Gauge Invariant Quark and Gluon Spin and Orbital Angular Momentum Distributions*

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Typically in (semi)-inclusive deep inelastic scattering the Bjorken variable $x_{\rm Bj}$ is fixed, and only forward hadron matrix elements are accessible. In order to accommodate these experimental conditions, we propose to describe a hadron in terms of those observables Γ that are diagonal in the basis formed by quark and gluon partons. We then construct gauge invariant $x_{\rm Bj}$-distributions associated with such an observable. The gauge covariant definition of parton states in fully interacting QCD are chosen so that the parton distributions are given by the physical structure functions $q(x_{\rm Bj})$ and $g(x_{\rm Bj})$. To satisfy these requirements, quark and gluon partons must be eigenstates of the generators of the covariant translations,

$$\begin{aligned} T^q_- &: \psi(x) \;\to U(x, x+a^-)\; \psi(x+a^-)\;, \\ T^g_- &: D_\lambda(x) \to U(x, x+a^-)\; D_\lambda(x+a^-)\; U(x+a^-, x)\;, \end{aligned} \tag{1}$$

and the operator of the observable Γ must commute with T^q_- and T^g_-.

A hadron angular momentum is completely described by four scale dependent $x_{\rm Bj}$-distributions. Two of them coincide with the polarized quark and gluon structure functions Δq and Δg. The other two,

$$f_{L_q}(x_{\rm Bj}) = \frac{\int {\rm d}\xi^- e^{ix_{\rm Bj}P^+\xi^-} \langle P| \int {\rm d}^2x^\perp \psi^\dagger_+(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)\psi_+(x^\perp + \xi^-)\, |P\rangle}{2\pi\sqrt{2}\,(\int {\rm d}^2x^\perp)}$$

and

$$f_{L_g}(x_{\rm Bj}) = \frac{\int {\rm d}\xi^- e^{ix_{\rm Bj}P^+\xi^-} \langle P| \int {\rm d}^2x^\perp F^{+\lambda}(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)F_\lambda{}^+(x^\perp + \xi^-)\, |P\rangle}{4\pi x_{\rm Bj}P^+(\int {\rm d}^2x^\perp)}$$

are naturally regarded as the $x_{\rm Bj}$ distributions of quark and gluon orbital angular momentum. They are well defined physical objects and are gauge invariant by means of the residual gauge covariant derivative $\mathcal{D}_i = \partial_i - ig\mathcal{A}_i$, where $\mathcal{A}$ is given by

$$\mathcal{A}_\lambda(x^+, x^1, x^2) = \frac{1}{\int {\rm d}x^-} \int {\rm d}x^- A_\lambda(x) \tag{2}$$

in $A^+ = 0$ gauge. They are observable in principle and can be calculated in models or within lattice QCD. However, it remains an open question whether the $x_{\rm Bj}$-distributions of quark and gluon orbital angular momentum which we have defined can be measured in a practical experimental process.

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GAUGE INVARIANT QUARK AND GLUON ANGULAR MOMENTUM DISTRIBUTIONS

(S. Bashinsky)

HEP-PH/9804397: S.B., R.L. JAFFE

- RECENT "BURST" OF THEORETICAL INTEREST:
  P. Hägler, A. Schäfe (9802362)
  A. Harindranath, R. Kundu (9802406)
  O.V. Teryaev (9803403)
  P. Hoodbhoy, X. Ji, W. Lu (9804337)

- Remember the naive parton model: $p^+ = x_{Bj} P^+$

p^+ characterizes transformation properties under $x^- \to x^- + a^-$:

$$\psi_{p^+} \to e^{-ip^+a^-} \psi_{p^+} ,$$
$$A^\lambda_{p^+} \to e^{-ip^+a^-} A^\lambda_{p^+}$$

- Generalize to interacting fields $\to$ States of <u>gauge-covariant</u>, <u>independent</u> t-n.

T_-^q: $\psi(x) \to U(x, x+a^-)\psi(x+a^-)$

T_-^g: $D_\lambda(x) \to U(x, x+a^-) D_\lambda(x+a^-) U(x+a^-, x)$

$$\left(U(x, x+a^-) \equiv P e^{ig\int_{x+a^-}^{x} d\xi^- A^+(\xi)} , \right.$$

$$\left. D_\lambda(x) = \partial_\lambda - igA_\lambda(x) \right)$$

- gauge covariance: ✓
- non-interacting limit: ✓*
- reproduce $q(x_{Bj})$, $g(x_{Bj})$: ✓*

*) T^q and T^g become ordinary translations in $A^+=0$

- Unphysical gauge degrees of freedom: Take $A^+=0$ gauge. Still have the residual gauge invariance:

$$\psi(x) \to e^{i\alpha(\tilde{x})}\psi(x)$$

$$A_\lambda(x) \to e^{i\alpha(\tilde{x})}\left(A_\lambda(x)+\frac{i}{g}\partial_\lambda\right)e^{-i\alpha(\tilde{x})}$$

$$\tilde{x}=(x^+, x^1, x^2)$$

As for the usual gauge invariance introduce $\mathcal{A}_\lambda(\tilde{x})$, s.t.

$$\mathcal{A}_\lambda(\tilde{x}) \to e^{i\alpha(\tilde{x})}\left(\mathcal{A}_\lambda(\tilde{x})+\frac{i}{g}\partial_\lambda\right)e^{-i\alpha(\tilde{x})}$$

$(\mathcal{D}_\lambda \equiv \partial_\lambda - ig\mathcal{A}_\lambda \leftarrow$ the res. g. cov. der.)

$$A(x)=\mathcal{A}(\tilde{x})+G(x), \quad G_\lambda(x) \to e^{i\alpha(\tilde{x})}G_\lambda(x)e^{-i\alpha(\tilde{x})}$$

DESCRIPTION OF NUCLEON PROPERTIES

- Restricted to sectors of $\tilde{\psi}$ and $\tilde{G}$ with $p^+ = x_{Bj} P^+$

  $\Rightarrow$ propose to work with observables, Γ, that are diagonal in $\{\tilde{\psi}(p^+), \tilde{G}(p^+)\}$

  $\Rightarrow$ Γ's are entirely specified by their $x_{Bj} \equiv P^+/p^+$ densities

  $$f_{\Gamma q}(x_{Bj}), \quad f_{\Gamma g}(x_{Bj})$$

Necessary & sufficient: $[\Gamma, T^q] = [\Gamma, T^g] = 0$

$$([\delta_\Gamma, \delta^q] = [\delta_\Gamma, \delta^g] = 0)$$

- Construct Γ x_{Bj}-distributions as $\Gamma \xrightarrow{1} \delta_\Gamma \xrightarrow{2} \delta_{\Gamma q,g}^{(x_{Bj})} \to f_{\Gamma q,g}(x_{Bj})$

EXAMPLE:

chirality $\xrightarrow{1} \delta\psi(x) = i\epsilon\gamma^5\psi(x) \xrightarrow{2}$

$\to \delta\tilde{\psi}(p^+) = i\epsilon\gamma^5\tilde{\psi}(x_{Bj}p^+)\,\delta(p^+/p^+ - x_{Bj}) \xrightarrow{3}$

$$\to \frac{1}{2\pi\sqrt{2}}\int d\xi^-\, e^{ix_{Bj}p^+\xi^-}\langle P|\psi_+^\dagger(0)\gamma^5\psi_+(\xi^-)|P\rangle$$

$$(\psi_+ \equiv \tfrac{1}{2}\gamma^-\gamma^+\psi)$$

- The result for $f_{\Gamma q}(x_{Bj})$, $f_{\Gamma g}(x_{Bj})$ is
  - gauge invariant (Wilson lines in an arbitr. gauge
  - Lorentz boost invariant
  - Properly normalized ($q+g$ 1st moments $=\langle\Gamma\rangle$, in free theory $f_{\Gamma q,g} = \Gamma^{q,g}\delta(1-x_{Bj})$)
  - Reproduce $q(x_{Bj})$, $g(x_{Bj})$ (also $\Delta q(x_{Bj})$, $\Delta g(x_{Bj})$)

ORBITAL ANGULAR MOMENTUM

Remember:

$$[\underline{\Gamma}, \underline{T}^q] = [\underline{\Gamma}, \underline{T}^g] = 0$$

$\Rightarrow$ work with γ^3:

$$\delta\psi = i\epsilon\left[\underline{\tfrac{1}{2}\gamma^0\gamma^3\gamma^5} + \underline{(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)}\right]\psi$$

$$\delta G_\lambda = i\epsilon\Big[\underbrace{-i(\delta^1_\lambda \delta^x_2 - \delta^2_\lambda \delta^x_1)}_{\underline{-i\varepsilon^{+-}{}_\lambda{}^x}} + \underline{(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)}\Big] G_\lambda$$

$$(\mathcal{D} = \partial - ig\mathcal{A})$$

Substituting the generators, obtain f_Σ, f_{Lq}, $f_{\Delta G}$, f_{Lg},

e.g.

$$f_\Sigma(x_{Bj}) = \frac{i}{4\pi x_{Bj} P^+}\int d\xi^- e^{i x_{Bj} P^+ \xi^-} \times$$
$$\times \langle P | F^{+\lambda}(0) F^{*+}_{\lambda}(\xi^-) | P\rangle$$

$$f_{Lq}(x_{Bj}) = \frac{1}{2\pi\sqrt{2}\left(\int d^2x^\perp\right)}\int d\xi^- e^{i x_{Bj} P^+ \xi^-} \times$$
$$\times \langle P | \int d^2x^\perp \psi_+^\dagger(x^\perp)\underline{(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)}\psi_+(x^\perp + \xi^-) | P\rangle$$

- Construct Γ x_{Bj}-distributions as $\Gamma \xrightarrow{1} \delta_\Gamma \xrightarrow{2} \delta^{(x_{Bj})}_{\Gamma q,g} \rightarrow f_{\Gamma q,g}(x_{Bj})$

EXAMPLE:

chirality $\xrightarrow{1} \delta\psi(x) = i\epsilon\gamma^5\psi(x) \xrightarrow{2}$

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$$\rightarrow \frac{1}{2\pi\sqrt{2}}\int d\xi^- e^{ix_{Bj}p^+\xi^-}\langle P|\psi_+^\dagger(0)\gamma^5\psi_+(\xi^-)|P\rangle$$

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