The Usefulness of Positivity in Spin Physics.

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We will emphasize the relevance of *positivity* in spin physics, which puts non-trivial model independant constraints on spin observables. These positivity conditions are based on the positivity properties of density matrix or Schwarz inequalities f or transition matrix elements in processes involving several particles carrying a non-zero spin. We will illustrate this important point by means of several examples chosen in different areas of particle physics.

First we consider longitudinally and transversely polarized nucleon-nucleon total cross sections, which involve three observables σ_{tot}, $\Delta\sigma_T$ and $\Delta\sigma_L$. Positivity leads to the following non-trivial inequality among them, $|\Delta\sigma_T| \leq \sigma_{tot} + \Delta\sigma_L/2$, fulfilled by low energy data. In polarized Deep-Inelastic scattering (DIS), we recall the positivity bound on the A_2 asymmetry, namely $|A_2| \leq \sqrt{R}$, where R is the usual ratio σ_L/σ_T. Given the small value of R, A_2 is expected to be not very large and, indeed, recent data from E143 at SLAC and SMC at CERN, do satisfy this condition. Next let us turn to some co nsiderations on the gluon distributions (GD). In addition to the two familiar GD, $G(x)$ and $\Delta G(x)$, off-shell gluons generate a longitudinal GD, denoted by $G_L(x)$ and a transverse GD, denoted by $\Delta G_T(x)$. Positivity provides an upper boun d on $\Delta G_T(x)$ in complete analogy with the previous case, which can be turned into a lower bound for $G_L(x)$. Another interesting case is that of a two-body exclusive reaction with spin-1/2 particles, i.e. $NN \to NN$, $p\bar{p} \to \Lambda\bar{\Lambda}$, etc... One can prove the existence of many quadratic relations among the spin observables and when some of them are not measured, one gets non-trivial inequalities. As an example, in $p\bar{p} \to \Lambda\bar{\Lambda}$ from the existing data on A_{LL} one can exclude a large value for D_{NN} at large angles. Returning to inclusive production, if we consider a reaction of the type $spin1/2 + spinless \to spin1/2 + anything$, which is described in terms of seven spin observables, one can derive several positivity conditions. Finally, the positivity properties of the *forward* qua rk-nucleon elastic amplitude led to a non-trivial bound for the quark chiral-odd distribution $h_1^q(x, Q^2)$, namely, $2|h_1^q(x, Q^2)| \leq q(x, Q^2) + \Delta q(x, Q^2)$, which is not affected by QCD corrections. This positivity bound can be generalized to the case of *off-forward* parton distributions and lead to new interesting constraints, very useful in the small x-region.

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K. ABE et al. PRL 76 (1996) 587

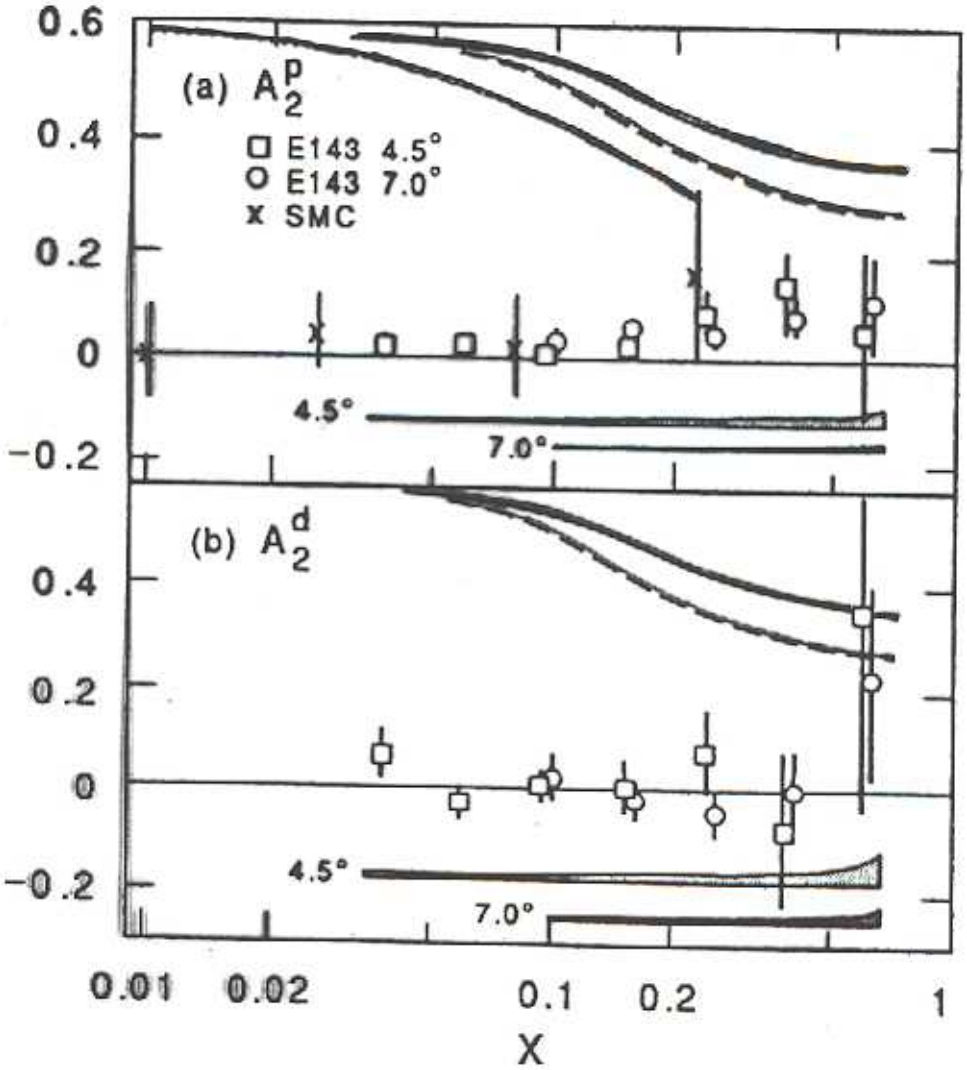

FIG. 1. Measurements for (a) A_2^p and (b) A_2^d from E143 (two data sets) and SMC as a function of x. Systematic errors are indicated by bands. The curves show the $\sqrt{R}$ [22] positivity constraints. The solid, dashed, and dotted curves correspond to the 4.5° E143, 7.0° E143, and SMC kinematics, respectively. Overlapping data have been shifted slightly in x to make errors clearly visible.

$|A_2| \leq \sqrt{R}$

3. LONGITUDINAL GLUON DISTRIBUTION

(J.S. AND O. TERYAEV, PHYS. LETT. B419 (1998) 400)

IN ADDITION TO THE TWO FAMILIAR GLUON DISTRIBUTIONS $G(x)$ AND $\Delta G(x)$, OFF-SHELL GLUONS GENERATE A LONGITUDINAL G.D. AND A TRANSVERSE G.D.

CONSIDER THE LIGHT-CONE GLUON MATRIX DENSITY

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p,s | A^\rho(0) A^\sigma(\lambda n) | p,s \rangle$$

$$\sim \frac{1}{2}\left(e^\rho_{1T} e^\sigma_{1T} + e^\rho_{2T} e^\sigma_{2T}\right) G(x) + \frac{i}{2}\left(e^\rho_{1T} e^\sigma_{1T} - e^\rho_{2T} e^\sigma_{2T}\right) \Delta G(x)$$

$$+ \frac{i}{2}\left(e^\rho_{1T} e^\sigma_{L} - e^\rho_{L} e^\sigma_{1T}\right) \Delta G_T(x) + e^\rho_L e^\sigma_L \, G_L(x)$$

ANALOGOUS TO g_2 FOR QUARKS

FOR THE TWIST-TWO PART OF $\Delta G_T(x)$ WE HAVE A W-W "TYPE" RELATION (J.S. AND O. TERYAEV, PR D56 (1997) R1353)

$$\Delta G_T(x) = \int_x^1 \frac{\Delta G(z)}{z} dz$$

A SIMPLE ANALOGY BETWEEN $\gamma^* p \to \gamma^* p$ AND $Gp \to Gp$ LEADS TO THE FOLLOWING POSITIVITY CONSTRAINT

$$|\Delta G_T(x)| \leq \sqrt{\tfrac{1}{2} G(x) G_L(x)}$$

IF WE CONSIDER THAT $G(x)$ AND $\Delta G_T(x)$ ARE KNOW WE CAN TURN THIS INEQUALITY INTO A LOWER BOUND FOR $G_L(x)$

$$G_L(x) \geq 2\lambda(x) G(x) \qquad \lambda(x) = \left(\frac{\Delta G_T(x)}{G(x)}\right)^2$$

IN TWIST-TWO APPROXIMATION FOR $x \sim 0.1$, $\lambda \sim 0.01$

$\Rightarrow G_L(x) \geq 0.5$ BUT LARGER FOR SMALLER x.

3 - LONGITUDINAL GLUON DISTRIBUTION

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FOR THE TWIST-TWO PART OF $\Delta G_T(x)$ WE HAVE A W-W "TYPE" RELATION (J.S. AND O. TERYAEV, PR D56 (1997) R1358)

$$\Delta G_T(x) = \int_x^1 \frac{\Delta G(z)}{z} dz$$

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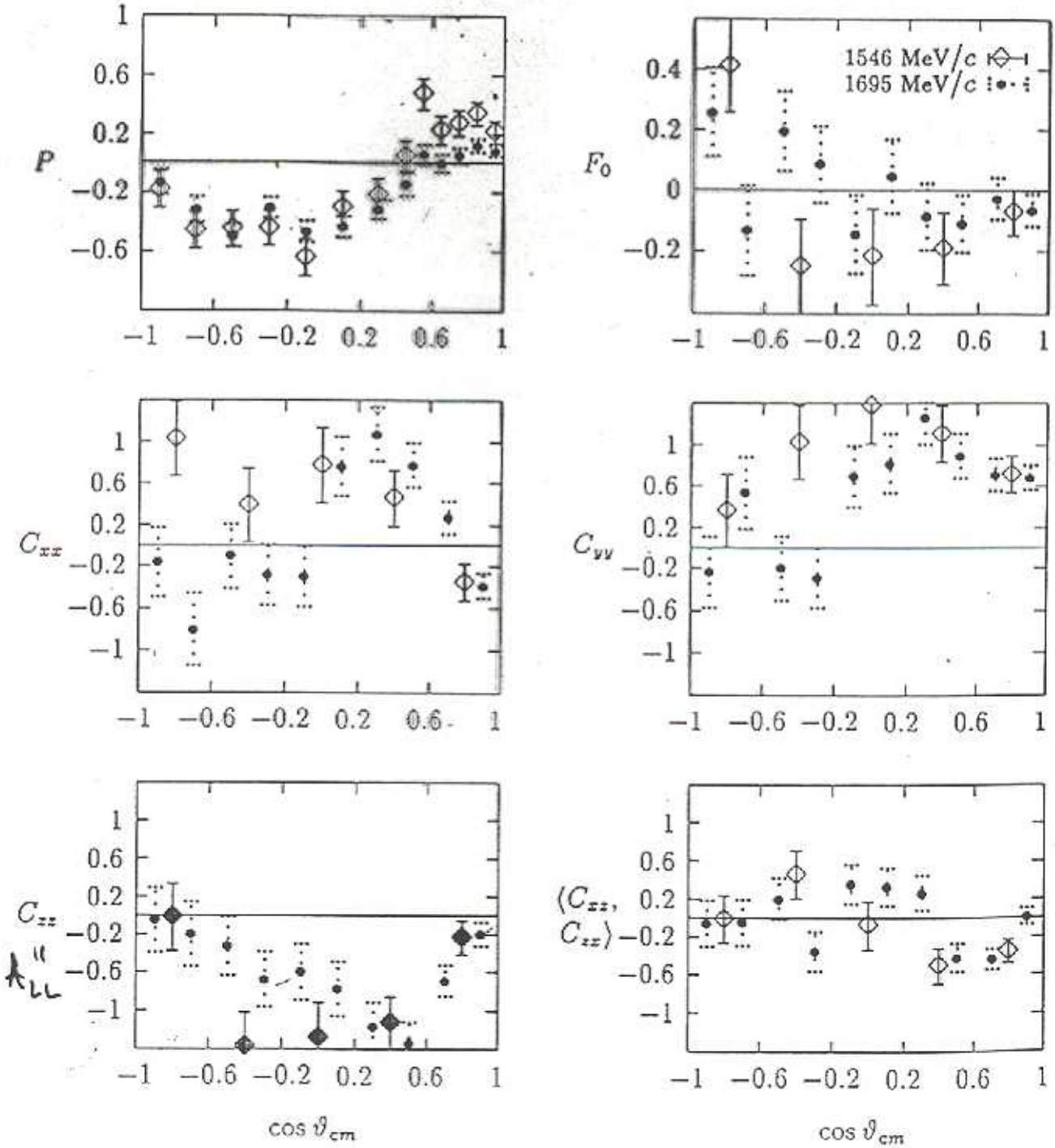
IN TWIST-TWO APPROXIMATION FOR $x \sim 0.1$, $\lambda \sim 0.01$

$\Rightarrow G_L(x) \geq 0.3$ BUT LARGER FOR SMALLER x

$\bar{p}p \to \bar{\Lambda}\Lambda$

$A_{LL}^2 + D_{NN}^2 \leq 1$



1. Polarisation P, spin-singlet fraction F_0, and spin-correlation coefficients C_{ij} for the reaction $\bar{p}p \to \bar{\Lambda}\Lambda$ at 1446 (open squ
5 MeV/c (black dots). Data are from Ref. [2].

5. POSITIVITY CONSTRAINTS FOR $pp \to \Lambda X$

(M.G. DONCEL and A. MÉNDEZ, PHYS. LETT. B 41 (1972) 83)

LET'S CONSIDER THREE TRANSVERSE SPIN OBSERVABLES

P_Λ Λ POLARIZATION

A_N ANALYZING POWER (OR LEFT-RIGHT ASYM.)

D_{NN} DEPOLARIZATION PARAMETER

POSITIVITY YIELDS THE FOLLOWING CONSTRAINTS WHICH MUST BE SATISFIED FOR ANY KINEMATIC POINT $(x_F, p_T, \sqrt{s})$

① $1 + D_{NN} \geq |P_\Lambda + A_N|$

② $1 - D_{NN} \geq |P_\Lambda - A_N|$

GIVEN D_{NN} ONE CAN FIND THE ALLOWED REGION IN THE PLANE P_Λ, A_N (AND VICE VERSA)

SEE GRAPHS FOR THREE VALUES $D_{NN} = 0, 1/3, 1/2$

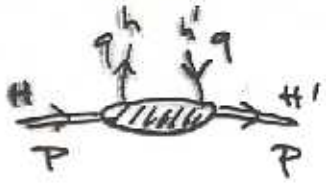
FOR $D_{NN} < 0$ SAME REGION BUT MUST EXCHANGE P_Λ AND A_N. IF $D_{NN} \to 1 \Rightarrow P_\Lambda$ AND $A_N \to 0$

E704 SATISFIES POSITIVITY

FOR EXAMPLE AT $p_T \sim 1$ GeV/c

$D_{NN} \sim 1/3$ AND THEY HAVE $A_N \sim -10\%$, $P_\Lambda \sim -30\%$

6 - POSITIVITY CONSTRAINTS FOR FORWARD PARTON DISTRIBUTIONS



LET'S CONSIDER THE THREE FORWARD QUARK DISTRIBUTIONS

$$q(x)\ ,\quad \Delta q(x)\ ,\quad h_1^q(x)$$

WHICH CAN BE RELATED TO THE FORWARD QUARK-NUCLEON ELASTIC AMPLITUDE

$$q_{h'}(q) + N_H(P) \to q_h(q) + N_{H'}(P)$$

h_1^q CORRESPONDS TO $H = h' = +1/2$, $H' = h = -1/2$

LET'S USE THE MATRIX ELEMENTS

$$a_X^{H,h}(P,q) = \langle N_H(P)\,|\,O\,|\,q_h(q), X\rangle$$

ONE CAN SHOW THAT

$$q_+(x) \equiv \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

$$= \frac{1}{2}\sum_X \left(|a_X^{+,+}|^2 + |a_X^{-,-}|^2\right)$$

$$h_1^q(x) = \frac{1}{2}\sum_X \left[(a_X^{+,+})^*(a_X^{-,-}) + (a_X^{+,+})(a_X^{-,-})^*\right]$$

SCHWARZ INEQUALITY LEADS IMMEDIATELY TO

$$2\left|h_1^q(x)\right| < q(x) + \Delta q(x)$$

(J.S. PHYS. REV. LETT. 74 (1995) 1292)

STRONGER THAN THE TRIVIAL ONE $\left|h_1^q(x)\right| \leq q(x)$

THIS SIMPLE RESULT IS NOT AFFECTED BY QCD CORRECTIONS UP TO NLO

(W. VOGELSANG , PHYS. REV. D57 (1998) 1886 ;
C. BOURRELY , J.S. , O. TERYAEV , PHYS. LETT. B420 (1998) 375)