NLO QCD Corrections to A_{LL}^{π}

Werner Vogelsang (RBRC/BNL)

collaboration with :

Barbara Jäger (Regensburg)

Marco Stratmann (Regensburg)

- **Accessing Δg at RHIC**
- **NLO QCD Corrections to A_{LL}^{π}**
- **Results**

major goal of RHIC spin program: measure $\Delta g(x)$

key advantages of RHIC:

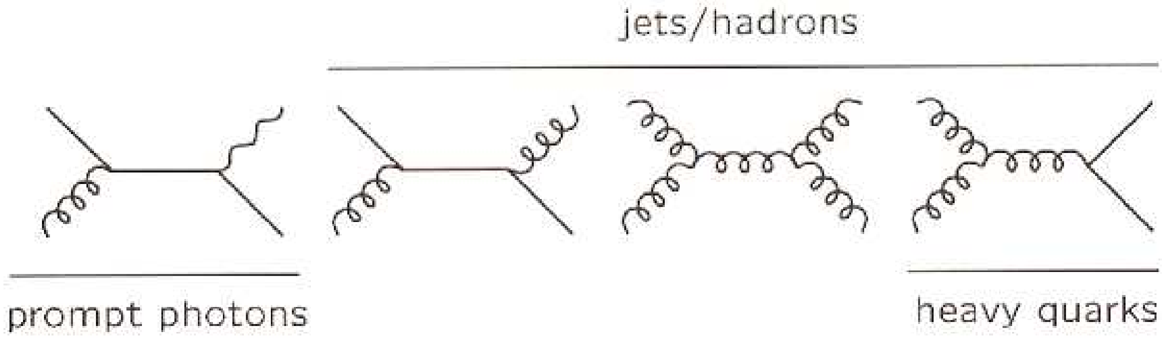
- Δg can be probed in *various* processes

  $\rightarrow$ can verify universality of pol. pdfs for the 1st time

  $\updownarrow$

  foundation for predictive power of pQCD

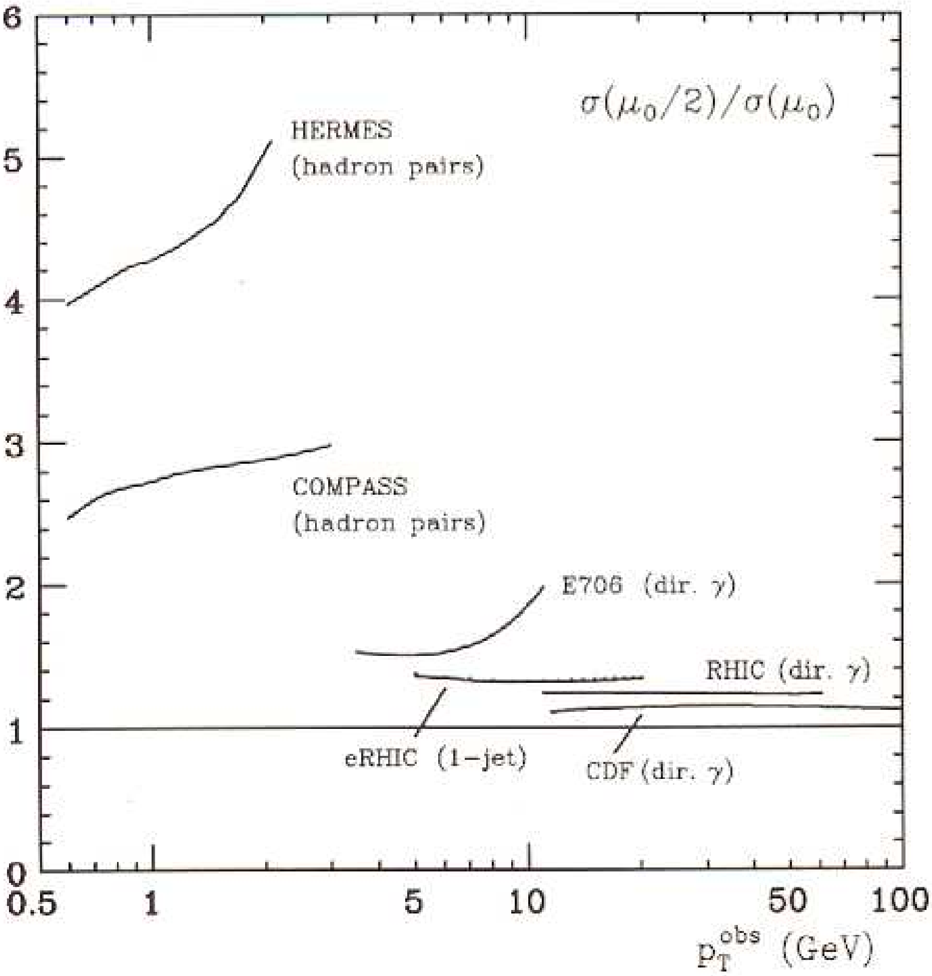
  candidates with a *dominant* gluon contribution in LO:



- large c.m.s. energy $\sqrt{S} \rightarrow$ high p_T accessible

  $\rightarrow$ pQCD should be applicable

scale dependence $\leftrightarrow$ measure for reliability of pQCD:



$\leftarrow$ ratio of x-secs for two choices of μ should be $\simeq 1$

$\uparrow$ $\times$ $\qquad$ $\uparrow$ $\checkmark$

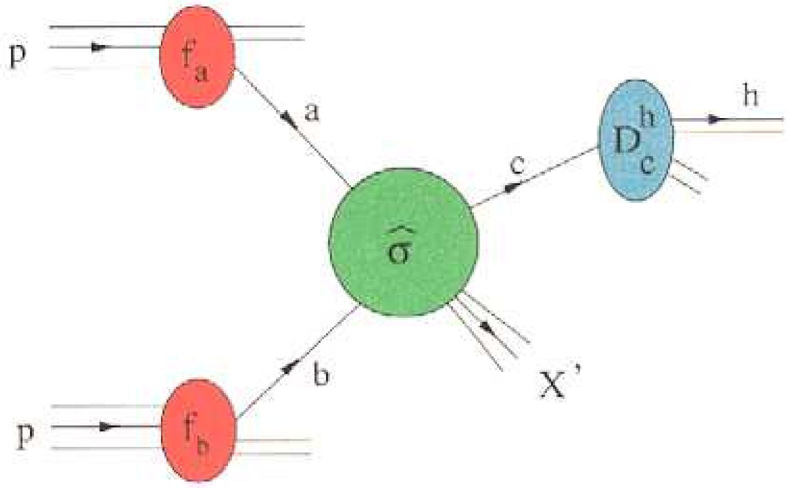
general framework for inclusive-pion production:

starting point: factorization theorem

Libby, Sterman; Ellis et al.; Collins et al.; ...

requirement: a hard scale, e.g., pions with high-p_T

$$d\sigma = \Bigg|$$



$$\Bigg|^2 + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\boxed{\begin{aligned}\frac{d\sigma^{pp\to\pi X}}{dp_T} = & \sum_{abc}\int dx_a\, dx_b\, dz_c\, f_a(x_a,\mu)\, f_b(x_b,\mu)\, D_c^\pi(z_c,\mu) \\ & \times \frac{d\hat{\sigma}^{ab\to cX'}}{dp_T}(x_a P_a, x_b P_b, P^\pi/z_c,\mu) \;+\; \text{Power corr.}\end{aligned}}$$

$$\uparrow$$

short-distance

pQCD: power series in α_s

- scale $\mu \sim p_T$: separates long- and short-dist. physics
- formula valid for polarized case as well :

$$f(x,\mu_f) \to \Delta f(x,\mu_f) \quad \text{and} \quad d\hat{\sigma}/dp_T \to d\Delta\hat{\sigma}/dp_T$$

experimentally relevant: double-spin asymmetry

$$\boxed{A_{\rm LL}^{\pi} \equiv \frac{d\sigma^{pp\to\pi X'}(++) - d\sigma^{pp\to\pi X'}(+-)}{d\sigma^{pp\to\pi X'}(++) + d\sigma^{pp\to\pi X'}(+-)}}$$

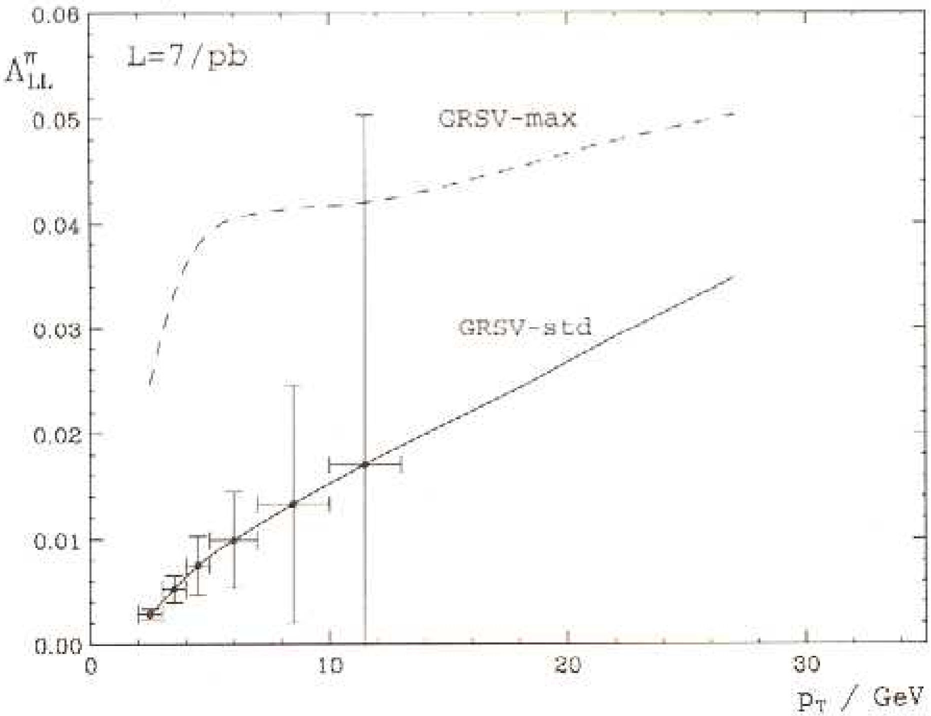
$\uparrow\uparrow$ $\uparrow\uparrow$

helicities of colliding protons

LO prediction for $A_{\rm LL}^{\pi}(p_T^{\pi})$ at $\sqrt{S} = 200\,\mathrm{GeV}$:

very moderate luminosity!

$\downarrow$

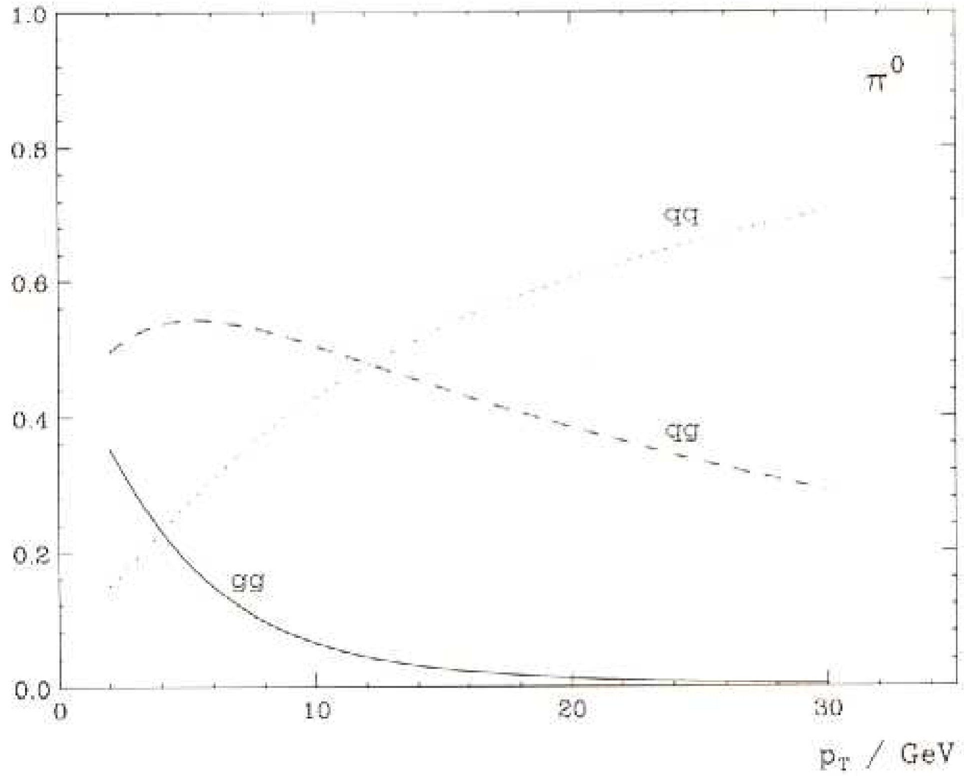


estimate of statistical errors: $\delta A \simeq \frac{1}{P_1 P_2} \times \frac{1}{\sqrt{\mathcal{L}\sigma_{\rm bin}\varepsilon_{\rm eff}}}$

[with $P_1 = P_2 = 0.5$ (beam pol.) and $\varepsilon_{\rm eff} = 1$ (detection efficiency)]

sensitivity to Δg (with $\mathcal{L} = 7/\mathrm{pb}$!) $\rightarrow$ very promising breakdown into contributions from different subprocesses:

$d\sigma^{\pi^0}/dp_T$ ($\sqrt{S} = 200\,\text{GeV}$)



[unpol. NLO: Aversa et al.; pdfs: CTEQ5M; frag-fcts: Kretzer]

→ gluon-induced processes relevant up to large p_T

polarized: $qq/qg/gg$ ratio depends strongly on Δg

NLO pQCD hard scattering works well at colliders $\rightarrow$

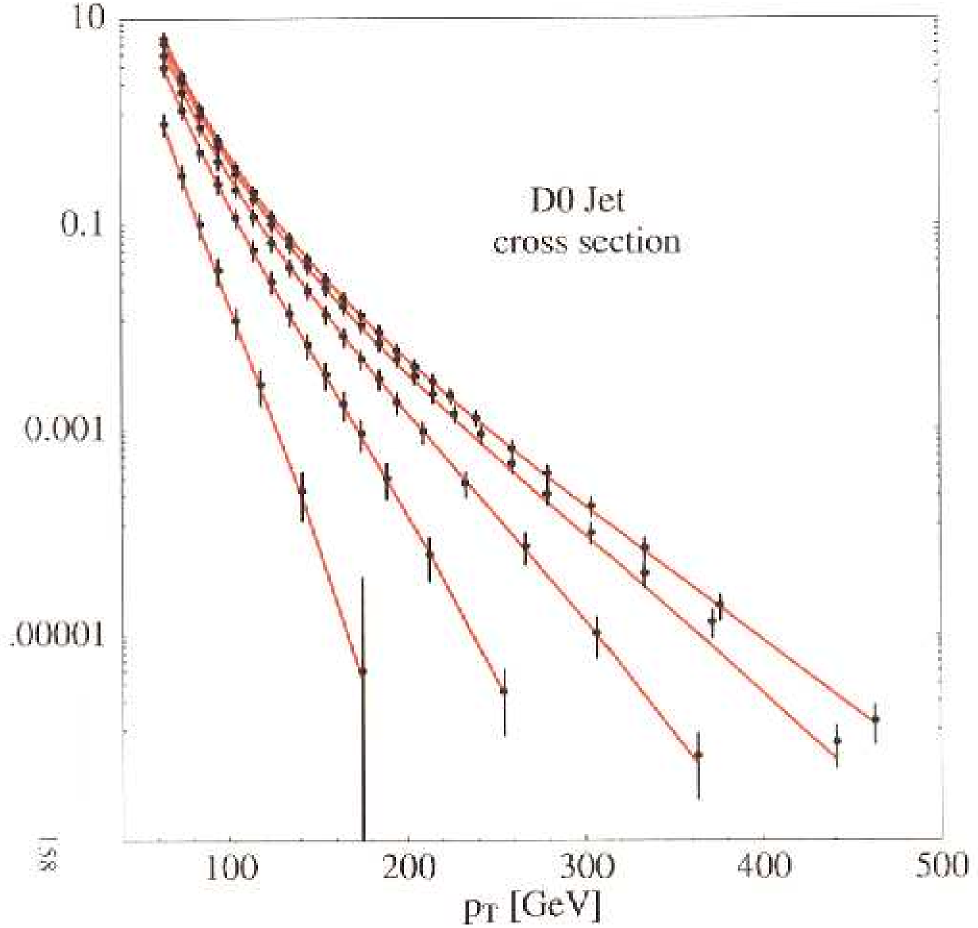
in general, NLO QCD corrections are a must :

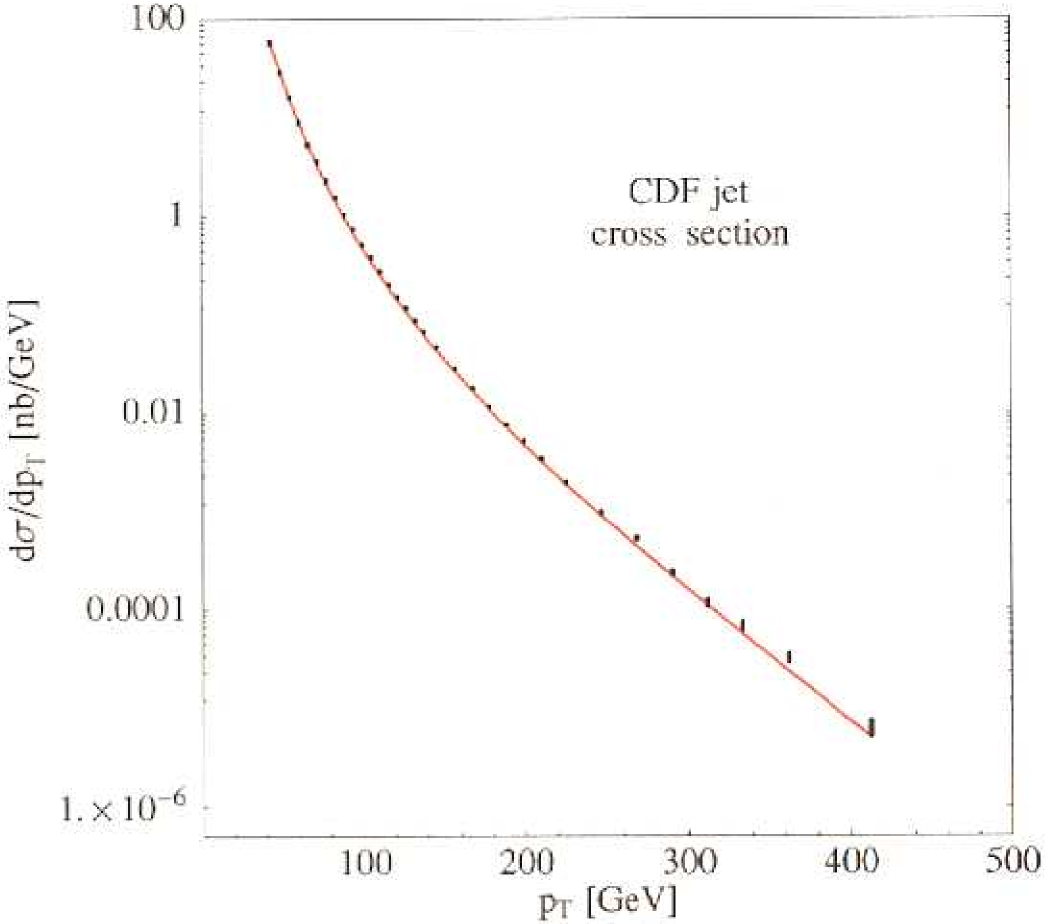
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} d\sigma_{\text{phys}} = 0$$

- however, $\neq 0$ in truncated preturbation theory
- dependence on unphysical scale μ strong in LO

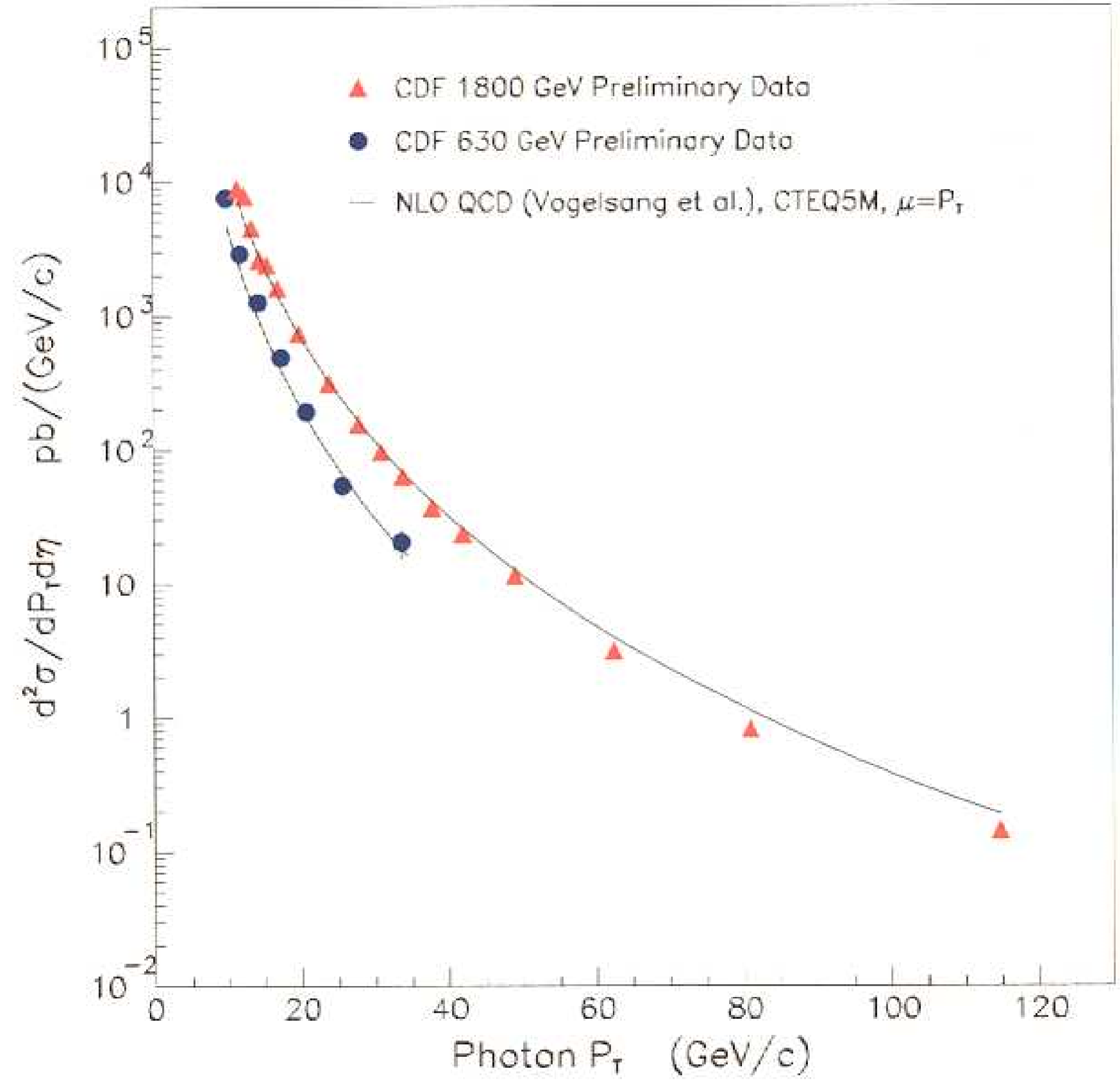
  $\rightarrow$ sizable theoretical uncertainties
- QCD corrections often important,
  in particular for polarized cross section
- more reliable angular / p_T distributions, jet def., ...

Example : High-p_T jets at the Tevatron :





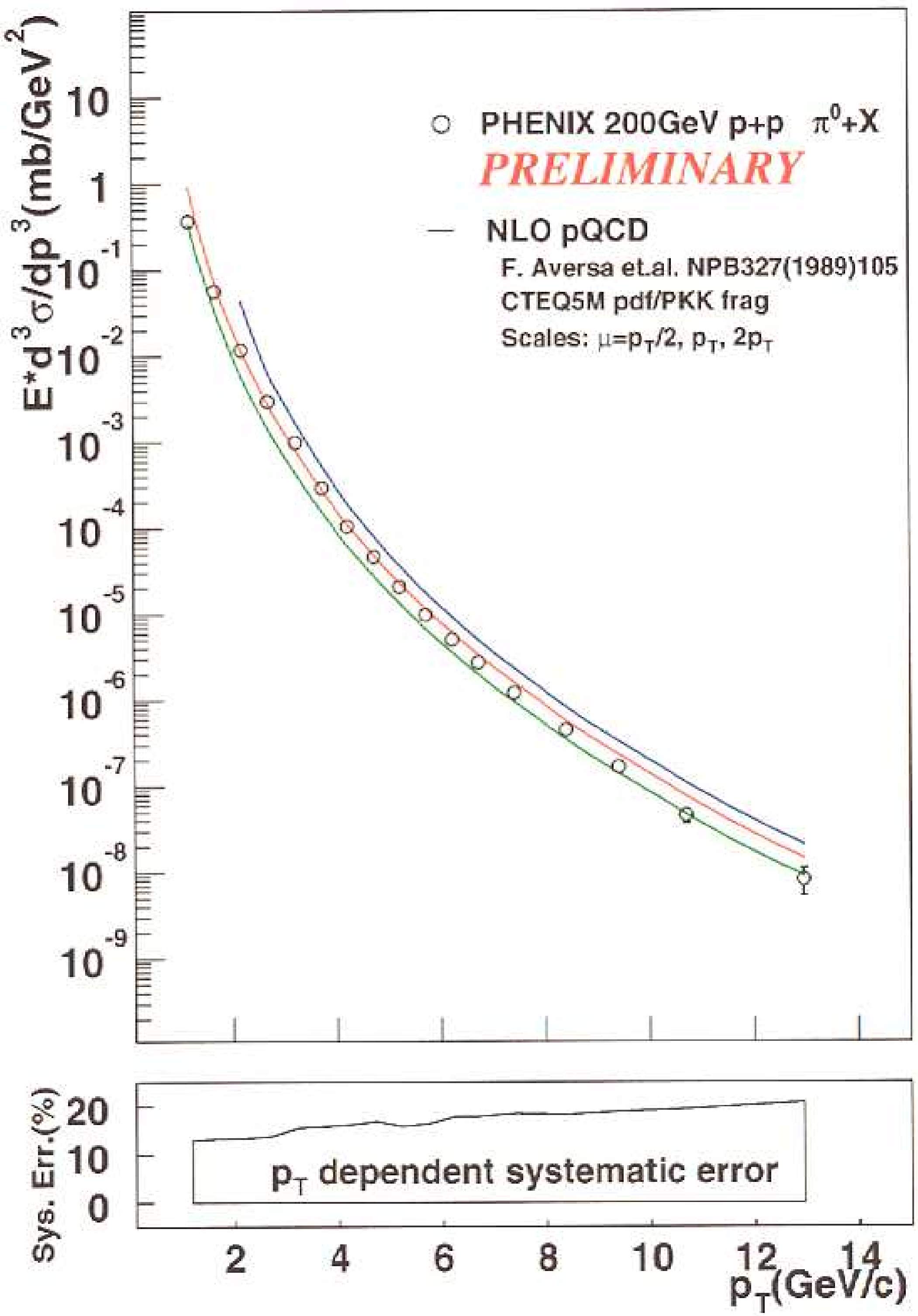
Example : High-p_T photons at the Tevatron :



AND :

$pp \longrightarrow \pi^0 X$ **by**
PHENIX

($\pm$30% normalization unc.)



NLO QCD corrections to $\mathrm{A}^{\pi}_{\mathrm{LL}}$ - outline:

at $\mathcal{O}(\alpha_s^2)$ one has:

all LO $2 \to 2$  parton-parton scattering processes

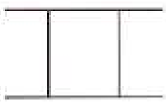
unpol.: 4 processes $qq' \to qq'$, $qq \to qq$, $q\bar{q} \to gg$, $gg \to gg$

all other processes related by crossing

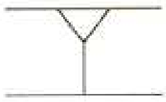
(however, need $\vec{q}\vec{g} \to qg$ etc.)

at $\mathcal{O}(\alpha_s^3)$ one has:

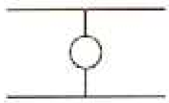
(1) 1-loop (virtual) corrections to all LO processes



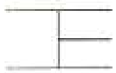
'box'



'vertex'



'selfenergy'

(2) all $2 \to 3$  parton-parton scattering processes

$qq' \to qq'g$, $q\bar{q} \to ggg$, $gg \to ggg$, etc.

all contributions separately singular

$\Rightarrow$ choose $d = 4 - 2\epsilon$ dimensions

Two strategies for NLO calculations :

(1) 'Monte-Carlo approach'

✓ different observables, exp. cuts

✓ "smaller amount of work"

× delicate numerics

× relatively slow in evaluation

→ D. de Florian

$$I = \lim_{\epsilon \to 0} \left(\int_0^1 \frac{dx}{x} x^{\epsilon} F(x) - \frac{1}{\epsilon} F(0) \right)$$

$$I = F(0) \ln \delta + \int_\delta^1 \frac{dx}{x} F(x)$$

"slicing method"

$$I = \int_0^1 \frac{dx}{x} [F(x) - F(0)]$$

"subtraction method"

(previous application : $\vec{p}\vec{p} \to \mathrm{jets}X$ de Florian, Frixione, Signer, WV)

(2) 'analytical method'

× 'only' single-incl. cross section

✓ numerically stable

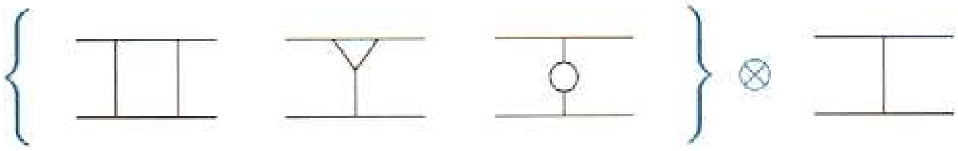
✓ fast → useful for global fits

→ our approach

(previous application : $\vec{p}\vec{p} \to \gamma X$ Gordon, WV; Contogouris et al.)

technical details (I) - 1-loop virtual corrections:

$\mathcal{O}(\alpha_s^3)$: only interference of 1-loop and Born amplitudes contributes:



IR+UV divergencies → work in $4-2\varepsilon$ dimensions

can extensively make use of available results

we use two different methods:

(1) renormalized propagators and vertices

Nowak, Praszalowicz, Slominski

 UV-divergent → tabulated in NPS

 UV-finite → calculate from scratch

(2) one-loop renormalized helicity amplitudes

Kunszt, Signer, Trocsanyi

some 'gymnastics' required to obtain desired results [e.g., 'color-linked matrix elements' Kunszt, Soper]

✓ results for methods (1) and (2) fully agree

✓ unpolarized results agree with Ellis, Sexton

technical details (II) - $2 \to 3$ contributions:

aim: calculation of *single-inclusive* pion cross section:

e.g. $gg \to q \; (\bar{q}g)$

fragments: $q \to \pi X$ integrated out

phase space integration performed in rest frame of the two unobserved partons

(parametrized by two angles $\theta_{1,2}$)

$$d\Delta\hat{\sigma}_{2\to3} \sim \ldots \int d\theta_1 d\theta_2 \sin^{1-2\varepsilon}\theta_1 \sin^{-2\varepsilon}\theta_2 \, |\Delta M_{2\to3}|^2$$

calculation requires extensive partial fractioning to get

$$I^{(k,l)} = \int \frac{d\theta_1 \, \sin^{1-2\varepsilon}\theta_1 \, d\theta_2 \, \sin^{-2\varepsilon}\theta_2}{(1+\cos\theta_1)^k(1+A\cos\theta_1+B\sin\theta_1\cos\theta_2)^l}$$

which can be done *analytically*

subtlety in polarized calculation: γ_5 in $4-2\varepsilon$ dimensions

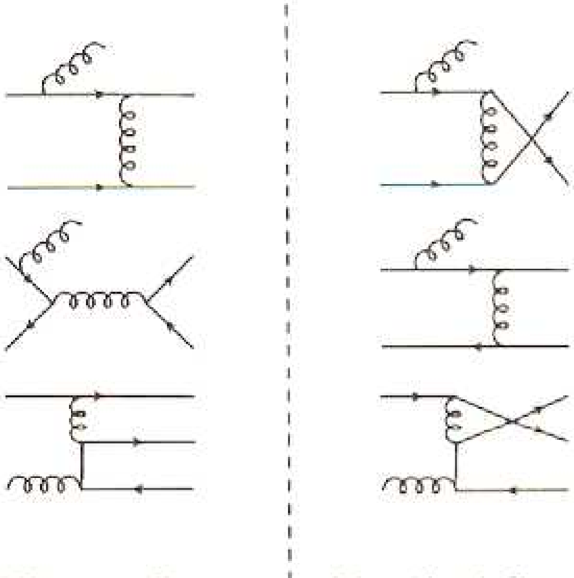
- γ_5 (and $\epsilon_{\mu\nu\rho\sigma}$) are genuine 4-dim. $\to$ use HVBM prescription :

$\{\gamma^\mu, \gamma_5\} = 0 \quad (\mu = 0,1,2,3) \qquad [\gamma^\mu, \gamma_5] = 0$ otherwise

✓ all $2 \to 3$ matrix elements computed . . .
(agreement unpol. case with Ellis, Sexton)

✓ . . . and integrated

some typical NLO $2 \to 3$ Feynman diagrams:



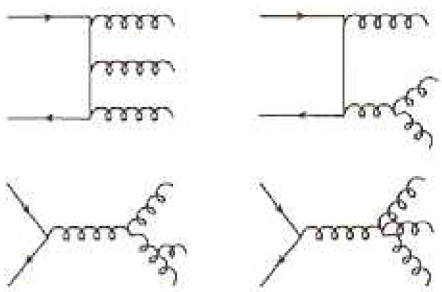
different flavors | identical flavors

$q \neq q'$ | $q = q'$

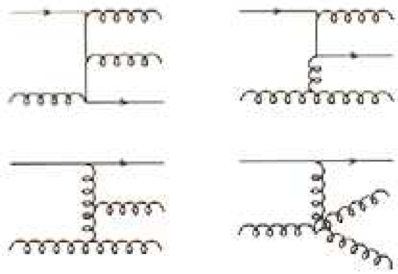
$qq' \to qq'g$

$q\bar{q} \to q'\bar{q}'g$

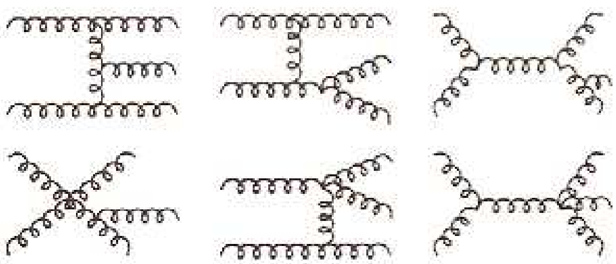
$qg \to qq'\bar{q}'$



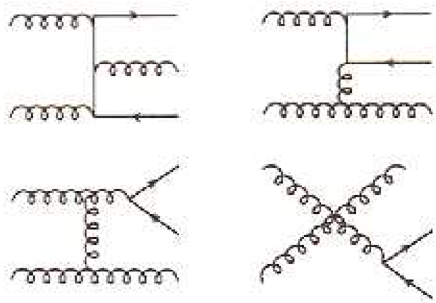
$q\bar{q} \to ggg$



$qg \to qgg$



$gg \to ggg$



$gg \to q\bar{q}g$

technical details (III) - cancellation of divergencies:

final step: adding up all real and virtual contributions

before taking the limit $\varepsilon \to 0$ all poles have to cancel:

UV $1/\varepsilon$-singularities

removed by renormalization of α_s

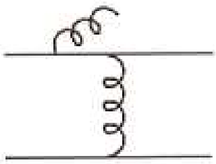
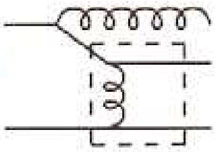
introduce arbitrary renormalization scale μ_r.

IR singularities ($1/\varepsilon^2$, $1/\varepsilon$)

cancel in sum of 1-loop and $2 \to 3$ contributions

collinear $1/\varepsilon$-singularities

have to be removed by factorization

e.g.:  $\rightarrow$  $\sim \frac{1}{\varepsilon} \int dx \Delta P_{qq}(x) \Delta\hat{\sigma}_{qq \to qq}$

introduce two arbitrary factorization scales μ_f and μ_f'

initial-/final-state singularities

$\swarrow \qquad \searrow$

$\Delta f(x, \mu_f) \qquad D_f^{\pi}(z, \mu_{f'})$

Final answer finite ! $\quad \rightarrow$ good check of results

final results (I) - $\mathcal{O}(\alpha_s^3)$ parton-parton processes:

16 different inclusive cross sections contribute:

fragmenting parton

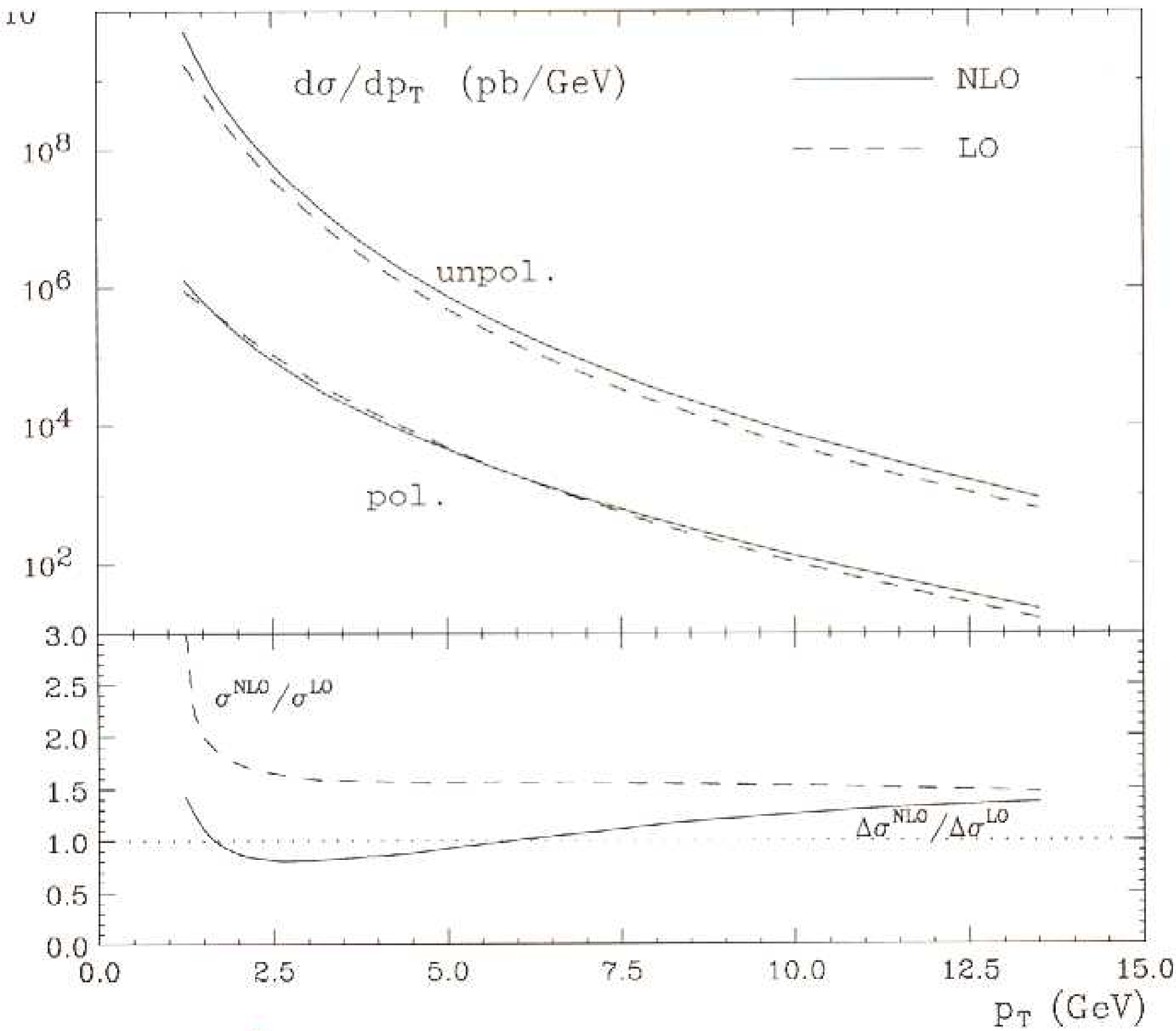
$$
\begin{array}{lcl}
 & & \downarrow \\
qq' & \to & q + X \\
 & \to & g + X \\
q\bar{q}' & \to & q + X \\
 & \to & g + X \\
q\bar{q} & \to & q' + X \\
 & \to & q + X \\
 & \to & g + X \\
qq & \to & q + X \\
 & \to & g + X \\
qg & \to & q' + X \\
 & \to & \bar{q}' + X \\
 & \to & \bar{q} + X \\
 & \to & q + X \\
 & \to & g + X \\
gg & \to & g + X \\
 & \to & q + X
\end{array}
$$

$\checkmark$: all done & unpol. results agree with Aversa et al.

$\Rightarrow$ full NLO results available

final results (II) - importance of NLO corrections:

$\sqrt{S} = 200\,\text{GeV}$



lower part: K-factor: σ^{NLO}/σ^{LO}, $\Delta\sigma^{NLO}/\Delta\sigma^{LO}$

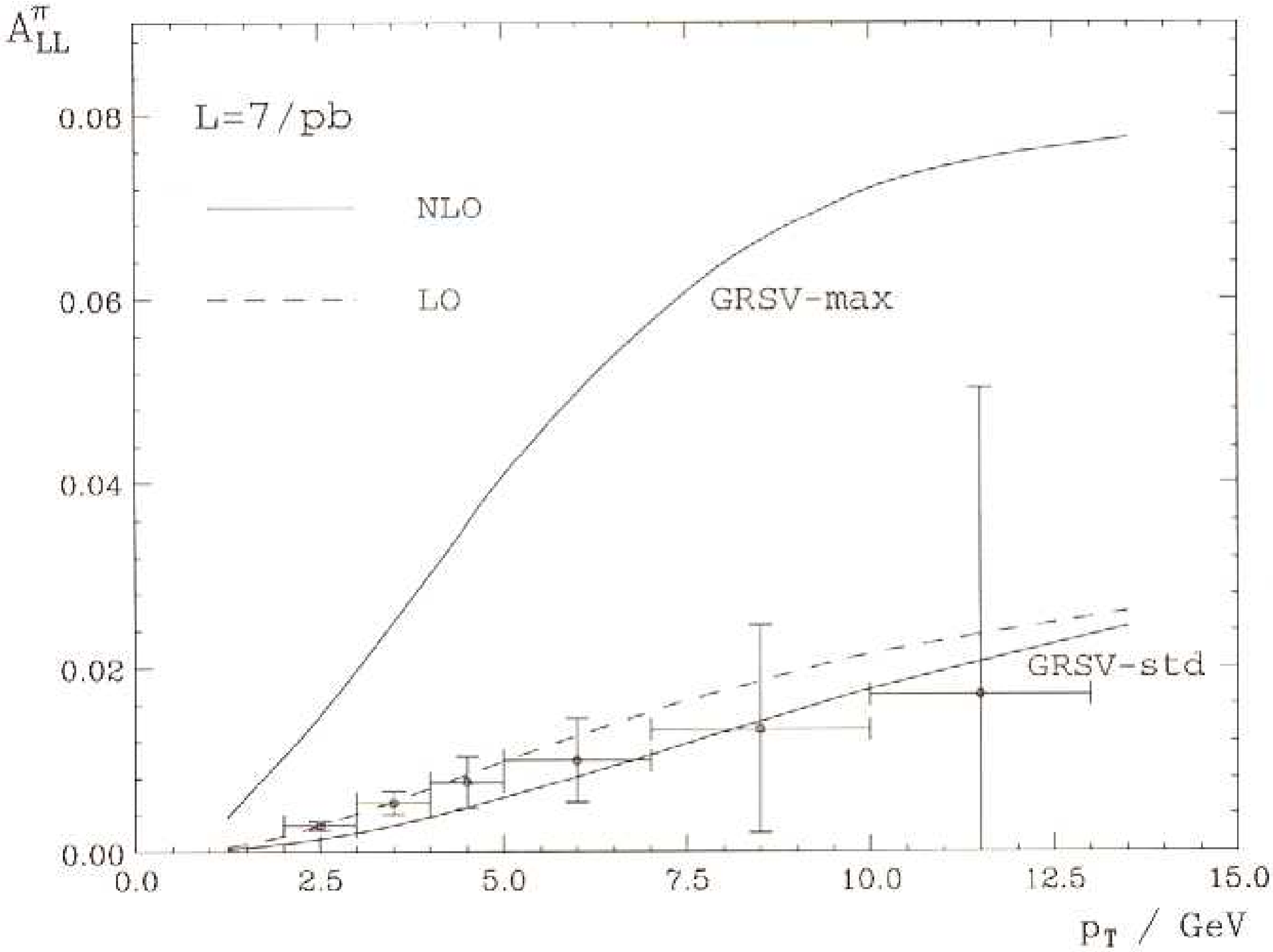
pdfs: CTEQ 5M (unpol.), GRSV std. (pol.)

frag. fcts: KKP

final results (III) - A_{LL}^{π} in NLO:

very moderate luminosity!

$\sqrt{S} = 200\,\text{GeV}$



pdfs: CTEQ 5M (unpol.), GRSV std. (pol.)

frag. fcts: KKP

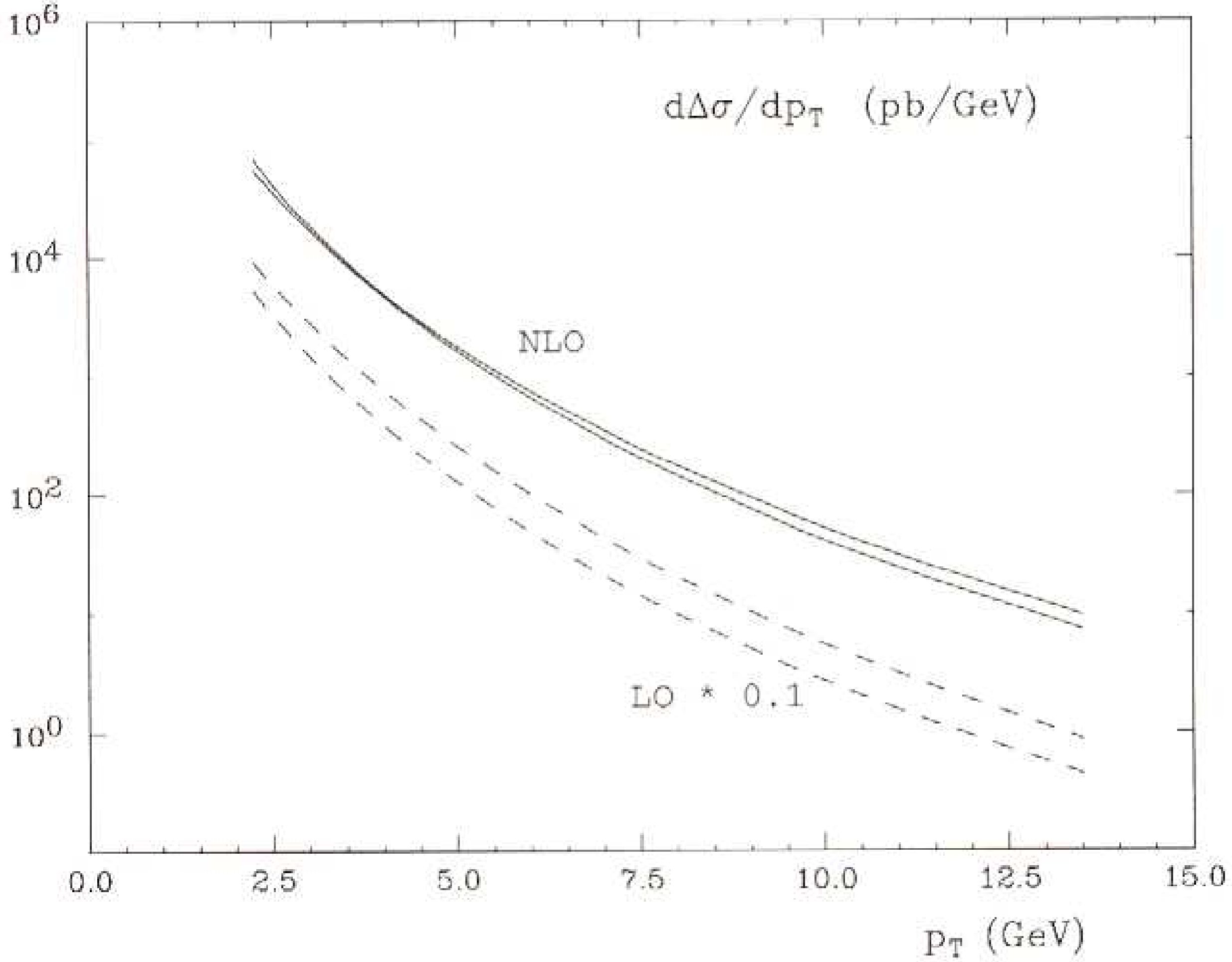
estimate of statistical errors: $\delta A \simeq \frac{1}{P_1 P_2} \times \frac{1}{\sqrt{\mathcal{L}\sigma_{\text{bin}}\varepsilon_{\text{eff}}}}$

[with $P_1 = P_2 = 0.4$ (beam pol.) and $\varepsilon_{\text{eff}} = 1$ (detection efficiency)]

good sensitivity to Δg even with $\mathcal{L} = 7/\text{pb}$!

final results (IV) - scale dependence:

$\sqrt{S} = 200\,\mathrm{GeV}$



variation of scales: $\mu_f = \mu'_f = \mu_r = p_T \ldots 2p_T$

pdfs: GRSV std.; frag. fcts: Kretzer

$\Downarrow$

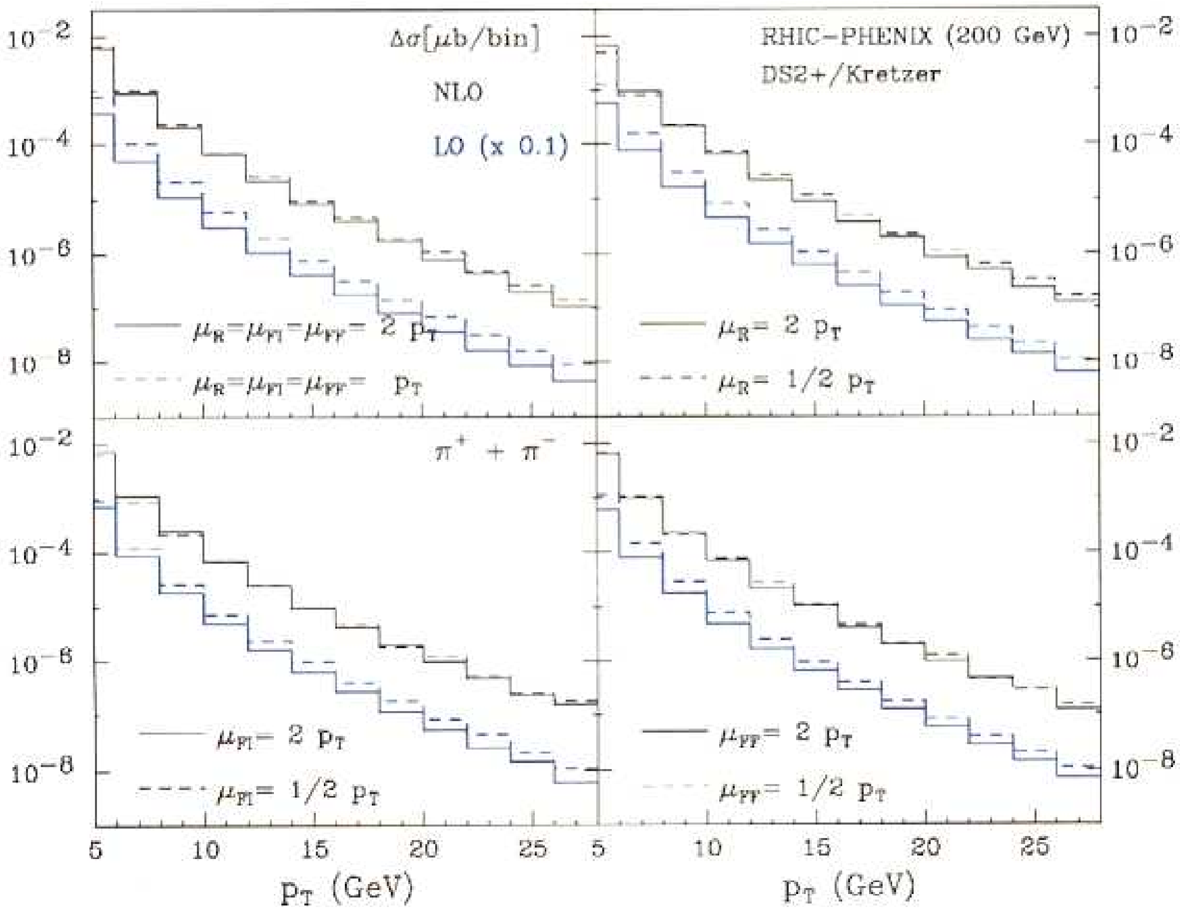
NLO results much more reliable

comparison with other calculations :

also very recently:

NLO QCD MC-code for hadron production at RHIC

D. de Florian



[from de Florian's talk at "Current and future directions at RHIC"]

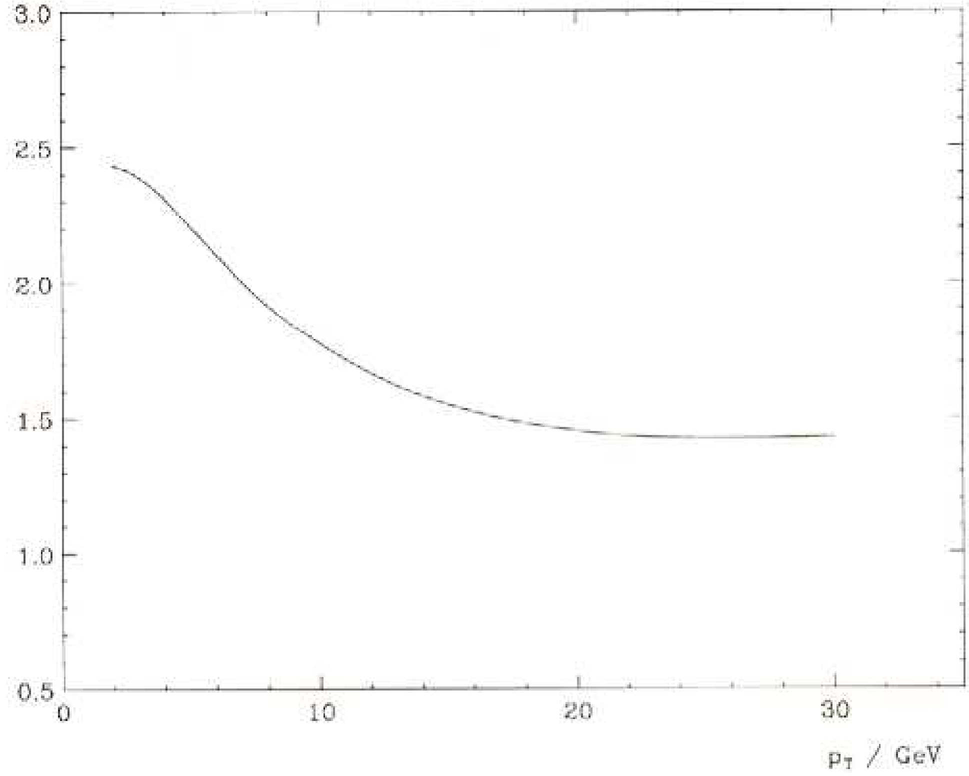
so far:

gross features of both calculations look very similar

- detailed comparisons are under way
- more quantitative results will be available soon

dependence of $\frac{d\sigma}{dp_T}$ and A_{LL}^{π} on fragmentation functions:

$d\sigma^{KKP}/d\sigma^{Kretzer}$



$A_{\mathrm{LL}}^{\pi}(p_T)$ (dashed: KKP; solid: Kretzer)