Bounds on Helicity Amplitudes at low Momentum Transfer

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In the context of electromagnetic hadronic interference for elastic scattering, a study of bounds on helicity amplitudes provides important information related to the evaluation of polarization and the understanding of spin-dependent reactions.

Notation for spin-$\frac{1}{2}$ spin-$\frac{1}{2}$ non-identical particle elastic reactions, proton-spin-$\frac{1}{2}$ ion collisions for example, is introduced by exhibiting the high s Born approximation Coulomb helicity amplitudes with Pauli form factor normalization $F_2(0) = \mu - 1$. The rôle played by single-flip and double-flip amplitudes in the asymmetry maximum near the interference region is similar to that for identical particle elastic scattering.

In the case of pp collisions, the asymmetry maximum is sensitive to the forward imaginary part of $\phi_2 = \langle + + |\phi| - - \rangle$ which may be bounded up to 6 GeV energies by the known transverse spin-polarized total cross section difference $\Delta\sigma_T = \sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}$. The maximum is more sensitive to the ratio of the helicity single-flip amplitude to the averaged *imaginary* non-flip amplitude $r_5 = \left(2m/\sqrt{-t}\right) \phi_5 \,/\, \mathrm{Im}\,(\phi_1 + \phi_3)$, imaginary values of which, according to a wide range of models incorporating—in various forms—the Pomeron, quarks, gluons and instantons, all have magnitude less than 20%. Several studies of the limited high energy elastic data in the interference region reveal that $\mathrm{Im}\, r_5 \approx 0.2 + 0.3$ with something similar for proton carbon collisions.

Rigorous bounds on the helicity ratio for spin-0 spin-$\frac{1}{2}$ collisions based on partial wave unitarity unfortunately merely indicate that the flip nonflip ratio $|\mathrm{Im}\, r| \leq 2.3$, at the high energies of interest. A positivity analysis of the absorptive element of the differential cross section in higher spin elastic cases also leads to a comparable bound on the helicity flip nonflip ratio, where the helicity-dependent hadronic slope parameters are supposed not to depart too greatly from the prominent t variation associated with the slope parameter b of the dominant spin-averaged amplitude.

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IMPORTANCE OF HELICITY FLIP AMPLITUDES

For the elastic scattering of non-identical particles, $NN' \to NN'$, of spin $\frac{1}{2}$ the one photon exchange asymptotic amplitudes

$$\varphi_1(++;++) \qquad \frac{\alpha s}{t} F_1 F_1' \qquad \varphi_3(+-;+-)$$

$$\varphi_2(++;--) \qquad \frac{\alpha s}{4mm'} F_2 F_2' \qquad -\varphi_4(+-;-+)$$

$$\varphi_5(++;+-) \qquad \frac{-\alpha s}{2m'\sqrt{-t}} F_1 F_2' \qquad -\varphi_6(++;-+)$$

$$(m \leftrightarrow m',\ F \leftrightarrow F')$$

are significant near interference, t_c.

Isospin invariance indicates $\varphi_5 = -\varphi_6$ for $np \to np$ but em breaks this, so $\varphi_5^{\gamma} \neq -\varphi_6^{\gamma}$ here.

In terms of helicity amplitudes, asymmetry

$$\frac{k^2 s}{\pi} A_N \frac{d\sigma}{dt} = \mathrm{Im}\left[(\varphi_1+\varphi_3)^* \varphi_5 - (\varphi_2-\varphi_4)^* \varphi_6\right]$$

is studied for polarization evaluation.
(B, Gotsman, Leader, Phys. Rev. D18, 694, 1978)

ASYMMETRY FOR ELASTIC $pp \to pp$

Identical particles, so $\varphi_5 = -\varphi_6$

and near interference, $t_c = 8\pi\alpha/\sigma_{tot}$

$$\frac{m}{\sqrt{-t}} A_N \doteq \frac{\left(2\,\mathrm{Im}\,r_5 - \kappa_p\right)\frac{t_c}{t} + \left(2\rho\,\mathrm{Im}\,r_5 - 2\,\mathrm{Re}\,r_5\right)}{1 + \left(t_c/t + \rho + \delta\right)^2}$$

(Kopeliovich and Lapidus) Bethe phase δ,

where the possible presence of the normalized

single-flip / nonflip ratio at high energies

$$r_5 = \frac{m}{\sqrt{-t}} \frac{\phi_5}{\mathrm{Im}\,\phi_+}, \qquad \phi_+ = \tfrac{1}{2}(\varphi_1 + \varphi_3)$$

is of concern for proton polarimetry.

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studied the hadron spin-flip question.

Bounds on ϕ_5 are reviewed and

examined, in addition to bounds on φ_+, φ_2.

TRANSVERSE SPIN – LOW MOMENTUM TRANSFER

Coefficient of t_c/t in A_N relates to the maximum asymmetry at interference.

$$2\,\mathrm{Im}\,r_5 - \kappa_p - \tfrac{1}{2}\kappa_p\,\mathrm{Im}\,r_2$$

(VI Blois)

is more accurate and includes a term involving the transverse-spin total cross section difference $\Delta\sigma_{tot}^{T}$, where

$$r_2 = \frac{\phi_2}{\mathrm{Im}\,\phi_+} \quad \text{and} \quad \mathrm{Im}\,r_2\Big|_{t=0} = -\frac{\Delta\sigma_{tot}^{T}}{\sigma_{tot}}$$

The difference $\Delta\sigma_{tot}^{T}$ drops from 6 mb at 2 GeV/c to 0.5 mb at 6 GeV/c

It would be vital to confirm this continuing decrease at higher energies.

An improvement on the bound for $\mathrm{Im}\,\varphi_2$

$$\left|\mathrm{Im}\,r_2(s,0)\right| < 1\%$$ is desirable.

Hadronic Flip / Nonflip $|r_5|$

MODELS	Landshoff $\frac{1}{2}\lvert\mu_p - 1 - \mu_n\rvert$	0.06
	Berger, (data > 3 GeV)	0.09
	Irving, Sorenson	0.03
	R parameter, 45 GeV pp	0.07 ± 0.04
	B.Z. Kopeliovich, 2 gluon	0.10
	Golosokov, $q\mathbb{P}$	~ 0.20, $-t = .3$
	Anselmino, Forte	~ 0.1
DATA	T.L. Trueman, $\mathrm{Im}\, r_5$	0.2 ± 0.3
	Akchurin, B, Penzo (150-300) "	0.08 ± 0.18
	E 704 (200 GeV/c only) "	0.15 ± 0.30
	Kopeliovich (pC, 200) $\mathrm{Im}\, r$	0.22 ± 0.26

Hadronic Double-flip / Nonflip

$\sigma_{tot}^{\uparrow\uparrow} / \sigma_{tot}^{\uparrow\downarrow}$ (A. Martin)

- 3/4 → 4/3 ISR
- 1/2 → 2 $Sp\bar{p}S$

BOUND ON HELICITY FLIP AMPLITUDE

For spin 0 – spin $\frac{1}{2}$ elastic collisions described by non flip $F_{++}(s,t)$ and flip $F_{+-}(s,t)$ amplitudes, the forward ratio is bounded

$$\frac{m}{\sqrt{-t}} \left| \frac{\mathrm{Im}\, F_{+-}}{\mathrm{Im}\, F_{++}} \right|_{t=0} \leq \frac{3m}{2} \sqrt{b} \left(\frac{16\pi}{15} \frac{b\, \sigma_{el}}{\sigma_{tot}^2} \right)^{\frac{1}{2}}$$

or, $$\left| \mathrm{Im}\, r \right| \leq 0.87\, m \sqrt{b}$$

where $2b$ is approximately the $\frac{d\sigma}{dt}$ slope

and

$$b = \frac{d}{dt} \ln \mathrm{Im}\, F_{++} \Big|_{t=0} \approx \frac{\sigma_{tot}^2}{32\pi\, \sigma_{el}}$$

D. P. Hodgkinson, Phys. Lett. 39B, 640 (1972).

Over the range $\sqrt{s} \in [50, 500]$, the slope $2b \in [14.0, 15.5]$, and for proton mass m,

$$\left| \mathrm{Im}\, r \right| \leq 2.3,$$

significantly greater than the critical value 0.9

HIGHER SPIN BOUNDS

Bound on $\mathrm{Im}\,\phi_5(++;+-)$ from

$$\frac{d\sigma^A}{dt} = \sum_i \left(\mathrm{Im}\,\phi_i\right)^2$$

$$= \sum_n (2n+1)\, c_n\, P_n(\cos\theta)$$

Mahoux (1976) BLS Phys Rep. 59('80) 164

$$c_n \geq 0. \Rightarrow$$

$$d\sigma^A/dt\ (t=0) \geq d\sigma^A/dt\ (t<0)$$

$$\left(\mathrm{Im}\,\phi_5\right)^2/|t| \leq \tfrac{1}{2}\left(b + \zeta_L^2 b_- + \tfrac{i}{2}\zeta_T^2 b_2\right)$$

$$|\mathrm{Im}\,\Gamma_5| \lesssim 2.3$$

S. M. Roy has improved bound
PL 70B (1977) 213.

Mennessier, Roy, Singh N.C. 50A, 443 (1979)

CONCLUSIONS

1. Confirm $\mathrm{Im}\,\phi_2(s,0) \propto \Delta\sigma_T$ is small

2. Many models indicate $\mathrm{Im}\,r_5 \approx 0.1$ suggesting CNI polarimetry at 10%

3. Rigorous bounds $\Rightarrow |\mathrm{Im}\,r_5| < 2.5$ mainly for spin 0 - spin $\frac{1}{2}$ collisions

4. Higher spin bounds leading to similar conclusions are available with additional assumptions.

5. Important to confirm that $\rho = \frac{\mathrm{Re}\,\phi_4}{\mathrm{Im}\,\phi_4}$ agrees with analytic expectations

6. Measurements at small angles with polarized protons give considerable information on the helicity amplitudes of elastic scattering.