Theory topics

in year-1 and beyond

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Outline :

I. Introduction : "Basic theory"

II. $A_{\rm LL}^{\pi}$ at RHIC

III. "Global analysis"

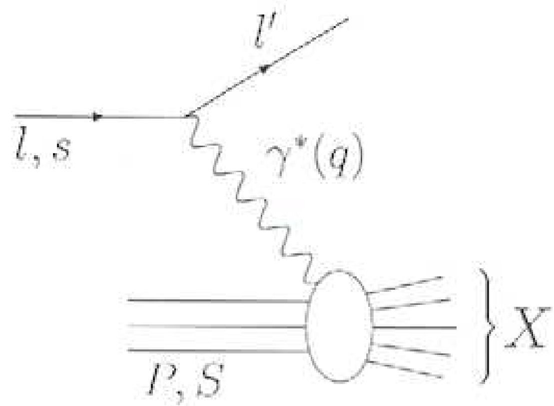
IV. Towards high orders

V. Transverse *two*-spin asymmetries

VI. $A_{\rm N}^{\pi}$ at RHIC

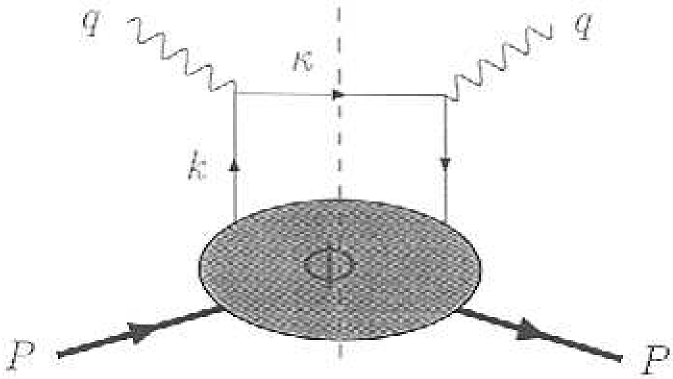
I. Introduction : "Basic theory"

polarized DIS :



$$\mathcal{W}_A^{\mu\nu}(P,q,S) = i\,M\,\varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\,q}\, g_1(x,Q^2) + \frac{S_\sigma(P\,q) - P_\sigma(S\,q)}{(P\,q)^2}\, g_2(x,Q^2) \right]$$

Parton model :



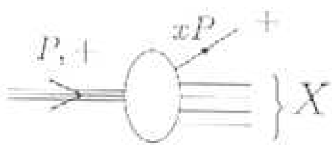
$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta\big((k+q)^2\big)\, \mathrm{Tr}\left[\Phi\, \gamma^\mu (\not k + \not q) \gamma^\nu\right]$$

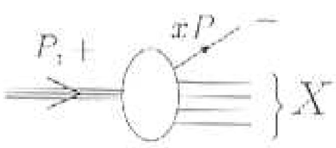
$$\Phi_{ij}(k,P,S) \;=\; \int \mathrm{d}^4\xi\; \mathrm{e}^{\mathrm{i}k\cdot\xi}\; \langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle$$

leading contributions, $k^\mu \sim x\,P^\mu$

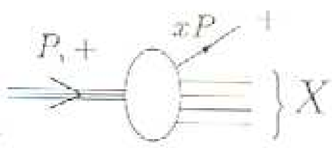
$$\Phi \;=\; \frac{1}{2}\left[\, q(x)\,\not{P} + \lambda\,\Delta q(x)\,\gamma_5\,\not{P} + \delta q(x)\,\not{P}\,\gamma_5\,\not{S}_\perp \right]$$

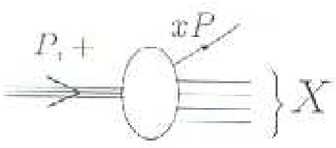
where

$q(x) = \Big|$ 

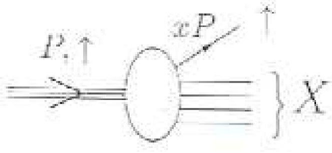
$\Big|^2 + \Big|$ 

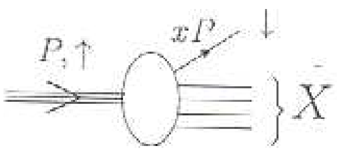
$\Big|^2$

$\Delta q(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

$\delta q(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

Probabilities –
map of nucleon spin structure at leading power !

These can be expressed as

$$\begin{aligned}
q\,(x) &= \frac{1}{4\pi}\int d\xi^-\, \mathrm{e}^{i\xi^- xP^+}\, \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\,(0,\xi^-,0_\perp)\,|P,S\rangle \\
\Delta q\,(x) &= \frac{1}{4\pi}\int d\xi^-\, \mathrm{e}^{i\xi^- xP^+}\, \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\,(0,\xi^-,0_\perp)\,|P,S\rangle \\
\delta q\,(x) &= \frac{1}{4\pi}\int d\xi^-\, \mathrm{e}^{i\xi^- xP^+}\, \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\,\gamma_5\,\psi\,(0,\xi^-,0_\perp)\,|P,S\rangle
\end{aligned}$$

In full QCD :

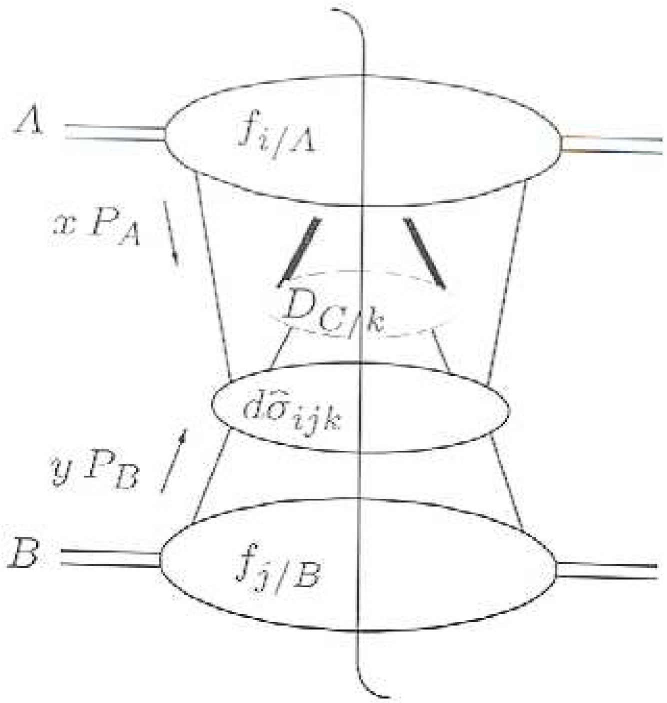
- definitions of pdf's (essentially) same
- gluon distributions $\sim$ F.T.$(G^{+j}\,G_{+j})$ etc.
- in $A^+ = 0$ gauge; to be made gauge-invariant
- "moments" $\rightsquigarrow$ local operators,
  e.g. $\int_0^1 dx\,\Delta q(x) \;\propto\; \langle P,S\,|\bar{q}\,\gamma^\mu\,\gamma^5\,q|\;P,S\rangle$
- operators involved require renormalization
  $\Rightarrow$ scale-dependent $q(x,\mu),\ldots$: "evolution"
- definitions vital in proofs of factorization theorems
- a measure of nucleon structure !

Collinear factorization :

(Sterman,Libby; Ellis et al.; Amati et al.; Curci et al.; Collins,Soper,Sterman; Collins)

Hard scale ($M = Q,\ p_T,\ m_{\rm HQ},\ \ldots$)

$$d\sigma_{AB\to C(M)X} = \sum_{ijk}\int dx\, dy\, dz\ f_{i/A}(x,\mu)\ f_{j/B}(y,\mu)$$
$$\times\ d\hat{\sigma}_{ijk}(xP_A, yP_B, P_C/z, \mu, \alpha_s(\mu))\ D_{C/k}(z,\mu)$$



... up to inverse powers in hard scale M

- partonic cross sections $d\hat{\sigma}_{ijk}$ are perturbative, and predicted by QCD

$$d\hat{\sigma}_{ijk} = d\hat{\sigma}^{(0)}_{ijk} + \frac{\alpha_s}{2\pi} d\hat{\sigma}^{(1)}_{ijk} + \ldots$$

- pdfs are *universal* : same in all reactions (holds separately for unpolarized, longitudinally pol., transversity)

- $\Rightarrow$ notion of "nucleon structure" meaningful

- allows tests of QCD

- enables us to look for / study new things
  - fragmentation (spin) effects
  - polarized photon structure
  - physics beyond the Standard Model

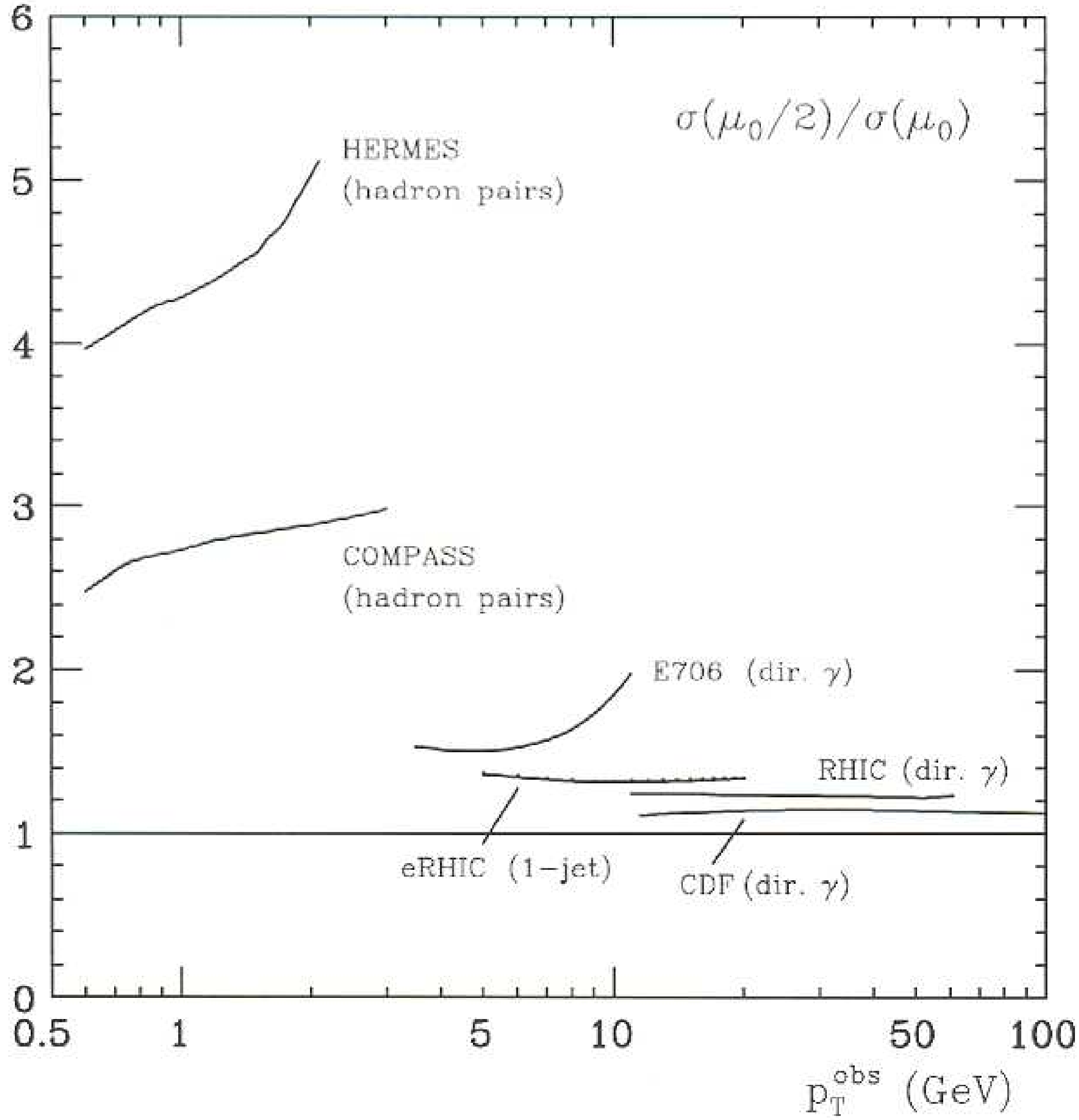
- scale μ is arbitrary; should be $\sim M$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} d\sigma_{AB} = 0$$

at *finite* order, residual scale dependence; to decrease with each new order of PT

Example :

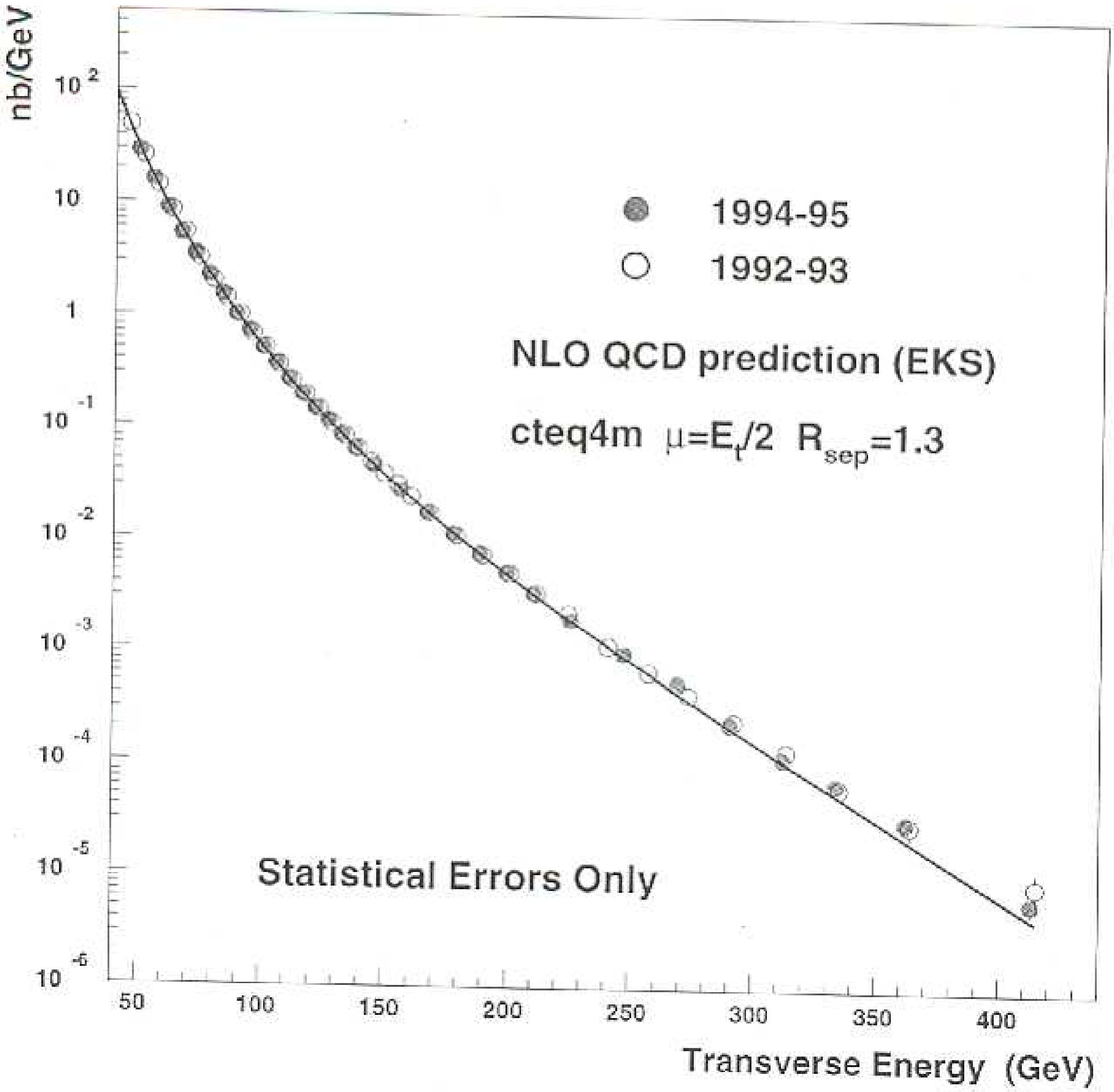
scale dependence of unpolarized cross sections relevant for measurements of gluon density



(de Florian,Stratmann,WV)

II. A_{LL}^{π} at RHIC

High-p_T jets at the Tevatron :



(among other things)

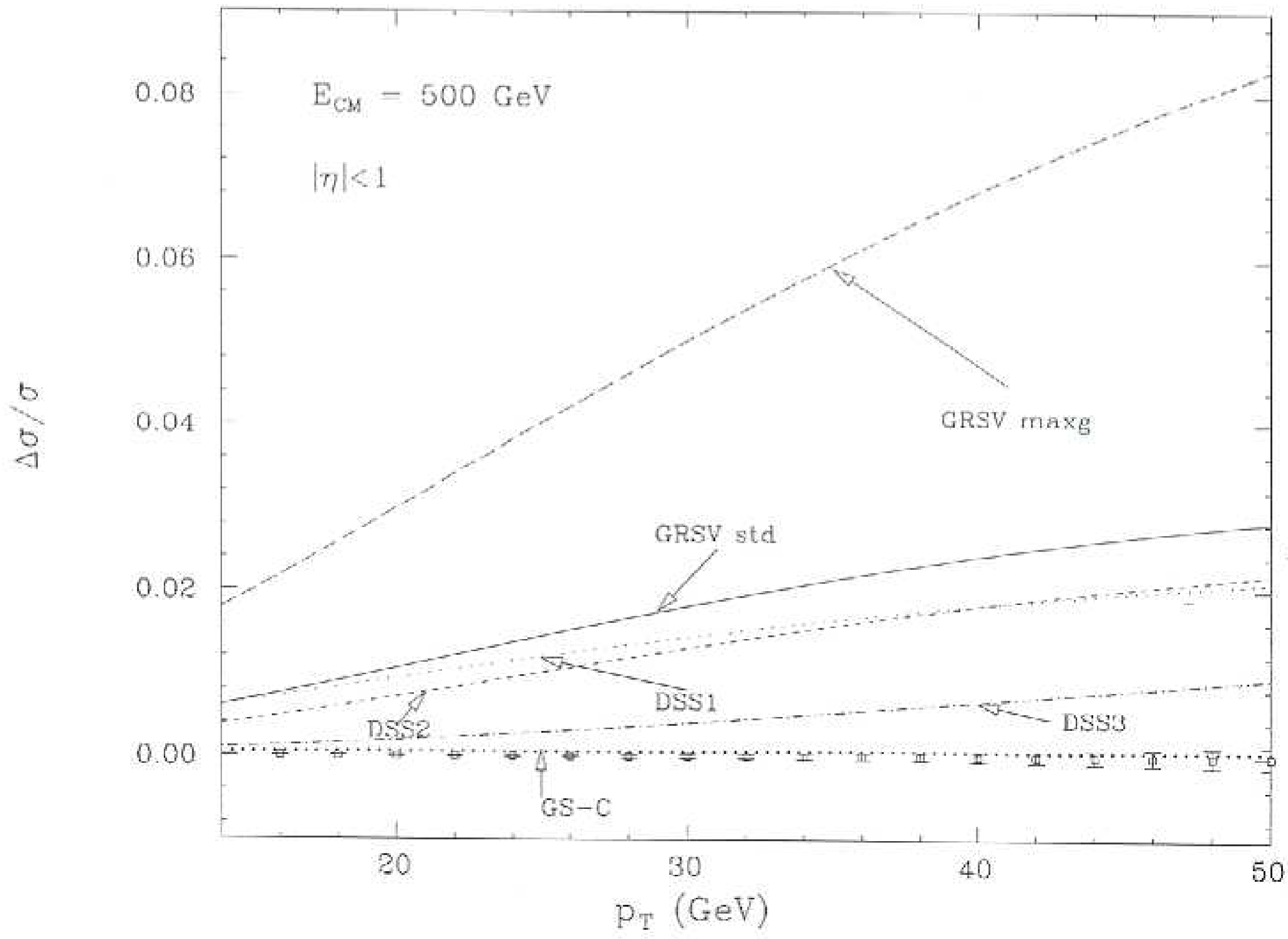
- source of information on gluon density

Jet production at RHIC :

Spin asymmetry

$$A_{LL}^{\text{jet}} = \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}} \equiv \frac{d\Delta\sigma}{d\sigma}$$

gives access to Δg :

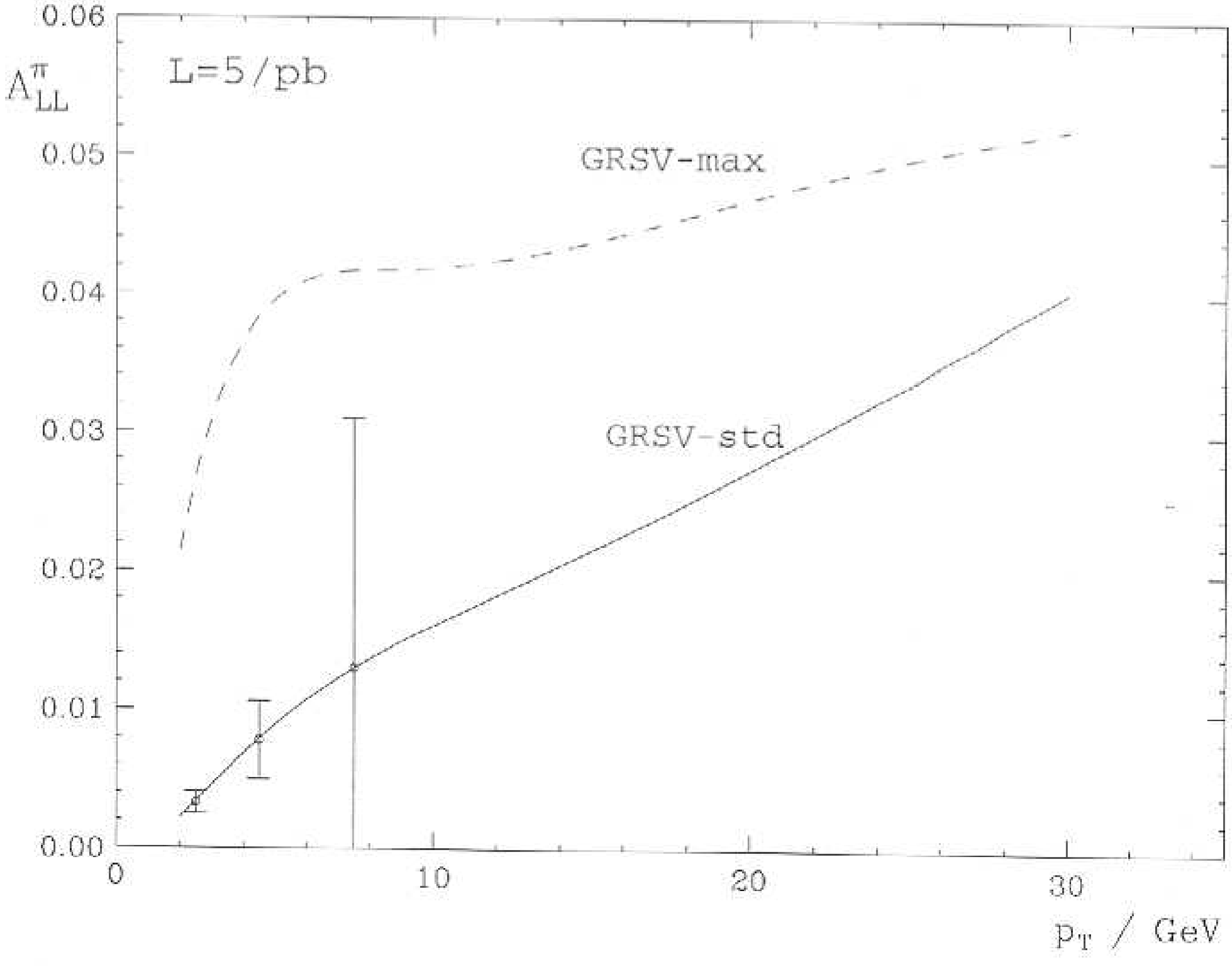


NLO (de Florian, Frixione, Signer, W.V.)

leading pions as jet surrogates :

- useful at lower energies
- can go to lower p_T ($\sim$ few GeV)
- do not require "4π" angular coverage

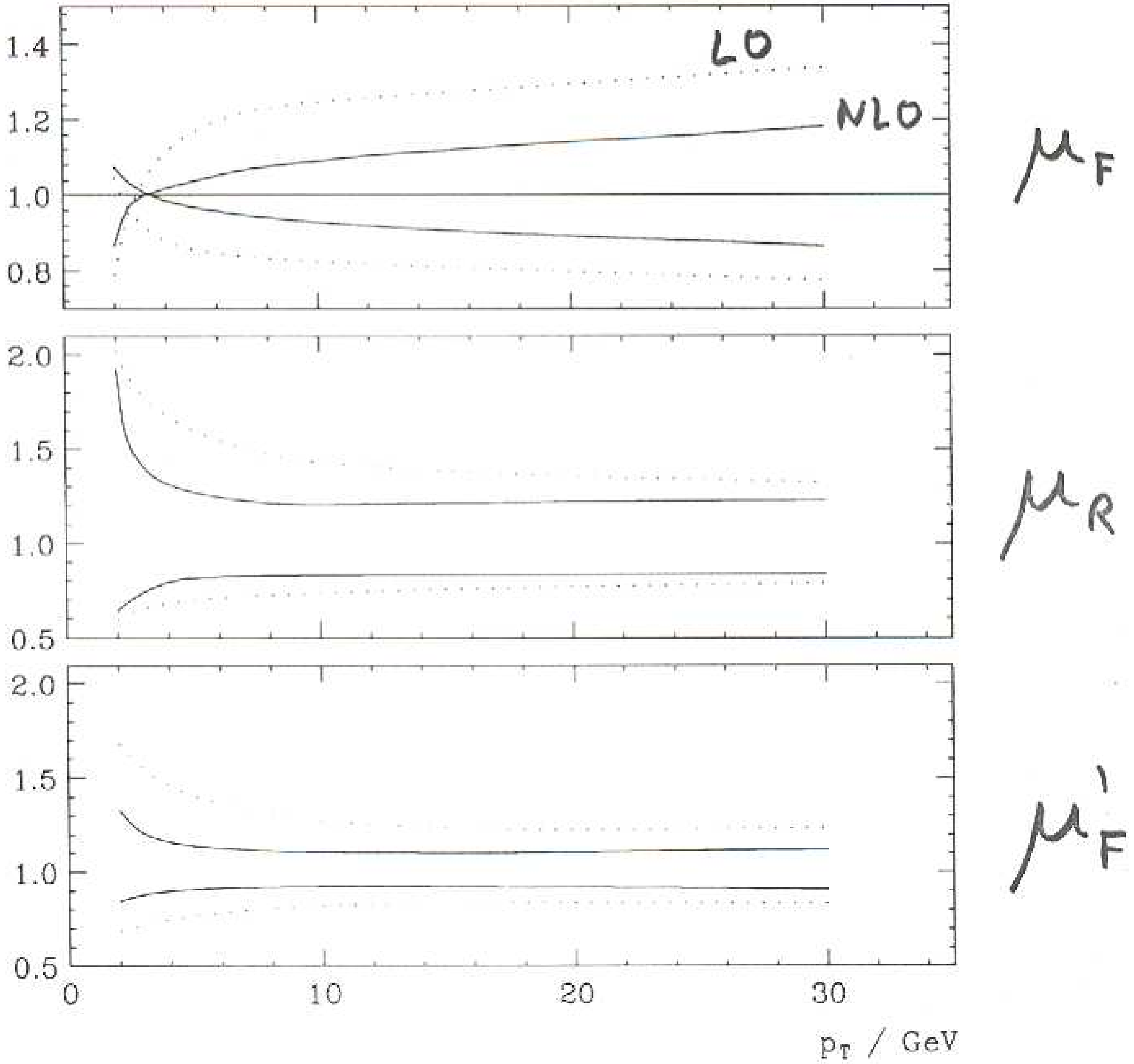
Pion production at RHIC
spin asymmetry sensitive to Δg :



$P_T/2 \;,\; 2P_T$

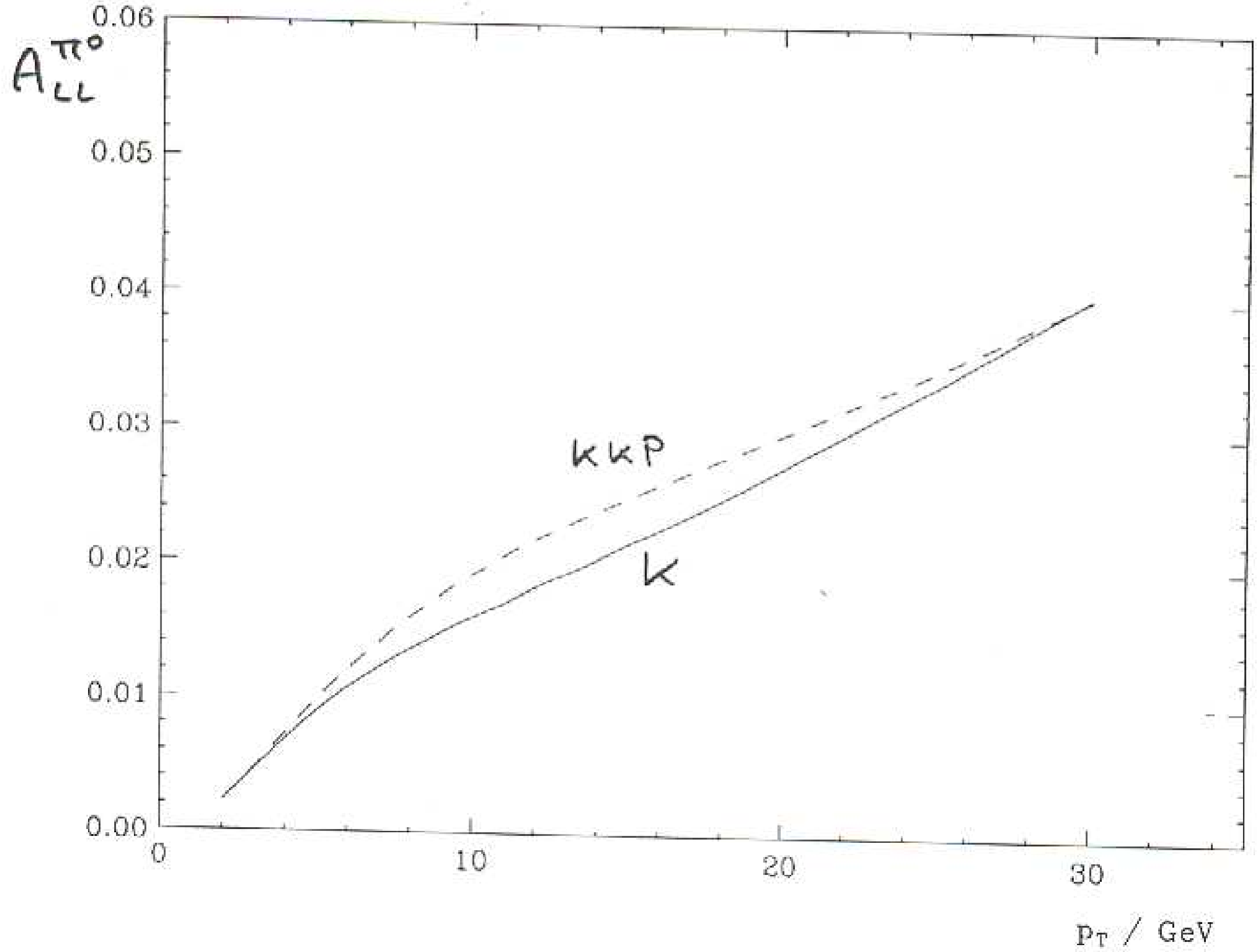
$\sigma(\mu) / \sigma(\text{all } \mu = P_T)$

Scale dependence

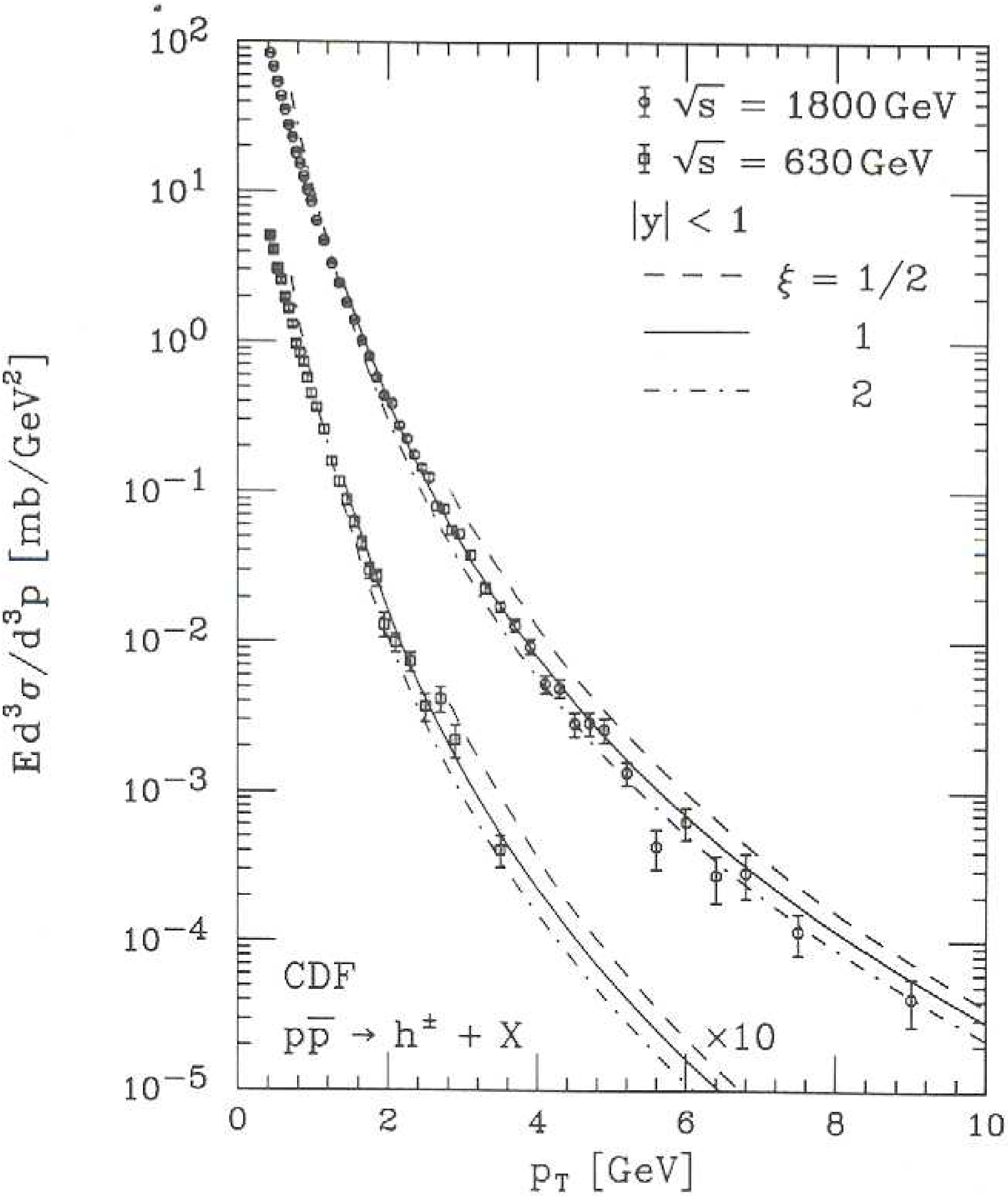


- same in polarized case ?

Spin asymmetry and fragmentation fcts. :



Comparison to data from unpol. colliders :
Kniehl, Kramer, Pötter



III. "Global analysis"

To learn from data need to efficiently evaluate

$$\sigma = \sum_{ijk} f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k$$

- a lesson from unpolarized case
- need "global analysis"
  - input pdfs at scale μ_0 in terms of ansatz with free parameters
  - evolve to scale μ relevant to a data point
  - compare to data and assign χ^2 value
  - vary parameters and minimize χ^2
- requires typically 1000's of evaluations of the cross section
- want $\hat{\sigma}_{ijk}$ at order "as high as possible"
  - theoretical uncertainties decrease
  - but already NLO often numerically involved and time-consuming
- *very* hard to reconcile

unpolarized case :

- gross features of pdfs known
- often ok to use LO $\to$ NLO 'K' factors
- even here, not always the case
  (gluon distribution at large x)

polarized case :

- pdfs known with *much less* accuracy
- pdfs and partonic cross sections
  may have zeros !
  $\Rightarrow$ locally large NLO corrections possible
  (bad convergence of fits based on K fact.)
- want fast and practical way of using exact
  higher-order cross sections in global fits

$\to$ recent work with Marco Stratmann

(similar efforts : AAC, M. Hirai et al.)

"Mellin technique"

(earlier ideas : Berger, Graudenz, Hampel, Vogt; Kosower)

Moments of a function $f(x)$:

$$f^{n} \equiv \int_0^1 dx\, x^{n-1}\, f(x)$$

Consider general cross sec. for producing final state H with observed variable O

$$\frac{d\sigma^H}{dO} = \sum_{a,b,c} \int_{\text{exp-bin}} dT \int_{x_a^{\text{min}}}^1 dx_a \int_{x_b^{\text{min}}}^1 dx_b \int_{z_c^{\text{min}}}^1 dz_c$$

$$\times\, f_a(x_a,\mu_F)\, f_b(x_b,\mu_F)\, D_c^H(z_c,\mu_F')$$

$$\times\, \frac{d\hat{\sigma}_{ab}^c}{dOdT}(x_a P_A, x_b P_B, P_H/z_c, T, \mu_R, \mu_F, \mu_F')\ ,$$

Express pdfs by their Mellin inverses :

$$f_a(x_a,\mu_F) = \frac{1}{2\pi i}\int_{\mathcal{C}_n} dn\, x_a^{-n}\, f_a^{n}(\mu_F)$$

$$f_b(x_b,\mu_F) = \frac{1}{2\pi i}\int_{\mathcal{C}_m} dm\, x_b^{-m}\, f_b^{m}(\mu_F)$$

Find :

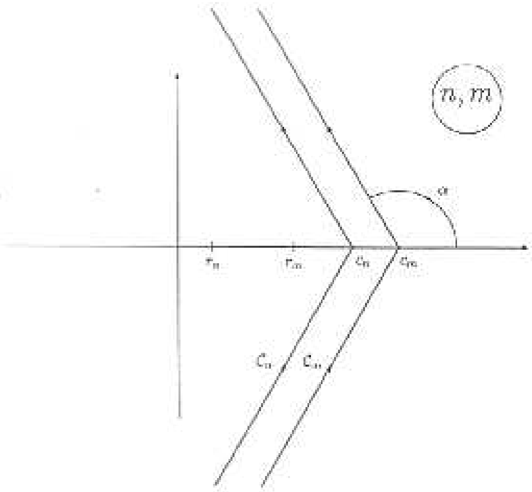
$$\frac{d\sigma^H}{dO} = \frac{1}{(2\pi i)^2}\sum_{a,b,c}\int_{C_n} dn \int_{C_m} dm\, f_a^n(\mu_F)\, f_b^m(\mu_F)$$

$$\times \int_{\text{exp-bin}} dT \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 dx_b \int_{z_c^{\min}}^1 dz_c\; x_a^{-n} x_b^{-m}\; D_c^H(z_c, \mu'_F)$$

$$\times \frac{d\tilde{\sigma}_{ab}^c}{dOdT}(x_a P_A,\, x_b P_B\,,\, P_H/z_c, T,\, \mu_R, \mu_F, \mu'_F)$$

$$\equiv \sum_{a,b}\int_{C_n} dn \int_{C_m} dm\; f_a^n(\mu_F)\, f_b^m(\mu_F)\; \tilde{\sigma}_{ab}^H(n, m, O, \mu_R, \mu_F)$$

- $\tilde{\sigma}_{ab}^H(n, m, O, \mu_R, \mu_F)$ is cross section for "dummy" pdfs $x_a^{-n} \times x_a^{-m}$
- contains all tedious integrations
- can be pre-calculated on a suitable grid in n, m
- for optimal contours, *exponential decrease* of x_a^{-n}, x_a^{-m} along contours
- pdfs fall off at least as fast as $1/|n|^4$, $1/|m|^4$



Finally, n, m integrations are all that's left !

Example : Prompt γ at RHIC

$$\frac{d\Delta\sigma^{\gamma}}{dp_T} = \sum_{a,b} \int_{\eta-\text{bin}} d\eta \int_{x_a^{\min}}^{1} dx_a \int_{x_b^{\min}}^{1} dx_b \, \Delta f_a(x_a,\mu_F)\, \Delta f_b(x_b,\mu_F)$$

$$\times \frac{d\Delta\hat{\sigma}^{\gamma}_{ab}}{dp_T d\eta}(x_a P_A,\, x_b P_B\,, p_T, \eta,\, \mu_R, \mu_F)$$

$$d\Delta\hat{\sigma}^{\gamma}_{ab} = \underbrace{d\Delta\hat{\sigma}^{\gamma,(0)}_{ab}}_{\text{LO}} + \left(\frac{\alpha_s}{\pi}\right) \underbrace{d\Delta\hat{\sigma}^{\gamma,(1)}_{ab}}_{\text{NLO}} + \dots$$

Toy analysis :

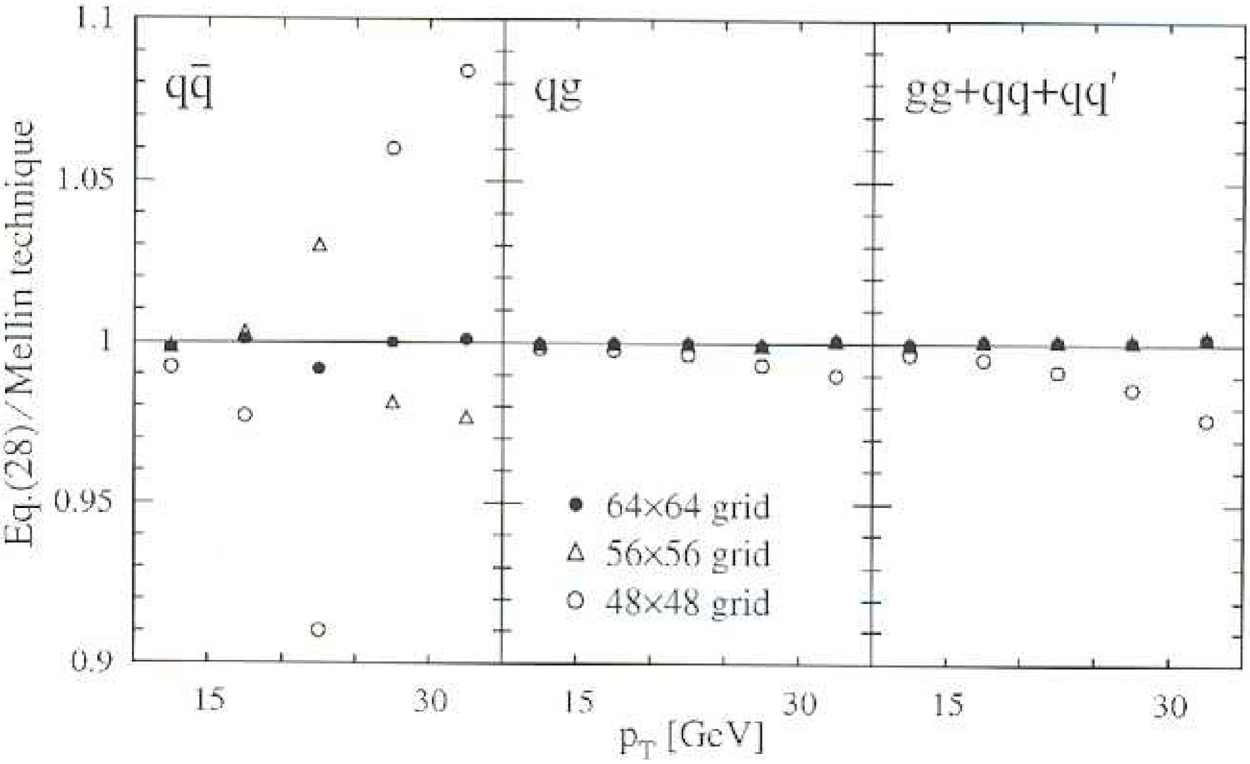
- NLO, scales $\mu_F = \mu_R = p_T$
- $\sqrt{S} = 200$ GeV, $|\eta| < 0.35$, isolated cr. sec.
- "ficticious" data points at
  $p_T = 12.5, 17.5, 22.5, 27.5, 32.5$ GeV
  calc. with GRSV $\oplus$ random Gaussian 1σ shift
- fit to DIS *and* prompt photon "data"
  ansatz for gluon density :

  $$\Delta g(x,\mu_0) = N\, x^{\alpha}\, (1-x)^{\beta}\, (1+\gamma\, x)\; g(x,\mu_0)$$

- perform large number of fits;
  allow for $\Delta\chi^2 = 4$ to obtain "error band"

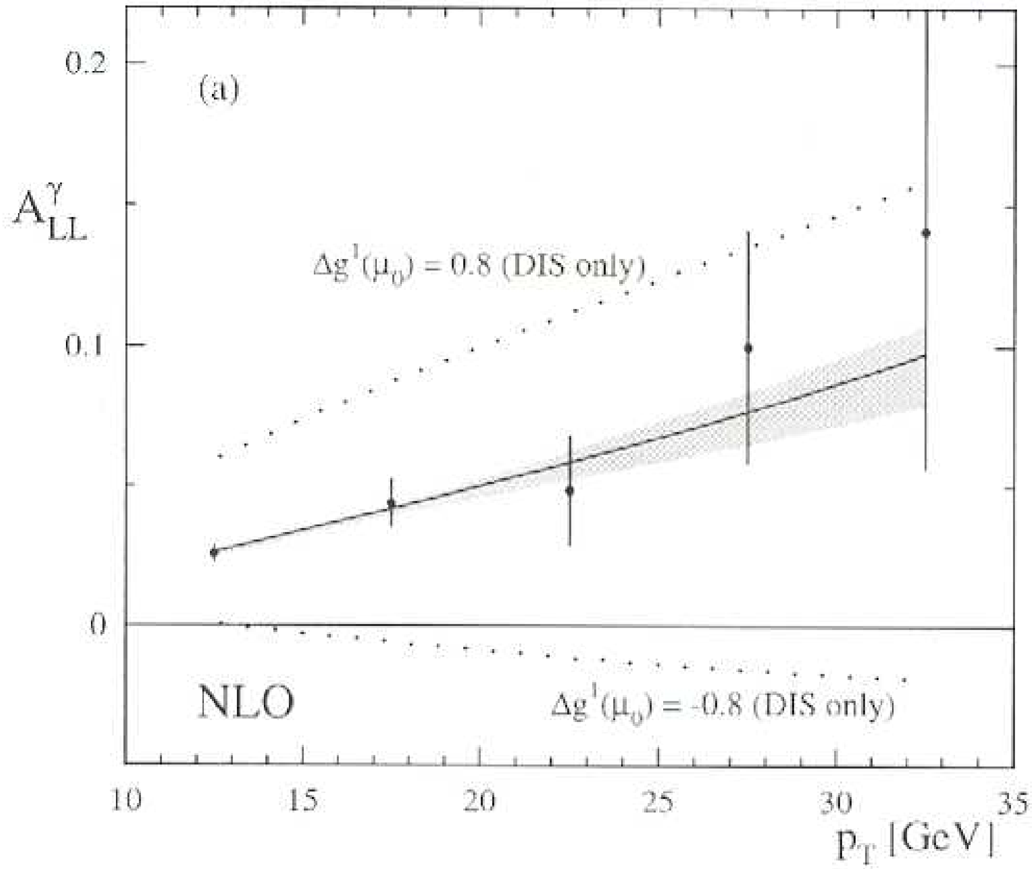
Accuracy of method :

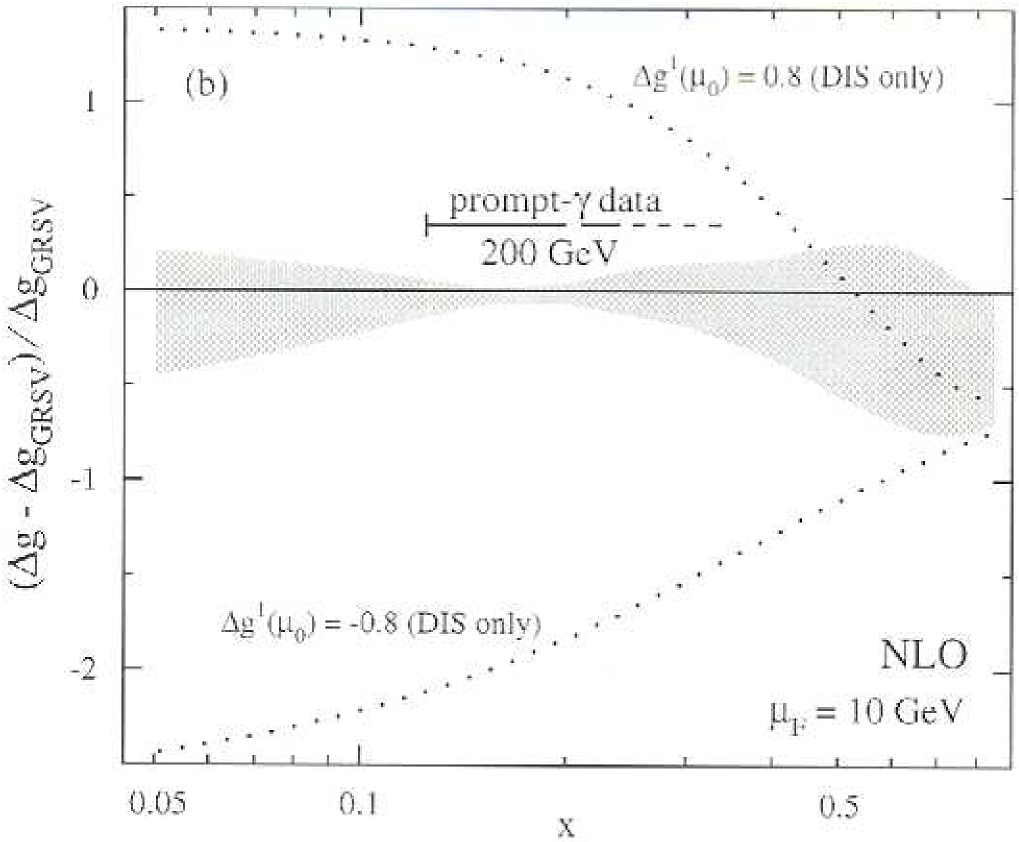
(Stratmann,WV)



Evaluation of cross section extremely fast :

- generation of grids in n, m takes $\sim$ 5 hrs.
- after that :
  1000 evaluations of cr. sec. in $\sim$ 10 sec.





(further constraints by data at $\sqrt{S} = 500$ GeV)

V. Transverse *two*-spin asymmetries

Factorization formalism valid for cross sections with two transversely polarized beams (Collins)

- involves transversity densities
- probably not preferred way of measuring δq; promising : "interference fragmentation fcts." (Jaffe,Jin,Tang; Grosse-Perdekamp et al.)

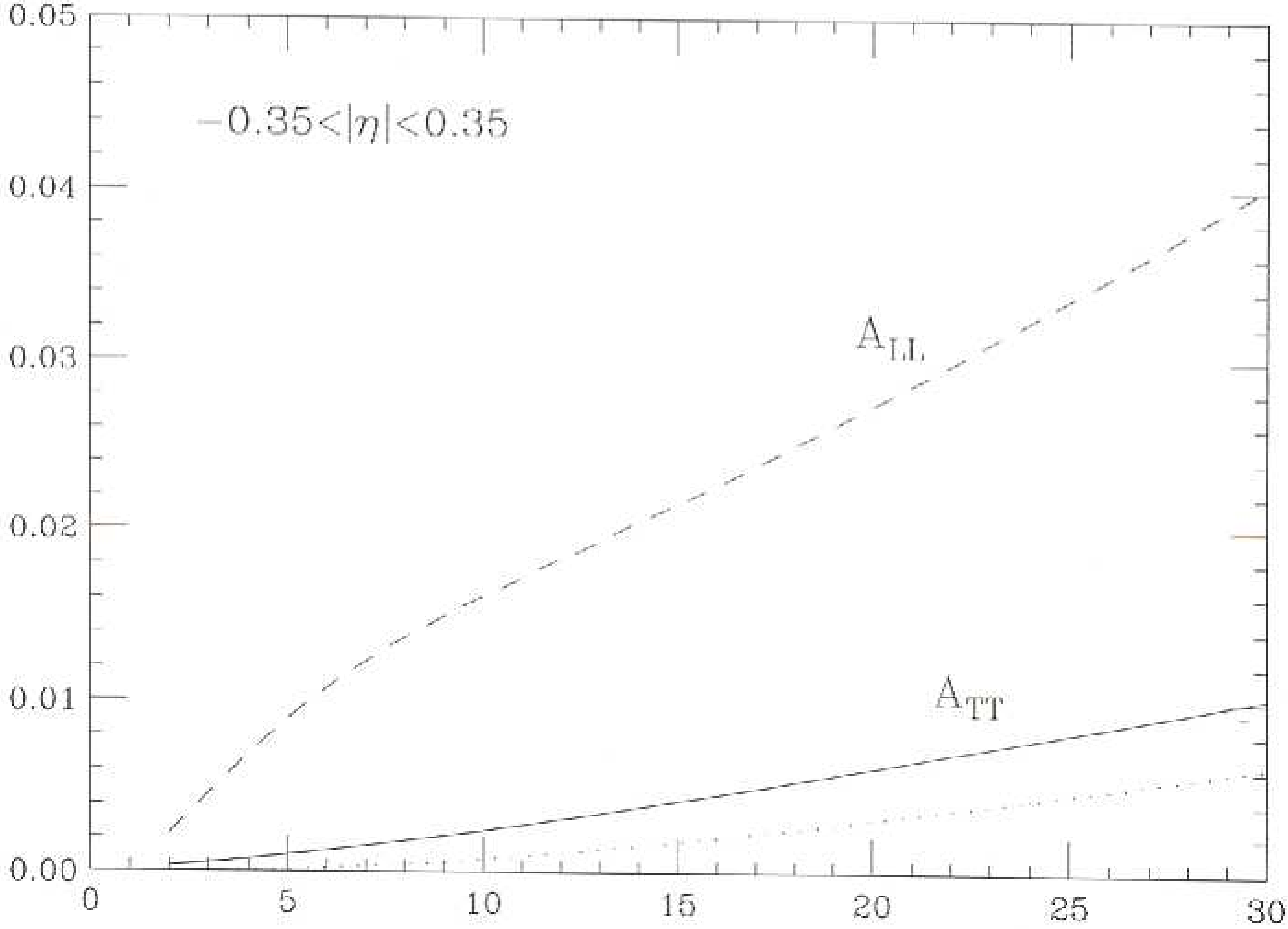
Re-emphasize : A_{TT} is expected small,
"$A_{TT} \ll A_{LL}$" (Jaffe,Saito)

- Drell-Yan process in $\vec{p}\vec{p}$:
  - presumably $\delta\bar{q}(x)$ small
  - in addition, low rates
- dir. photons, jets, inclusive hadrons,... :
  - no gluon transversity, however, gluon contribution to unpolarized cross section !
  - relevant hard scattering cross sections typically color-suppressed
  - Soffer's inequality limits size of δq

  \+ rates can be substantial

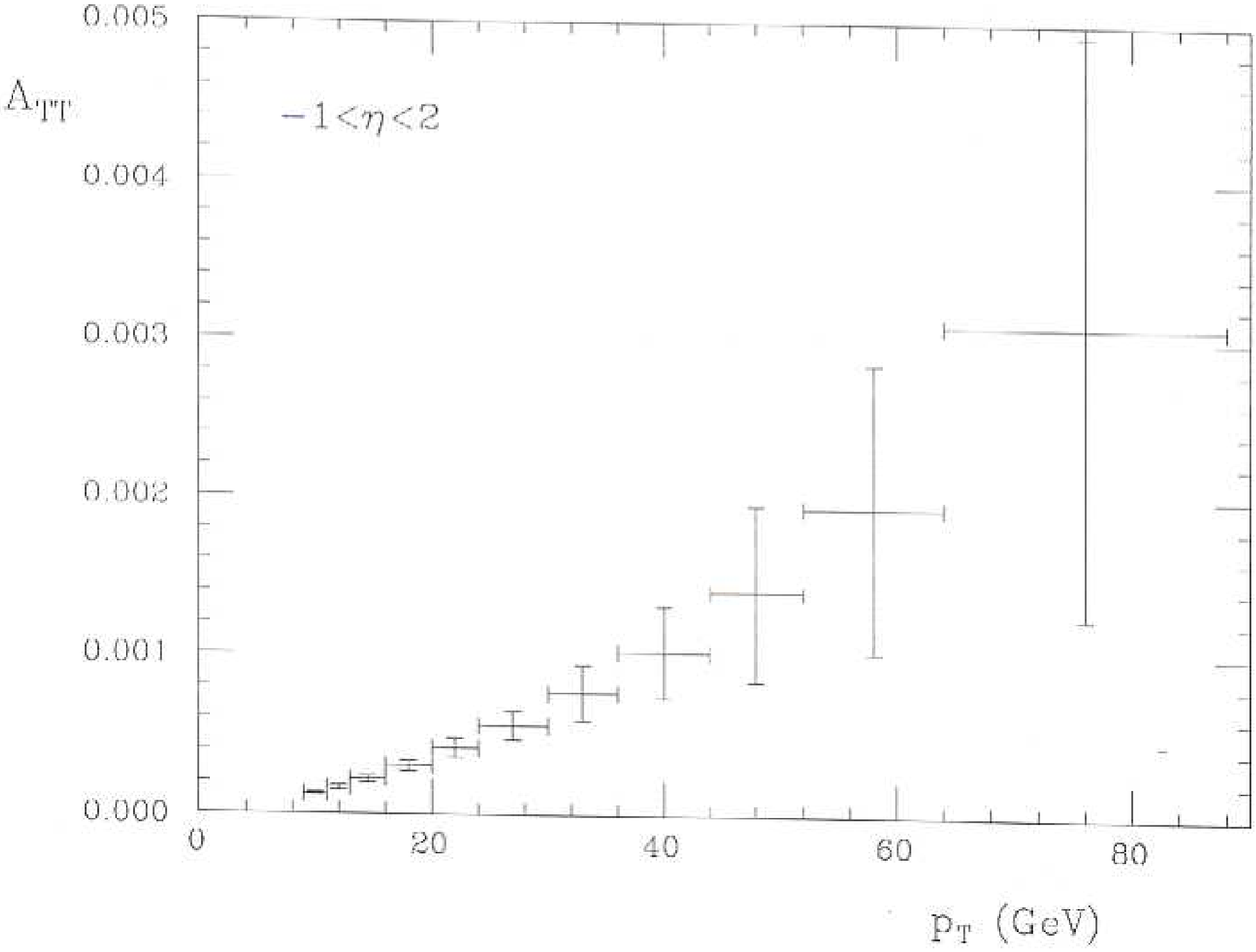
  $\Rightarrow$ small asymmetries *may* be measurable

Estimate "maximal" A^{π}_{TT}, based on Soffer's inequality :



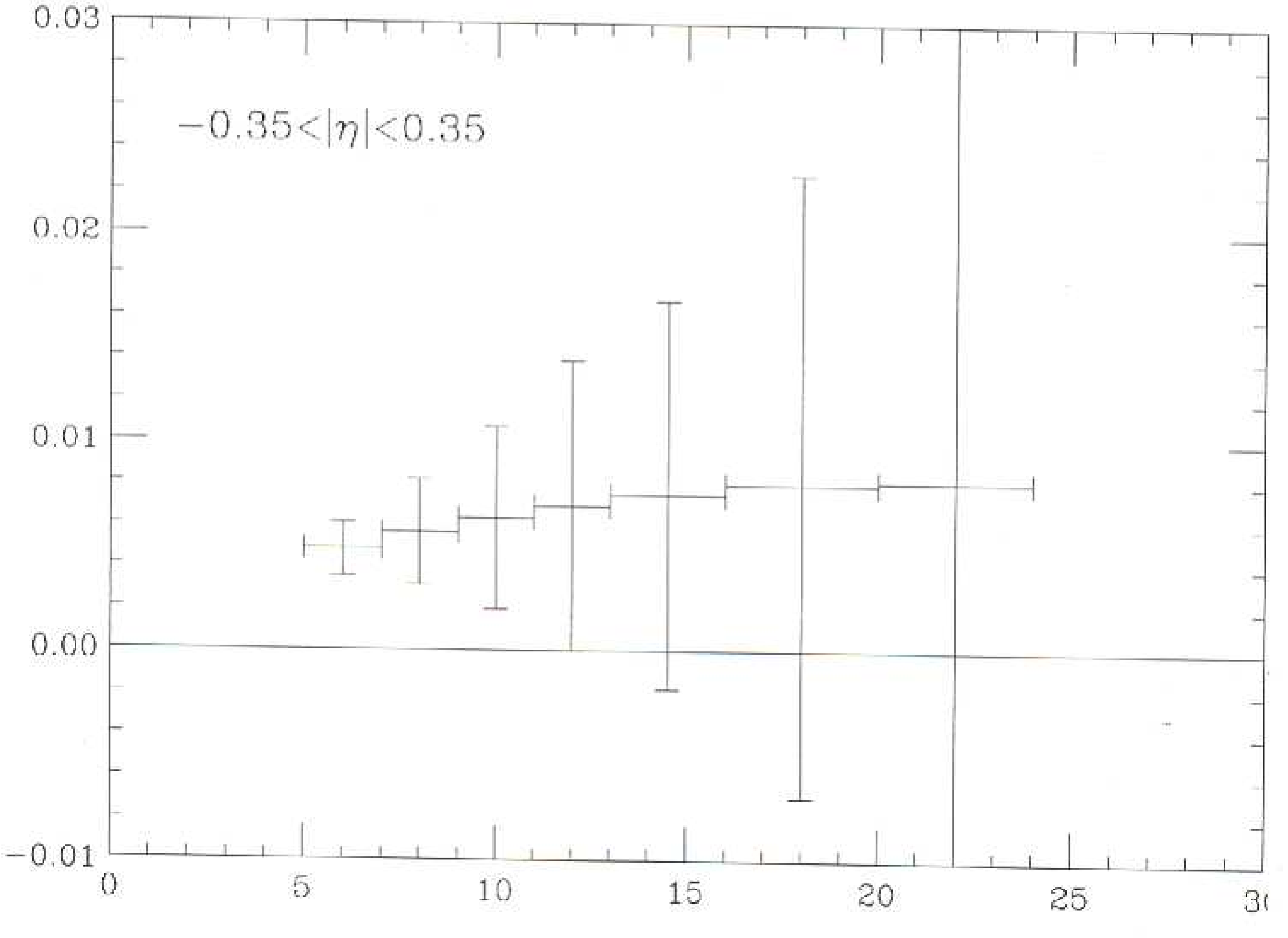
(dotted : $\delta q(x, \mu_0) = \Delta q(x, \mu_0)$)

A_{TT} in single-inclusive jet production :
($\sqrt{S} = 500$ GeV)



(de Florian, Stratmann, WV)

A_{TT} for prompt photon production :
($\sqrt{S} = 200$ GeV)



(Soffer, Stratmann, WV)

VI. A_N^{π} at RHIC

- sizable A_N seen in fixed-target expts.
  E581/704...
- pQCD : A_N to vanish at leading power
  (Kane,Pumplin,Repko)

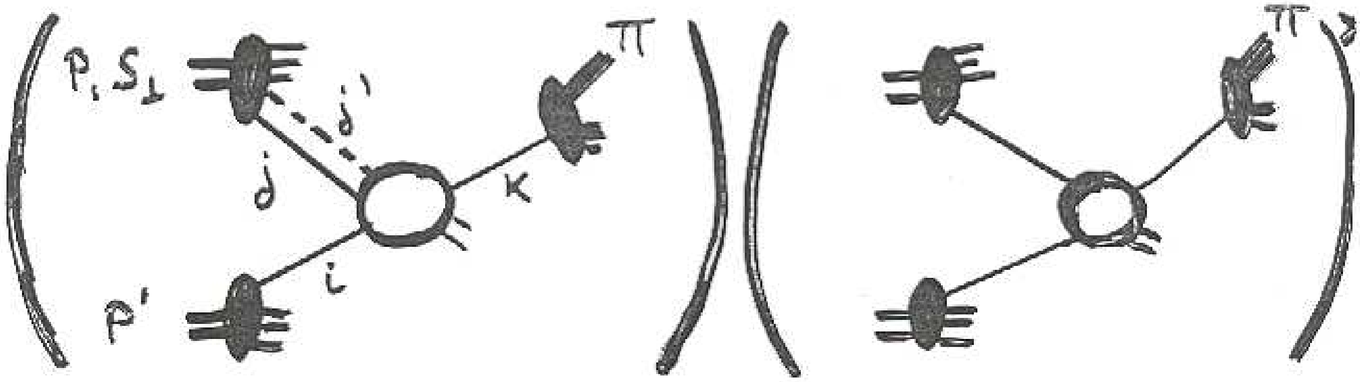
Qiu & Sterman : can A_N be understood as a nonleading-power effect in pQCD ?

- prove factorization to first non-leading power in unpol. / pol. hadron-hadron collisions
  (earlier work in DIS : Vainshtein,Shuryak; Ellis,Furmanski,Petronzio; Jaffe,Soldate; Qiu)
- typical structure for A_N is :

$$\Delta\sigma_{A+B\to\pi}(\vec{s}_T)$$

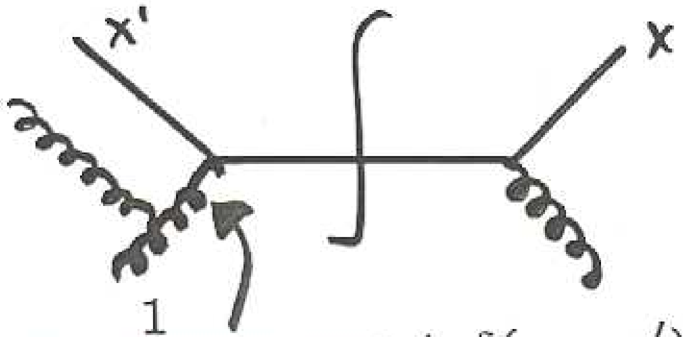
$$= \sum_{i(jj')k} \phi^{(3)}_{jj'/A}(x,x',\vec{s}_T) \otimes \phi_{i/B}(y) \otimes H_{i(jj')\to k}(\vec{s}_T) \otimes D_{k\to\pi}(z)$$

+ other terms + higher power corrections



- $T_F(x,x') = \text{F.T.}\langle P,S|\bar{\psi}(0)\,\gamma^+ F_\alpha^+(\xi_2^-)\psi\left(\xi_1^-\right)|P,S\rangle$
- factorization $\Rightarrow$ universality
- interference $\Rightarrow$ interpretation not easy

- gluon propagator :



$$\frac{1}{x - x' + i\epsilon} \longrightarrow i\pi\delta(x - x') + \text{real}$$

$$\Rightarrow \text{need } T_F(x,x)\ , \text{ and } \frac{d}{dx}T_F(x,x)$$

- argue

$$\frac{d}{dx}T_F(x,x) \gg T_F(x,x) \quad (\text{large-}x)$$

- calculable short-distance cross sections :

$$\hat{\sigma} \sim \sigma_{\text{born}} \times \left(A\,\frac{p_T}{-\hat{u}} + B\,\frac{p_T}{-\hat{t}} \right)$$

$$\sim \frac{\lambda p_T}{S} \qquad \sim \lambda/p_T$$

- no evolution yet
- other approaches :
  - — Sivers, Anselmino et al.; Boer et al.; Leader...

  $k_\perp$-dependent distribution/fragmentation fcts.
  - — Boros et al.