**but transport properties are not known for** $T/T_c \sim 1 - 2$

# $\sim$ 2000-01: Ideal hydro

**minimum-bias Au+Au at RHIC** Kolb, Heinz, Huovinen et al ('01), nucl-th/0305084



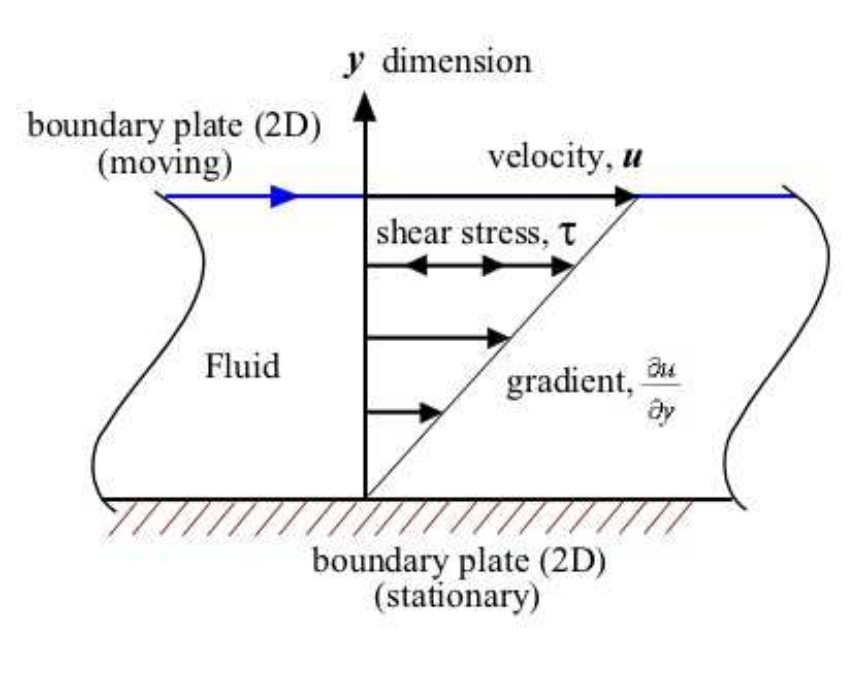

$\tau_{therm} = 0.6$ **fm** , $\langle e \rangle_{init} \sim 5 GeV/fm^3$, $T_{freezeout} = 120$ **MeV**

**equation of state with QGP (Q) favored over hadron gas (H)**

# Shear viscosity

**1687 - I. Newton (Principia)**

$$T_{xy} = -\eta \frac{\partial u_x}{\partial y}$$




**acts to reduce velocity gradients**

**1985 - quantum mechanics:** $\Delta E \cdot \Delta t \geq \hbar/2$

**+ kinetic theory:** $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz, PRD 31 ('85)

$$\eta \approx 4/5 \cdot T/\sigma_{tr} \ , \qquad \textbf{entropy } s \approx 4n$$

$\Rightarrow$ **minimal viscosity:** $\eta/s = \frac{\lambda_{tr} T}{5} \geq \hbar/15$

**2004 - string theory** AdS/CFT: $\eta/s \geq \hbar/4\pi$ Policastro, Son, Starinets, PRL87 ('02) Kovtun, Son, Starinets, PRL94 ('05)

$\eta/s \geq 4\hbar/(25\pi)$ Brigante et al, arXiv:0802.3318

**strongly coupled $\mathcal{N}=4$ SYM in 4D $\Leftrightarrow$ weakly-coupled gravity on AdS$_5$**

**not QCD, but key insights, in large 't Hooft coupling $\lambda \equiv g_s^2 N_c \to \infty$ limit**

**only modest change to ideal gas thermodynamics** Gubser et al ('96), Buchel et al ('08)**:**

$$\frac{S}{S_{SB}} = \frac{3}{4}\left[1 + 4\delta + \mathcal{O}(\frac{\sqrt{\lambda}}{N_c^2}) + \mathcal{O}(\frac{1}{\lambda^{3/2}})\right]$$

**very small, but nonzero shear viscosity** Son, Policastro et al ('02), ('04), Buchel et al ('09)**:**

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \delta + \mathcal{O}(\frac{1}{\lambda^{3/2}})\right]$$

**short relaxation times** Baier et al arxiv:0712.2451, Chesler and Yaffe, arxiv:0812.2053**,**

$$\tau_\pi = \frac{2 - \ln 2}{2\pi T} \qquad\qquad \tau_{th} \sim \frac{\mathcal{O}(1)}{T_{eff}}$$

# Viscosity in QCD

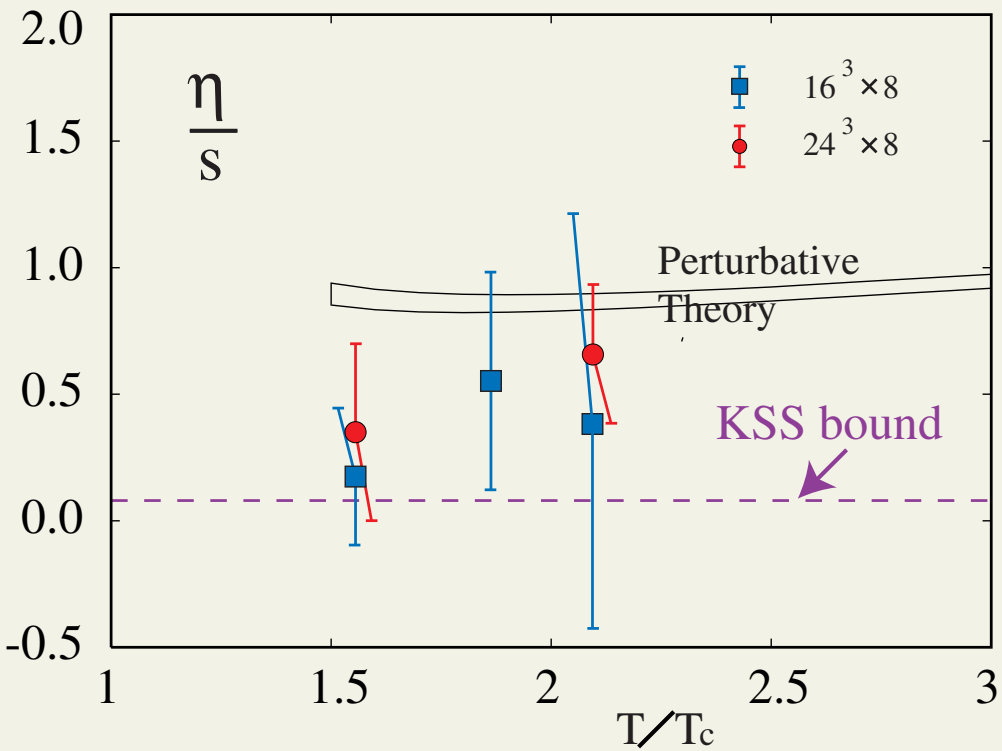
**shear:** $\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{-i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle \equiv \lim_{\omega\to 0} \frac{1}{2\omega} \sigma_\eta(\omega)$

**bulk:** $\zeta = \lim_{\omega\to 0} \frac{1}{18\omega} \int dt d^3x\, e^{-i\omega t} \langle [T^i_i(x,t), T^i_i(0,0)] \rangle \equiv \lim_{\omega\to 0} \frac{1}{2\omega} \sigma_\zeta(\omega)$

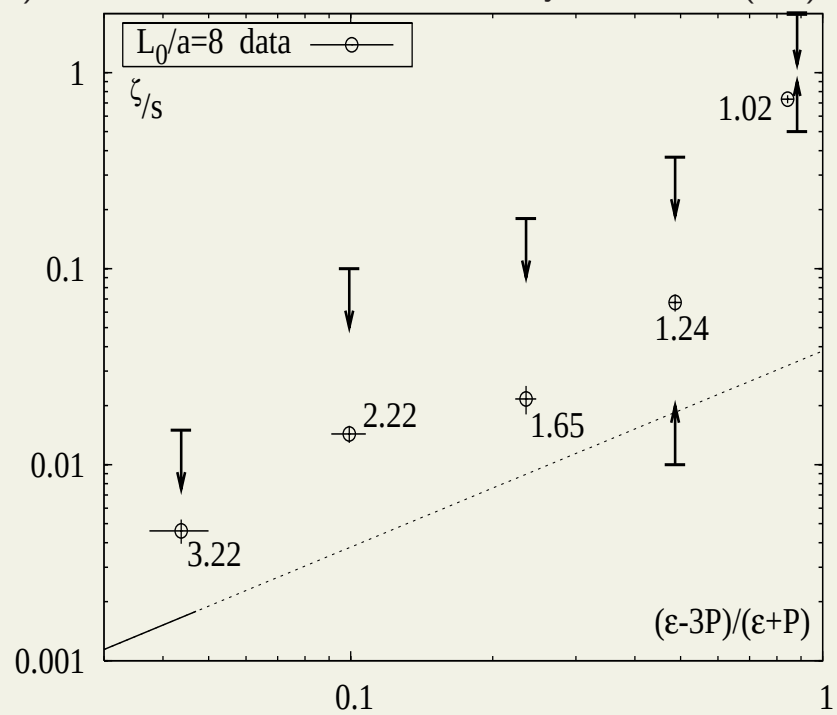
**perturbation theory:** $\eta/s \sim \mathcal{O}(1), \qquad \zeta/s \sim 0.02\alpha_s^2 \sim 0$

Arnold, Moore, Yaffe, JHEP 0305 ('03); Arnold, Dogan, Moore, PRD74 ('06)

**lattice:** **shear estimate** Nakamura & Sakai, NPA774 ('06)




**bulk estimate** Meyer, PRD76 ('07)



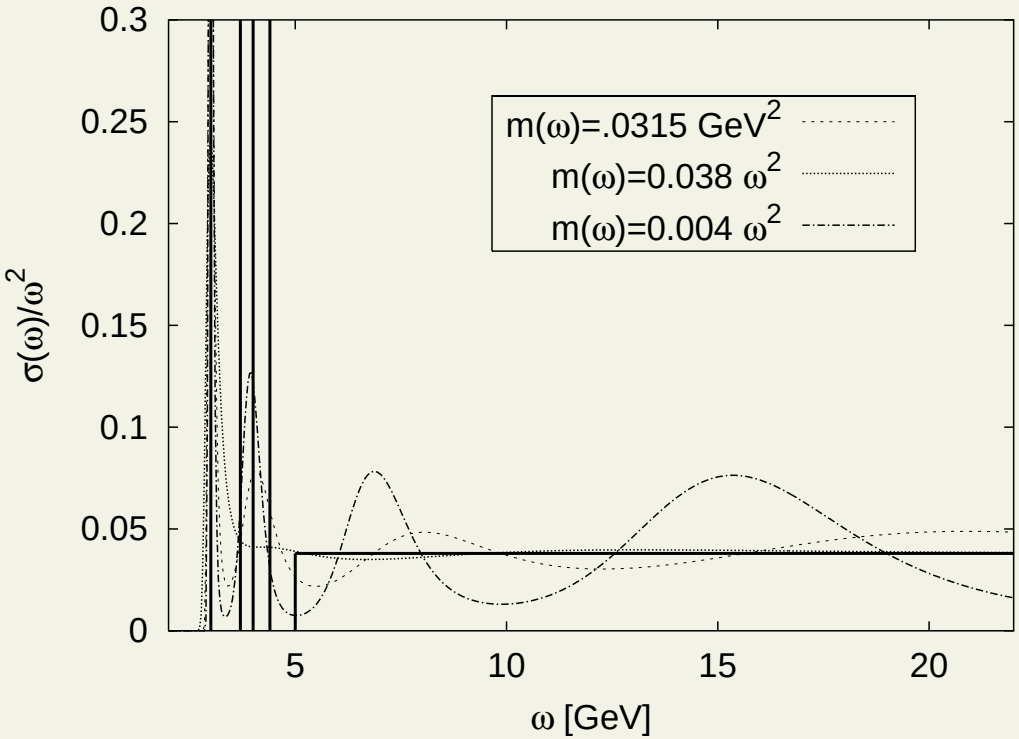
**VERY HARD:** - **need derivative of $\sigma(\omega)$ near $\omega = 0$**
- **underdetermined inversion problem on lattice**

$$G(\tau) = \int_0^\infty d\omega\, \sigma(\omega) K(\omega, \tau)$$

$\Rightarrow$ **feasibility and accuracy yet to be demonstrated**

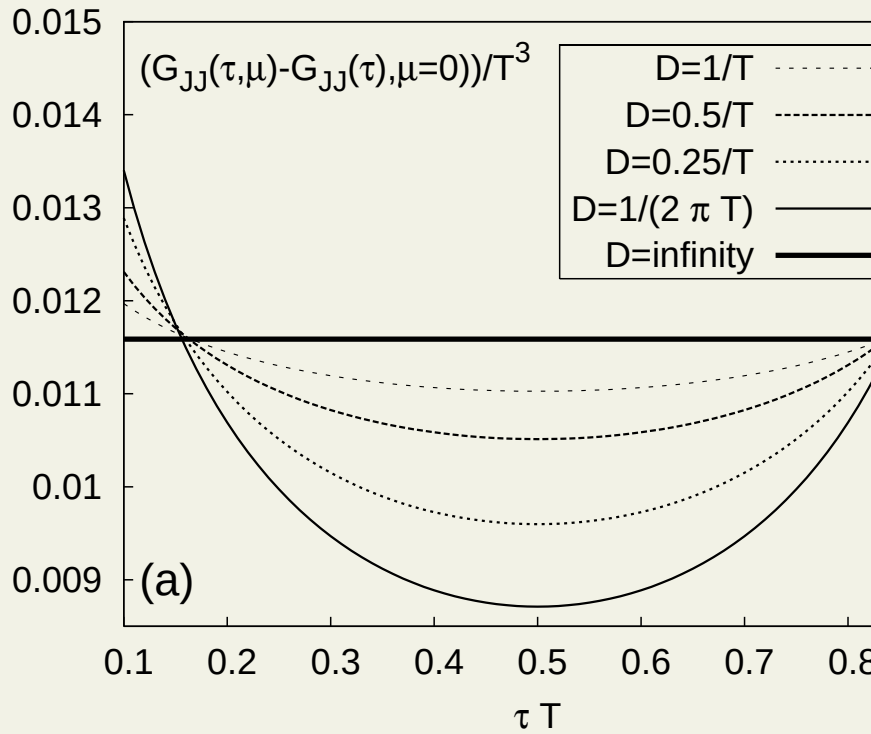
**e.g., charmonium - OK**

Petreczky & Jakovac, PRD75 ('07)




**but heavy-quark diffusion not promising**

Petreczky & Teaney, PRD73 ('05)



## Optimistically, we can

- **test the calculated QCD equation of state experimentally**
- **measure the (shear) viscosity in QCD**
- **test the universality of shear viscosity bound** $\eta/s \gtrsim 1/(4\pi)$

**requires some dissipative framework**

# Dissipative dynamical frameworks

- **causal relativistic hydrodynamics**

  Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al, DM & Huovinen

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad (\mu_B \to 0)$$

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - p g^{\mu\nu} + \pi^{\mu\nu} - \Pi \Delta^{\mu\nu}$$

$$\dot{\pi}^{\mu\nu} = F^{\mu\nu}(e, u, \pi, \Pi) \quad , \qquad \dot{\Pi} = G(e, u, \pi, \Pi)$$

**e.g. Israel-Stewart theory**

- **covariant transport** Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

$$p^\mu \partial_\mu f = C_{2\to 2}[f] + C_{2\leftrightarrow 3}[f] + \cdots$$

**fully causal and stable**

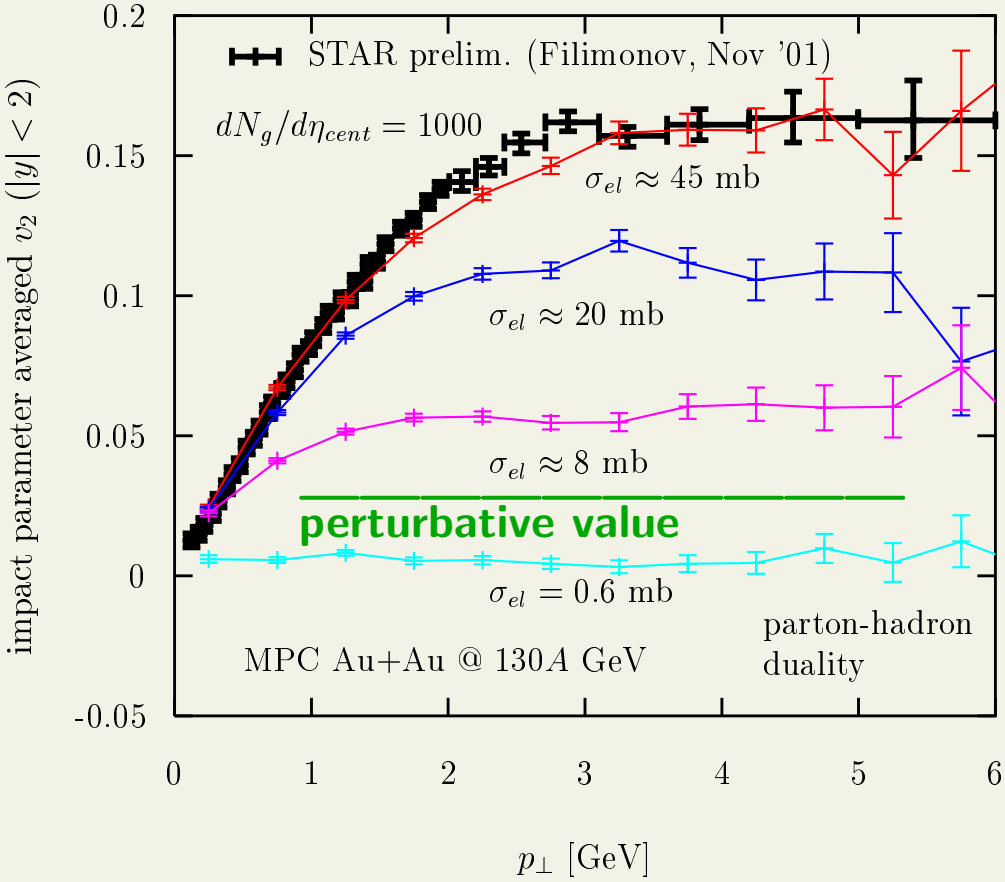
**near hydrodynamic limit, transport coefficients and relaxation times:**

$$\eta \approx 1.2 T/\sigma_{tr}, \qquad \tau_\pi \approx 1.2 \lambda_{tr}$$

# RHIC - not an ideal fluid

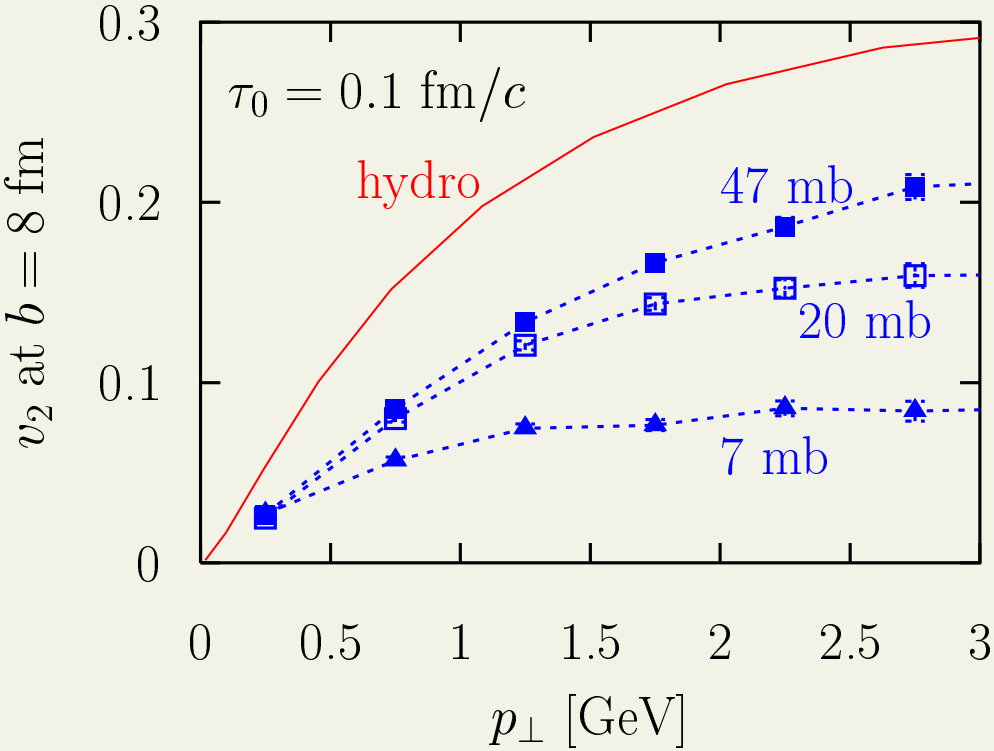
**$2 \to 2$ transport $v_2(p_T)$ vs data**

DM & Gyulassy, NPA 697 ('02)




**transport vs ideal hydro**

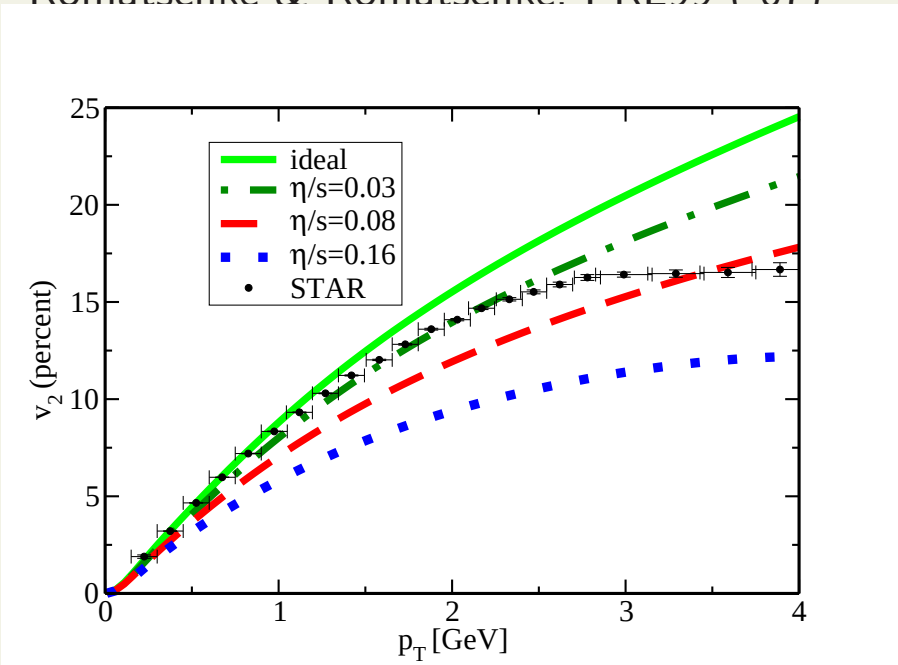
DM & Huovinen, PRL94 ('05)



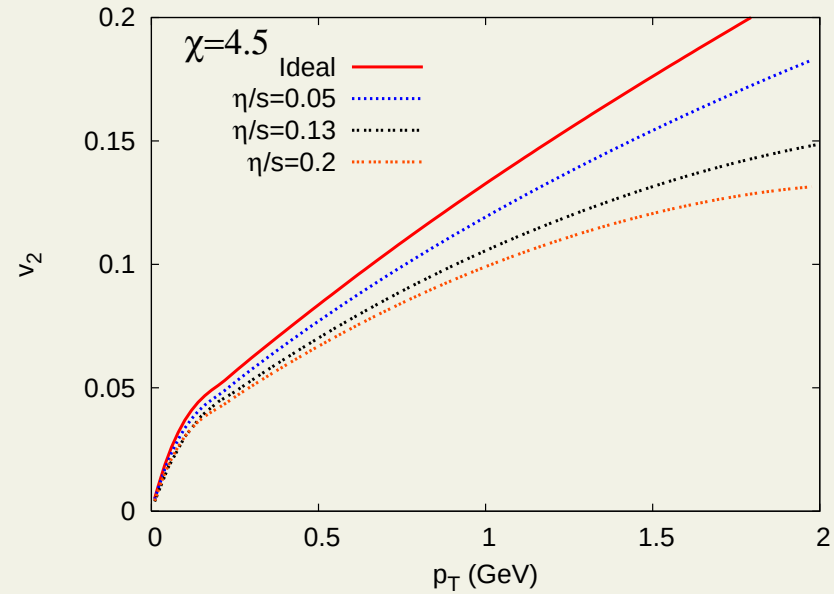

**dissipation reduces $v_2$ by $30 - 40$% even for very large $\sigma_{gg \to gg} \sim 50$ mb**

# Viscous hydro: $\sim 20-30\%$ effects for $\eta/s = 1/(4\pi)$ in Au+Au at RHIC.

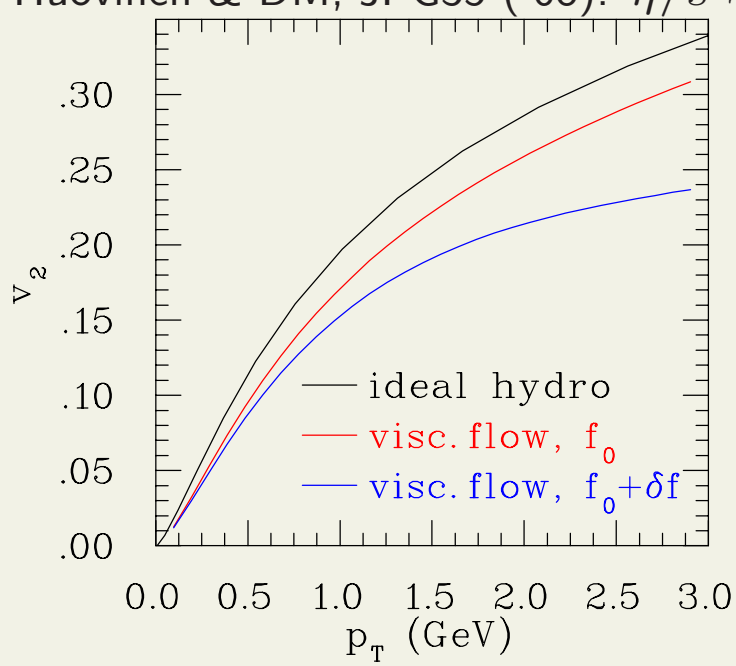
Romatschke & Romatschke, PRL99 ('07)



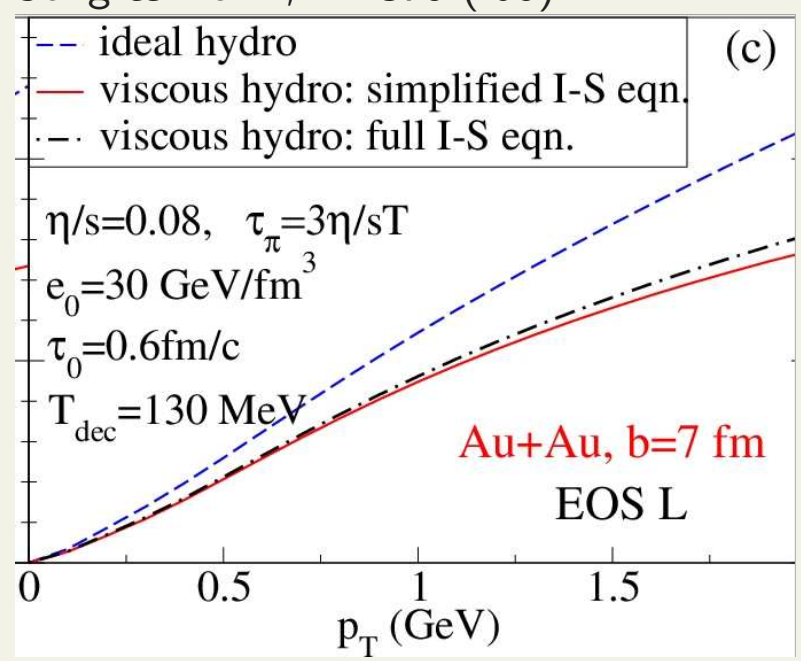

Dusling & Teaney, PRC77



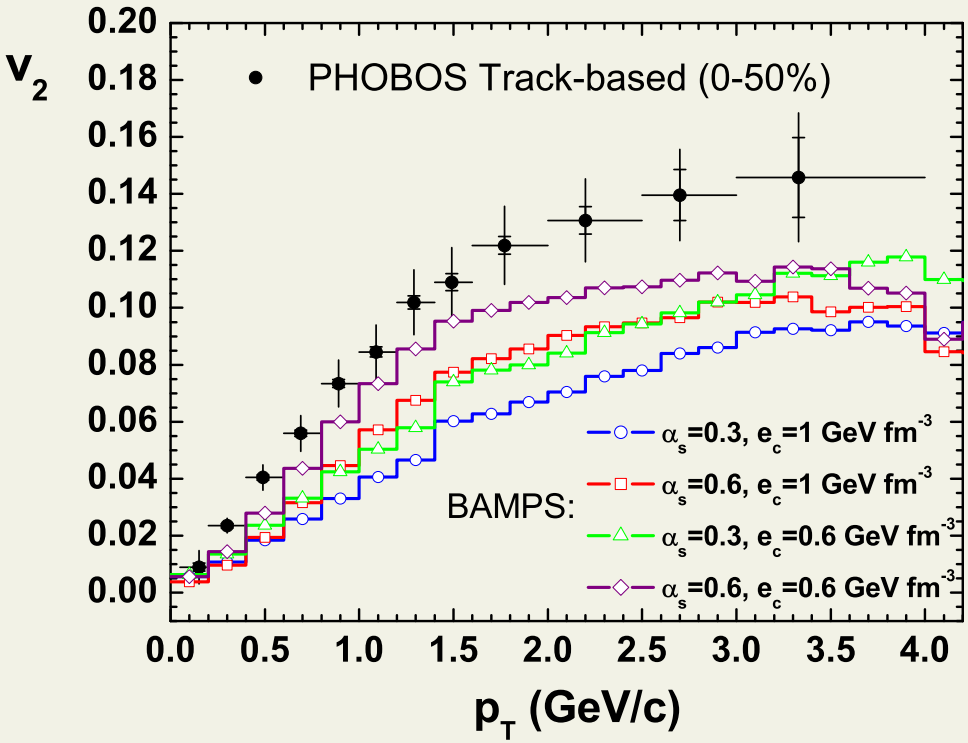

Huovinen & DM, JPG35 ('08): $\eta/s \approx 1/4\pi$

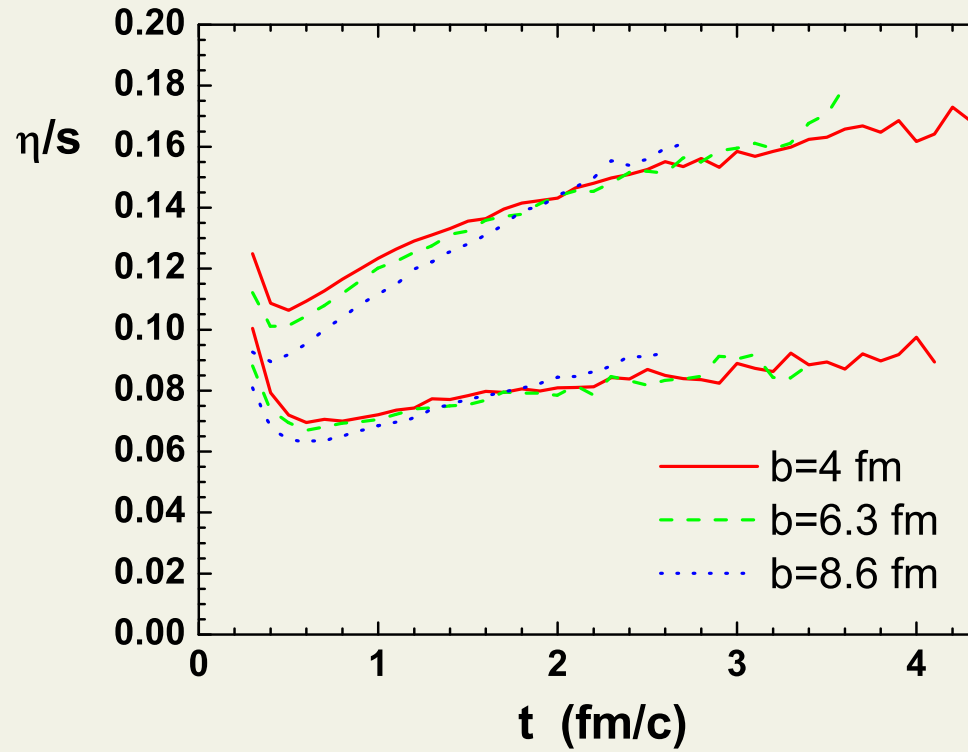



Song & Heinz, PRC78 ('08)



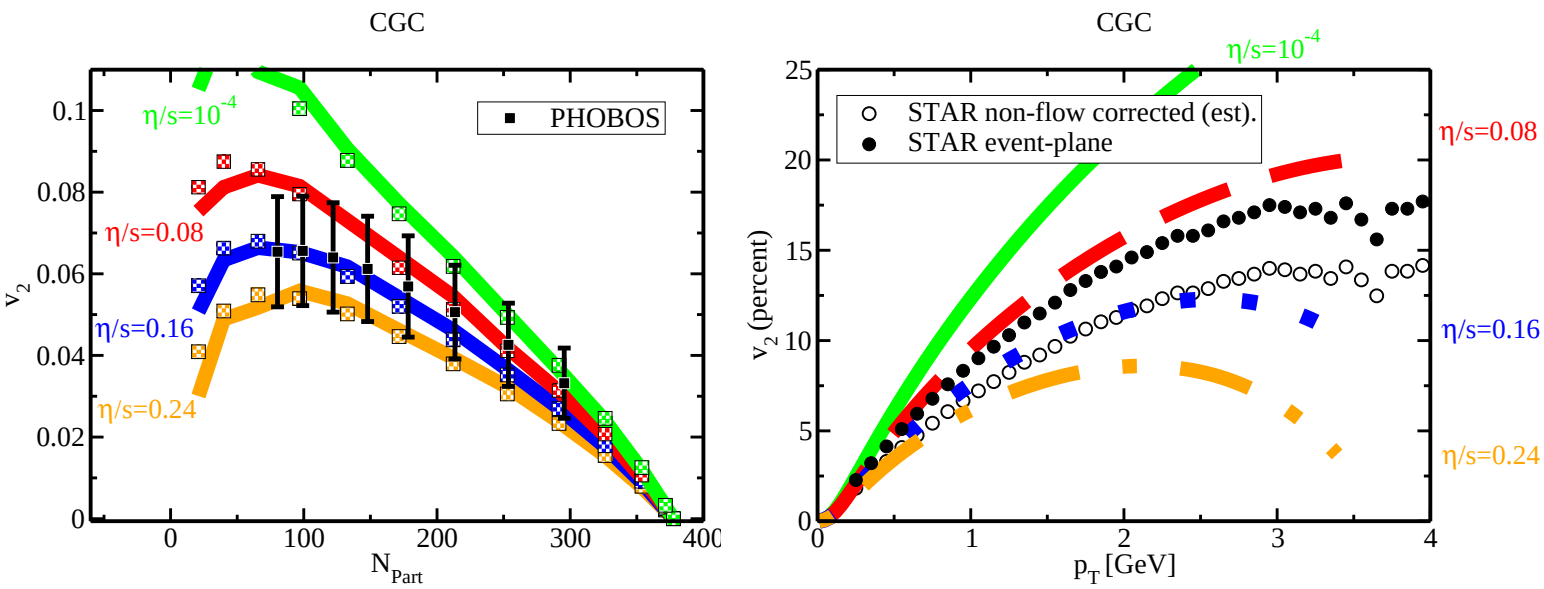
# radiative $3 \leftrightarrow 2$ transport Xu & Greiner, ('08)



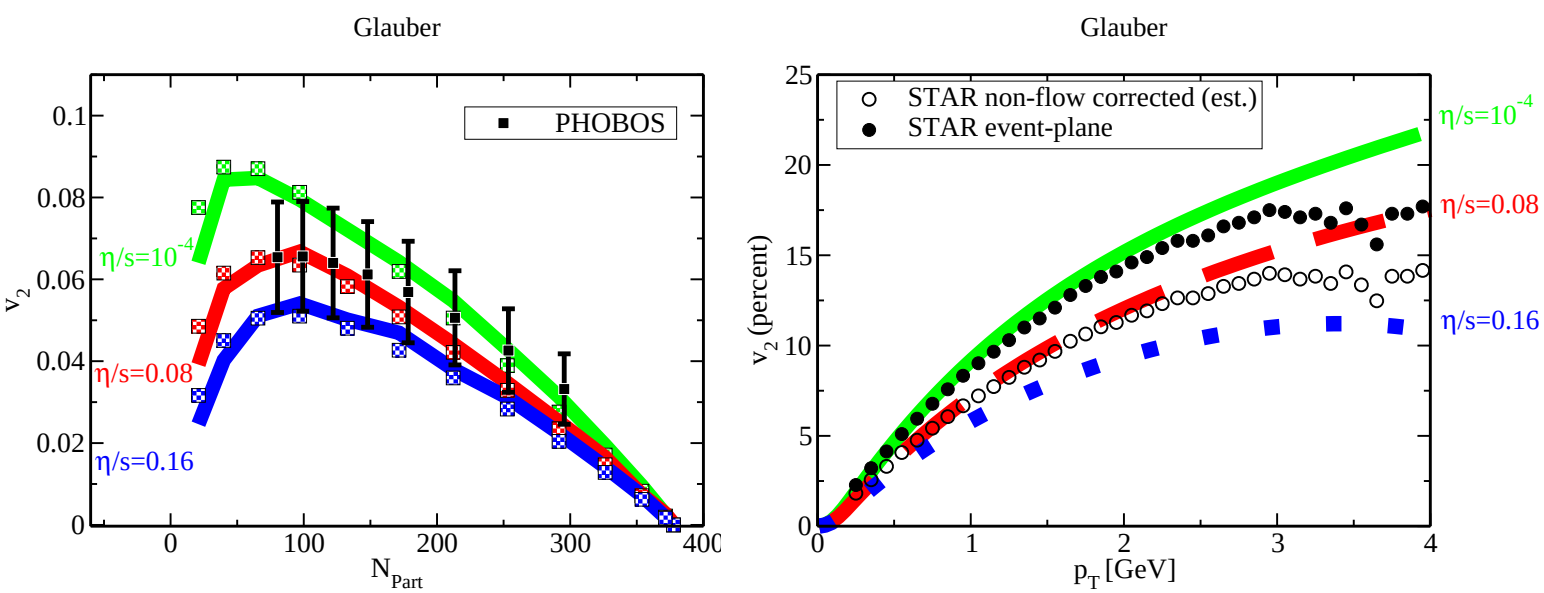




**similar conclusions - mainly the viscosity matters, not the microscopics**

# But, details matter: Romatschke & Luzum, PRC78 ('08): Au+Au, KLN profile




## binary collision profile - $\eta/s$ differs by factor of two?

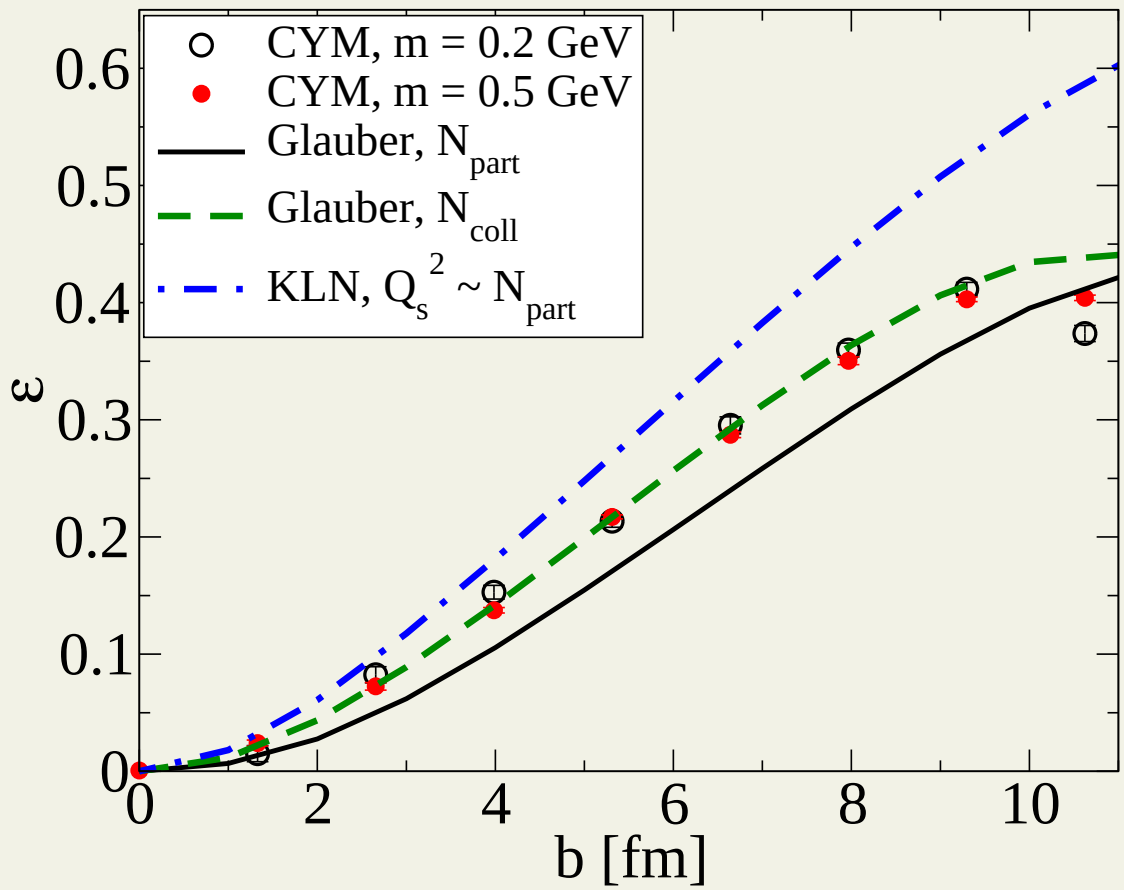


## why $\eta/s = 0.1 \pm 0.1(\pm 0.08)$ becomes $\eta/s \lesssim 0.5$... uncertainties:

- initial conditions $\rightarrow$ fit to subset of data
- equation of state, $\eta/s$ variations, bulk viscosity, relaxation times
- conversion of fluid to particles (freezeout prescription)
- validity/break-down of hydrodynamics - e.g., hadron transport

## Obviously, initial conditions matter - eccentricity crucial for $v_2$

Venugopalan & Lappi, PRC74 ('06)

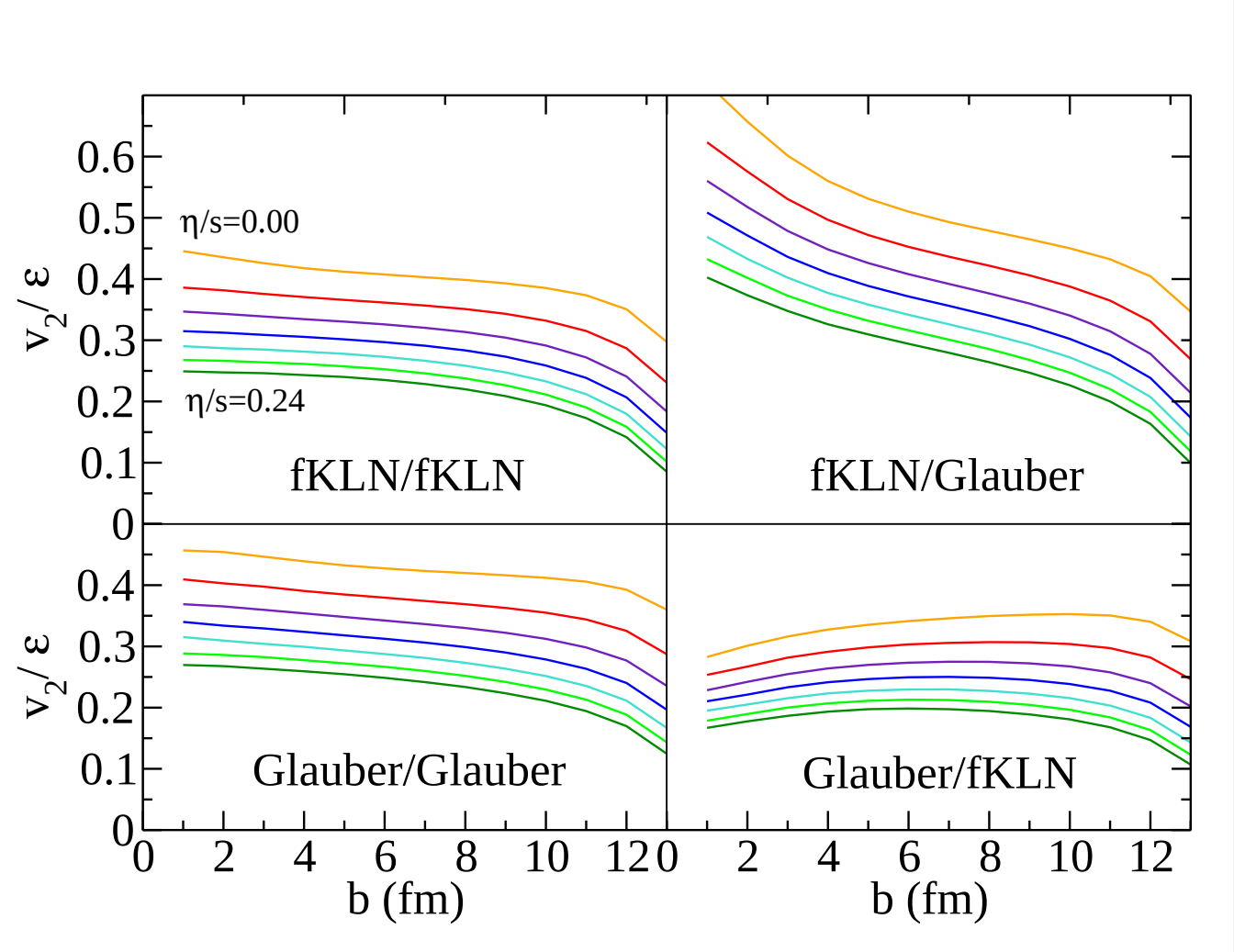



**Numerical classical Yang-Mills solutions close to binary collision profile = the theory prejudice at high energies.**

$$[D_\mu, F^{\mu\nu}] = j^\nu$$

# centrality dependence can help distinguish between eccentricity scenarios

Heinz et al, PRC80 ('09)




(Glauber = 85% wounded + 15% binary)

## How accurate is hydro?

- **hydrodynamics is an approximation, a truncation scheme** $\rightarrow T^{\mu\nu}$, $N^{\mu}$

- **validity and accuracy (beyond thumb rules)?**

## requires a truly nonequilibrium framework

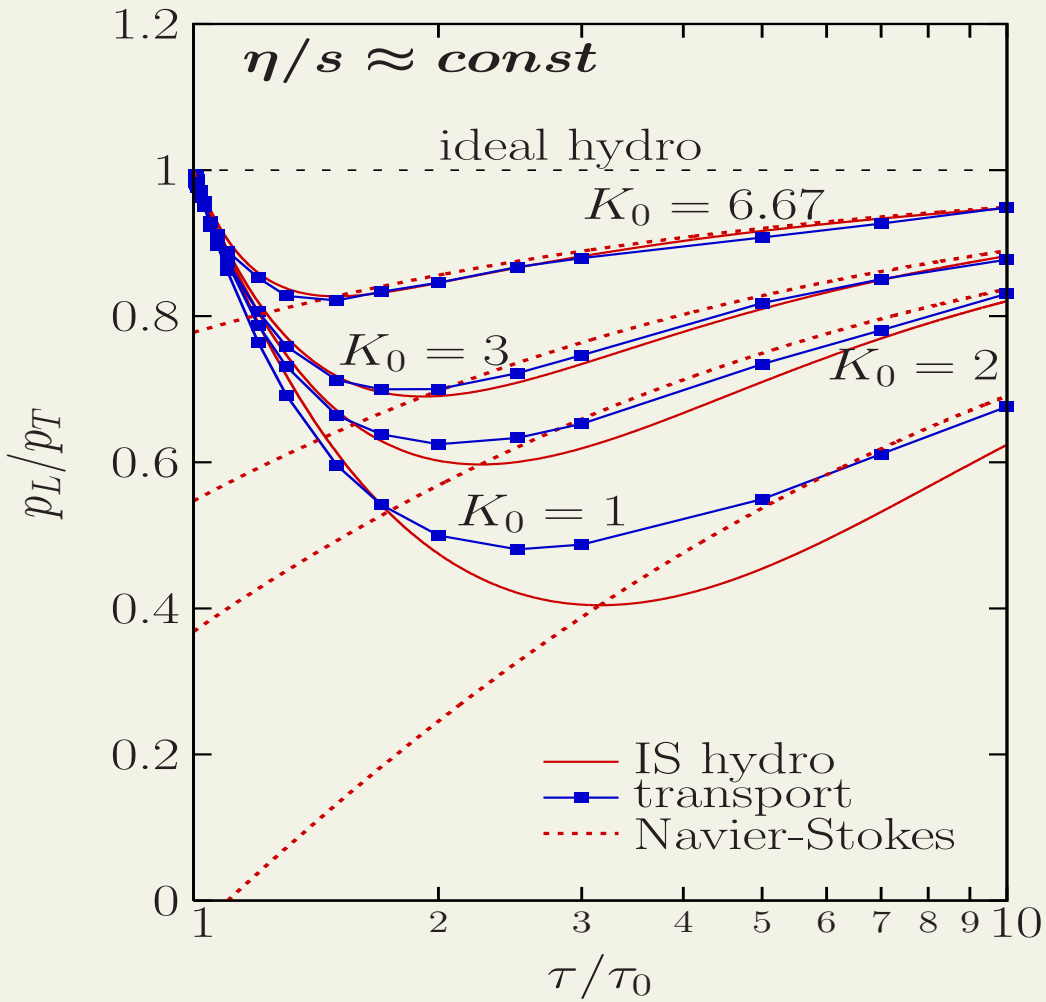
$\rightarrow$ **compare to covariant transport (has a hydro limit)**

# Validity of dissipative hydro

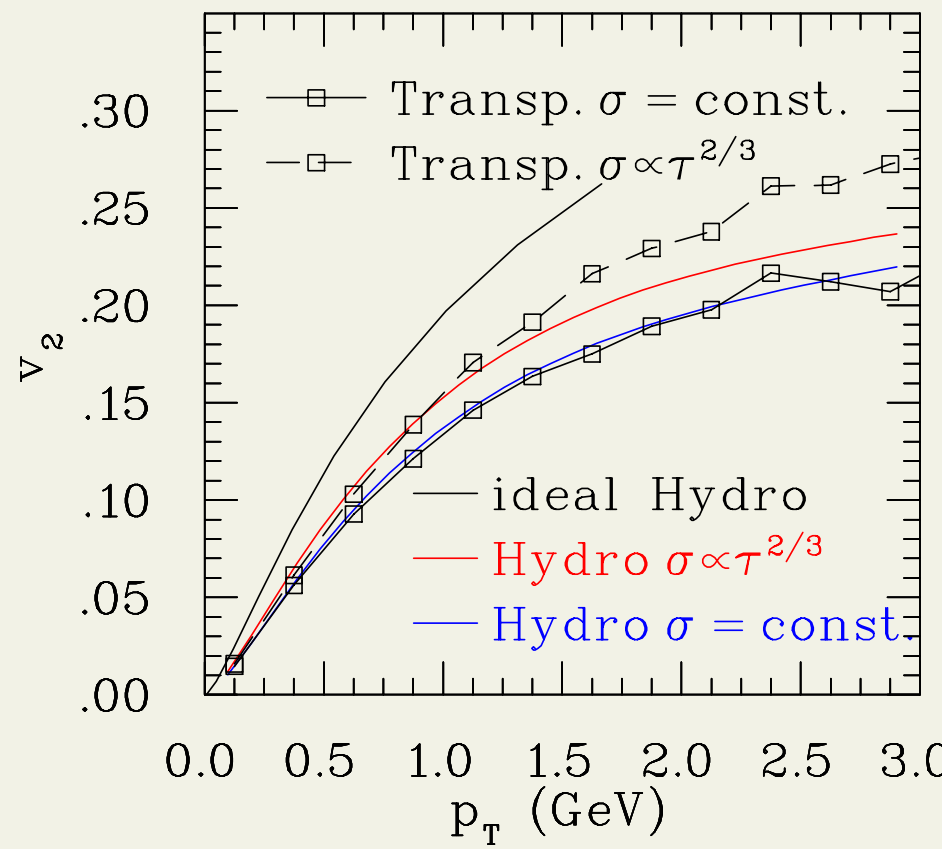
**Israel-Stewart hydro accurate within 10% at RHIC when** Huovinen & DM PRC79 ('08)

$$\frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \quad \Leftrightarrow \quad K_0 \gtrsim 2-3$$

**pressure anisotropy (0+1D Bjorken)**

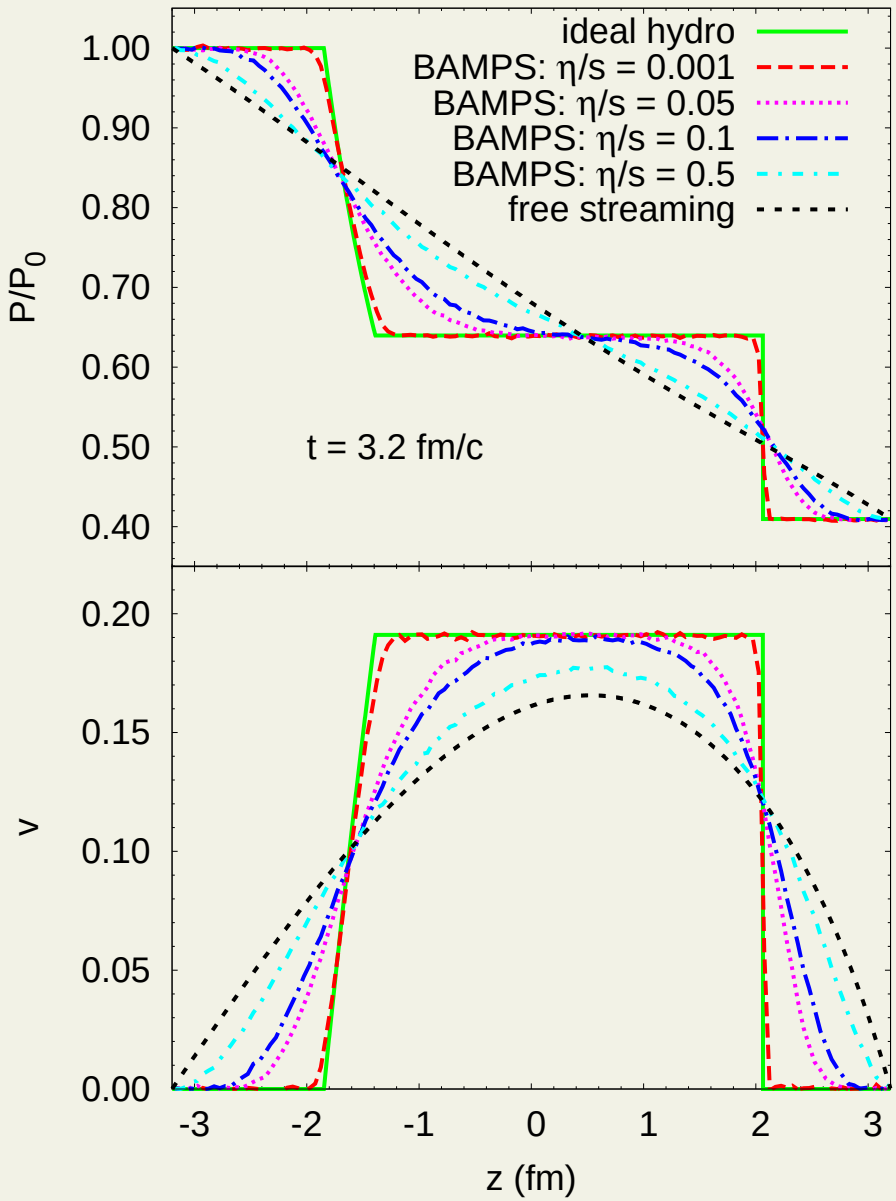



**$v_2$ in Au+Au ($b = 8$ fm)**




$\eta/s \approx 1/(4\pi) \quad \sigma_{gg} \approx 50mb$

# Another test - shock in a box




**shock propagation in 1+1D (Riemann problem)**

Bouras et al, arXiv:0902.1927:

**transport converges to ideal hydro as** $\eta/s \to 0$

# From viscous hydro to particles

**heavy-ion applications in the end must match hydrodynamics to a particle description**

- **in local equilibrium - one-to-one mapping**

$$T^{\mu\nu}_{LR} = diag(e, p, p, p) \qquad \Leftrightarrow \qquad f_{eq} = \frac{g}{(2\pi)^3} e^{-p^\mu u_\mu / T}$$

- **near local equilibrium - few-to-many mapping**

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \delta T^{\mu\nu} \qquad \Leftrightarrow \qquad f = f_{eq} + \delta f$$

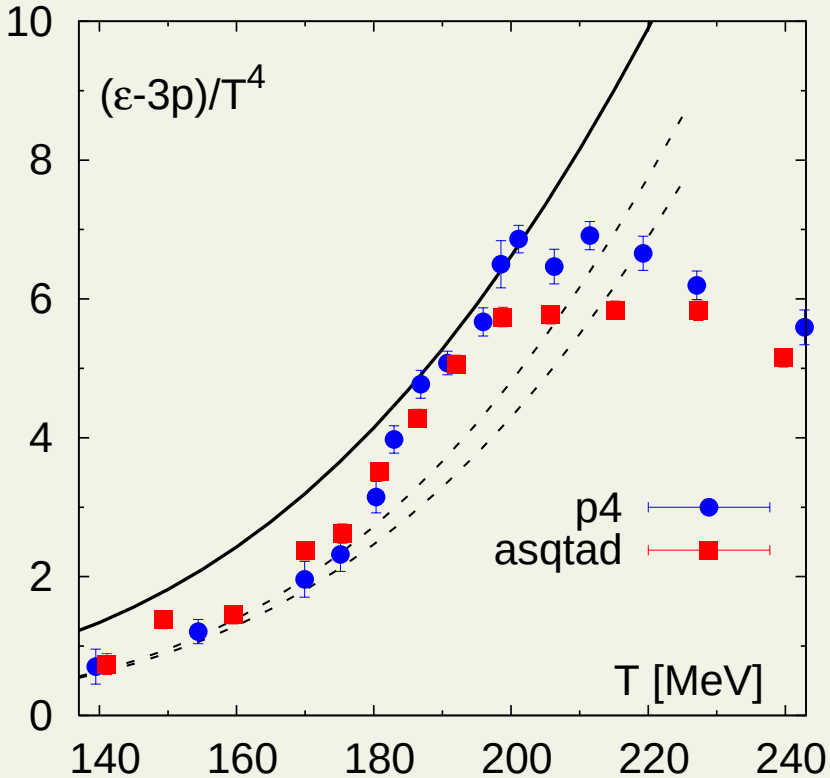
**correction $\delta f$ affects basic observables - spectra, elliptic flow $v_2(p_T)$, ...**

# Noninteracting "resonance gas" does seem to make sense

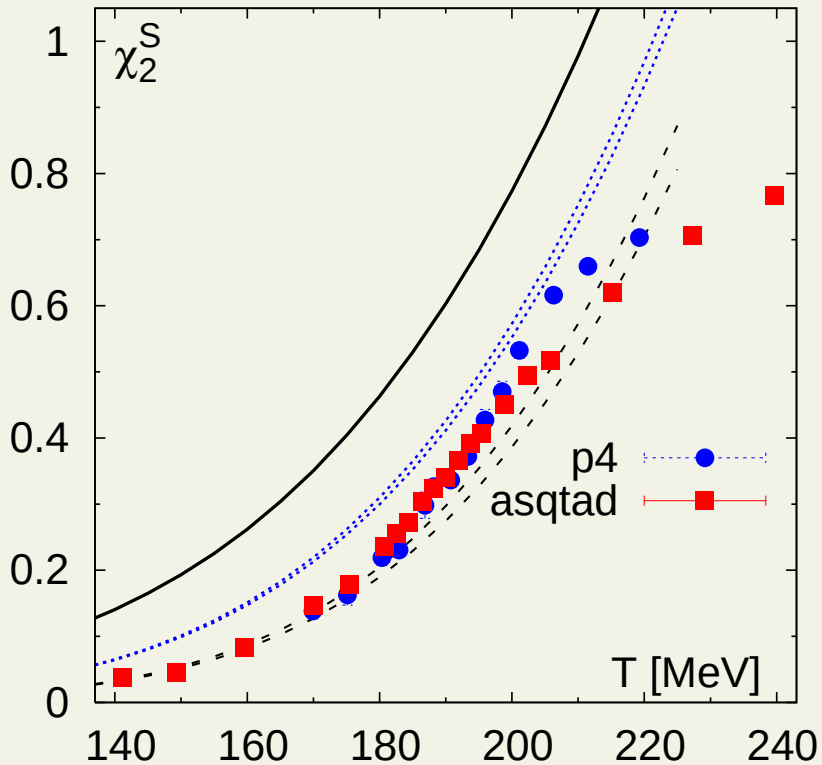
$$f_i(p) = \frac{g_i}{(2\pi)^3}\left[\exp[E_i(p) - \mu_i]/T \pm 1\right]^{-1}$$

**- agrees with lattice QCD (once lattice artifacts are accounted for)** Huovinen & Petreczky, NPA837 ('10)

**equation of state**




**strangeness fluctuations**



## Conversion to hadron gas - still work in progress:

- **need $\delta f_i$ for <u>each species</u> based on same local hydro fields**

- **so far, mostly "democratic" prescriptions - ignores interactions**

Romatschke et al... Monnai et al...

$$\delta f_i = f_{0,i} \frac{\pi^{\mu\nu} p_{\mu,i} p_{\nu,i}}{2T^2(e+p)}$$

**or linear response - driven by interactions** Teaney, Dusling, Moore ('09)

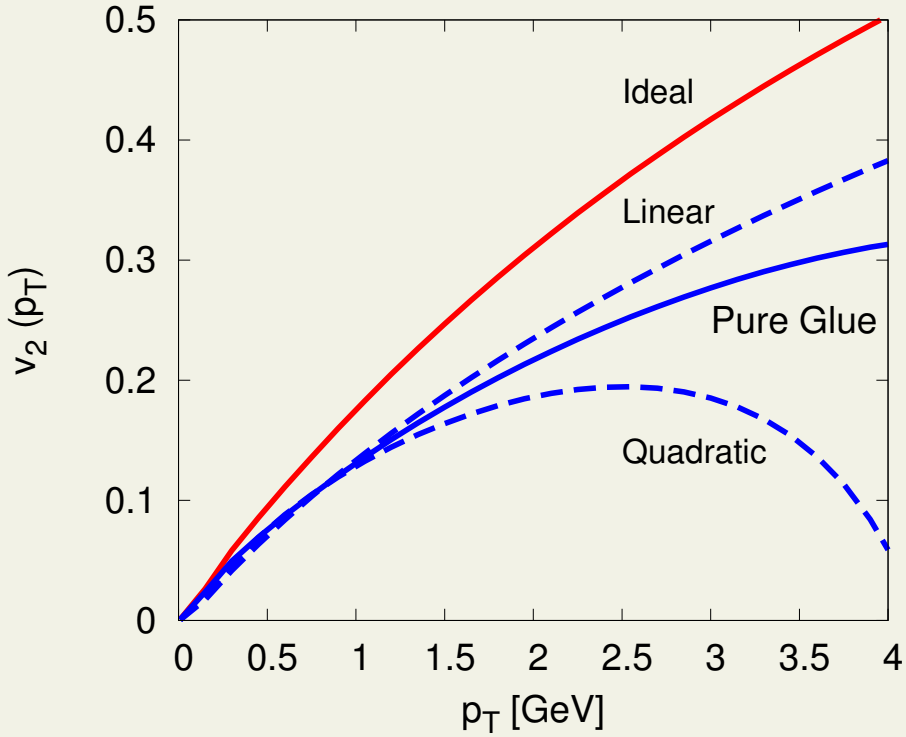
$$\delta f_i \equiv f_{eq,i} \times C(\chi)\, \pi^{\mu\nu} \frac{p_\mu p_\nu}{T^2} \chi_i(\frac{E}{T}) \ , \qquad \chi(x) \sim x^\alpha$$

- **neither ensures non-negative** $f = f_0 + \delta f \geq 0$

# Momentum dependence of $\delta f$ matters at high $p_T$ Teaney et al, ('09)

## Phenomenological Summary

pure glue, $\eta/s = 0.08$, $e_{\rm frz} = 0.6\,{\rm GeV/fm}^3$




pQCD is closer to a linear ($\tau_R =$ const) rather than a quadratic ansatz

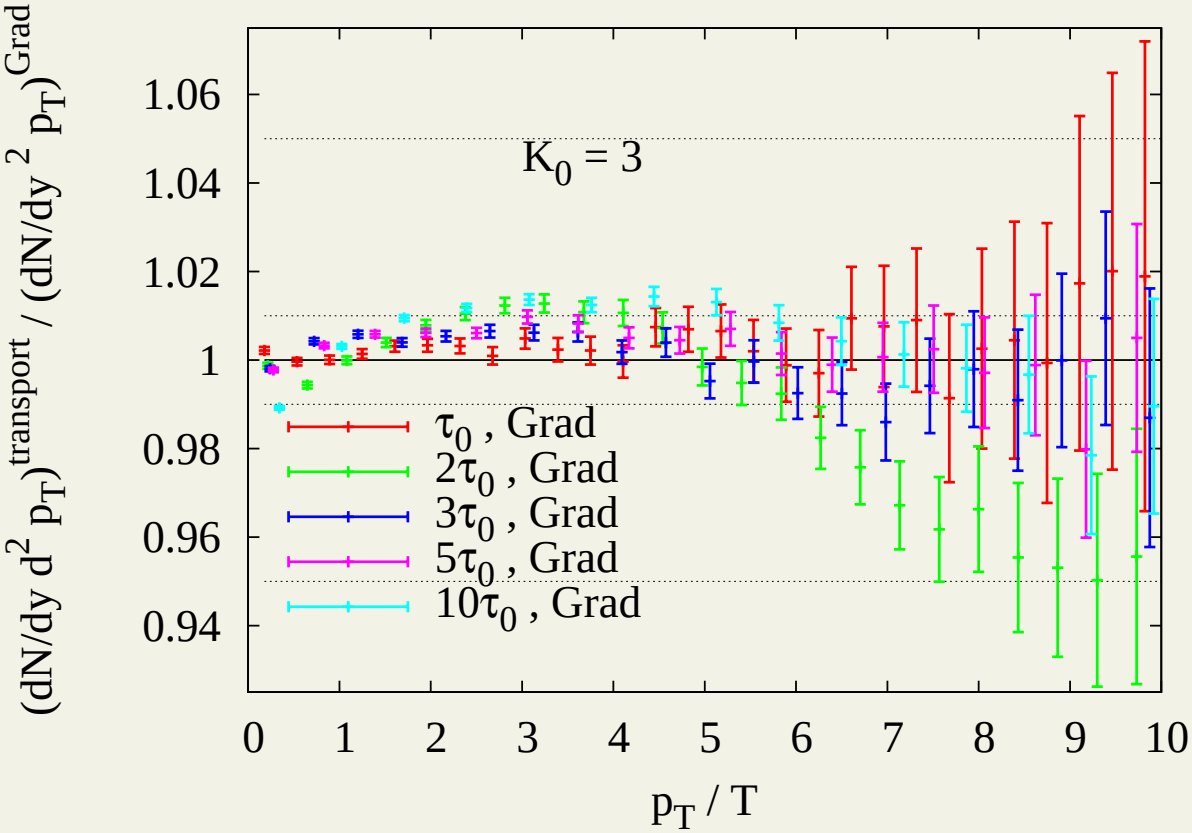
Teaney

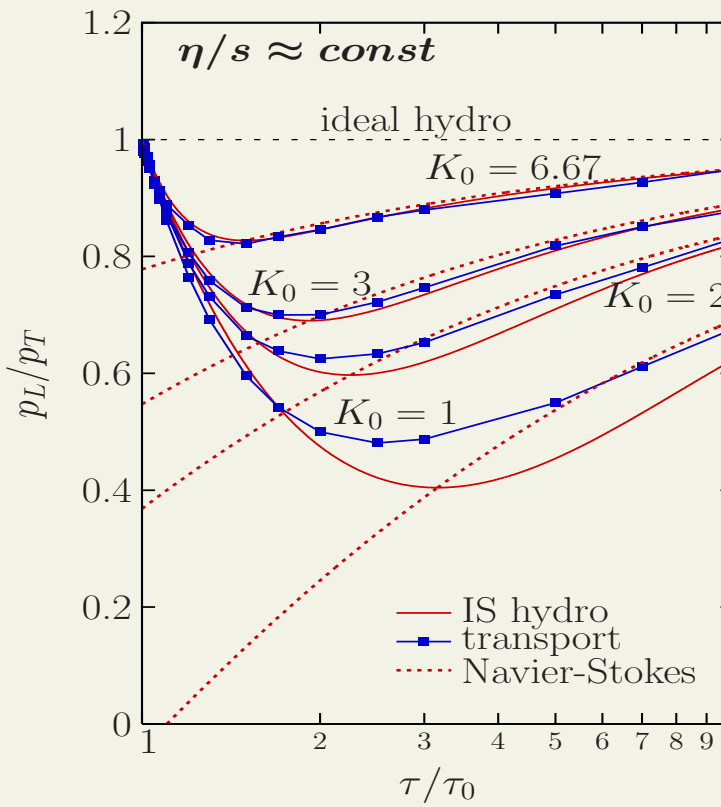
# $\delta f$ from full nonlinear transport

$2 \to 2$ **covariant transport (no linearization),** $f \geq 0$ **always**

**transport spectra / Grad approximation ratio -** $\eta/s \sim 0.1$**, 0+1D Bjorken**

DM ('09)

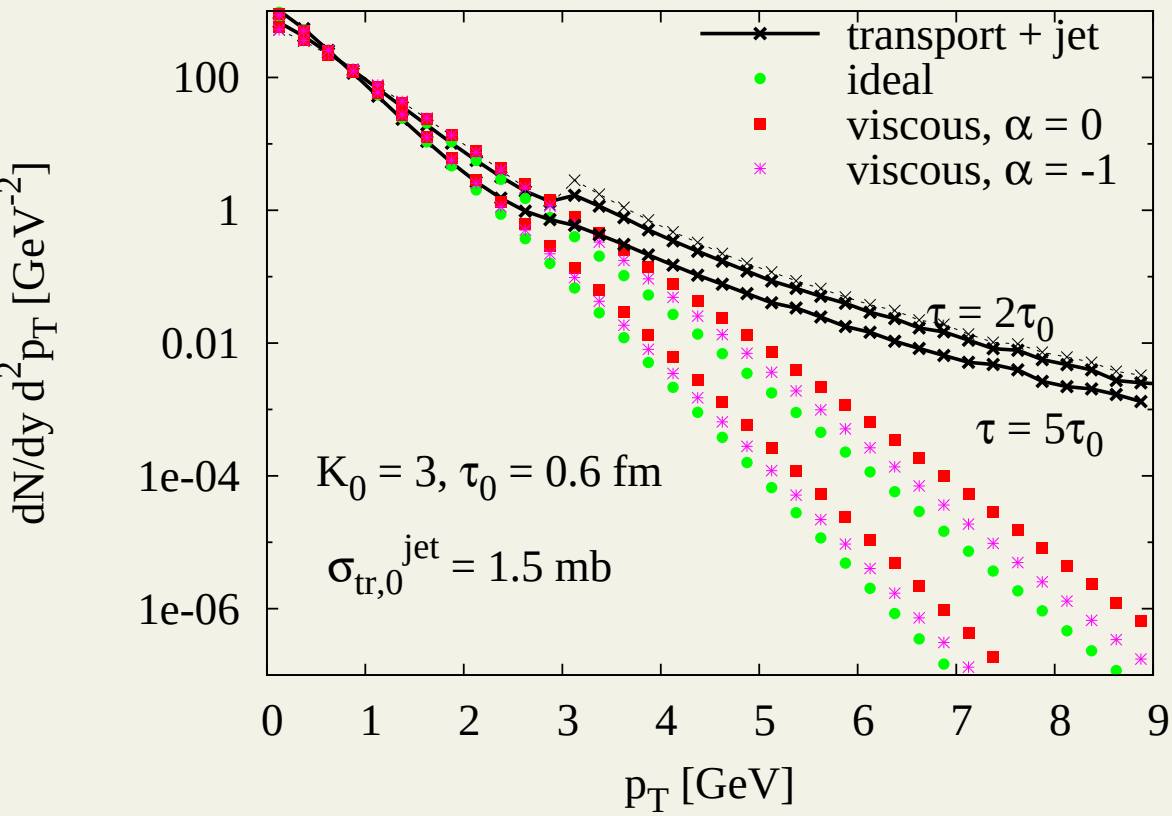


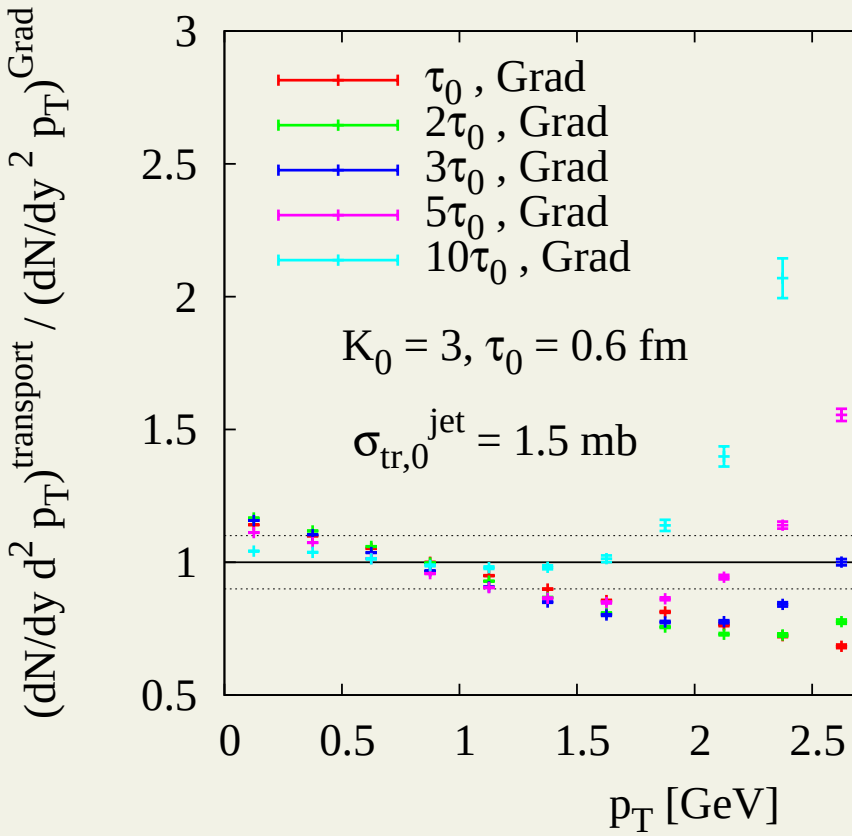




**Grad ansatz** ($\alpha = 0$) **works better than it should** $p_T/T \sim 6(!)$

**in Au+Au, however, high-pT tails do not thermalize, even for low $\eta/s \approx 0.1$**
**$\Rightarrow$ jets limit applicability of hydro**

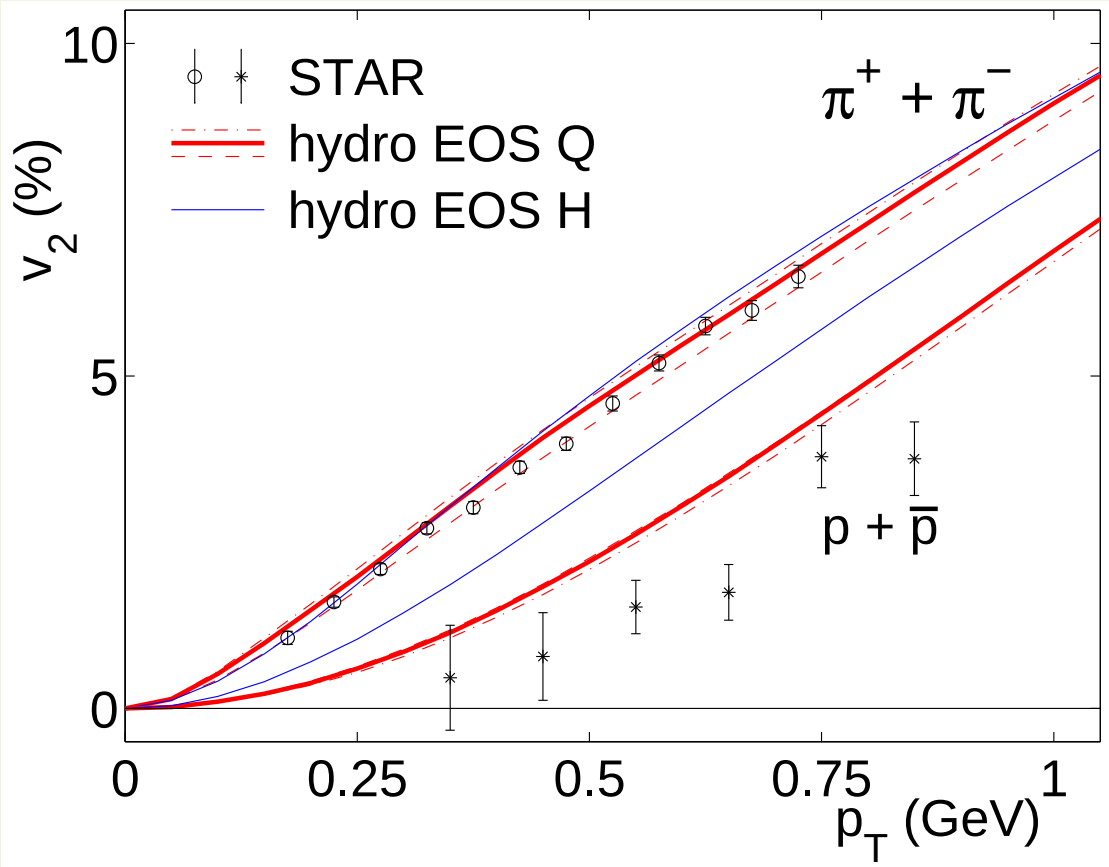
DM ('09):



# Should not ignore EOS

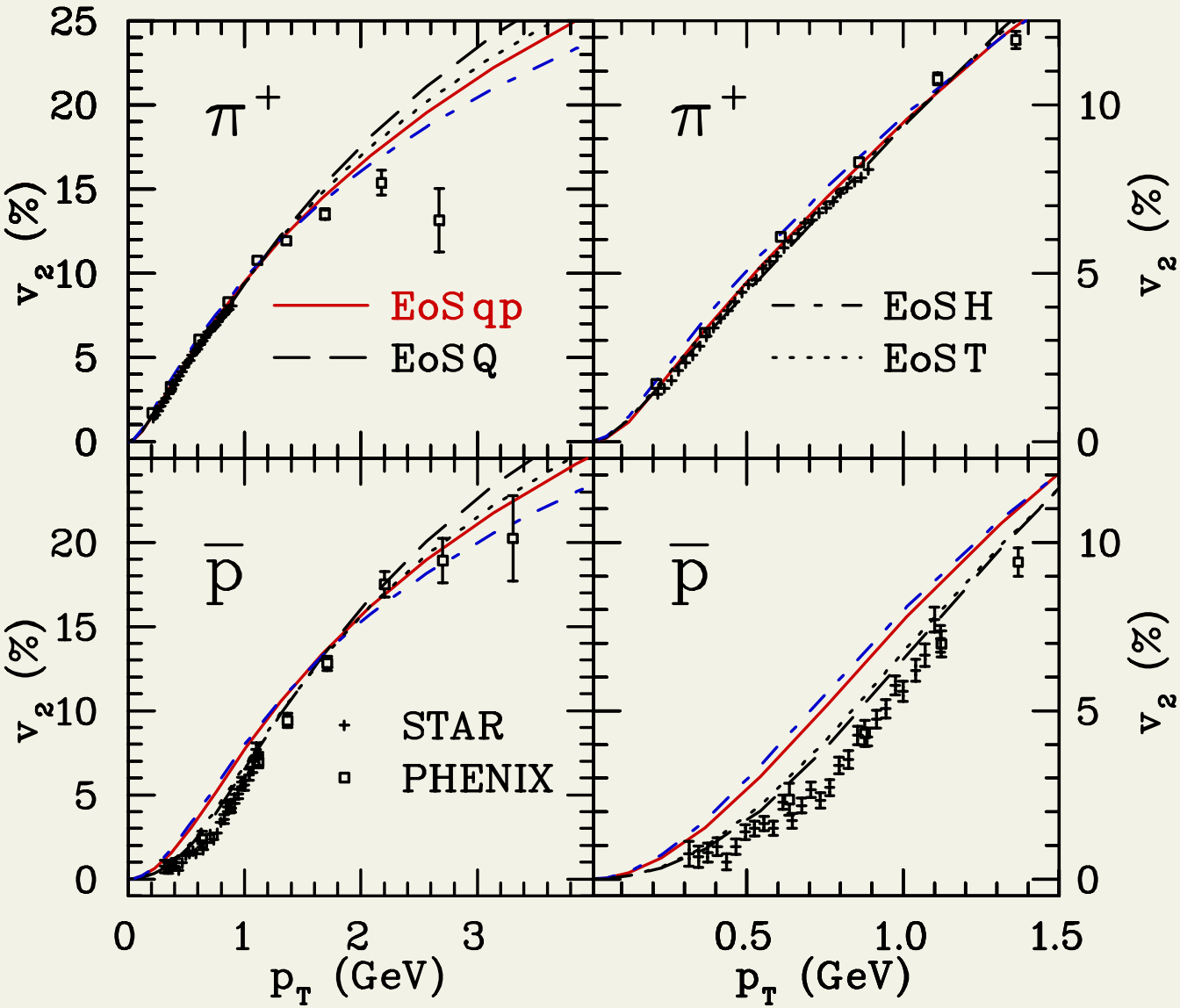
**2001 ideal hydro - minbias Au+Au at RHIC** Kolb, Heinz, Huovinen et al ('01), nucl-th/0305084




**EOS with 1st-order phase transition (Q) favored over hadron gas (H)**

# Ideal hydro + realistic EOS

**for realistic EOS proton $v_2$ same as for pure hadron gas?!**




Huovinen, NPA761, 296 ('05)

**Q: bag model**

**qp: lattice fit ($T_c = 170$ MeV)**

**H: hadron gas**

**T: interpolated $\varepsilon(T)$ between hadron gas and $\varepsilon \propto T^4$ plasma**

**MUST test particle species dependence from viscous hydro!**

# Multicomponent dissipative hydro - a mess: Denicol (Dec 2009)

## Closed Equations – Bulk and Shear

$$\frac{d\Pi_{(i)}}{d\tau} + \frac{\Pi_{(i)}}{\tau_{\Pi(i)}} + \sum_{j\neq i} \frac{\Pi_{(j)}}{\tau_{\Pi(i)(j)}} = -\left(\beta_{\zeta(i)} + \zeta_{\Pi\Pi(i)}\Pi_{(i)}\right)\theta + \zeta_{\Pi\pi(i)}\pi^{\mu\nu}_{(i)}\sigma_{\mu\nu}$$
$$-\zeta_{\Pi n(i)}\partial_\mu n^\mu - \alpha_{\Pi n(i)} n^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi n(i)} n^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$
$$-\zeta_{\Pi q(i)}\partial_\mu q^\mu - \alpha_{\Pi q(i)} q^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi q(i)} q^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$

$$\frac{d\pi^{\langle\mu\nu\rangle}_{(i)}}{d\tau} + \frac{\pi^{\mu\nu}_{(i)}}{\tau_{\pi(i)}} + \sum_{j\neq i} \frac{\pi^{\mu\nu}_{(j)}}{\tau_{\pi(i)(j)}} = 2\left(\beta_{\eta(i)} + \eta_{\pi\Pi(i)}\Pi_{(i)}\right)\sigma^{\mu\nu} - 2\eta_{\pi\pi(i)}\pi^{\langle\mu}_{\alpha(i)}\sigma^{\nu\rangle\alpha} + 2\pi^{\langle\mu}_{\alpha(i)}\omega^{\nu\rangle\alpha} - \left(\frac{1}{2} + \frac{7}{6}\eta_{\pi\pi(i)}\right)\pi^{\mu\nu}_{(i)}\theta$$
$$+2\eta_{\pi n(2)(i)}\nabla^{\langle\mu} n^{\nu\rangle}_{(i)} + 2\beta_{\pi n(i)} n^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi n(i)} n^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$
$$+2\eta_{\pi q(2)(i)}\nabla^{\langle\mu} q^{\nu\rangle}_{(i)} + 2\beta_{\pi q(i)} q^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi q(i)} q^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$

Eqs. will depend on $\Pi_{(i)}, q^\mu_{(i)}, \pi^{\mu\nu}_{(i)}, \nu^\mu_{(i)}$ **MORE variables!**

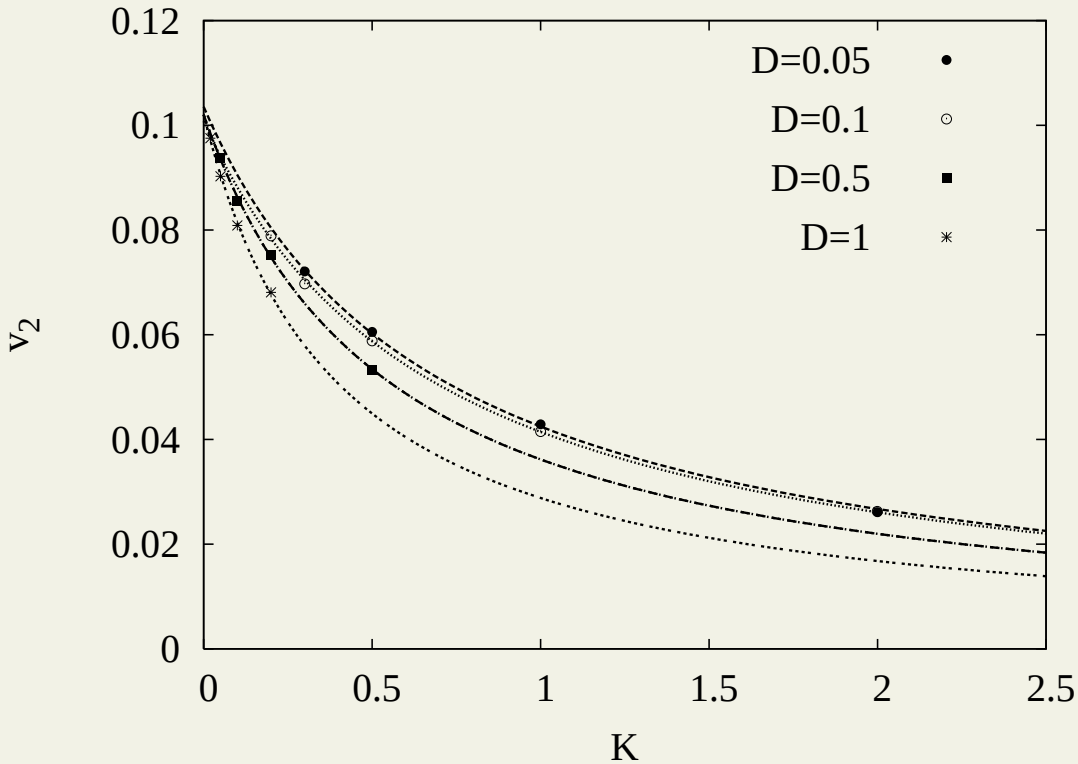
$\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu$ (possible?)

G. Denicol

Tech-qm

# Knudsen fits

Gombeaud & Ollitrault, nucl-th/0702075: $v_2(K \propto \lambda_{MFP}/R)$




$$v_2(K) = \frac{v_2^{max}}{1 + K/K_0}$$

**not too informative - in a realistic scenario, the (local) Knudsen number varies with coordinate and time**

**e.g.,** $\eta/s = const \Leftrightarrow K \sim \tau^{-2/3}$ **during longitudinal 0+1D expansion stage**

**(also** $K_0$ **has been computed for a static, 2D world)**

**Much more useful to interpolate for real dynamical models**

**e.g., take a viscous hydro and fit:**

$$v_2(b, \eta/s) = \frac{v_2^{max}(b)}{1 + const(b) \times \eta/s}$$

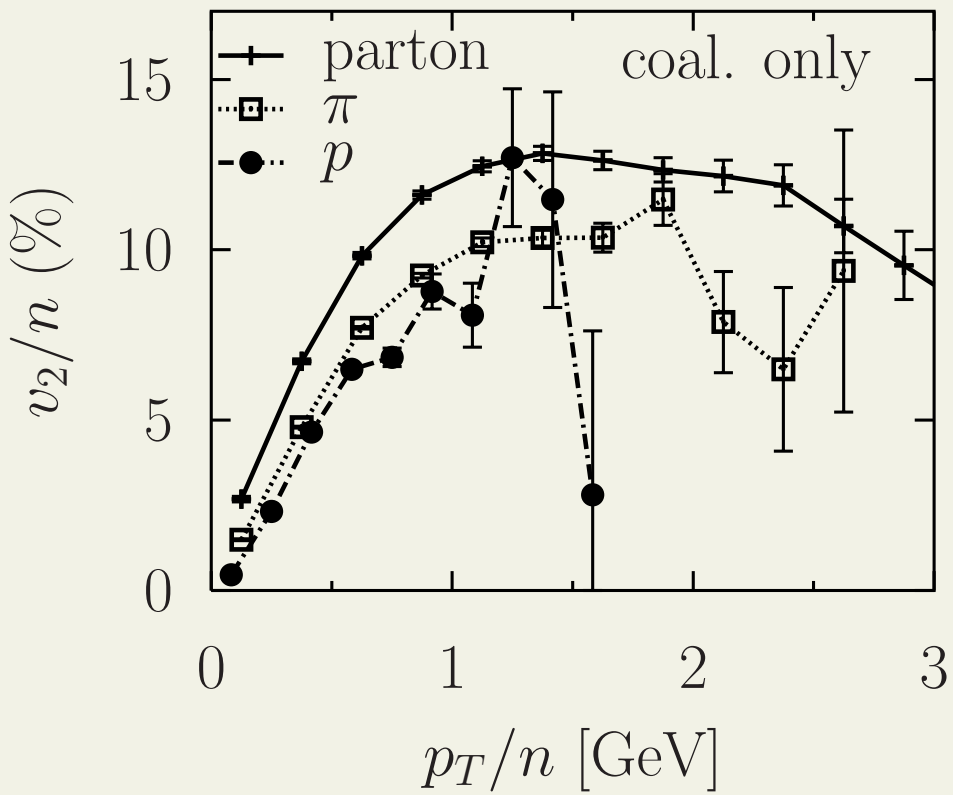
**unless you want to rely on kinetic theory...**

# On quark number scaling

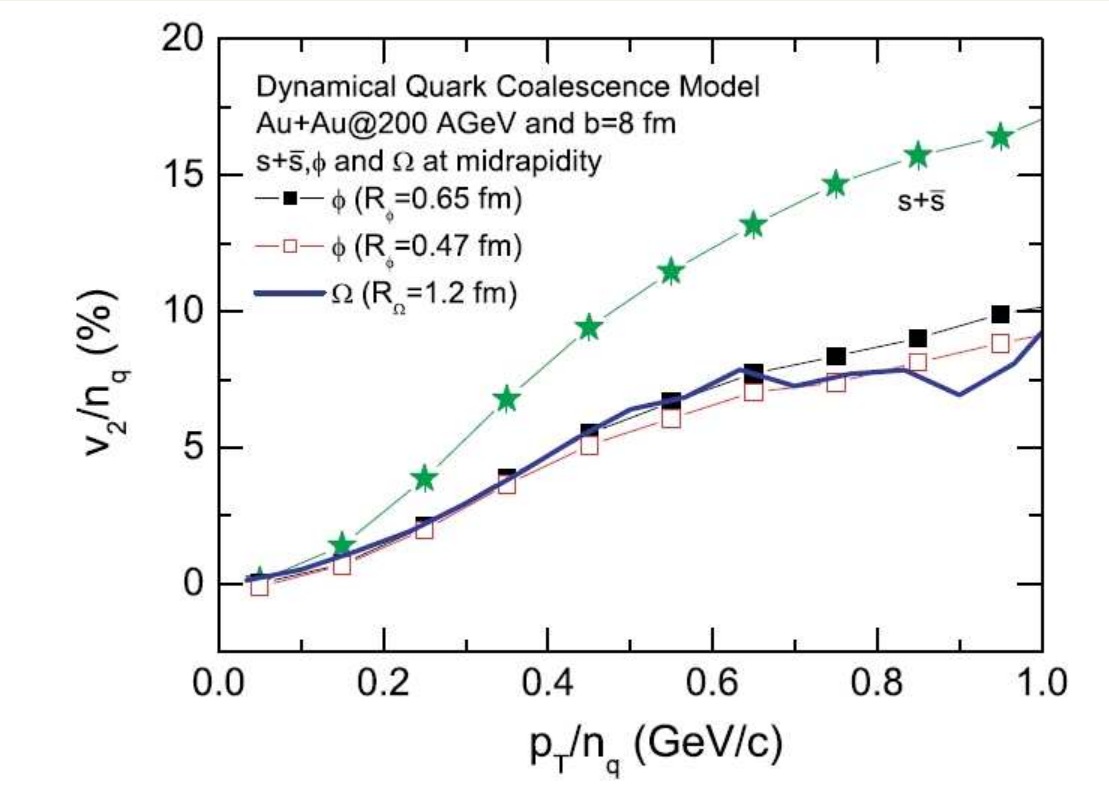
**quark number scaling implies nonequilibrium phenomena** DM, nucl-th/0408044

**but simple scaling formulas do <u>not</u> arise from a dynamical approach**

DM, nucl-th/0406066




Chen & Ko, PRC73 ('06)



# Summary

**We have come a long way during the past few years:**

- **we can solve relativistic dissipative hydro (2+1D)**
- **we can solve covariant transport near its hydro limit (full 3D)**
- **hydro applicability at RHIC looks good (from transport) for** $\eta/s \lesssim 0.2$
- **support for resonance gas model based on lattice QCD**
- **based on dissipative hydro + Cooper-Frye ansatz:** $\eta/s \sim 0.1 - 0.2$ **at RHIC**

  $\eta/s \gtrsim 0.5$ **very hard to accommodate in either transport or hydro**

**Still more work needs to be done:**

- **dissipative hydrodynamics of particle mixtures - identified particles(!)**
- **hadronic afterburner lacking**
- **initial conditions, fluctuations - size, centrality, and** $\sqrt{s}$ **systematics**