Recent results in QCD resummation, and on A_{TT}

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Outline :

I. Introduction : why resum ?

II. Soft emission in QED

III. In QCD . . .

IV. "Joint" resummation

V. Resummation for polarized scattering

VI. (Old and) new results on A_{TT}

I. Introduction : why resum ?

QCD at short distances

- prerequisite : asymptotic freedom $\Rightarrow$ pert. theory

- **"infrared safe" observables**

$$Q^2\,\sigma_{\text{phys}}(Q) \;=\; \sum_n c_n\,\alpha_s^n(Q) \;+\; \mathcal{O}\left(\frac{1}{Q^p}\right)$$

examples : $e^+e^- \to$ hadrons, jets, event shapes, ...

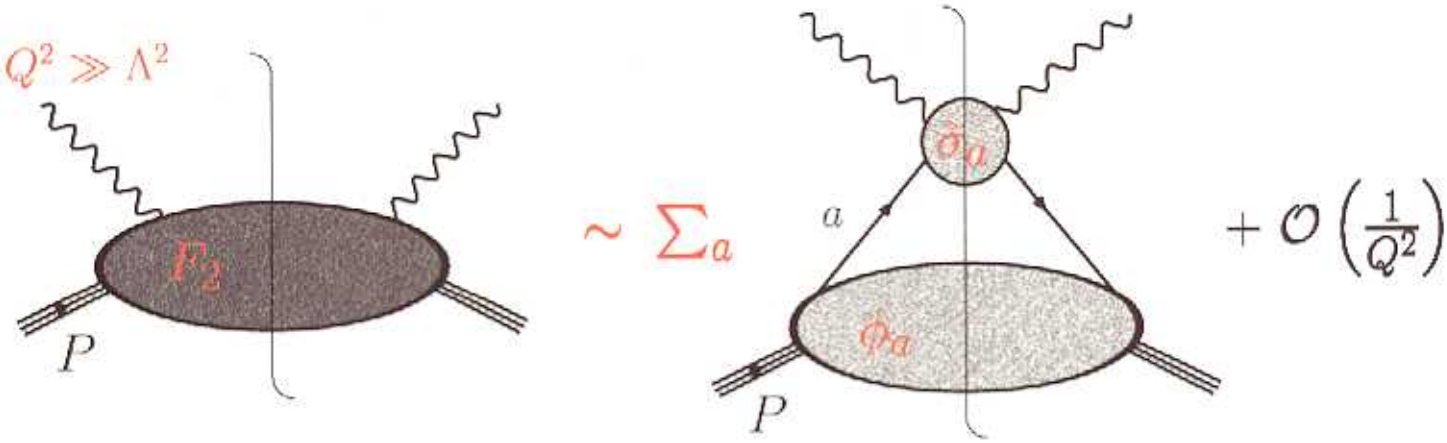
- extension : **"factorizable" observables**

$$Q^2\,\sigma_{\text{phys}}(Q,m) \;=\; \mathcal{H}\left(\frac{Q}{\mu},\alpha_s(\mu)\right) \otimes \mathcal{S}\left(\frac{\mu}{m},\alpha_s(\mu)\right) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

  - short-distance "hard" part $\mathcal{H}$:
    perturbative, specific to process
  - long-distance "soft" part $\mathcal{S}$:
    incalculable (at present), universal
  - μ – factorization scale

examples : DIS $ep \to e'X$, $p\bar{p} \to$ jets + X, γ + X, ...

example : deeply-inelastic scattering



$$F_2 \sim \sum_a \int_x^1 \frac{d\xi}{\xi}\, \hat{\sigma}_a\left(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu)\right)\, \phi_a\left(\xi, \frac{\mu}{m}, \alpha_s(\mu)\right)$$

$$\equiv \sum_a \hat{\sigma}_a \otimes \phi_a$$

- PDF's $\longleftrightarrow$ operator matrix elements :

e.g. $$q(\xi) = \frac{1}{4\pi} \int dy^-\, e^{iy\,\xi P}\, \langle P, S \,|\, \overline{\psi}(0)\, \gamma^+\, \psi\left(0, y^-, \mathbf{0}_\perp\right) \,|\, P, S\rangle$$

$\longrightarrow$ nucleon structure

- factorization theorems :

  extension to $pp \to$ jet $+ X$, Drell-Yan, etc.

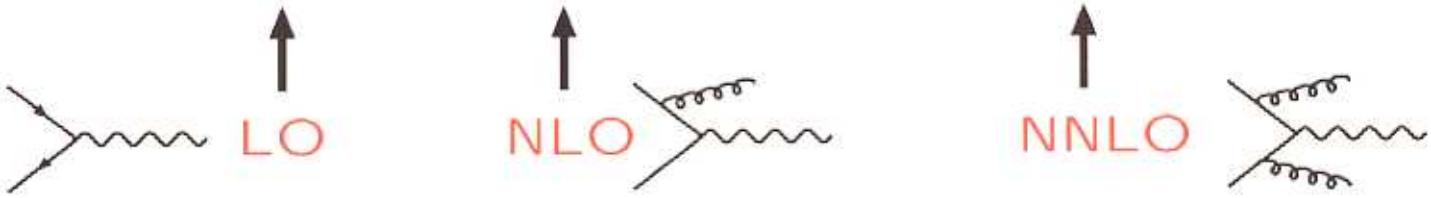
(Sterman,Libby; Ellis et al.; Amati et al.; Curci et al.; Collins,Soper,Sterman; Bauer,Fleming,Pirjol,Rothstein,Stewart)

The full picture : e.g. $pp \to ZX$

$$\sigma_{pp}^{Z} = \sum_{a,b} \phi_a(\mu) \otimes \phi_b(\mu) \otimes \hat{\sigma}_{ab}^{Z}\left(\frac{M_Z}{\mu}, \alpha_s(\mu)\right)$$

- perturbative cross sections :

$$\hat{\sigma}\left(\frac{M_Z}{\mu}, \alpha_s(\mu)\right) = \hat{\sigma}^{(0)} + \alpha_s(\mu)\, \hat{\sigma}^{(1)}\left(\frac{M_Z}{\mu}\right) + (\alpha_s(\mu))^2\, \hat{\sigma}^{(2)}\left(\frac{M_Z}{\mu}\right) + \dots$$



to be calculated for each process !

- "DGLAP" evolution of PDF's :

$$\mu \frac{\partial}{\partial \mu} \phi_i(\mu) = \sum_j \phi_j(\mu) \otimes \gamma_{ij}(\alpha_s(\mu))$$

$$\gamma_{ij}(\alpha_s) = \frac{\alpha_s}{2\pi}\gamma_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \gamma_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \gamma_{ij}^{(2)} + \dots$$

LO NLO NNLO

universal (= same in all processes)

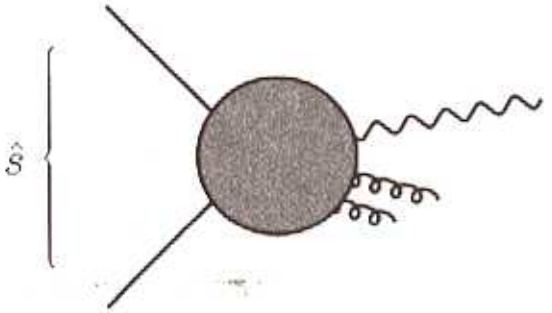
"state-of-the-art" : NLO

" fixed-order perturbation theory "

Despite presence of large scale Q, and despite being infrared-finite :

fixed-order perturbation theory for $\hat{\sigma}$ may not always be adequate

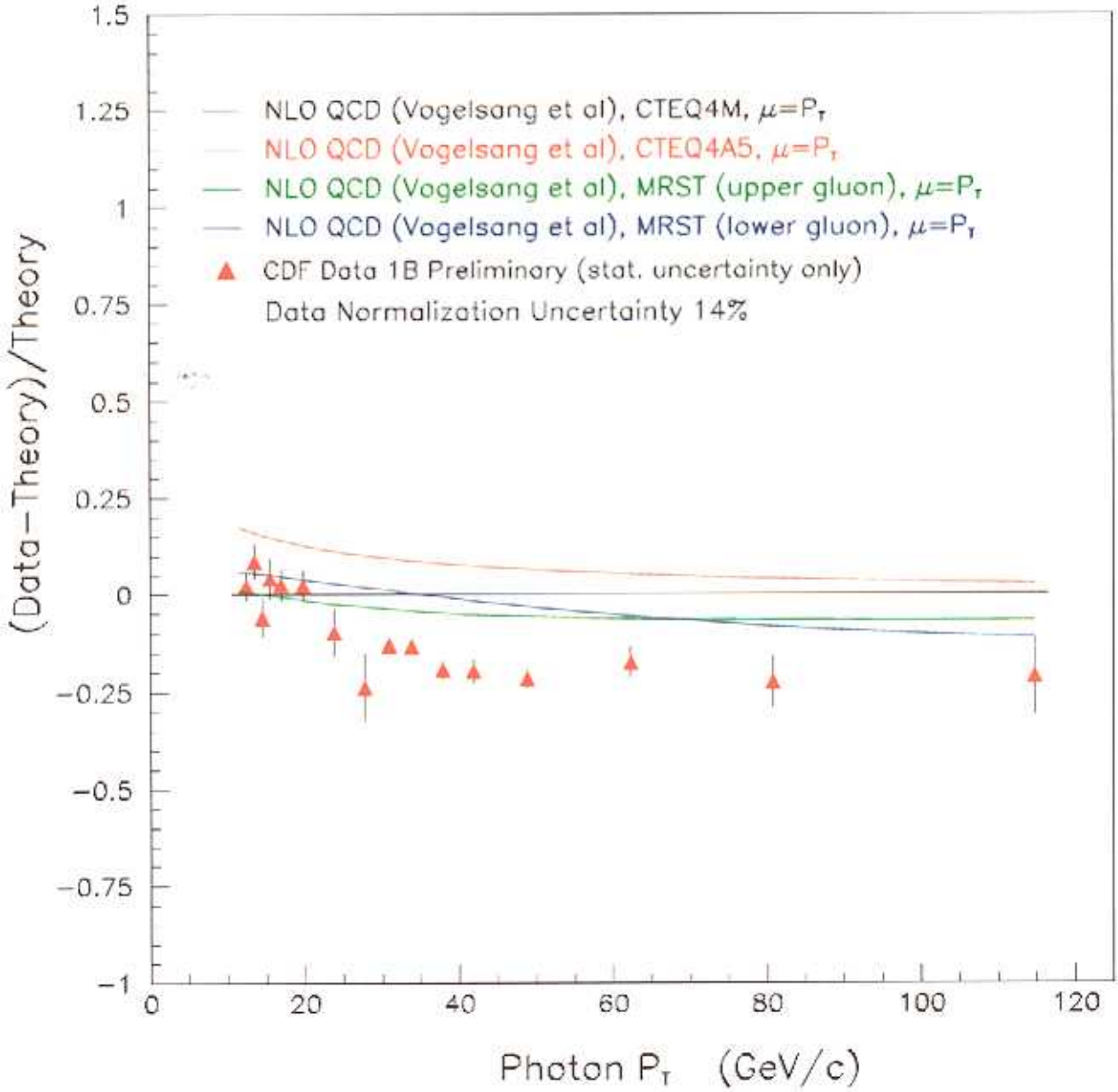
- example : Z production via $q\bar{q} \to Z + X$



$Q \approx M_Z$, tr. mom. q_T

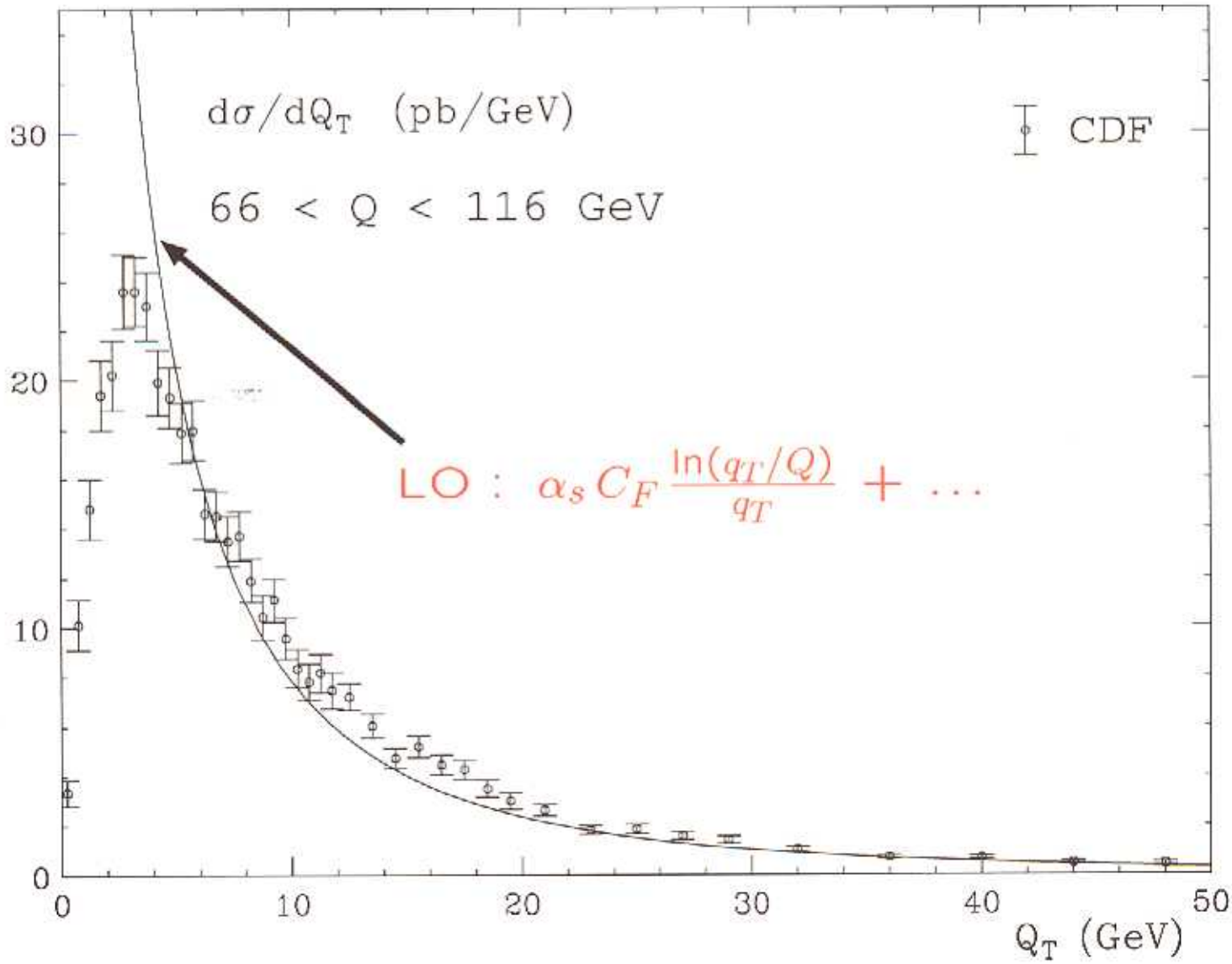
- when $\hat{s} \to Q^2$: $\hat{\sigma} \sim \alpha_s^k \frac{\ln^{2k-1}(\hat{s}-Q^2)}{\hat{s}-Q^2} + \ldots$
- "threshold logs" – just enough partonic energy
- origin : suppression of gluon radiation
- when $q_T \to 0$: $\frac{d\hat{\sigma}}{dq_T} \sim \alpha_s^k \frac{\ln^{2k-1}(q_T/Q)}{q_T} + \ldots$
- "q_T logs" – recoil against soft radiation
- real-virtual IR cancellations leave large logs
  $\to$ may spoil expansion in $\alpha_s(Q)$
- such corrections associated with soft and/or collinear emission
  $\to$ can often be treated to all orders
- = resummation !

CDF, Run-1B



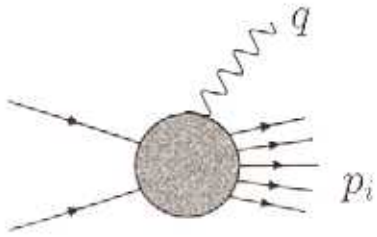
– a case for resummation ?

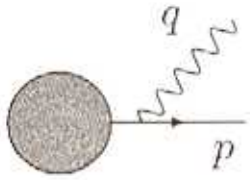
Classic Example : Transverse-momentum distribution of Z bosons at the Tevatron



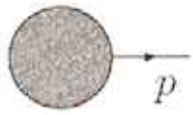
II : Soft emission in QED

$M^\mu(p_1, \ldots, p_k; q) =$





$\approx_{q\to 0}$



$\cdot\, e\, \frac{p^\mu}{p\cdot q}\, \epsilon_\mu(q)$

$\Uparrow$

eikonal factor

- single emission off all legs :

$$|M(p_1, \ldots, p_k; q)|^2 \approx_{q\to 0} |M(p_1, \ldots, p_k)|^2 \cdot d\rho_1(q)$$

$$d\rho_1(q) = \frac{\alpha}{\pi} \sum_{\text{legs } k} e_k^2 \frac{d\omega}{\omega} \frac{d\theta_{kq}^2}{\theta_{kq}^2}$$

- n soft photons emitted:

$$d\rho_n(q_1, \ldots q_n) = \frac{1}{n!} \prod_{i=1}^{n} d\rho_1(q_i)$$

$$d\rho_n(q_1, \ldots q_n) \;=\; \frac{1}{n!} \prod_{i=1}^{n} d\rho_1(q_i)$$

- factorization of dynamics

- exponentiation *if* photon phase space symmetric

- sometimes achievable by integral transforms :

$$\delta\Big(2P\cdot\sum_j k_j - m^2\Big) \;=\; \frac{1}{2\pi\, i\, Q^2} \int_C dN\, \mathrm{e}^{-N\left(2P\cdot\sum_j k_j - m^2\right)/Q^2}$$

$$\delta\Big(\vec{q}_T - \sum_j \vec{k}_T^{\,j}\Big) \;=\; \frac{1}{(2\pi)^2} \int d^2 b\, \mathrm{e}^{i\vec{b}\cdot\left(\vec{q}_T - \sum_j \vec{k}_T^{\,j}\right)}$$

- $\Rightarrow$ phase space factorizes :
  "factorization of kinematics"

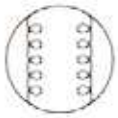
III. In QCD ...

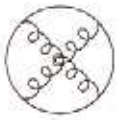
- an emitted soft gluon carries color :

$$|M(\{p_i\},q\rangle \;\approx_{q\to 0}\; \epsilon_\mu(q)\,\mathcal{J}_a^\mu\;|M(\{p_i\}\rangle$$

$$\mathcal{J}_a^\mu(q) \quad = \quad \sum_i T_a^i\,\frac{p_i^\mu}{p_i\cdot q} \qquad \text{matrix}$$

- still, eikonal cross sections exponentiate in terms of "webs" (Gatheral; Frenkel, Taylor) schematically :

$1 + C_{\text{①}}$ $+ C_{\text{⑪}}$  $+ C_{\otimes}$ $+ \ldots$

$= \exp\Big[\, C_{\text{①}}$ $+ \,(\, C_{\otimes} - C_{\text{⑪}}\,)$  $+ \ldots \Big]$

- in addition, running coupling

IV : "Joint" resummation

Laenen, Sterman, WV :

logarithms at threshold and at $q_T = 0$ can be resummed *simultaneously*

- "jointly" resummed cross section :

$$\frac{d\sigma^{\text{res}}}{dQ^2\, dq_T^2} \propto \sum_q \int_C dN \left(\frac{Q^2}{S}\right)^{-N} \int d^2\mathbf{b}\; e^{-i\,\bar{\mathbf{q}}_T\cdot\bar{\mathbf{b}}}\; f_q^N(Q)\; f_{\bar{q}}^N(Q)$$

$$\times\; \exp\left\{2\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, A_a(\alpha_s(k_\perp^2))\left[J_0(b\,k_\perp)\, K_0\left(\frac{2Nk_\perp}{Q}\right) + \ln\left(\frac{\bar{N}k_\perp}{Q}\right)\right]\right\}$$

- *two* inverse transforms $(N,\, b)$
- exponent approximated by

$$2\int_{Q^2/\chi^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, A_q\left(\alpha_s(k_\perp^2)\right)\, \ln\left(\frac{k_\perp \bar{N}}{Q}\right)$$

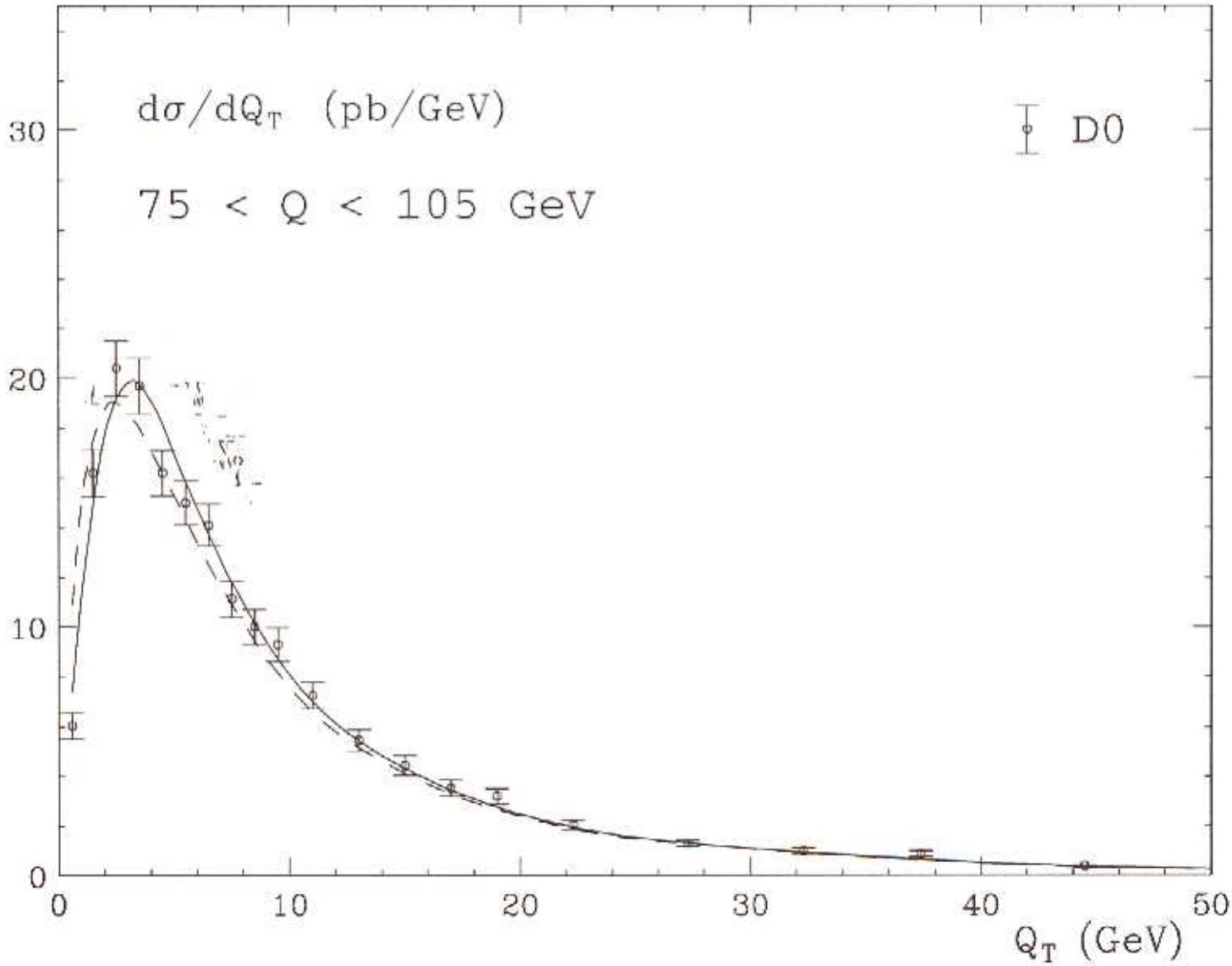
$$\chi \equiv \chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \bar{b}/4\bar{N}}$$

- at very low $q_T \to 0$: expect non-pert. effects exponent gives guide to form of corrections :

$$\sim \left(\frac{b^2}{4} - \frac{N^2}{Q^2}\right)\int_0^\lambda dk_\perp\, k_\perp\, \alpha_s(k_\perp^2)\, \ln\left(\frac{Nk_\perp}{Q}\right)$$

(Kulesza,Sterman,WV)

(CTEQ5M pdfs)

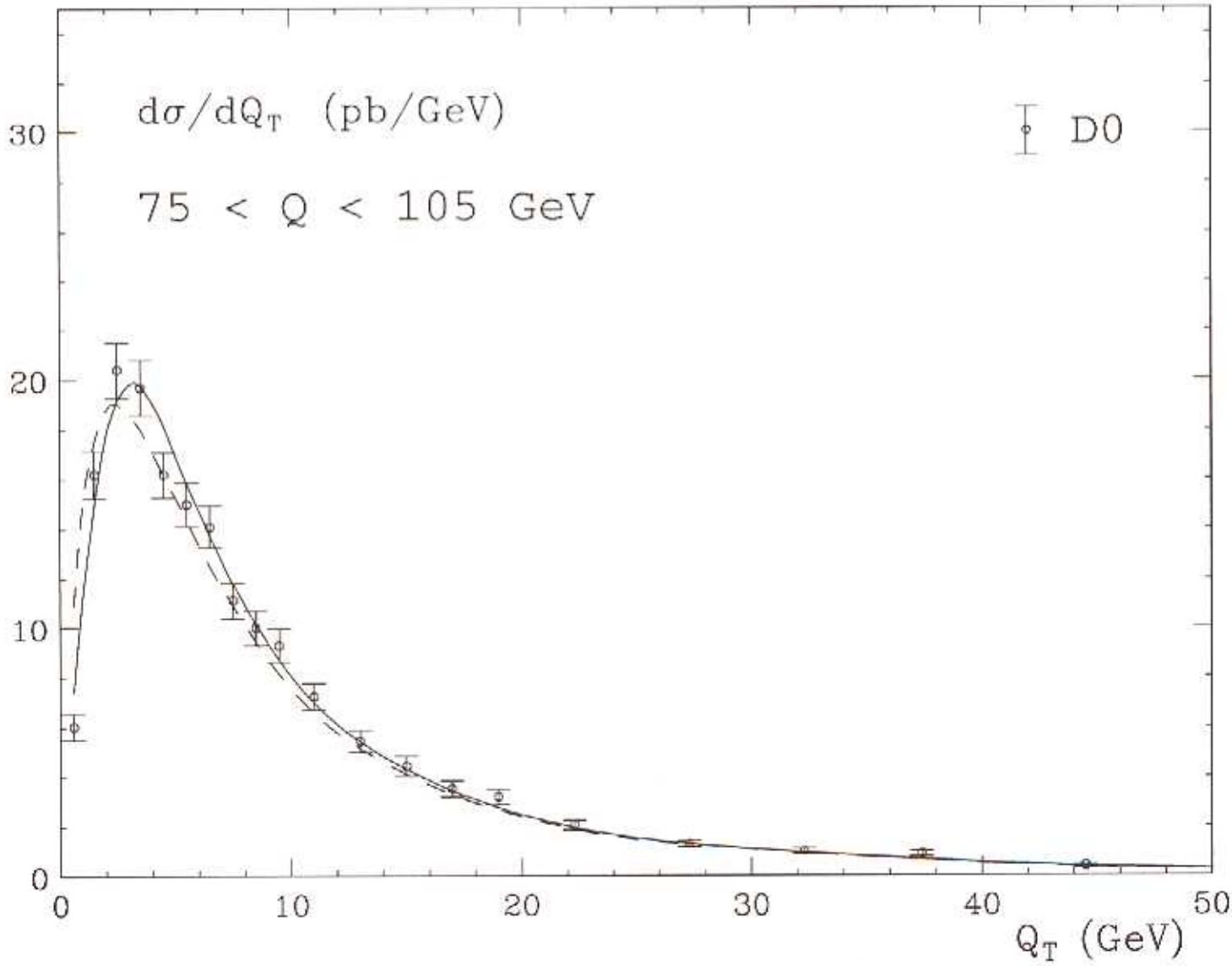


dashed : "purely perturbative" resummed

solid : Gaussian smearing $-g\,b^2$ with $g = 0.8$ GeV^2

(Kulesza,Sterman,WV)

(CTEQ5M pdfs)



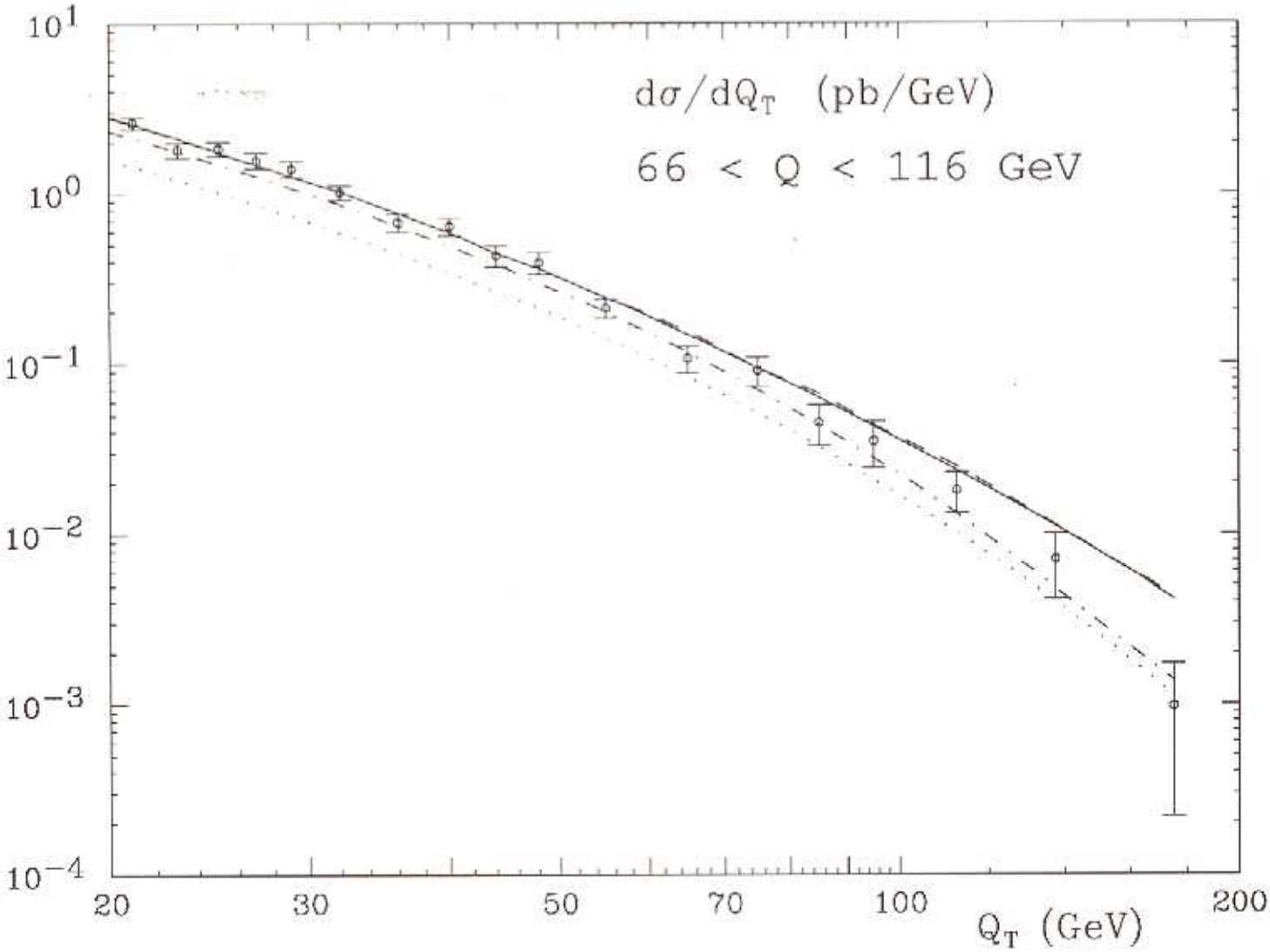
dashed : "purely perturbative" resummed

solid : Gaussian smearing $-g\,b^2$ with $g = 0.8$ GeV^2

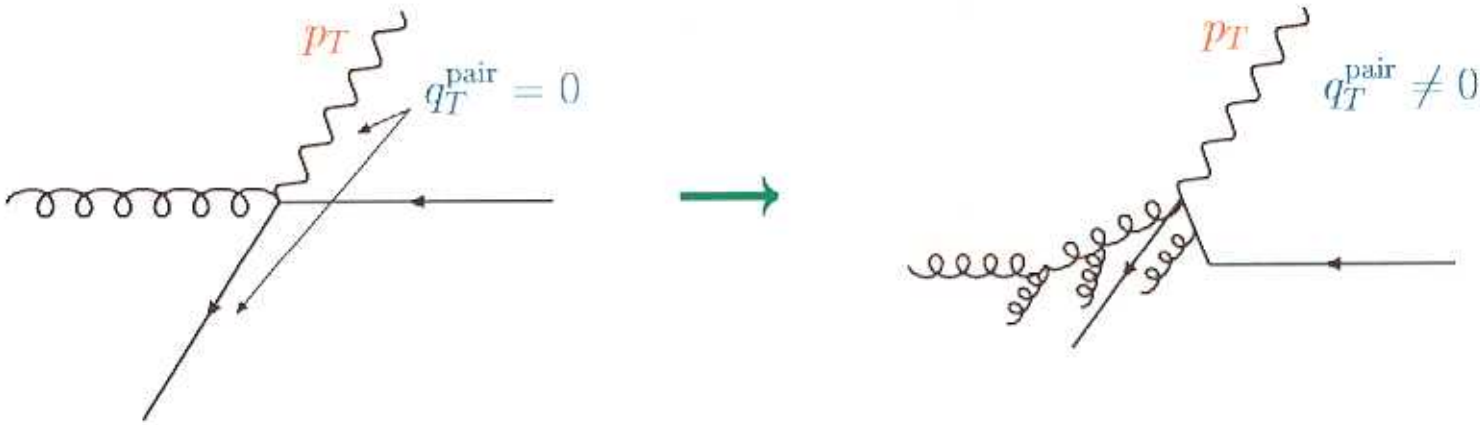
- at large $q_T \sim Q$, expect resummation to fail
  fixed-order more appropriate
  "matching" :

$$\frac{d\sigma}{dQ^2 dq_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2 dq_T^2} - \underbrace{\frac{d\sigma^{\text{exp(k)}}}{dQ^2 dq_T^2}}_{\text{expansion of res.}} + \underbrace{\frac{d\sigma^{\text{fixed(k)}}}{dQ^2 dq_T^2}}_{\text{fixed o.}}$$

$\rightarrow$ no double-counting



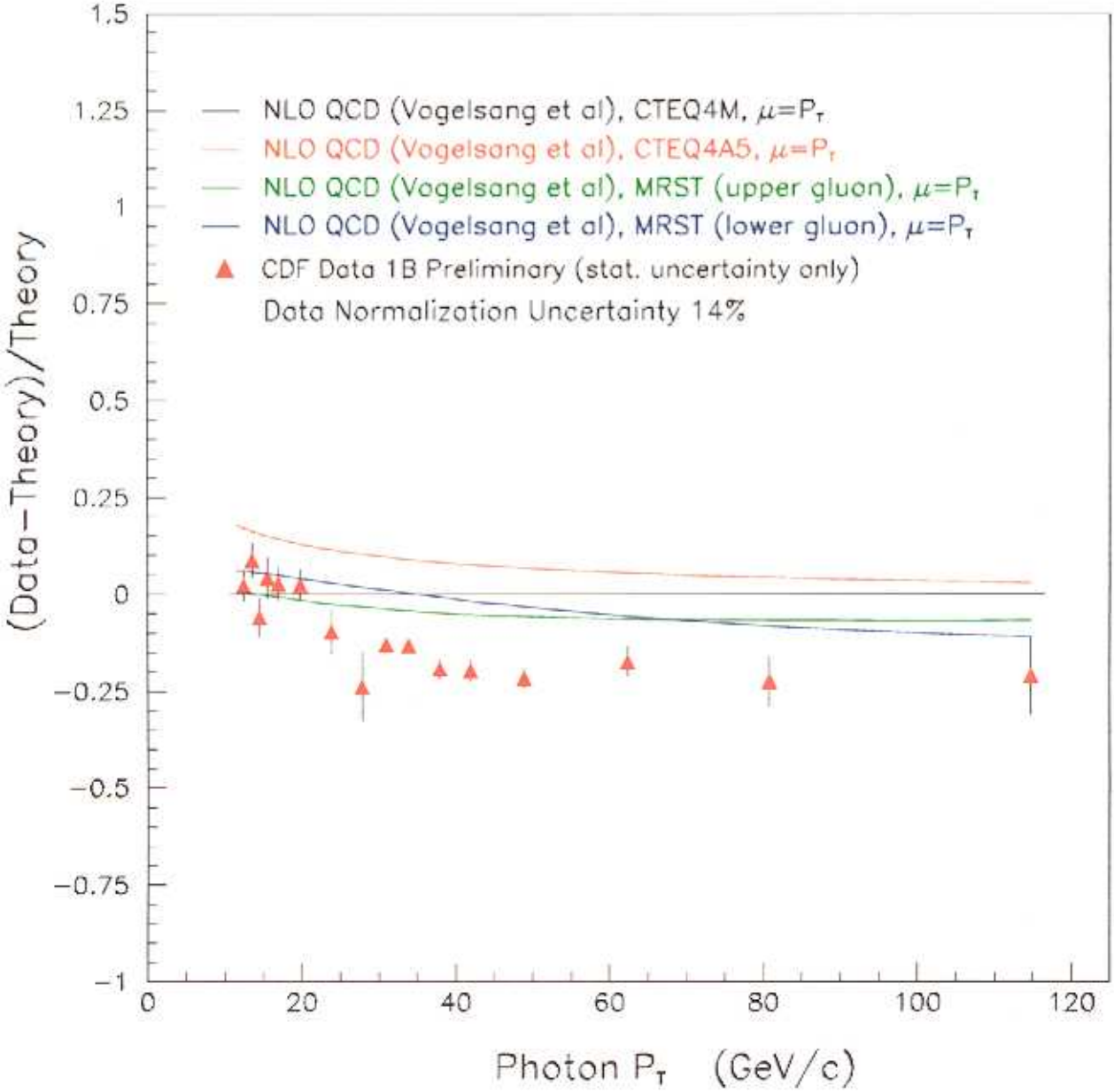
- extension to single-inclusive cross sections , for example direct photons $pp \to \gamma X$



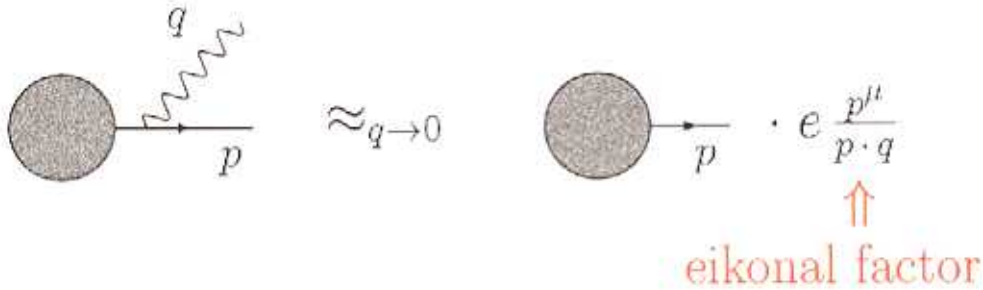
  * motivated by phenomenologically observed "intrinsic-k_T smearing" (Apanasevich et al.)
  * q_T and/or "joint" resummations for photons : Laenen,Sterman,WV; Li; Fink,Owens

- phenomenological studies of these ideas in progress
- large effects likely, promising
- need to put "matching" on firmer basis
- non-perturbative effects also here

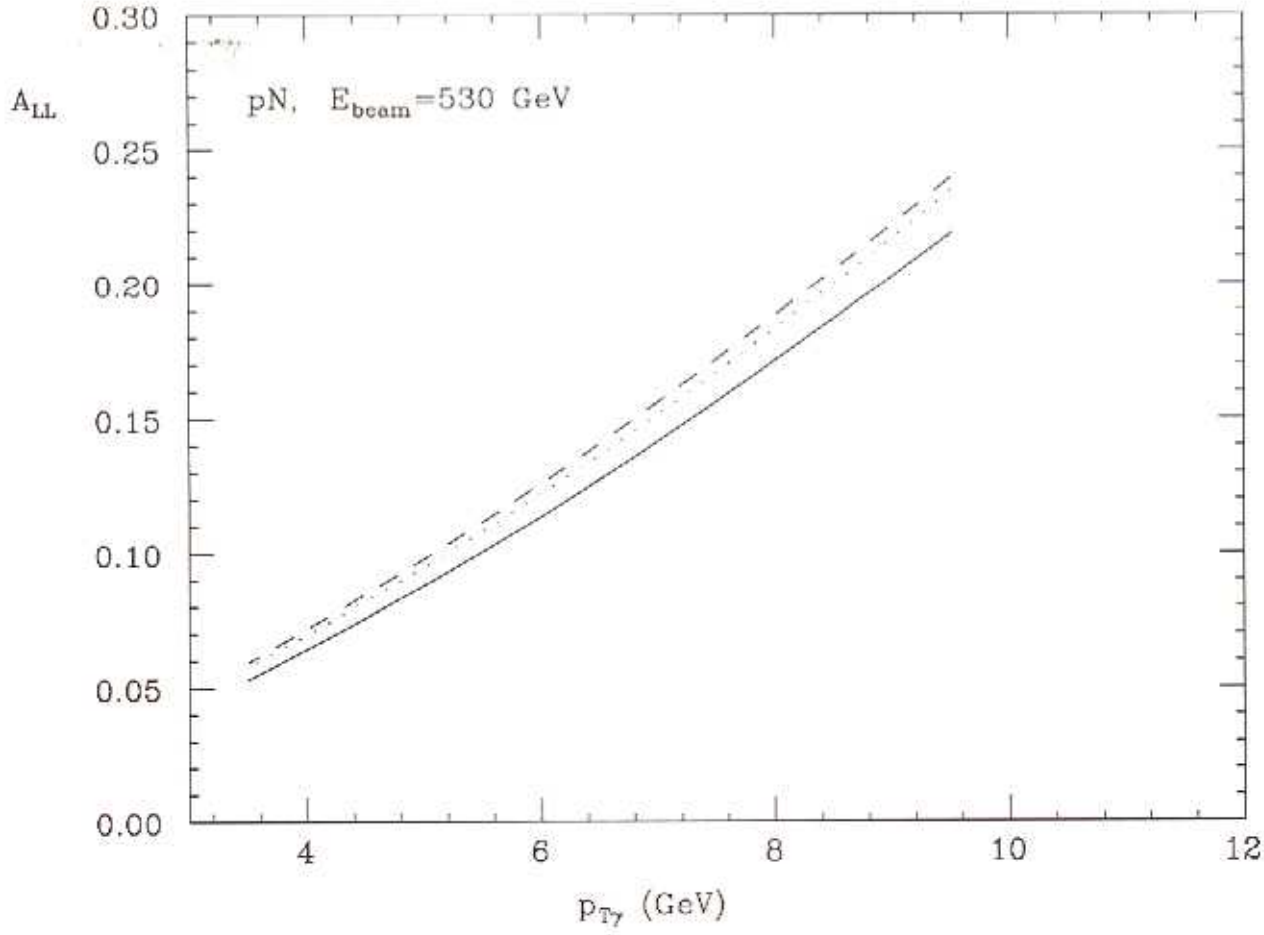
CDF, Run-1B


NLO QCD (Vogelsang et al), CTEQ4M, $\mu=P_T$
NLO QCD (Vogelsang et al), CTEQ4A5, $\mu=P_T$
NLO QCD (Vogelsang et al), MRST (upper gluon), $\mu=P_T$
NLO QCD (Vogelsang et al), MRST (lower gluon), $\mu=P_T$
CDF Data 1B Preliminary (stat. uncertainty only)
Data Normalization Uncertainty 14%
(Data−Theory)/Theory
Photon P_T (GeV/c)
1.5
1.25
1
0.75
0.5
0.25
0
−0.25
−0.5
−0.75
−1
0
20
40
60
80
100
120

V. Resummation for polarized scattering



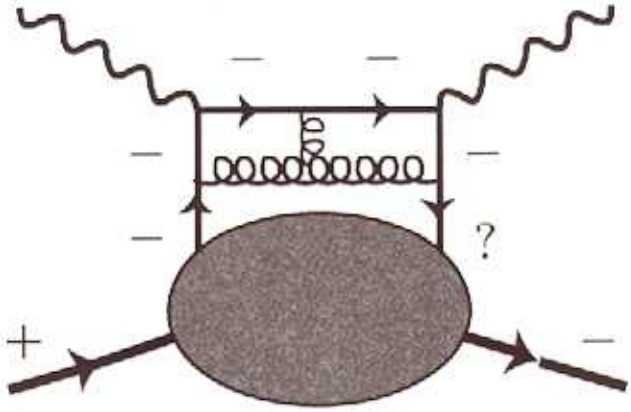
eikonal factor : spin-*in*dependent



- dotted : LO, dashed : NLO
- solid : threshold-resummed

VI. New results for A_{TT}

Helicity flip required $\Rightarrow$ δq not in incl. DIS :



Possibilities (among others) :

- Collins effect, interference fragmentation, etc.
  - require independent measurement of a fragmentation function
  - often not the only mechanism involved (see recently Brodsky,Hwang,Schmidt; Collins)

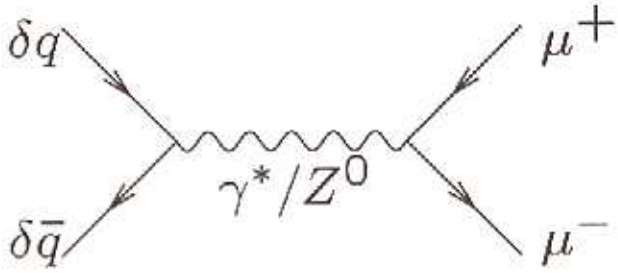
- collisions of transversely pol. protons at RHIC

$$A_{TT} = \frac{d\sigma^{p^\uparrow p^\uparrow} - d\sigma^{p^\uparrow p^\downarrow}}{d\sigma^{p^\uparrow p^\uparrow} + d\sigma^{p^\uparrow p^\downarrow}}$$

Drell-Yan dimuon production, $pp \to \mu^+\mu^- X$

$+$: LO $q\bar{q} \to \gamma^* \to \mu^+\mu^-$

$-$: presumably $\delta\bar{q}$ small



- dir. photons, jets, inclusive hadrons,... :

  Re-emphasize : A_{TT} is expected small,
  "$A_{TT} \ll A_{LL}$" (Jaffe,Saito)

  – no gluon transversity, however, gluon contribution to unpolarized cross section !

  – relevant hard scattering cross sections typically color-suppressed

  – Soffer's inequality limits size of δq

  + rates can be substantial

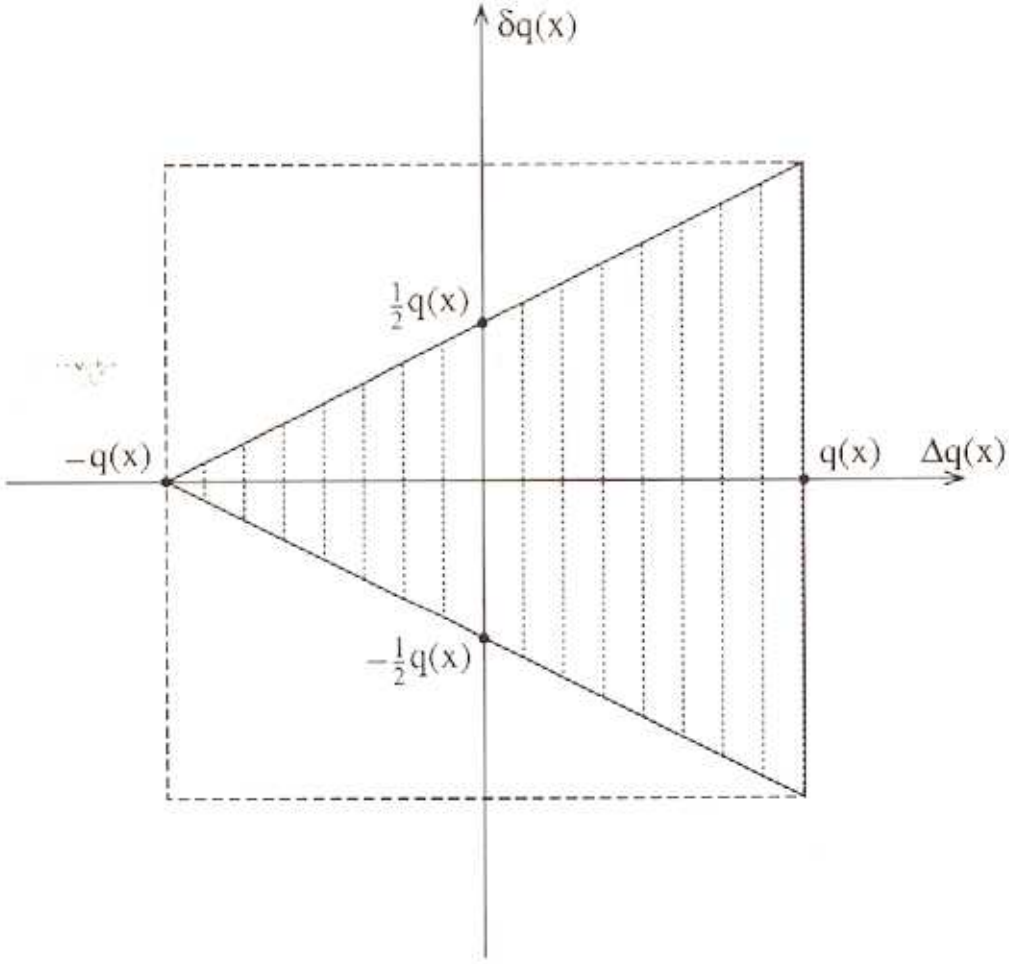
  $\Rightarrow$ small asymmetries *may* be measurable

- estimate "upper bounds" on A_{TT} by saturating Soffer's inequality at $\mu_0 \sim 0.6$ GeV (Soffer,Stratmann,WV)

- "hard to imagine" that A_{TT} could be much bigger !

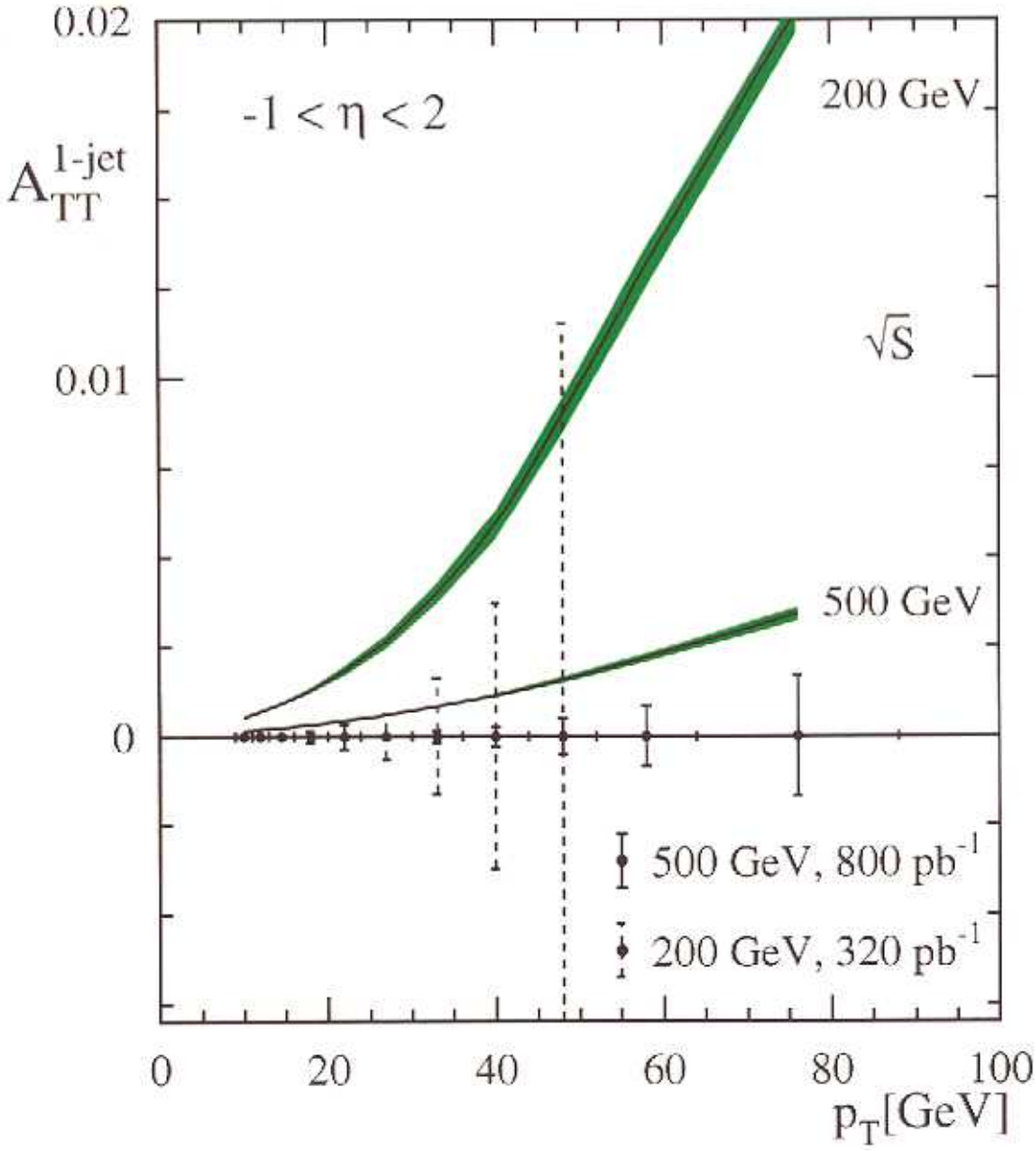
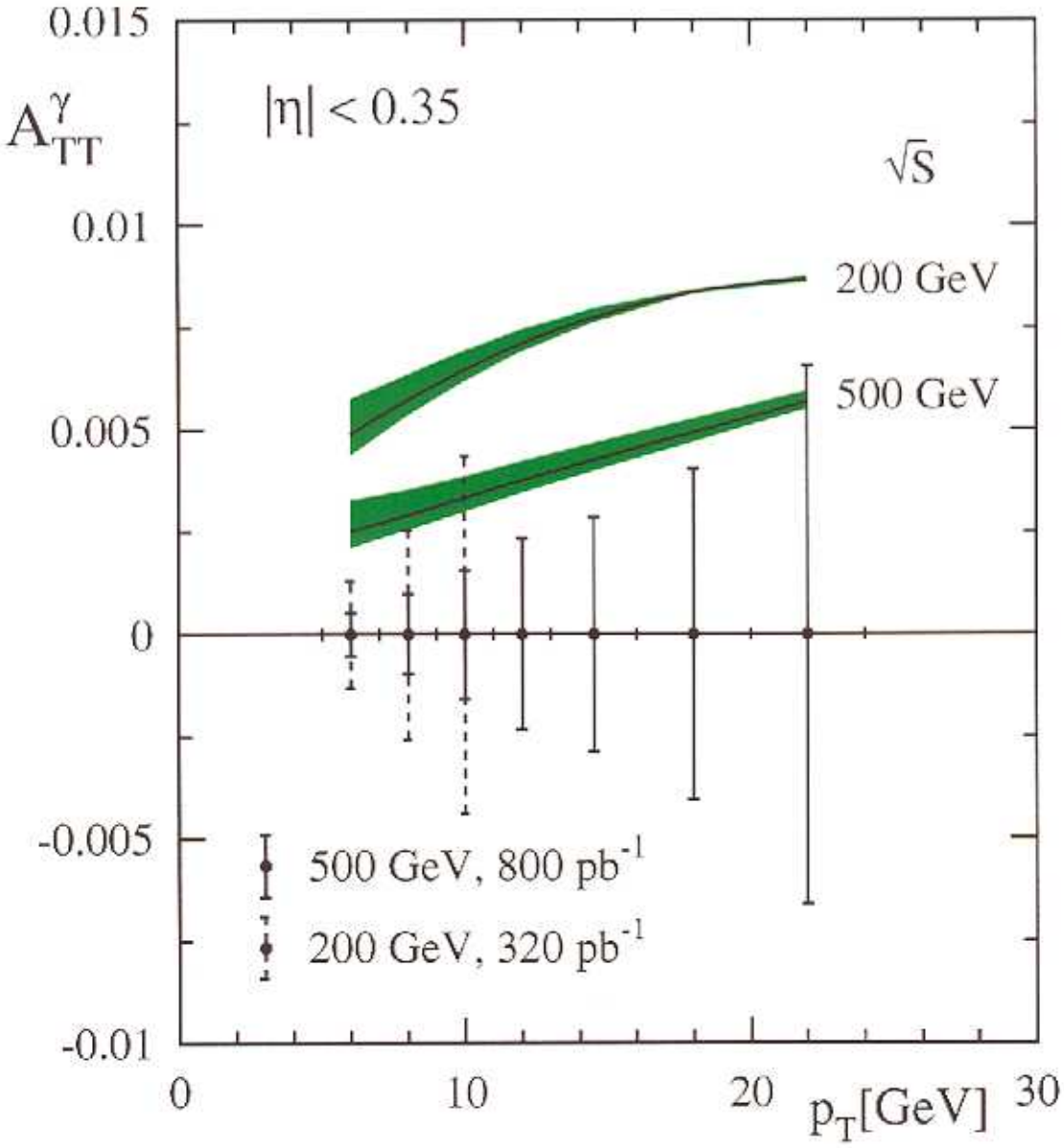
From helicity structure :
Soffer's inequality

$$2\,|\delta q(x,\mu)| \;\leq\; q(x,\mu) + \Delta q(x,\mu)$$



- for all flavors
- constraint for models for transversity
- preserved under QCD evolution
  (Barone; Bourrely,Soffer,Teryaev;
  Martin,Schäfer,Stratmann,WV)

(Soffer,Stratmann,WV)



(Soffer,Stratmann,WV)