

# Recent results in QCD resummation, and on $A_{TT}$

Werner Vogelsang

RIKEN-BNL Research Center  
and Nuclear Theory, BNL

RSC meeting, June 17, 2002

work with A. Kulesza, E. Laenen, G. Sterman  
J. Soffer, M. Stratmann

## Outline :

I. Introduction : why resum ?

II. Soft emission in QED

III. In  $\text{QCD}$  . . .

IV. “Joint” resummation

V. Resummation for polarized scattering

VI. (Old and) new results on  $A_{TT}$

# I. Introduction : why resum ?

## QCD at short distances

- prerequisite : asymptotic freedom  $\Rightarrow$  pert. theory
- “infrared safe” observables

$$Q^2 \sigma_{\text{phys}}(Q) = \sum_n c_n \alpha_s^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

examples :  $e^+e^- \rightarrow$  hadrons, jets, event shapes, ...

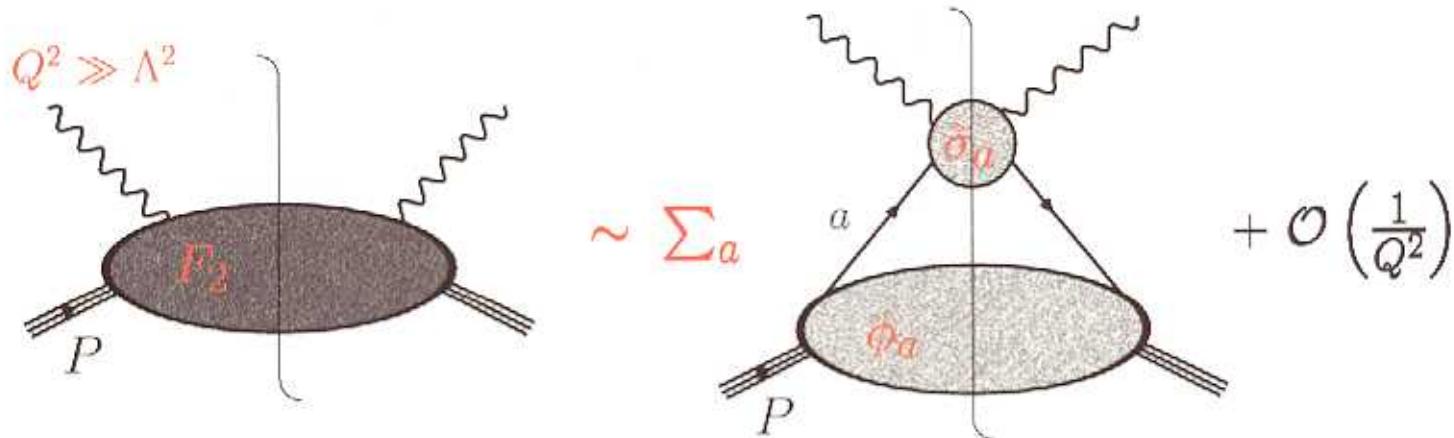
- extension : “factorizable” observables

$$Q^2 \sigma_{\text{phys}}(Q, m) = \mathcal{H}\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \otimes \mathcal{S}\left(\frac{\mu}{m}, \alpha_s(\mu)\right) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- short-distance “hard” part  $\mathcal{H}$  : perturbative, specific to process
- long-distance “soft” part  $\mathcal{S}$  : incalculable (at present), universal
- $\mu$  – factorization scale

examples : DIS  $ep \rightarrow e'X$ ,  $p\bar{p} \rightarrow$  jets + X,  $\gamma + X$ , ...

**example :** deeply-inelastic scattering



$$F_2 \sim \sum_a \int_x^1 \frac{d\xi}{\xi} \hat{\sigma}_a \left( \frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu) \right) \phi_a \left( \xi, \frac{\mu}{m}, \alpha_s(\mu) \right)$$

$$\equiv \sum_a \hat{\sigma}_a \otimes \phi_a$$

- PDF's  $\longleftrightarrow$  operator matrix elements :

e.g.  $q(\xi) = \frac{1}{4\pi} \int dy^- e^{iy^- \xi P} \langle P, S | \bar{\psi}(0) \gamma^+ \psi(0, y^-, 0_\perp) | P, S \rangle$   
 → nucleon structure

- factorization theorems :

extension to  $pp \rightarrow \text{jet} + X$ , Drell-Yan, etc.

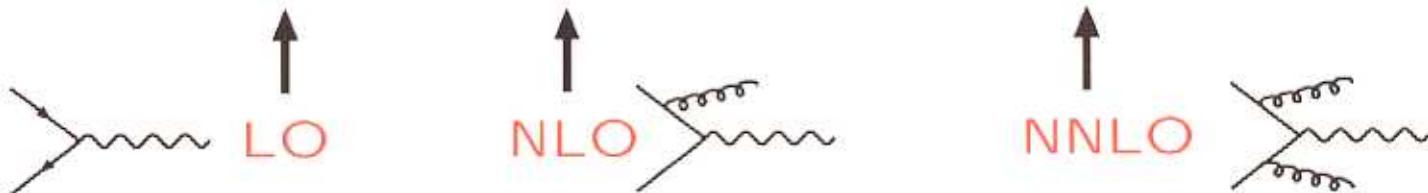
(Sterman, Libby; Ellis et al.; Amati et al.; Curci et al.;  
 Collins, Soper, Sterman; Bauer, Fleming, Pirjol, Rothstein, Stewart)

## The full picture : e.g. $pp \rightarrow ZX$

$$\sigma_{pp}^Z = \sum_{a,b} \phi_a(\mu) \otimes \phi_b(\mu) \otimes \hat{\sigma}_{ab}^Z \left( \frac{M_Z}{\mu}, \alpha_s(\mu) \right)$$

- perturbative cross sections :

$$\hat{\sigma} \left( \frac{M_Z}{\mu}, \alpha_s(\mu) \right) = \hat{\sigma}^{(0)} + \alpha_s(\mu) \hat{\sigma}^{(1)} \left( \frac{M_Z}{\mu} \right) + (\alpha_s(\mu))^2 \hat{\sigma}^{(2)} \left( \frac{M_Z}{\mu} \right) + \dots$$



to be calculated for each process !

- “DGLAP” evolution of PDF's :

$$\mu \frac{\partial}{\partial \mu} \phi_i(\mu) = \sum_j \phi_j(\mu) \otimes \gamma_{ij}(\alpha_s(\mu))$$

$$\gamma_{ij}(\alpha_s) = \frac{\alpha_s}{2\pi} \gamma_{ij}^{(0)} + \left( \frac{\alpha_s}{2\pi} \right)^2 \gamma_{ij}^{(1)} + \left( \frac{\alpha_s}{2\pi} \right)^3 \gamma_{ij}^{(2)} + \dots$$



universal (= same in all processes)

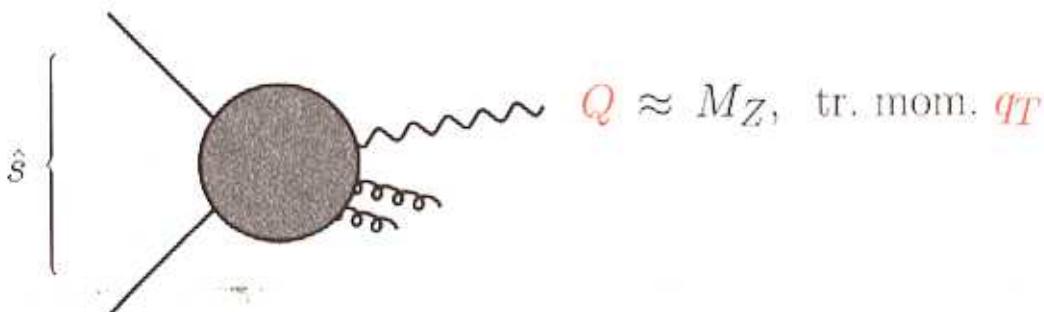
“state-of-the-art” : NLO

“fixed-order perturbation theory”

Despite presence of large scale  $Q$ , and despite being infrared-finite :

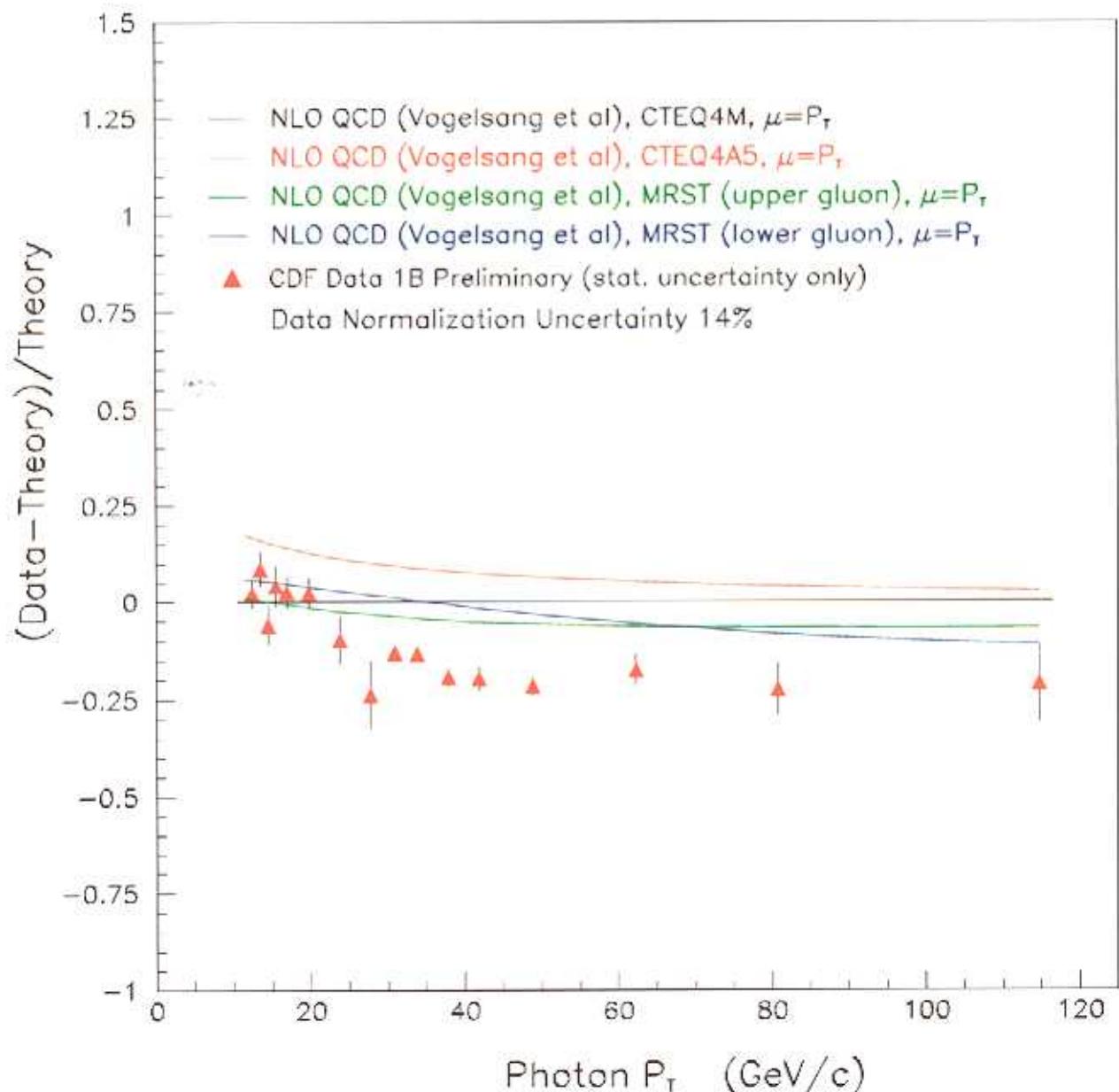
fixed-order perturbation theory for  $\hat{\sigma}$  may not always be adequate

- example :  $Z$  production via  $q\bar{q} \rightarrow Z + X$



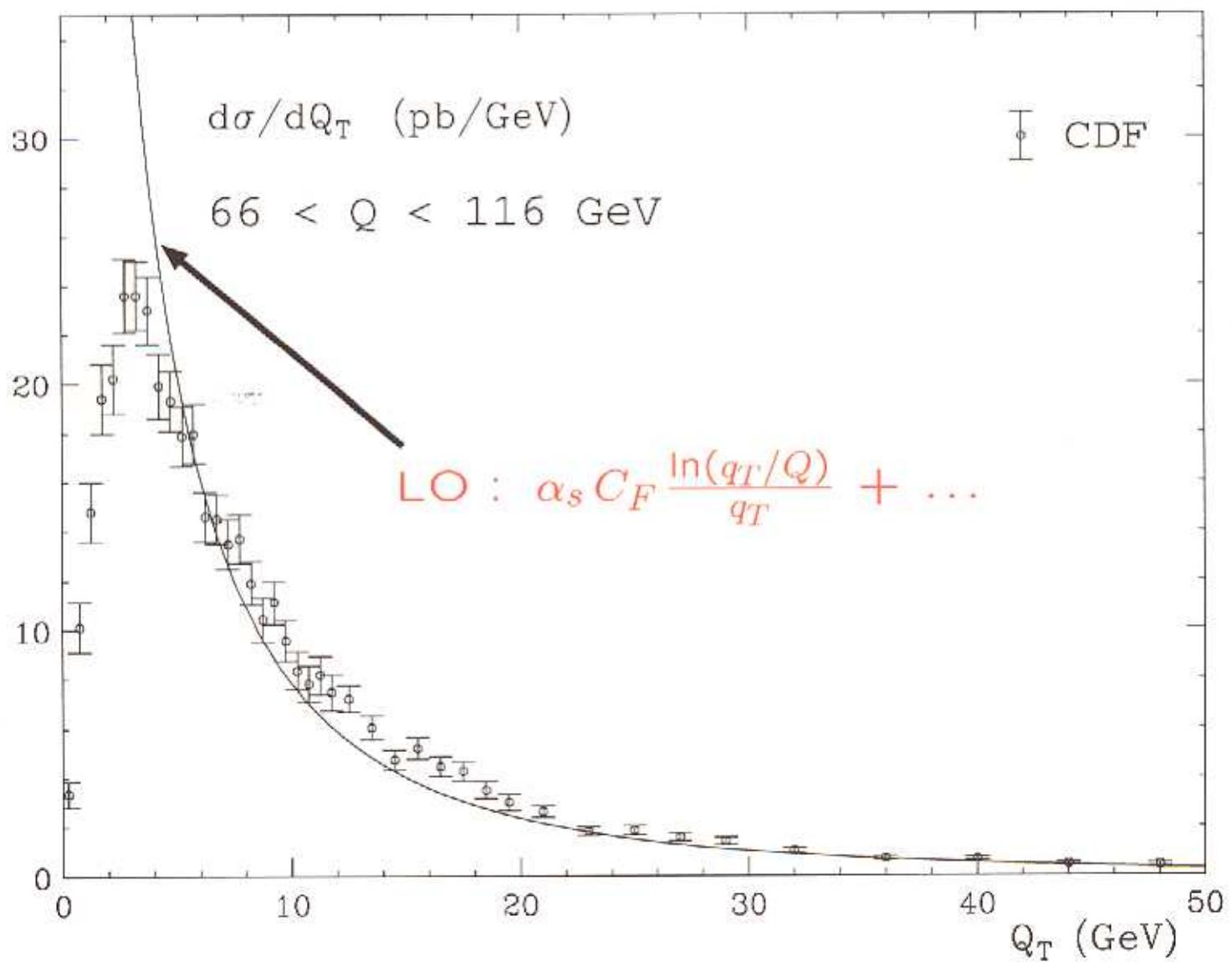
- when  $\hat{s} \rightarrow Q^2$  :  $\hat{\sigma} \sim \alpha_s^k \frac{\ln^{2k-1}(\hat{s}-Q^2)}{\hat{s}-Q^2} + \dots$
- “threshold logs” – just enough partonic energy
- origin : suppression of gluon radiation
- when  $q_T \rightarrow 0$  :  $\frac{d\hat{\sigma}}{dq_T} \sim \alpha_s^k \frac{\ln^{2k-1}(q_T/Q)}{q_T} + \dots$
- “ $q_T$  logs” – recoil against soft radiation
- real-virtual IR cancellations leave large logs  
→ may spoil expansion in  $\alpha_s(Q)$
- such corrections associated with soft and/or collinear emission  
→ can often be treated to all orders
- = resummation !

## CDF, Run-1B



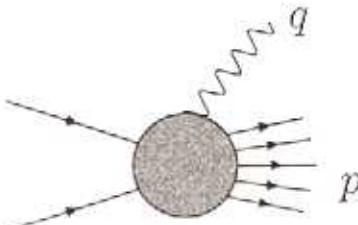
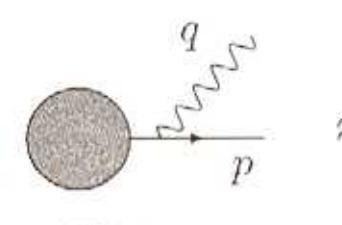
– a case for resummation ?

## Classic Example : Transverse-momentum distribution of Z bosons at the Tevatron



## II : Soft emission in QED

$$M^\mu(p_1, \dots, p_k; q) = \text{Diagram of a shaded circle with a wavy outgoing leg labeled } q \text{ and several straight outgoing legs labeled } p_i.$$


 $\approx_{q \rightarrow 0}$ 

 $\cdot e^{\frac{p^\mu}{p \cdot q} \epsilon_\mu(q)}$ 
↑  
eikonal factor

- single emission off all legs :

$$|M(p_1, \dots, p_k; q)|^2 \approx_{q \rightarrow 0} |M(p_1, \dots, p_k)|^2 \cdot d\rho_1(q)$$

$$d\rho_1(q) = \frac{\alpha}{\pi} \sum_{\text{legs } k} e_k^2 \frac{d\omega}{\omega} \frac{d\theta_{kq}^2}{\theta_{kq}^2}$$

- $n$  soft photons emitted:

$$d\rho_n(q_1, \dots, q_n) = \frac{1}{n!} \prod_{i=1}^n d\rho_1(q_i)$$

$$d\rho_n(q_1, \dots q_n) = \frac{1}{n!} \prod_{i=1}^n d\rho_1(q_i)$$

- factorization of dynamics
- exponentiation *if* photon phase space symmetric
- sometimes achievable by integral transforms :

$$\delta\left(2P \cdot \sum_j k_j - m^2\right) = \frac{1}{2\pi i Q^2} \int_C dN e^{-N\left(2P \cdot \sum_j k_j - m^2\right)/Q^2}$$

$$\delta\left(\vec{q}_T - \sum_j \vec{k}_T^j\right) = \frac{1}{(2\pi)^2} \int d^2 b e^{i \vec{b} \cdot (\vec{q}_T - \sum_j \vec{k}_T^j)}$$

- $\Rightarrow$  phase space factorizes :  
“factorization of kinematics”

### III. In QCD ...

- an emitted soft gluon carries color :

$$|M(\{p_i\}, q\rangle \approx_{q \rightarrow 0} \epsilon_\mu(q) \mathcal{J}_a^\mu |M(\{p_i\}\rangle$$

$$\mathcal{J}_a^\mu(q) = \sum_i T_a^i \frac{p_i^\mu}{p_i \cdot q} \quad \text{matrix}$$

- still, eikonal cross sections exponentiate in terms of “webs” (Gatheral; Frenkel, Taylor) schematically :

$$1 + C_{\textcircled{1}} \text{ (diagram)} + C_{\textcircled{0}} \text{ (diagram)} + C_{\textcircled{*}} \text{ (diagram)} + \dots$$

$$= \exp \left[ C_{\textcircled{1}} \text{ (diagram)} + (C_{\textcircled{*}} - C_{\textcircled{0}}) \text{ (diagram)} + \dots \right]$$

- in addition, running coupling

## IV : “Joint” resummation

Laenen, Sterman, WV :

logarithms at threshold and at  $q_T = 0$  can be resummed *simultaneously*

- “jointly” resummed cross section :

$$\frac{d\sigma^{\text{res}}}{dQ^2 dq_T^2} \propto \sum_q \int_C dN \left( \frac{Q^2}{S} \right)^{-N} \int d^2 b e^{-i \bar{q}_T \cdot \bar{b}} f_q^N(Q) f_{\bar{q}}^N(Q)$$

$$\times \exp \left\{ 2 \int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2)) \left[ J_0(b k_\perp) K_0 \left( \frac{2N k_\perp}{Q} \right) + \ln \left( \frac{N k_\perp}{Q} \right) \right] \right\}$$

- two inverse transforms ( $N, b$ )
- exponent approximated by

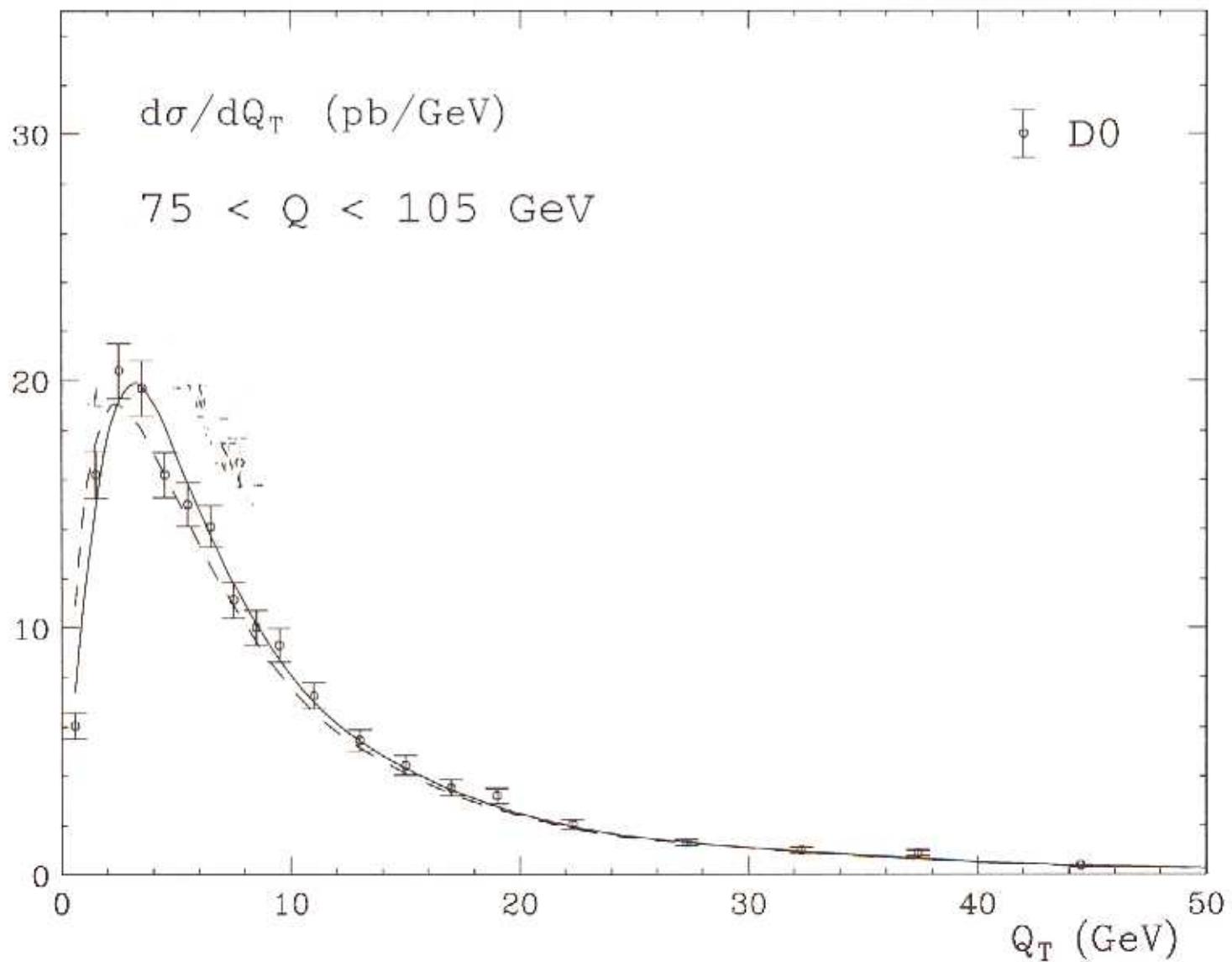
$$2 \int_{Q^2/\chi^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2)) \ln \left( \frac{k_\perp N}{Q} \right)$$

$$\chi \equiv \chi(N, \bar{b}) = \bar{b} + \frac{N}{1 + \bar{b}/4N}$$

- at very low  $q_T \rightarrow 0$  : expect non-pert. effects exponent gives guide to form of corrections :

$$\sim \left( \frac{b^2}{4} - \frac{N^2}{Q^2} \right) \int_0^\lambda dk_\perp k_\perp \alpha_s(k_\perp^2) \ln \left( \frac{N k_\perp}{Q} \right)$$

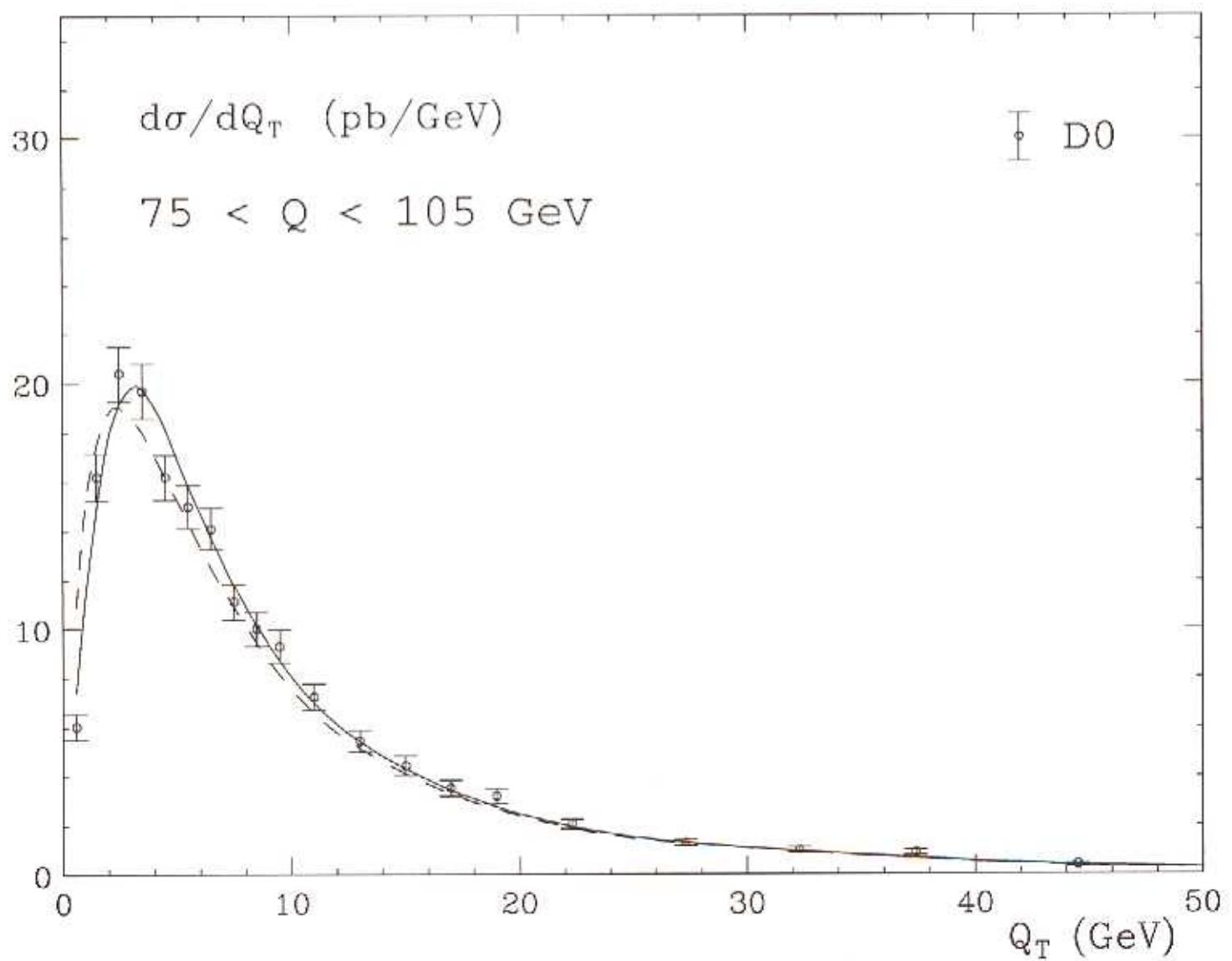
(Kulesza,Sterman,WV)  
(CTEQ5M pdfs)



dashed : “purely perturbative” resummed

solid : Gaussian smearing  $-gb^2$  with  $g = 0.8$  GeV<sup>2</sup>

(Kulesza, Sterman, WV)  
(CTEQ5M pdfs)



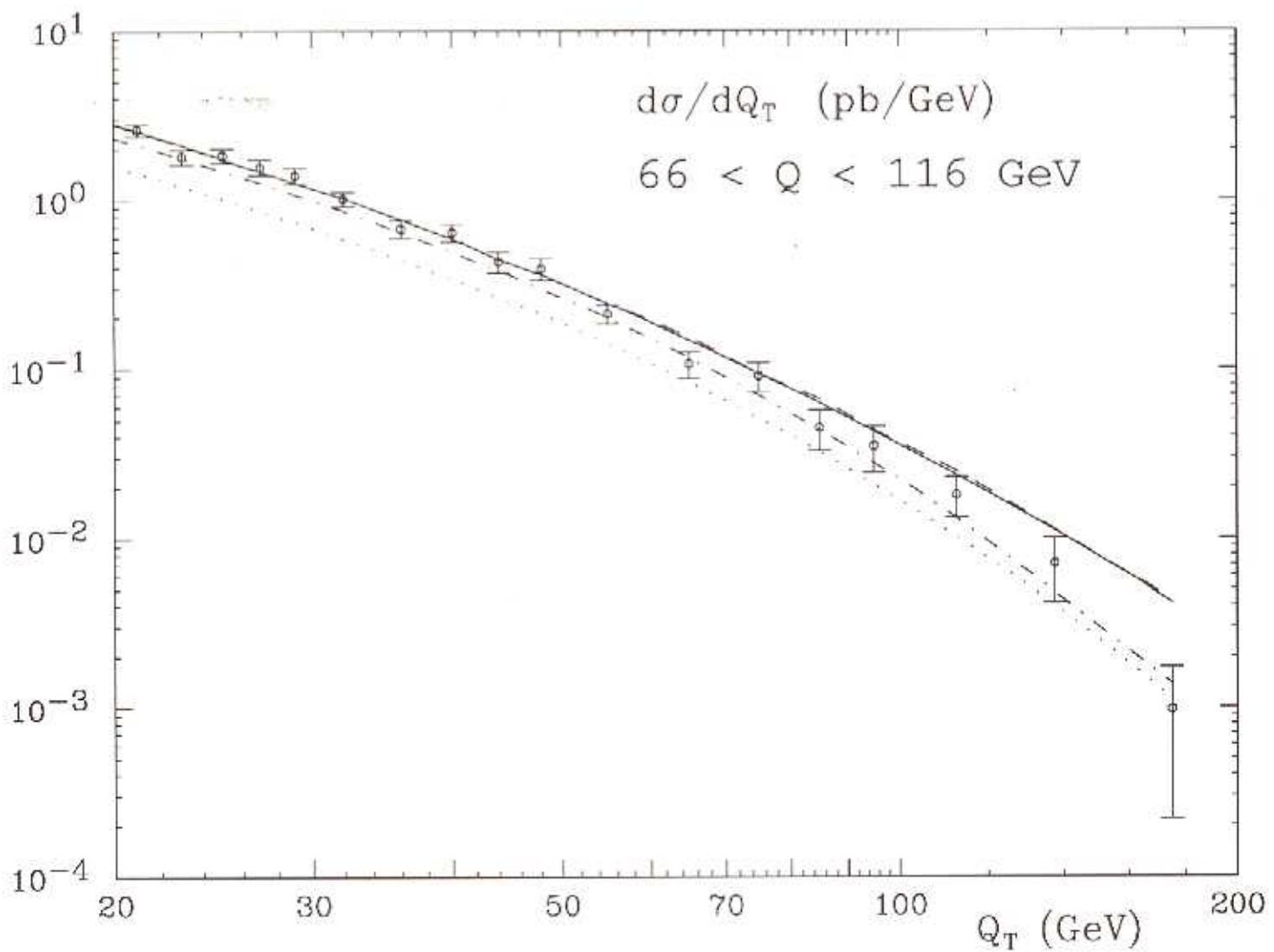
dashed : "purely perturbative" resummed

solid : Gaussian smearing  $-gb^2$  with  $g = 0.8$  GeV $^2$

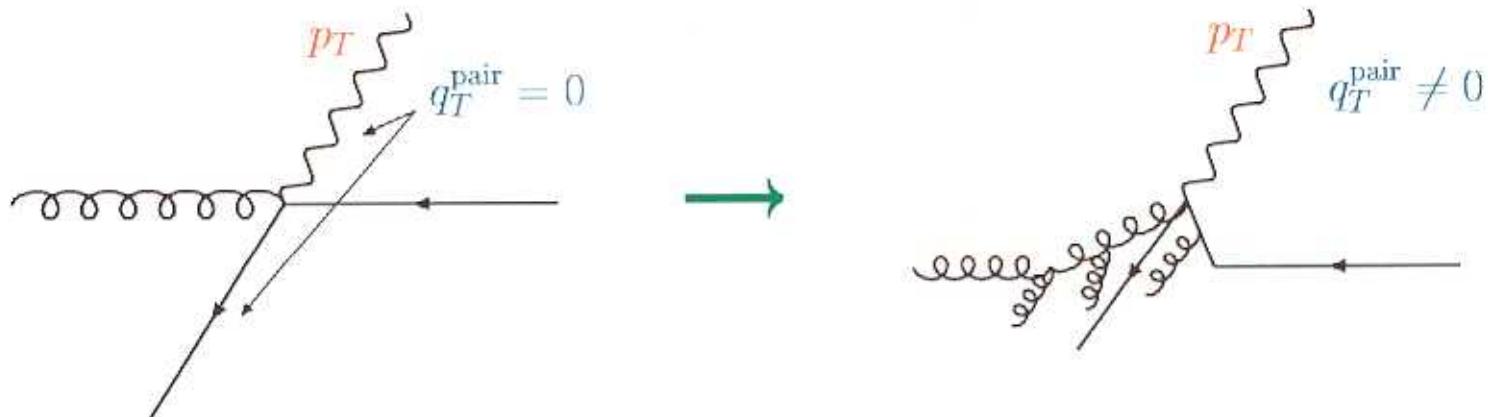
- at large  $q_T \sim Q$ , expect resummation to fail  
fixed-order more appropriate  
“matching” :

$$\frac{d\sigma}{dQ^2 dq_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2 dq_T^2} - \underbrace{\frac{d\sigma^{\text{exp(k)}}}{dQ^2 dq_T^2}}_{\text{expansion of res.}} + \underbrace{\frac{d\sigma^{\text{fixed(k)}}}{dQ^2 dq_T^2}}_{\text{fixed o.}}$$

→ no double-counting

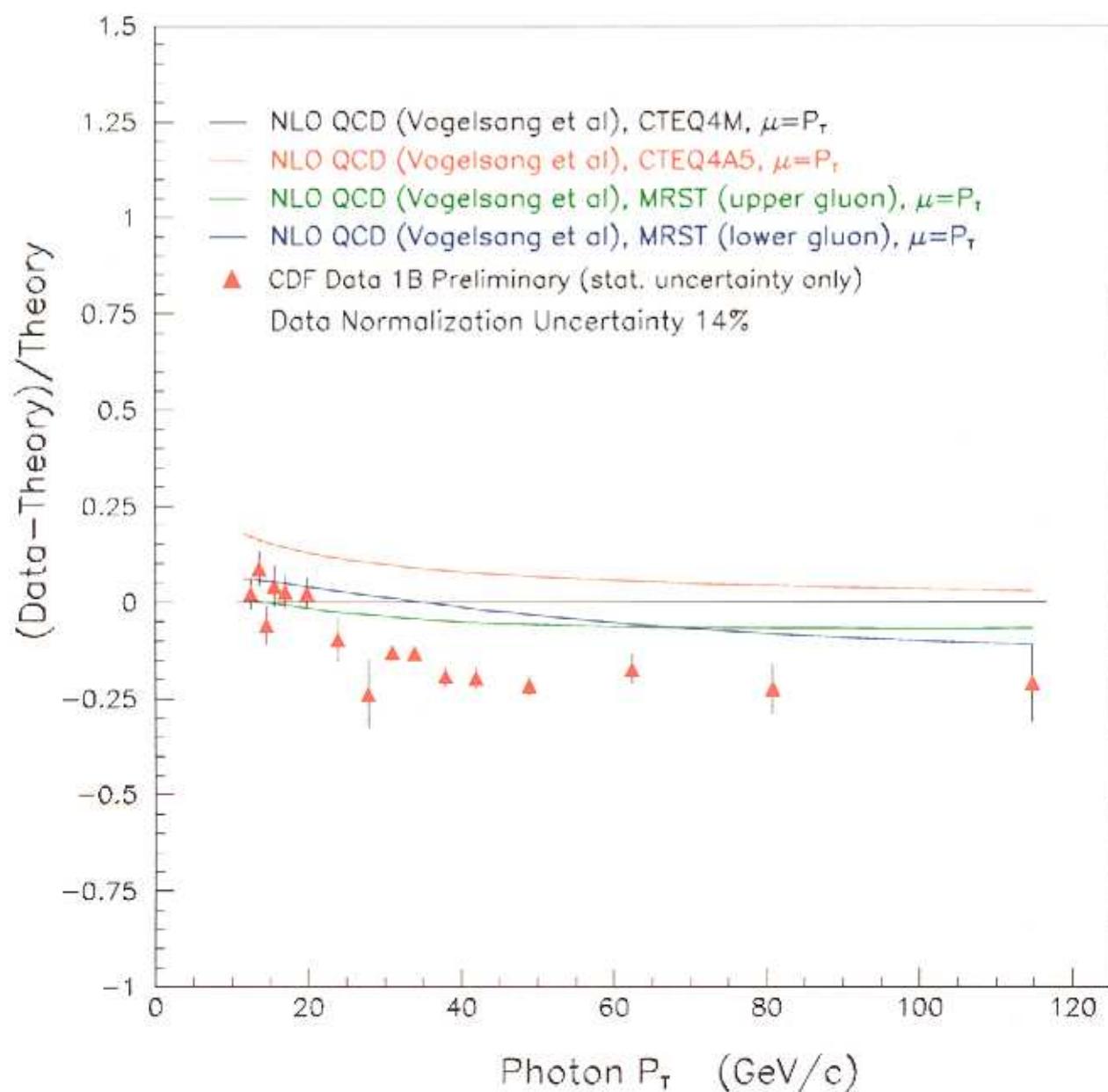


- extension to single-inclusive cross sections ,  
for example direct photons  $pp \rightarrow \gamma X$

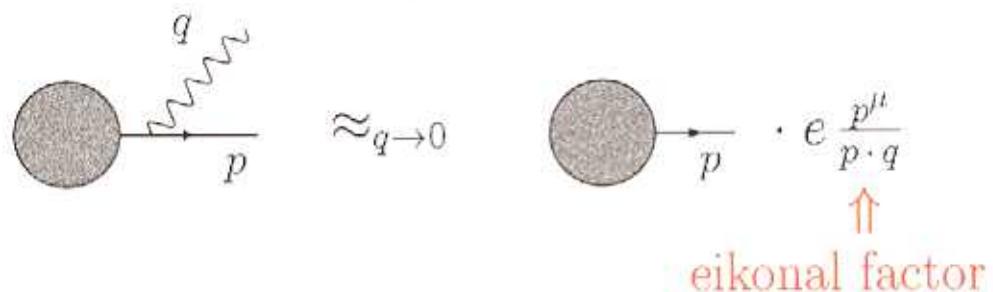


- \* motivated by phenomenologically observed  
“intrinsic- $k_T$  smearing” (Apanasevich et al.)
- \*  $q_T$  and/or “joint” resummations for photons :  
Laenen, Sterman, WV; Li; Fink, Owens
- phenomenological studies of these ideas in progress
- large effects likely, promising
- need to put “matching” on firmer basis
- non-perturbative effects also here

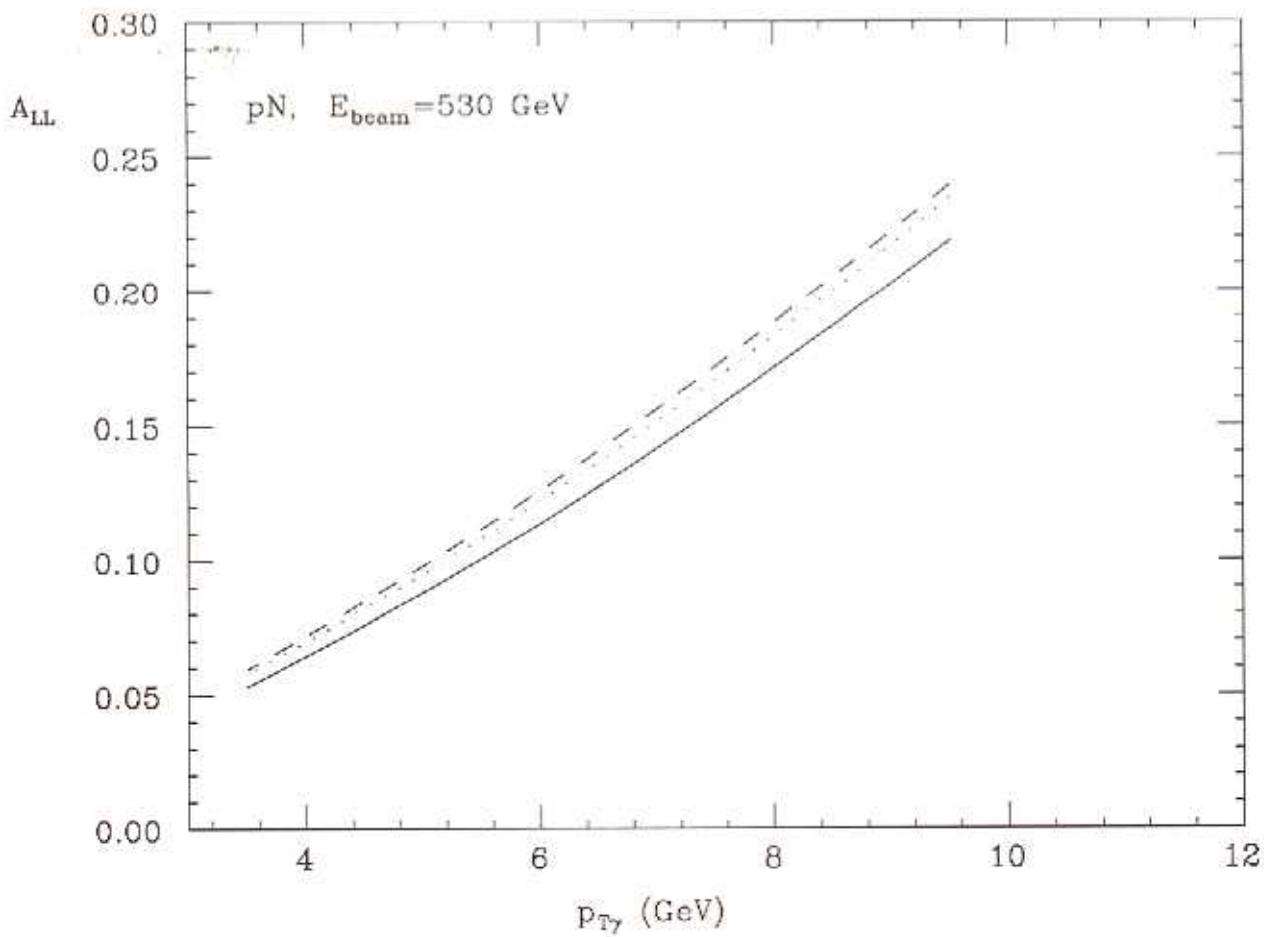
## CDF, Run-1B



## V. Resummation for polarized scattering



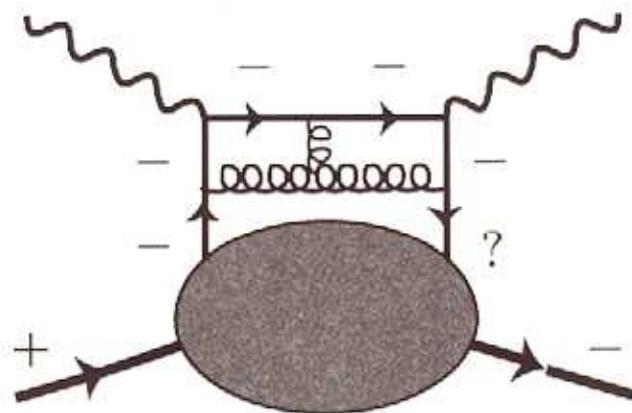
eikonal factor : spin-*independent*



- dotted : LO,      dashed : NLO
- solid : threshold-resummed

## VI. New results for $A_{TT}$

Helicity flip required  $\Rightarrow \delta q$  not in incl. DIS :

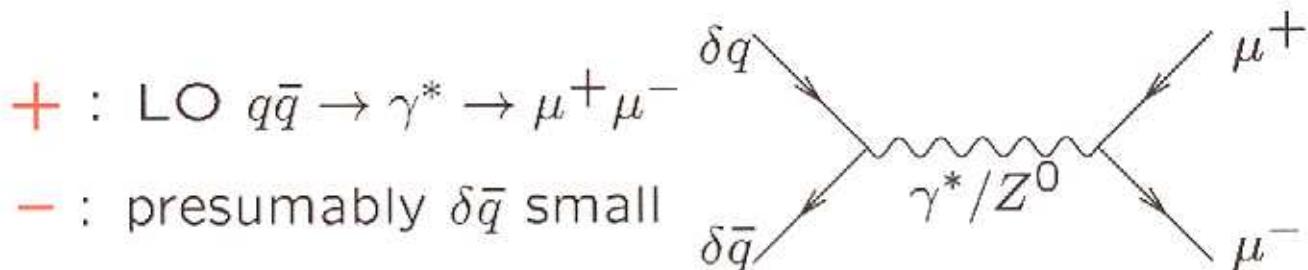


Possibilities (among others) :

- Collins effect, interference fragmentation, etc.
  - require independent measurement of a fragmentation function
  - often not the only mechanism involved  
(see recently Brodsky,Hwang,Schmidt; Collins)
- collisions of transversely pol. protons at RHIC

$$A_{TT} = \frac{d\sigma^{p\uparrow p\uparrow} - d\sigma^{p\uparrow p\downarrow}}{d\sigma^{p\uparrow p\uparrow} + d\sigma^{p\uparrow p\downarrow}}$$

Drell-Yan dimuon production,  $pp \rightarrow \mu^+ \mu^- X$



- dir. photons, jets, inclusive hadrons, ... :

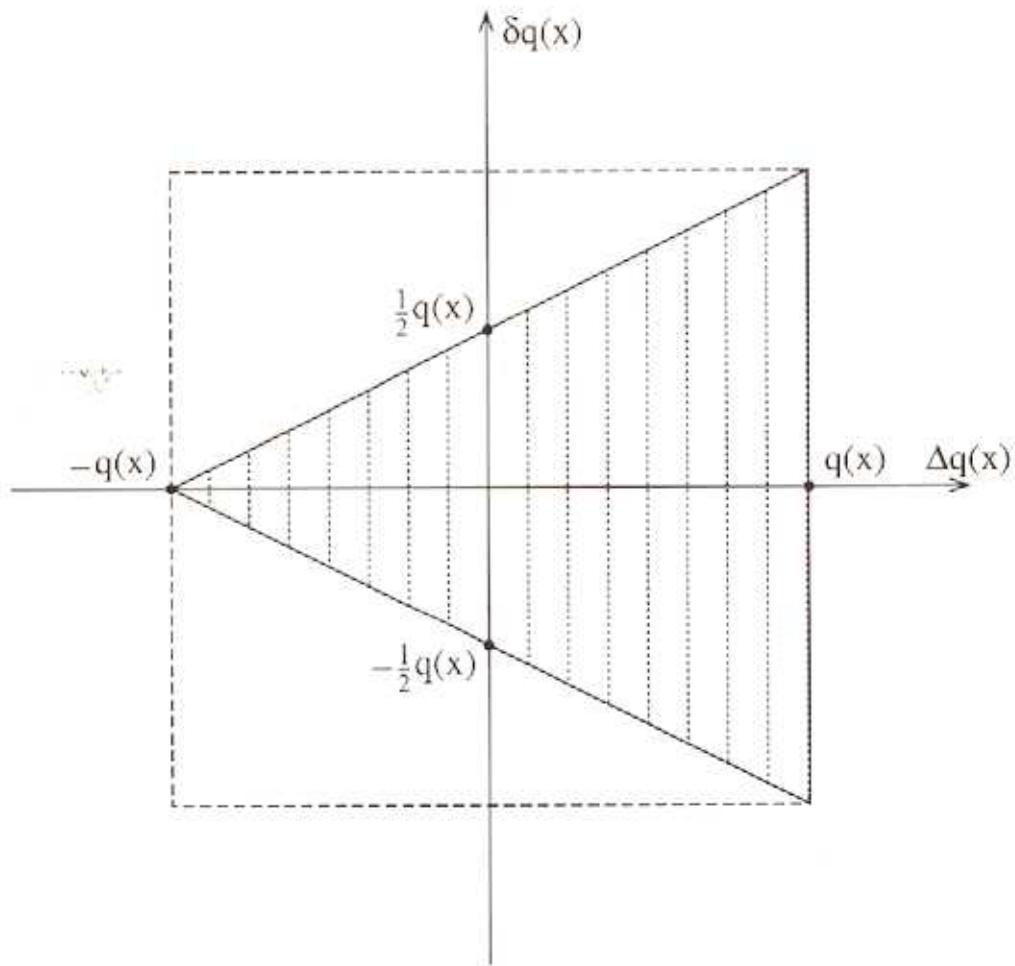
*Re-emphasize* :  $A_{TT}$  is expected small,  
 $"A_{TT} \ll A_{LL}"$  (Jaffe,Saito)

- no gluon transversity, however, gluon contribution to unpolarized cross section !
- relevant hard scattering cross sections typically color-suppressed
- Soffer's inequality limits size of  $\delta q$ 
  - + rates can be substantial  
⇒ small asymmetries *may* be measurable

- estimate “upper bounds” on  $A_{TT}$  by saturating Soffer's inequality at  $\mu_0 \sim 0.6$  GeV (Soffer,Stratmann,WV)
- “hard to imagine” that  $A_{TT}$  could be much bigger !

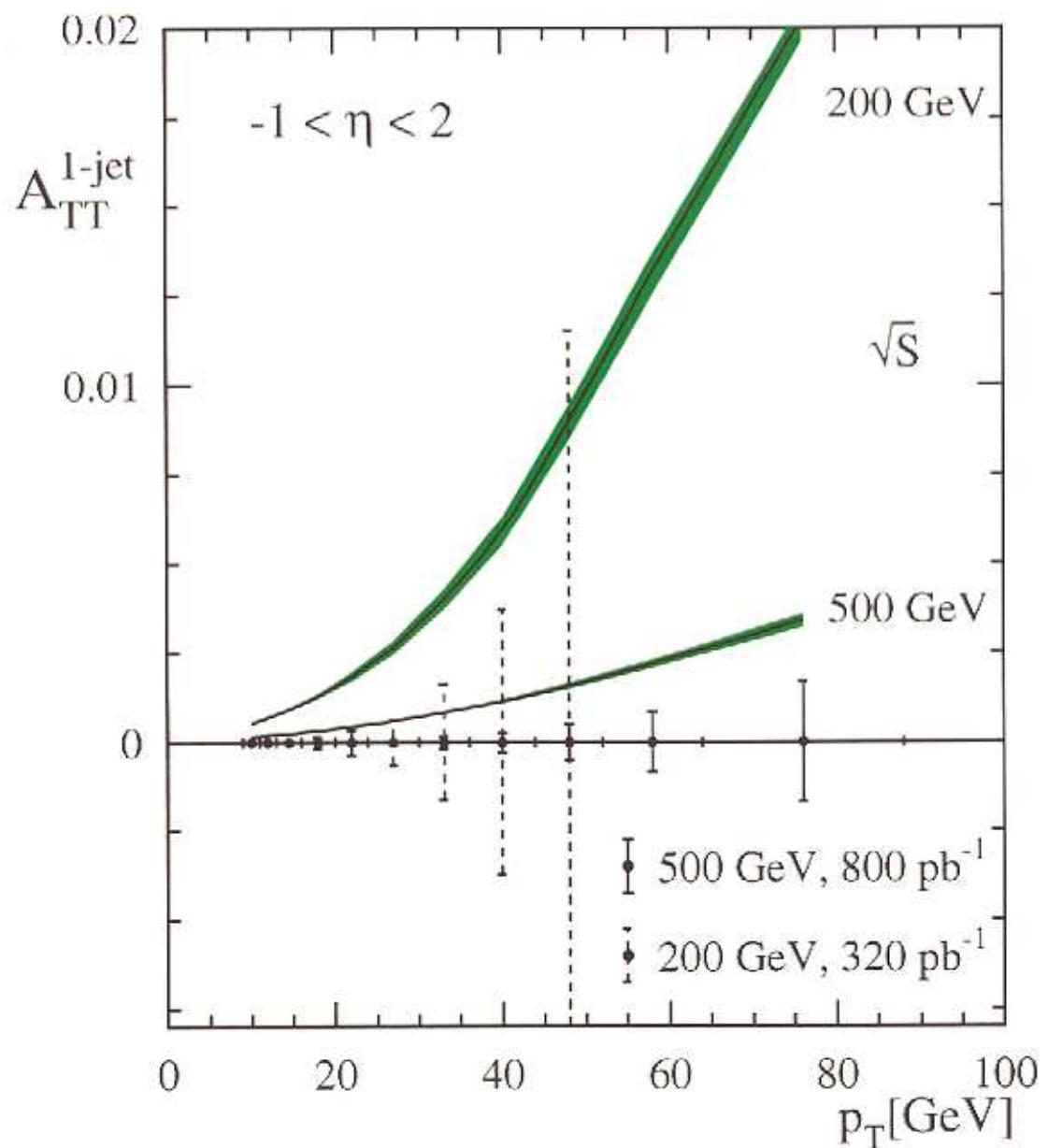
From helicity structure :  
Soffer's inequality

$$2 |\delta q(x, \mu)| \leq q(x, \mu) + \Delta q(x, \mu)$$



- for all flavors
- constraint for models for transversity
- preserved under QCD evolution  
(Barone; Bourrely, Soffer, Teryaev;  
Martin, Schäfer, Stratmann, WV) 297

(Soffer,Stratmann,WV)



(Soffer,Stratmann,WV)

