Electron Acceleration by Photon Absorption in a Magnetic Field

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It is shown that continuously resonant multi–photon absorption on magnetically confined electrons can be used in the configuration of a high gradient linear accelerator with efficient energy conversion. A method is proposed that utilizes the same laser pulse to accelerate electron bunches while providing synchronous magnetic flux compression to achieve continuous gradients of $10GeV/m$ at 100T. Possible applications include compact and high energy accelerators, photon factories and boosted muon injection into colliders, second harmonic photon generation and electron beam polarization.

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In a seminal paper [1], Lawson spelled out in 1982 the options available for pushing accelerator techniques toward higher energies. He concluded that efficient energy conversion at much higher electric fields was needed, and pointed out that the absorption of real single photons on particles was forbidden in terms of simultaneous energy and momentum conservation. Acceleration in a practical geometry, using intense laser-induced fields, has recently been demonstrated [2].

In the present paper it is shown that a radical departure from previously considered acceleration methods, which does not rely on electric fields but contrives to absorb real photons on magnetically confined electrons, offers high acceleration gradients and efficient "wall plug" energy conversion. With this new concept, using Doppler shifted laser photons, $1GeV$ electrons can in principle be obtained from a compact accelerator only $10cm$ long. The acceleration is continuous, such that $10TeV$ could be reached over $1km$. While a linear electron-positron collider based on such a scheme may itself be cost effective, the losses imposed by light lepton beamstrahlung at high energy and useful luminosities, prompt the idea of applying this acceleration concept to the production of highly boosted relativistic muon pairs for efficient injection into a storage ring collider.

The photon absorption accelerator (PAA) consists of a constant and uniform magnetic field solenoid into which electrons and photons are coinjected axially, in the same direction. The photons are absorbed on the electrons in a way that is analogous with photon absorption by atoms [3]. In the case of the PAA the "atoms" are formed by electrons that are bound in orbits normal to the axial magnetic field. Photon absorption causes the electrons to move in a spiral, accomodating the angular momentum of the absorbed photons in orbital motion while the absorbed photon linear momentum gives axial motion to the electrons.

It turns out that such a system exhibits the remarkable property that the "excited states" of these magnetically bound electrons form an infinite ladder of energy levels that are equally spaced in the laboratory frame. That is to say that, if the absorption conditions are chosen to be resonant for a particular photon wave length, then the orbiting electron continues to absorb resonantly any arbitrary number of photons of the same wave length. This resonance condition involves not only the energy spacing, but also the angular momentum deposited by the absorbed photons, which is accomodated through the combination of mechanical orbital angular momentum and canonical angular momentum residing in the magnetic flux enclosed within the electron orbit. That this combined resonance condition is rigorously maintained for any arbitrary number of photon absorptions can be seen from the full relativistic description of the system (see below). Qualitatively, the maintenance of resonance can be understood in terms of the Doppler shift of photons arriving in the moving reference frame of the electron orbit. Increasing axial electron momentum lengthens the wave length of the photons arriving in the orbit frame. At the same time there is a decrease in the cyclotron frequency as the transverse electron mass increases with absorbed photon energy and angular momentum. These effects compensate each other exactly. The relevant relationships can be derived as shown in the following section.

Let $\Omega = eB/m$ be the cyclotron frequency of electrons in a magnetic field B. Consider now the axial injection, into a solenoid magnetic field B, of a laser beam of photons of energy $\hbar\omega$ merged with a beam of electrons of energy $\gamma_o mc^2$. By choosing the laser photon angular frequency to be $\omega = \Omega\left(\gamma_o + \sqrt{\gamma_o^2 - 1}\right)$, the photons arriving in the moving electron frame will be redshifted into resonance. Alternatively, given ω and B, the electron injection energy can be chosen to be $\gamma_o mc^2$, where

$$\gamma_o = \frac{1}{2}\left(\Omega/\omega + \omega/\Omega\right).$$

What is required, in order to absorb resonantly on each electron an arbitrarily large number N of these laser photons, is a ladder of quantum states of the orbiting electron such that each step absorbs a redshifted photon and at the same time accomodates exactly one additional unit of angular momentum $\hbar$. Energy conservation in the lab frame demands $\gamma, = \gamma_o + N\hbar\omega/mc^2$ where , refers to the rotational energy within the axially moving frame and γ describes the axial motion. The mechanical angular momentum is given by $J_m = r\sqrt{,^2 - 1}mc$ where $r = \sqrt{,^2 - 1}mc/eB$ is the radius of the orbit. Together these relations yield $J_m = \left(,^2 - 1\right) m^2c^2/eB$. The canonical angular momentum of the orbiting electron in the magnetic field can be written $J_B = -\frac{1}{2}r^2eB = -\frac{1}{2}\left(,^2 - 1\right) m^2c^2/eB$. But the total angular momentum must accomodate the spin of N photons: $J = J_m + J_B = \frac{1}{2}\left(,^2 - 1\right) m^2c^2/eB = N\hbar$. Substitution of this result in the energy conservation relation derived above, leads to

$$\gamma = \frac{1}{2}\left(\Omega/,\omega + ,\omega/\Omega\right).$$

It can be seen that this expression for γ has the same form as the resonance condition derived above for γ_o at injection, with the cyclotron frequency in the axially moving frame being reduced to $\Omega/,$ due to rotational energy. Since this result is independent of N, the resonance condition is exactly maintained for any arbitrary number of photon absorptions in a constant magnetic field B.

Given, for example, a CO_2 laser with $\lambda = 10.6\mu m$ and a $5T$ solenoid, the electron kinetic energy at injection would have to be $51.12MeV$ to achieve resonance. The advantages to be gained from injecting electron beams which are already mildly relativistic can be appreciated as follows. Given a 5T solenoid, acceleration of electrons from rest would require microwave photons of $2.1419mm$ wave length (140 GHz). Presently available technology does not provide power levels at this frequency that would be comparable to GW outputs achieved with lasers. Alternatively, acceleration from rest with $10.6\mu m$ photons would require a magnetic field of $1010T$, which is well beyond forseeable technology.

In the frame moving with the electron orbit the photon wave length at resonance is $\Lambda = 2\pi c,/\Omega$, while the orbit radius is given by $r = \sqrt{,^2 - 1}c/\Omega \simeq c,/\Omega$. Thus the wave length $\Lambda \simeq 2\pi r$ has dimensions comparable to those of the electron orbit. At resonance [4], the photon absorption cross section is $\Lambda^2/2\pi \geq 2\pi r^2$. All photons of resonance wave length that pass through the electron orbit are thus absorbed, irrespective of the number of photons in this flux, since the resonance condition is strictly maintained through any arbitrary number of absorptions. This process is to be distinguished from non-resonant multi-photon absorption [5] in which the combination of a particular set of simultaneous photons is required to bridge the energy and angular momentum gap between two quantum levels. In essence, we assume that the absorption process at resonance is independent of photon intensity. The acceleration gradient is then determined by the distance of travel in the lab frame that is required in order to allow a progressively red-shifted photon bunch to be absorbed at the electron orbit.

In the laboratory frame a distance element in the direction of the axial magnetic field is given by $dz = c\sqrt{1 - \gamma^{-2}}dt$, where t denotes time elapsed in the lab. In the reference frame moving with the electron orbit, the period (and hence the time during which the photon field acts during the crossing of the plane of the orbit) is $\tau = 2\pi,/\Omega$, so that $d\tau = (2\pi/\Omega)\,d,$. In the laboratory frame this time element appears lengthened by the factor γ, so that $dt = \gamma d\tau$. Using the relation $2\pi c/\omega = \lambda$ (the laser wave length) and defining the constant $x \equiv \Omega/\omega$ we get: $dz = \lambda x^{-1}\sqrt{\gamma^2 - 1}d,$. Integrating over γ from γ_o to γ, the distance traveled in the lab becomes:

$$z = \frac{\lambda}{2x}\left[\gamma, - \gamma_o - x\ln\left(,\gamma/\gamma_o\right)\right].$$

The accelerator length required to reach a given final energy $U = \gamma,mc^2$ can be regarded as a function $z = z(B)$ through the B dependence of x and γ_o. From the leading term it follows that z has roughly a $1/B$ behavior, such that $z \simeq \pi U/eBc$. The theoretical aborption efficiency from an unpolarized photon bunch is 50% since half the photons have the correct helicity for a given magnetic field direction. Acceleration to $1TeV$ then requires $0.32\mu J$ per electron. In terms of the range of technically feasible fields, the accelerator length approaches an asymptotic minimum at a solenoid field of the order of $100T$ (see Fig. 1). At that field strength an energy of $1GeV$ is reached in $10cm$. Such magnetic fields have been achieved in pulse mode [6] but the strength of materials limits the durability of these devices to an inadequate number of cycles [7]. The length of a $1TeV$ PAA would be only $100m$, but the provision of a conventional pulsed $100T$ magnet of this length is unattractive. One possible solution is to compromise with a superconducting DC solenoid of much greater length. A better approach is proposed below.

Very large thermal electron currents have been obtained through electron emission from laser generated plasma surfaces [8]. Such currents are self-limited through the build up of retarding electric fields due to positive ions left behind in the plasma surface. A more effective way of harnessing the electron flow out of such a hot surface is proposed here. The idea is to use the same laser pulse, that drives the acceleration, to generate a synchronously travelling plasma ring on the inner surface of the accelerator tube (diameter $d = 4mm$, for example, in the case of a $1TeV$ PAA driven with a $248nm$ KrF laser in a $100T$ field). This would require a laser pulse divergence of d/z. A bias magnetic field, provided by a solenoid surrounding the tube, would cause electrons expelled from the hot surface to keel over into trajectories that plunge back into the surface at locations displaced in azimuthal angle, all in the same direction. The resulting azimuthal current encircles the bore of the accelerator tube, generating a magnetic field that enhances the applied bias field at radii smaller than the location of the plasma surface. The net result of this current is to draw the bias flux into the accelerator tube. That reduces the bias field at the plasma surface and leads to quenching of the current when all the flux has been compressed into the inner region. This occurs when $B = B(bias)D^2/d^2$, where D is the inner diameter of the bias coil. A $40mm$ diameter coil of $1T$ would suffice to supply the flux required to reach $100T$ in the central region. The quenching effect may be used to exploit the stability of a modest DC superconducting bias coil to "flat-top" the magnetic pulse in the central region.

Previous results from laser experiments suggest that adequate electron currents can be obtained to reach $100T$ with realistic pulse energies. In the case of the thermal electron surface current loops proposed here, no azimuthal *emf* is involved and retarding electric fields do not develop in the absence of charge separation. The process can be thought of as a thermally driven surface Hall current. It is expected that much lower electron temperatures will be effective in this geometry, leading to enhanced efficiency compared to previous laser demonstrations. Of crucial importance is the ability of this scheme to operate at the extremely short pulse lengths that suffice for electron bunch acceleration. Material damage is then greatly reduced and mechanical strength does not pose limitations.

The synchrotron power radiated in the frame moving with the electron orbit is given by [9]: $\dot{S} = 2\left(,^2 - 1\right)\Omega^2 e^2/3c$. A photon of energy ϵ, emitted at an angle θ with respect to the axial motion of the electron, is boosted in the laboratory frame to $\epsilon\left(\gamma + \sqrt{\gamma^2 - 1}\cos\theta\right)$. The emission in the orbit frame is symmetrically distributed within the range $\pi/2 - 1/, \leq \theta \leq \pi/2 + 1/,$, so that the synchrotron quanta are on average boosted by exactly a factor γ. The energy radiated in the laboratory during the interval over which , increases to $, + d,$ is thus $dS = \left[4\pi\Omega e^2\gamma\left(,^2 - 1\right)/3c\right] d,$. Integration over , yields:

$$S = \frac{\pi e^2 \Omega}{6cx}\left[,^4 - 1 - 2\left(,^2 - 1\right)\left(1 - x^2\right) - 4x^2 \ln ,\right].$$

The retention of only the fourth order term in , provides an approximation to 1:10000, leading to the following expression for the fractional energy loss via synchrotron radiation: $s \equiv S/U \simeq \left(4\pi^2 e^2 B^2 \lambda U\right)/\left(3k_e^2\, m^2 c^4\right)$. It is found that the losses are negligible, amounting to less than 16 parts per million in the KrF laser acceleration of electrons to $10TeV$ in a $100T$ field.

Practical considerations limit current plans for heavy lepton colliders to muons. Thus far, proposals have been made to harvest muons from the decay of pions produced by high energy protons in fixed targets. The muon yield is expected to average one per $60GeV$ of proton energy, but muon beam formation poses challenges in terms of both longitudinal and transverse cooling [10]. For injection into muon colliders it is desirable to have highly boosted relativistic beams of μ^+ and μ^- of equal momenta and small emittance. The PAA might come into consideration as a muon injector meeting these objectives. This would require injecting electrons and positrons in the same direction, and utilizing photons of both helicities to accelerate these. The electrons and positrons rotate oppositely, so that muon pairs with cancelled transverse momenta can be obtained via the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$ from in-beam collisions at the point when the transverse energy of both e^+ and e^- has reached the muon mass. The cross section (neglecting weak interference) [11] for this reaction in terms of the center of mass energy $\Lambda_\mu c^2$ of each outgoing muon is:

$$\sigma_\mu = \frac{\pi e^4}{6,_\mu^2 m_\mu^2 c^4}\left[2 + ,_\mu^{-2}\right]\sqrt{1 - ,_\mu^{-2}}.$$

Differentiation with respect to $,_\mu$ shows that the threshold peak $\bar{\sigma}_\mu \simeq 1.008\mu b$ in the cross section occurs when $,_\mu = \left[5/\left(\sqrt{21} - 1\right)\right]^{0.5} \simeq 1.181$. The cross section σ_p for annihilation into photons [12]:

$$\sigma_p = \frac{\pi e^4}{m^2 c^4}\left[\frac{,^4 + ,^2 - \frac{1}{2}}{,^4\left(,^2 - 1\right)}\ln\left(, + \sqrt{,^2 - 1}\right) - \frac{\sqrt{,^2 - 1}}{2,^3}\right]$$

exceeds that for muon production by a factor twenty in the region above the muon threshold; at the threshold peak the branching to muons is 4.23%. It follows that such an arrangement would in effect be a high energy photon "factory", yielding highly boosted "byproduct" muon pairs (see example in Fig. 1). These would be injected into a collider after separation in a dipole magnet. Losses during acceleration in the injector can be prevented by applying simultaneous laser photon bunches of positive and negative helicity separately, each bunch containing the same number of photons required for the acceleration but differing in bunch length. In this way positrons, for example, could be made to reach their final orbit radius first and continue drifting until they are struck as the more slowly expanding electron orbits reach final radius.

The relations derived above are illustrated in Fig. 1 for the example of a CO_2 laser driven PAA for muon pair production at the threshold peak. The kinetic energy at injection K, the final orbit radius r, the accelerator length z and the fractional energy loss to synchrotron radiation s are shown as a function of the axial magnetic field B. At a field $B = 100T$, acceleration with $10.6\mu m$ photons achieves this condition at a final particle energy of $154GeV$, giving the muons a relativistic boost of 1234 within a divergence cone of $430\mu rad$. The length of the required PAA is $16.1m$, with $K = 2.1MeV$, $r = 4.16mm$ and $s = 10.3$ parts per million. As a comparison, threshold production of muons with a KrF laser driven PAA at $100T$ would lead to a boost of 45000 within a $12\mu rad$ cone and a final energy of $4.7TeV$ after an acceleration distance of $494m$, requiring $K = 109MeV$ but with the same r and s as for the CO_2 laser.

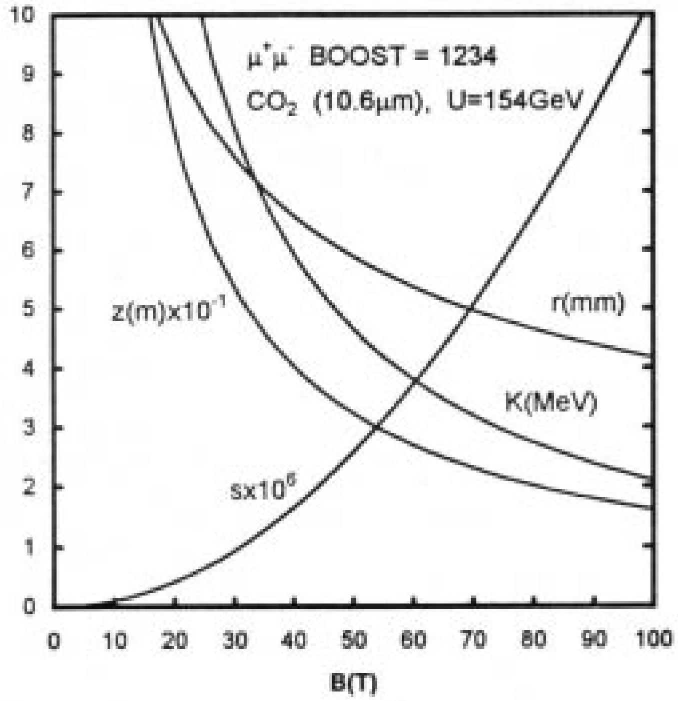

FIG. 1. Example of a CO_2 laser driven PAA with final transverse energy at the muon threshold peak. The parameters K (electron kinetic energy at injection), r (final orbit radius), z (accelerator length) and s (fractional energy loss by synchrotron radiation) are shown as a function of the axial magnetic field B.

Observations of the acceleration mechanism developed above, which can be termed continuously resonant multiphoton absorption (CRMPA) by magnetically confined electrons, do not appear in the literature. It is noteworthy that relativistic effects associated with gyromagnetic frequencies of charged particles subjected to plane electromagnetic waves in the presence of a magnetic field were discussed by Kolomenskii and Lebedev [13]. These ideas have been extended to tapered magnetic fields [14] and experimental work with gyrotron-like devices [15–17], but do not provide insight relevant to the phenomenon discussed in the present paper. The questions raised in relation to the CRMPA mechanism are reminiscent of recoilless nuclear absorption of gamma rays (which is at least in one sense the exact opposite); it was the experimental observation of the Mössbauer effect that forced accomodation within the quantum description of solids. Predictions related to CRMPA that need confirmation are that the absorption rate is constant and independent of photon intensity, and that photons of one helicity only are absorbed from an unpolarized beam, leaving a residue of 50% that is circularly polarized. Other issues are effects due to finite accelerator length and energy spread, off-axis injection of electrons into the solenoid, space charge, and convergent photon beams. The latter suggests the possibility of second harmonic generation by choosing a convergence cone of half-angle $\phi = \cos^{-1}\sqrt{1-\gamma_o^{-2}}$ and twice the magnetic field appropriate to parallel photon beams. Given axial symmetry, such a scheme would lead to 50% resonant foreward scattering at double frequency (normal incidence in moving frame), while the backscattered photons would restore energy to the electron beam. It may be possible to manipulate electron polarization on injection into a PAA by means of a spin-orbit resonance "bump" along the axial magnetic field, such that $\pm\triangle B \simeq g_e\hbar/r^2e$, where r is the electron orbit radius and g_e the electron g-factor. With a view to practical applications it is desirable to make magnetic flux compression work. Quartz accelerator tubes may provide oblique angle absorption of laser beam energy and minimize losses from flux propagation. Application to muon colliders would require optimal e^+e^- luminosity and economical e^+ production.

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