# RESUMMATION & $k_T$

George Sterman
BNL
MAY 11, 04
Werner Vogelsang

- Explicit & implicit resummation
- $k_T$ and joint resums
- Power corrections
- E dependence of the data
- Avoiding kinematic singularities
- E-dependence and $k_T$
  - threshold, $x_T \to 1$
  - low $x_T \to 0$
- Conclusions

## ⋆ Why Resum?

Every final state in hard scattering carries the imprint of QCD dynamics from at all distance scales

- **Phenomenological**
  - **Logarithmic corrections: explicit**

$$\frac{d\sigma(Q)}{dQ_1} \propto \frac{1}{Q_1} \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad \Lambda \ll Q_1 \ll Q$$

⋆ Z, H $p_T$, $e^+e^-$ event shapes, BFKL

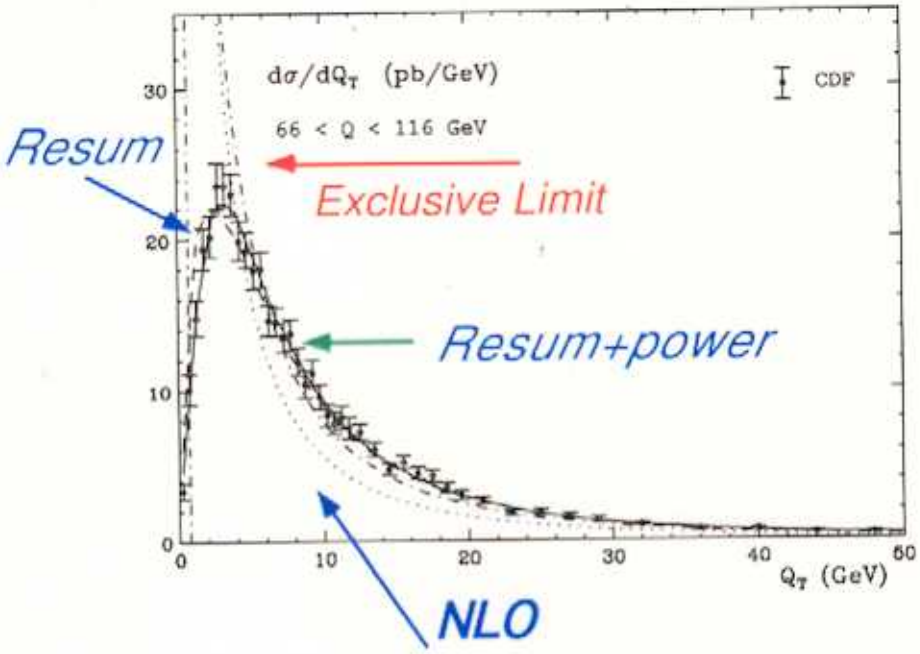
  - **Logarithmic corrections: implicit**

$$\sigma(Q) \propto \int \frac{dQ_1}{Q_1} F(Q_1) \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad F(0) = 0$$

⋆ Threshold resummations, 1PI high-$p_T$

- **Theoretical**
  - QED analog: soft photon radiation
    $\leftrightarrow$ correspondence to classical fields
  - Exploration of gauge theory
    - ∗ all orders predictions; strong coupling
    - ∗ guide to nonperturbative dynamics

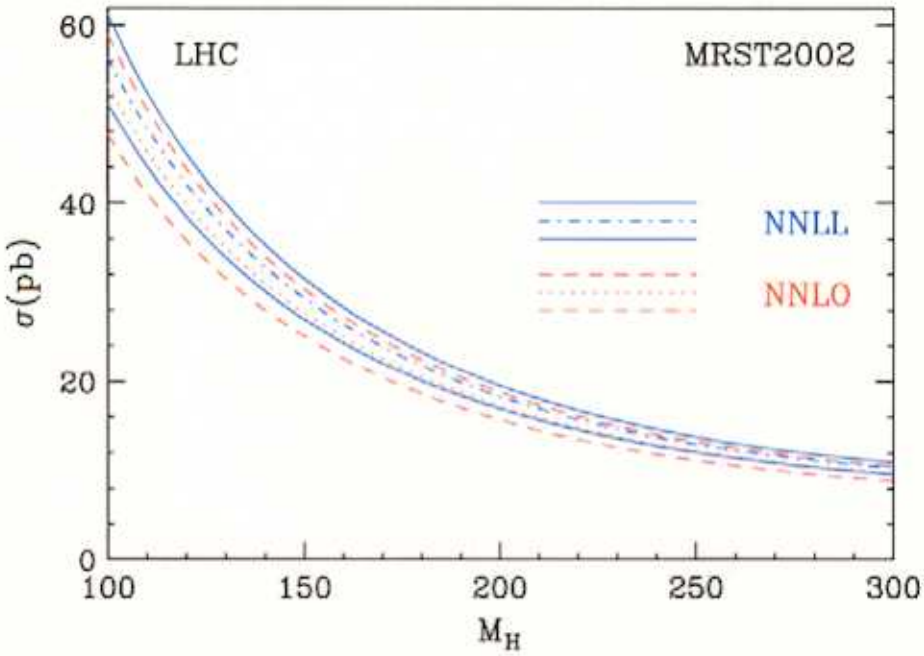
- **Explicit logs: Z $p_T$ at Run 1**




(from Kulesza, G.S., Vogelsang (2002))

- maximum then decrease near "exclusive" limit (parton model kinematics) replaces divergence
- Soft but perturbative radiation broadens distribution
- Typically NP correction necessary for quantitative description of data
- recover fixed order away from exclusive limit

- **Implicit logs: threshold resummation vs. fixed order for H at LHC**




(from Catani, de Florian, Grazzini, Nason (2003))

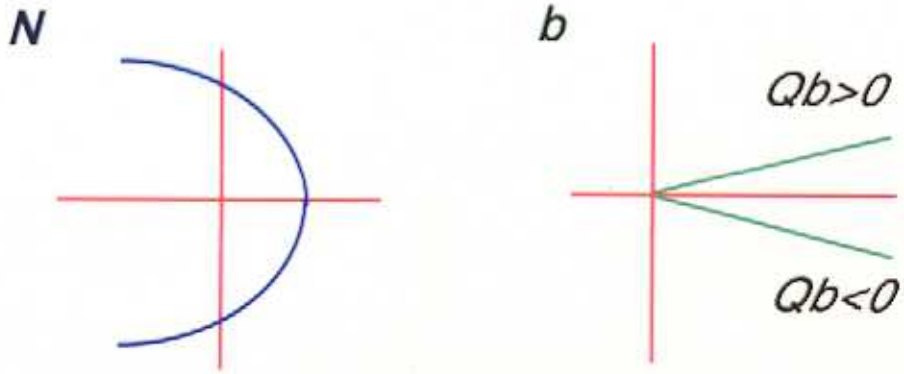
  - Modest change ↔ increased confidence
  - Modest decrease in scale dependence

## $\star$ $k_T$-Resummation

- Example: Joint Resummation for Z $Q_T$
- Double inverse transform

$$\frac{d\sigma^{\rm res}}{dQ^2 dQ_T^2} \sim \sigma_0^{a\bar{a}\to Z} \int_C dN\, \tau^{-N} \int_{C_b} d^2b\, e^{i\vec{Q}_T\cdot\vec{b}} \times \mathcal{C}_a(Q,b,N)\, e^{E_{a\bar{a}}(N,b,Q)}\, \mathcal{C}_{\bar{a}}(Q,b,N)$$

$$C_a(Q,b,N,\mu) = \sum_j C_{a/j}(N,\alpha_s(\mu))\, f_j(N, Q/(N+bQ))$$




- Perturbative exponent

$$E_{a\bar{a}}(N,b,Q) \sim \int_{[Q/(N+bQ)]^2}^{\mu^2} \frac{dm^2}{m^2} A_a(\alpha_s(m))\, \ln\left(\frac{Nm}{Q}\right)$$

- Incorporates energy and $p_T$-conservation $\rightarrow$ Average $p_T^2$ function of $\ln(1-x)Q$.

## * EXPONENT IN JOINT RESUM.
(closer look)

$$E_{ab}(N, b, Q)$$

$$= \int \frac{d^2 k_T}{(2\pi)^2} \frac{1}{k_T^2} \sum_{i=a,b} A(\alpha_s(k_T^2)) \times$$

(Laenen 01)

$$\times \left[ e^{-i \underline{b} \cdot \underline{k}_T} K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{Nk_T}{Q}\right) \right]$$

$$A_i(\alpha_s(k_T^2)) = C_i \frac{\alpha_s(k_T^2)}{\pi} + \dots$$

## * POWER CORRECTIONS

FROM $k_T^2 \to 0$

EVEN POWERS of $b$, $\frac{N}{Q}$

Identify NP parameters

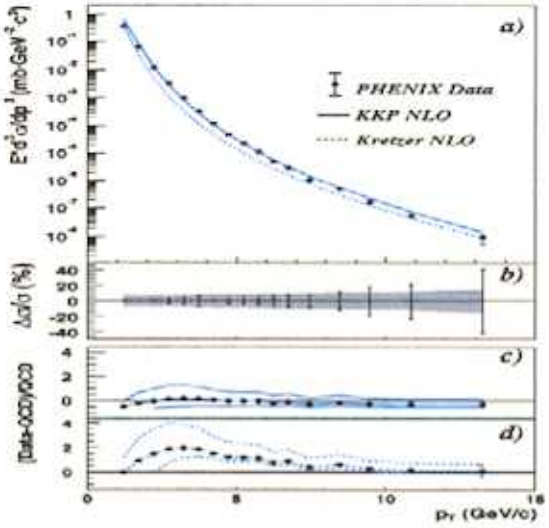
$$g_b \leftrightarrow \int_0^{\mu_{IR}} dk_T \, k_T \, A(\alpha_s(k_T^2)) \ln^b \frac{k_T}{\mu_{IR}}$$

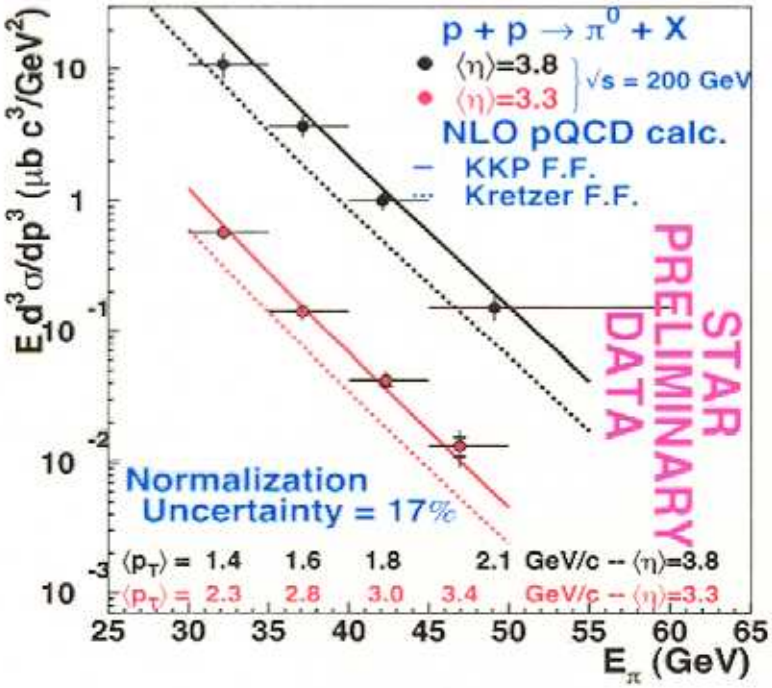
$$\Rightarrow e^{-\left(b^2 + \frac{2N^2}{Q^2}\right) g_1 \ln \frac{Q}{N} + \dots}$$

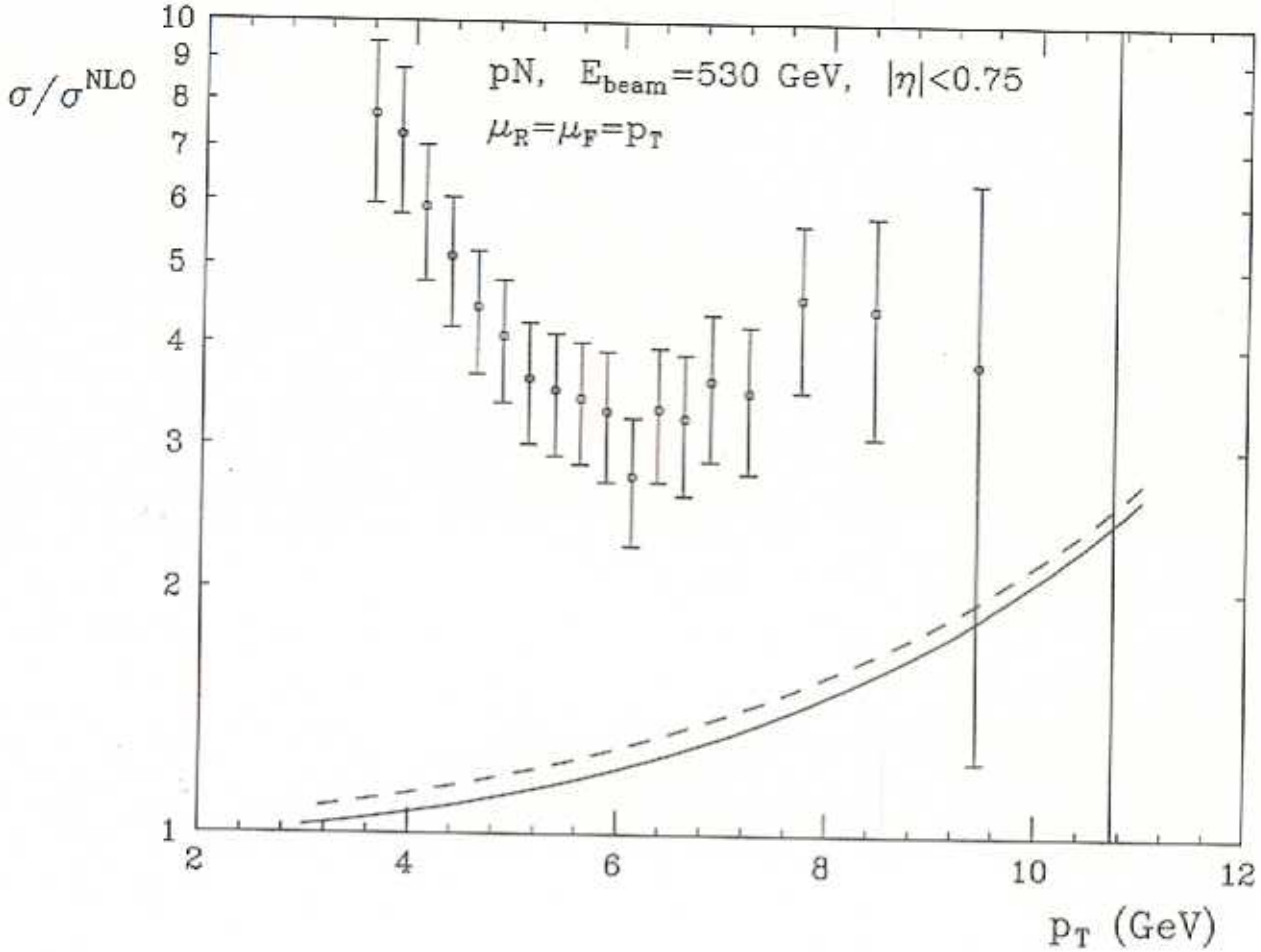
"shape function"

IMPLICIT LOGS

## * Particle Spectra: Fixed Target vs. Collider






Catani...
Vogelsang 99

- **Self-consistent recoil in Joint Resummation**

Laenen, GS, Vogelsang

- **Double inverse transform and approximation:**

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \int_{-\infty}^{\infty} d^2b \, d^2Q_T \, \mathrm{e}^{i\,\vec{Q}_T\cdot\vec{b}}$$

$$\times \; \tilde{\sigma}_{ab}^{(0)}(N) \, \mathrm{e}^{E(N,b,p_T)}$$

$$\times \quad \underbrace{\left(\frac{S}{4(\vec{p}_T - \frac{1}{2}\vec{Q}_T)^2}\right)^{N+1}}_{=\left(x_T^2\right)^{-N-1} \, \mathrm{e}^{N\vec{Q}_T\cdot\vec{p}_T/p_T^2}\left(1+\mathcal{O}(1/N,Q_T^2/p_T^2)\right)}$$

- **$Q_T$, $b$ integrals ($N$ imaginary) $\Rightarrow$**

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \, \tilde{\sigma}_{ab}^{(0)}(N) \, \left(x_T^2\right)^{-N-1}$$

$$\mathrm{e}^{E\mathrm{thr}(N,p_T)} \, \mathrm{e}^{\delta\, E\mathrm{recoil}(N,p_T)}$$

$$\delta\, E_{\mathrm{recoil(N,p_T)}} = \pi \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i\left(\alpha_s(k_T^2)\right)$$

$$\left[\left(I_0\left(\frac{Nk_T}{p_T}\right) - 1\right) K_0\left(\frac{Nk_T}{p_T}\right)\right]$$

- Isolate perturbative recoil; N NLL in $N$:

$$\delta\, E_{\rm recoil}(N, p_T) = \delta E_{\rm PT} + \delta\, E_{\rm np}$$
$$\delta E_{\rm PT} \propto \frac{\alpha_s(p_T^2/N^2)}{\pi}\,\frac{\zeta(2)}{2}$$

- isolate low scales $\leftrightarrow$ strong coupling

$$\delta\, E_{\rm recoil} \;=\; {\rm PT} \;+\; \lambda_{\rm ab}\,\frac{{\rm N}^2}{{\rm p}_{\rm T}^2}\,\ln\frac{{\rm p}_{\rm T}}{\rm N}$$

$$\lambda_{ab} \sim\; 2g_{\rm EW} \;\propto \int dk_T^2\,\alpha_s(k_T^2)$$

$$N \;\leftrightarrow\; \frac{1}{\ln x_T^2}$$

$$\delta\, E_{\rm recoil} \;=\; {\rm PT} \;+\; \lambda_{\rm ab}\,\frac{1}{{\rm p}_{\rm T}^2\ln^2\left(\frac{4{\rm p}_{\rm T}^2}{\rm S}\right)}\,\ln\left({\rm p}_{\rm T}\ln\left(\frac{4{\rm p}_{\rm T}^2}{\rm S}\right)\right)$$

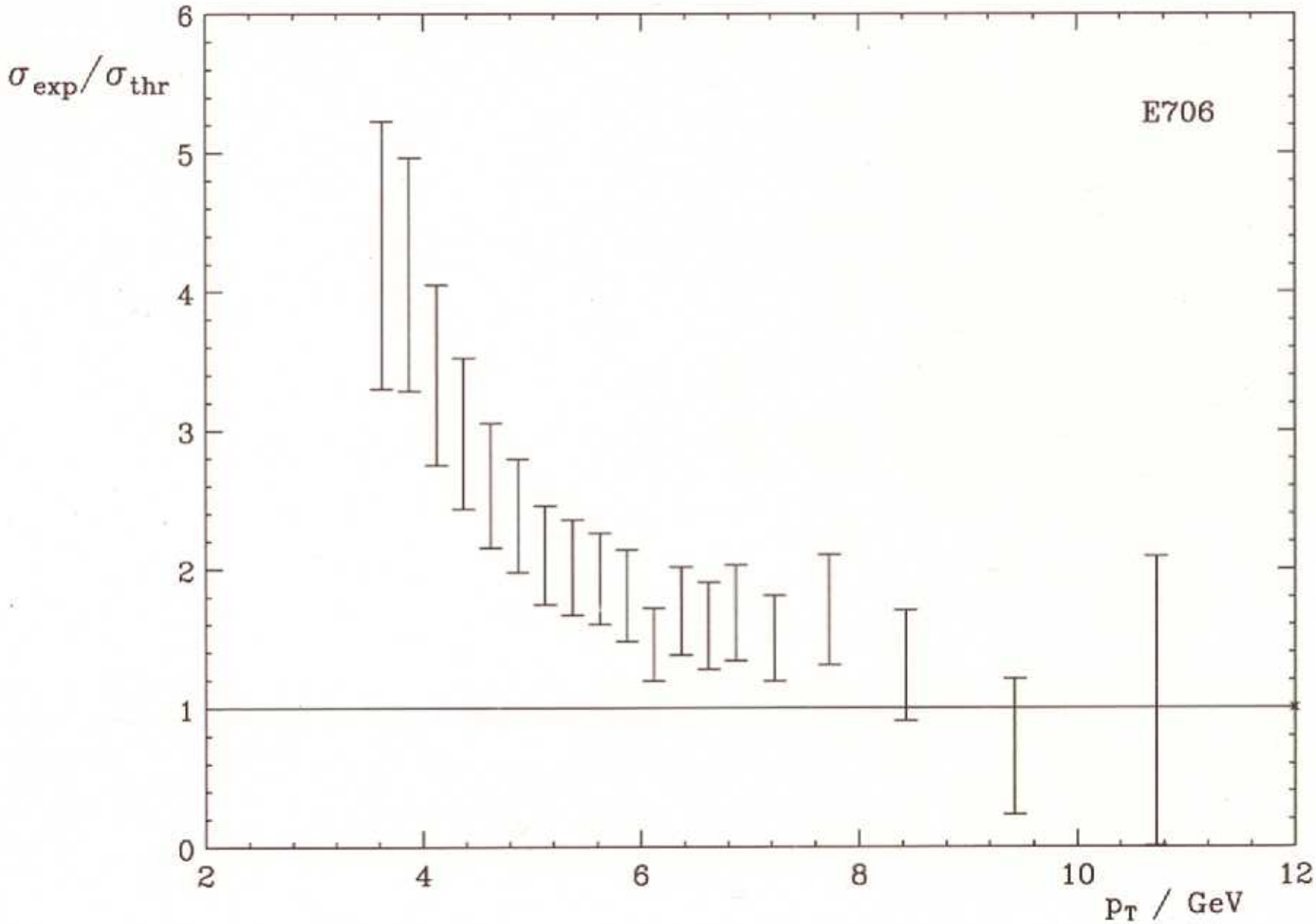
- **power suppressed in $p_T$**
- **decreases with $S$ at fixed $p_T$**
- **match to large- and small-$N$ behavior of Bessel functions $\rightarrow$**

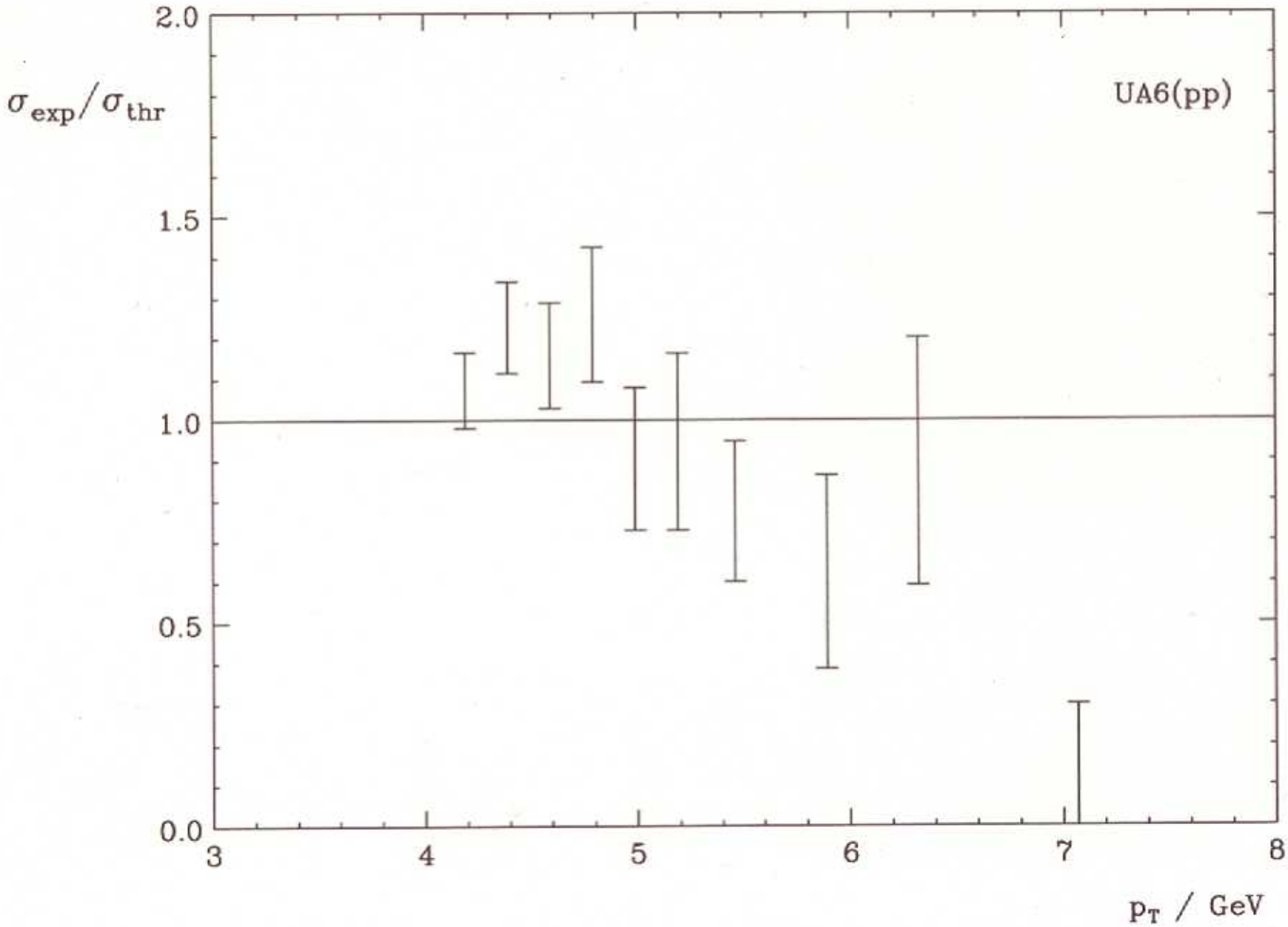
sample 'shape function' of $N/p_T$ only:

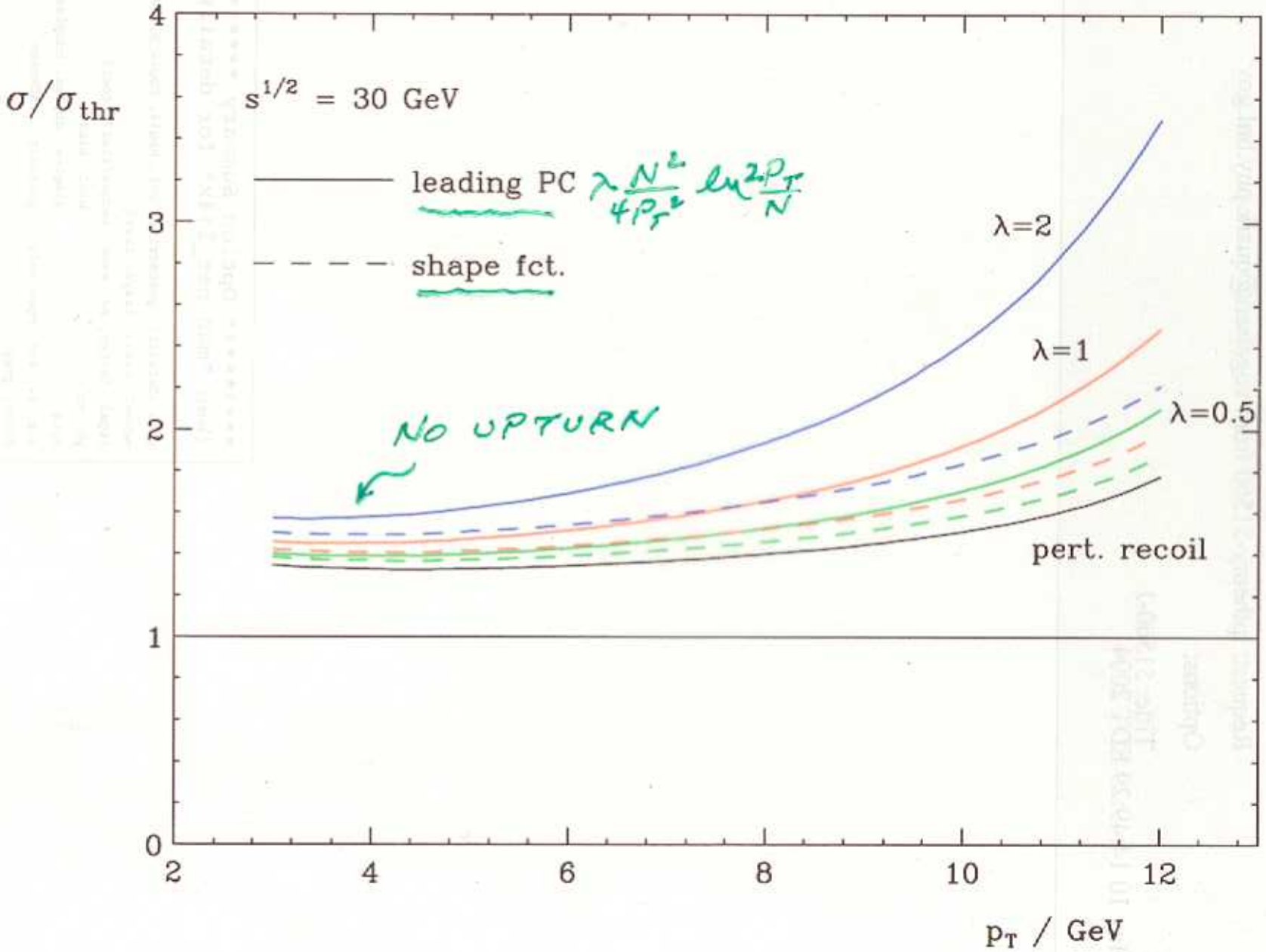
$$\delta E_{\rm np} = \frac{CN^2}{p_T^2}\,\frac{ln\left(1+\frac{2p_T}{N}\right)}{\left(1+\cancel{\frac{p_T}{N}}\right)^2}$$

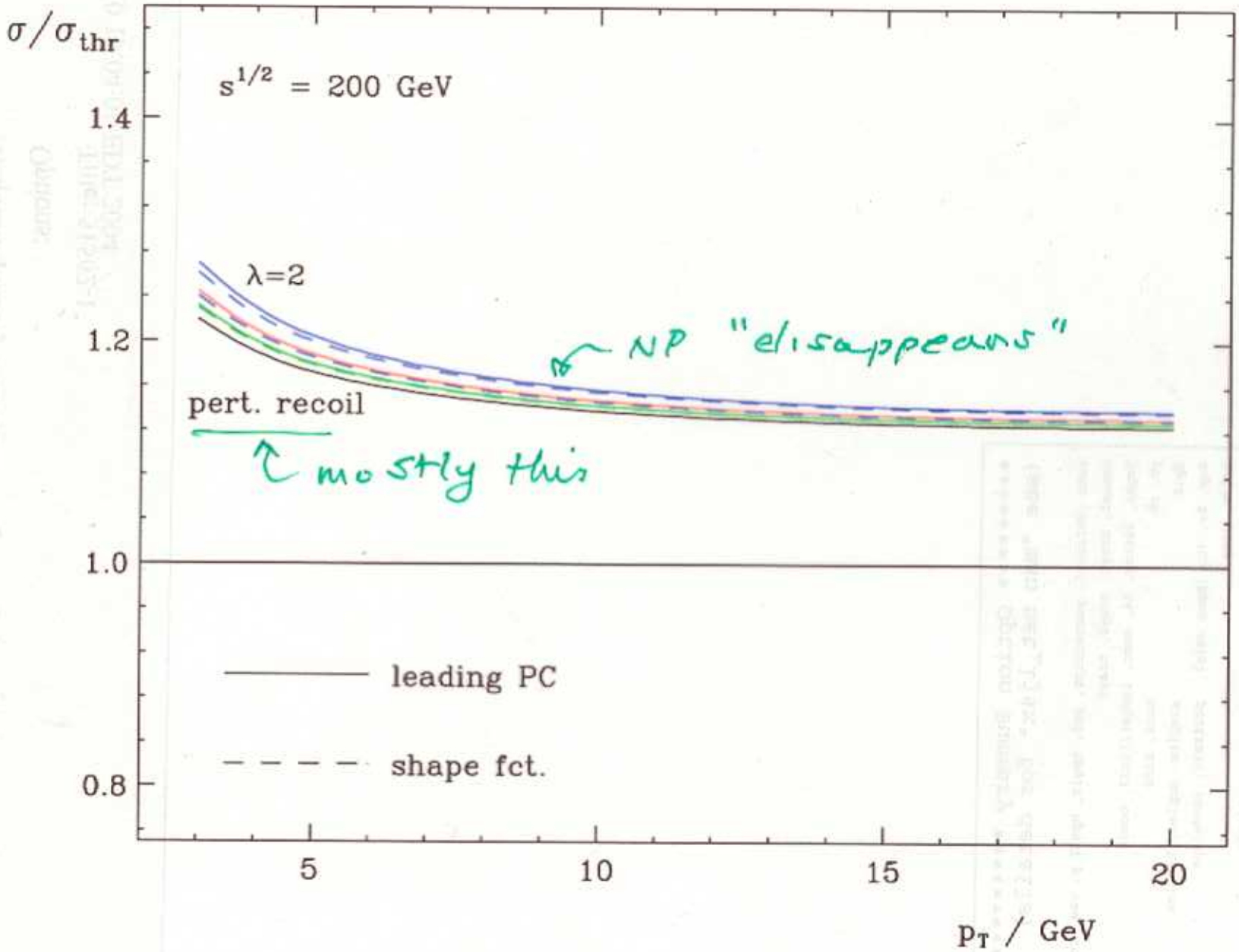
N/pT

($p_T$ in GeV)


$\sigma_{exp}/\sigma_{thr}$
E706
0
1
2
3
4
5
6
2
4
6
8
10
12
$p_T$ / GeV


2.0
1.5
1.0
0.5
0.0
$\sigma_{exp}/\sigma_{thr}$
UA6(pp)
3
4
5
6
7
8
$p_T$ / GeV


$\sigma/\sigma_{thr}$
$s^{1/2}$ = 30 GeV
leading PC $\lambda \frac{N^2}{4p_T^2} \ln^2 \frac{2p_T}{N}$
shape fct.
λ=2
λ=1
λ=0.5
NO UPTURN
pert. recoil
$p_T$ / GeV
0
1
2
3
4
2
4
6
8
10
12


σ/σthr
s^1/2 = 200 GeV
1.4
1.2
1.0
0.8
λ=2
NP "disappears"
pert. recoil
mostly this
leading PC
shape fct.
5
10
15
20
pT / GeV

E-dependence for $x_T \to 0$

- $N^2/p_T^2 \xrightarrow[x_T \to 0]{} 1/p_T^2$ but coeff. not large

- Enhanced upturn? E706 How?

**Desperation ⇒**

- An illustration (c.f. low-x 'unintegrated pdfs' Catani, Hautmann et al)

"Low-x" profile

$$E_{low} = C\int_0^{Q^2} \frac{dk_T^2}{k_T^2}\left(e^{ib\cdot k_T} - 1\right)\frac{\alpha_s(k_T^2)}{N-1}$$

$P_{gg}(N)$

$$\sim \frac{C}{N-1}\left[-\ln Q^2 b^2 - b^2\int dk_T^2\,\alpha_s(k_T^2)\right]$$

$b \to iN/2p_T$ ← $Q$

$$E_{low} \sim \frac{C}{N-1}\left[\ln N^2 + \frac{N^2}{Q^2}\int_0 dk^2\,\alpha_s(k^2)\right]$$

$N-1 \longleftrightarrow \ln\frac{1}{x_T}$

conjugate

$$E_{low} \sim \frac{C\left(\ln\frac{1}{x_T}\right)}{4p_T^2}$$ a 'hint'?

# CONCLUSIONS

- Resum $\rightarrow$ P.C.'s $e^{-F(x_T)/p_T^2}$ through $k_T$

- Modest large-N ($x_T \to 1$) power corrections consistent w/ E706. No obvious demand elsewhere.

- Low-$p_T$ rise requires extension of threshold/$k_T$ treatment
  - a hint of $e^{(\ln 1/x_T)/p_T^2}$