Fluctuations in the CGC: KNO scaling and higher-order eccentricities

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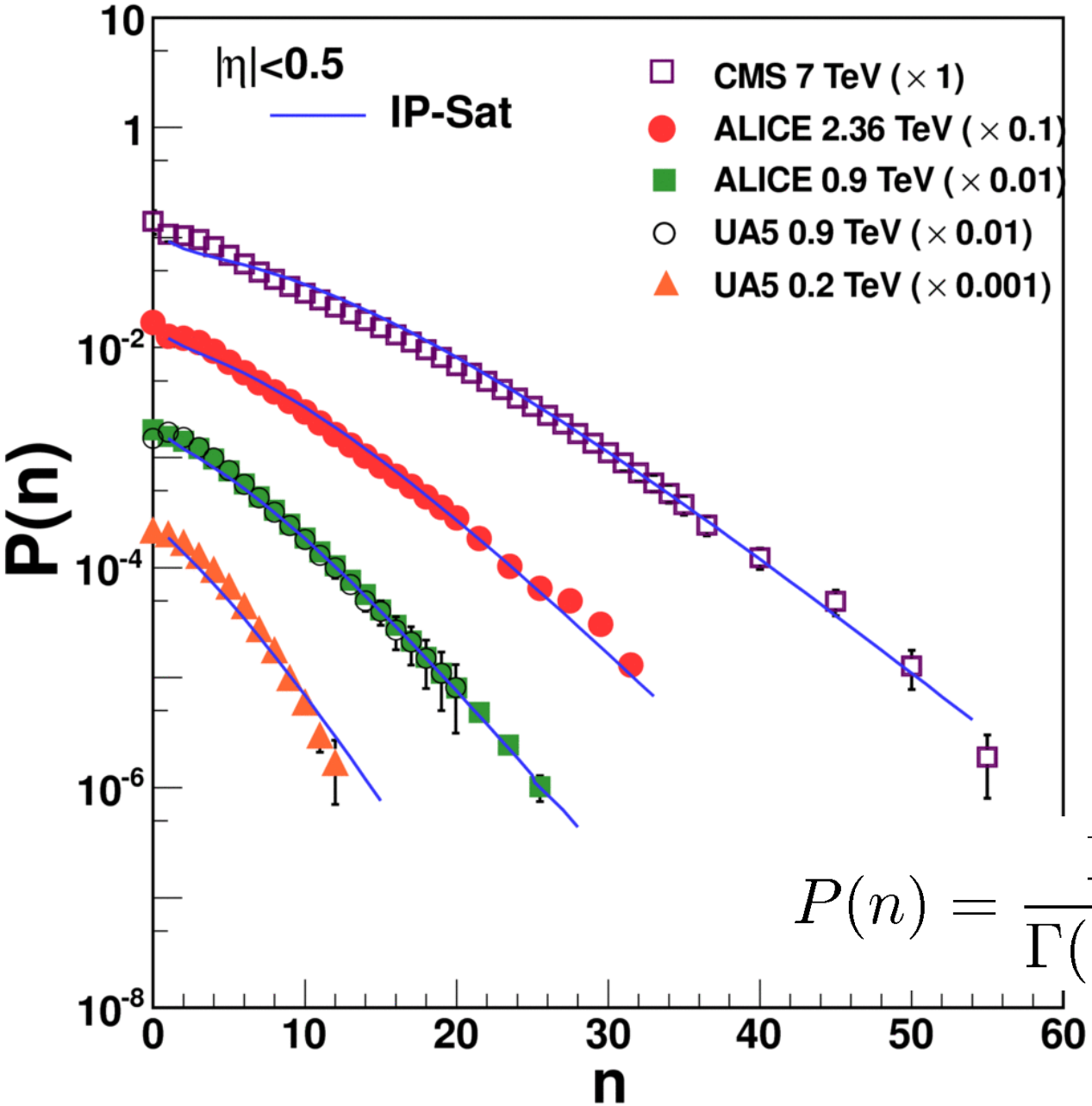
RHIC & AGS users meeting, June 12th, 2012

Multiplicity distributions in *pp* collisions

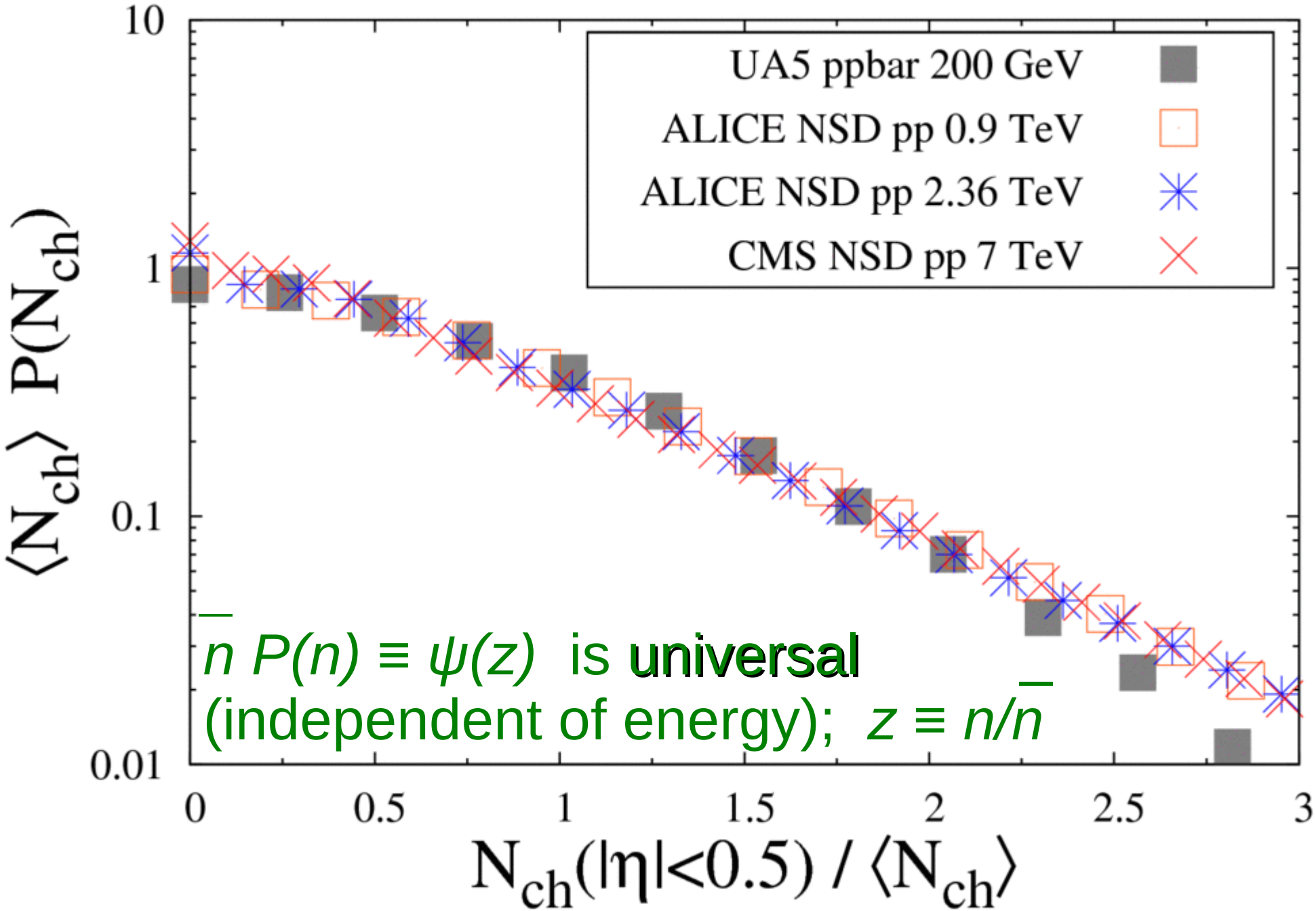


P(n): negative binomial distribution

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\langle n \rangle^n k^k}{(\langle n \rangle + k)^{n+k}}$$

Tribedy & Venugopalan: arXiv:1112.2445

KNO scaling in high-energy pp data



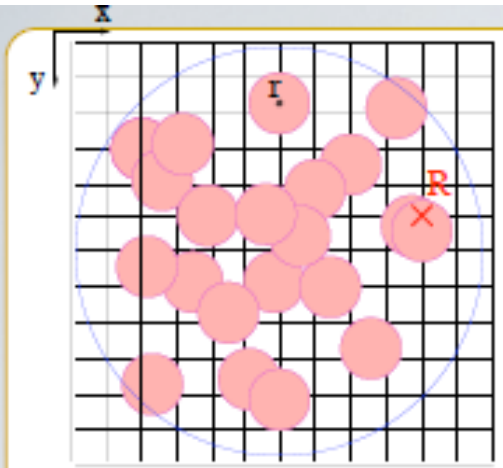
$\bar{n}\, P(n) \equiv \psi(z)$ is universal (independent of energy); $z \equiv n/\bar{n}$

Glauber / geometry fluctuations

(pA and AA collisions)

- fluctuations of transverse positions of valence charges in the colliding nuclei
- fluctuations due to finite probability of inelastic nucleon-nucleon interaction P(b)

fluctuations of valence partons in ⊥ plane



1. Initial conditions for the evolution (x=0.01)

$$N(\mathbf{R}) = \sum_{i=1}^{A} \Theta\left(\sqrt{\frac{\sigma_0}{\pi}} - |\mathbf{R} - \mathbf{r_i}|\right) \longrightarrow Q^2_{s0}(\mathbf{R}) = N(\mathbf{R})\, Q^2_{s0,\,\mathrm{nucl}}$$

$$\downarrow$$

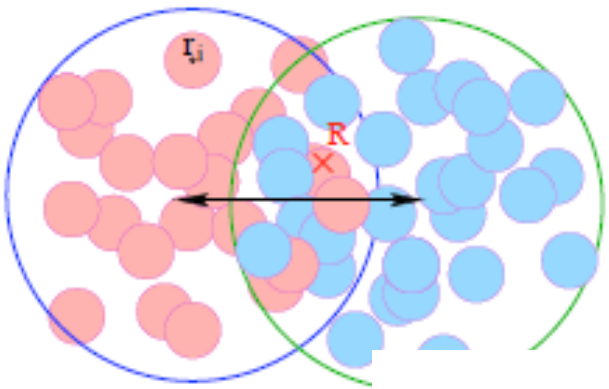
$$\varphi(x_0 = 0.01, k_t, R)$$

2. Solve local running coupling BK evolution at each transverse point

rcBK equation or KLN model

$$\downarrow$$

$$\varphi(x, k_t, R)$$



3 Calculate gluon production at each transverse point according to kt-factorization

INPUT: $\varphi(\mathbf{x} = \mathbf{0.01}, \mathbf{k_t})$ FOR A SINGLE NUCLEON:

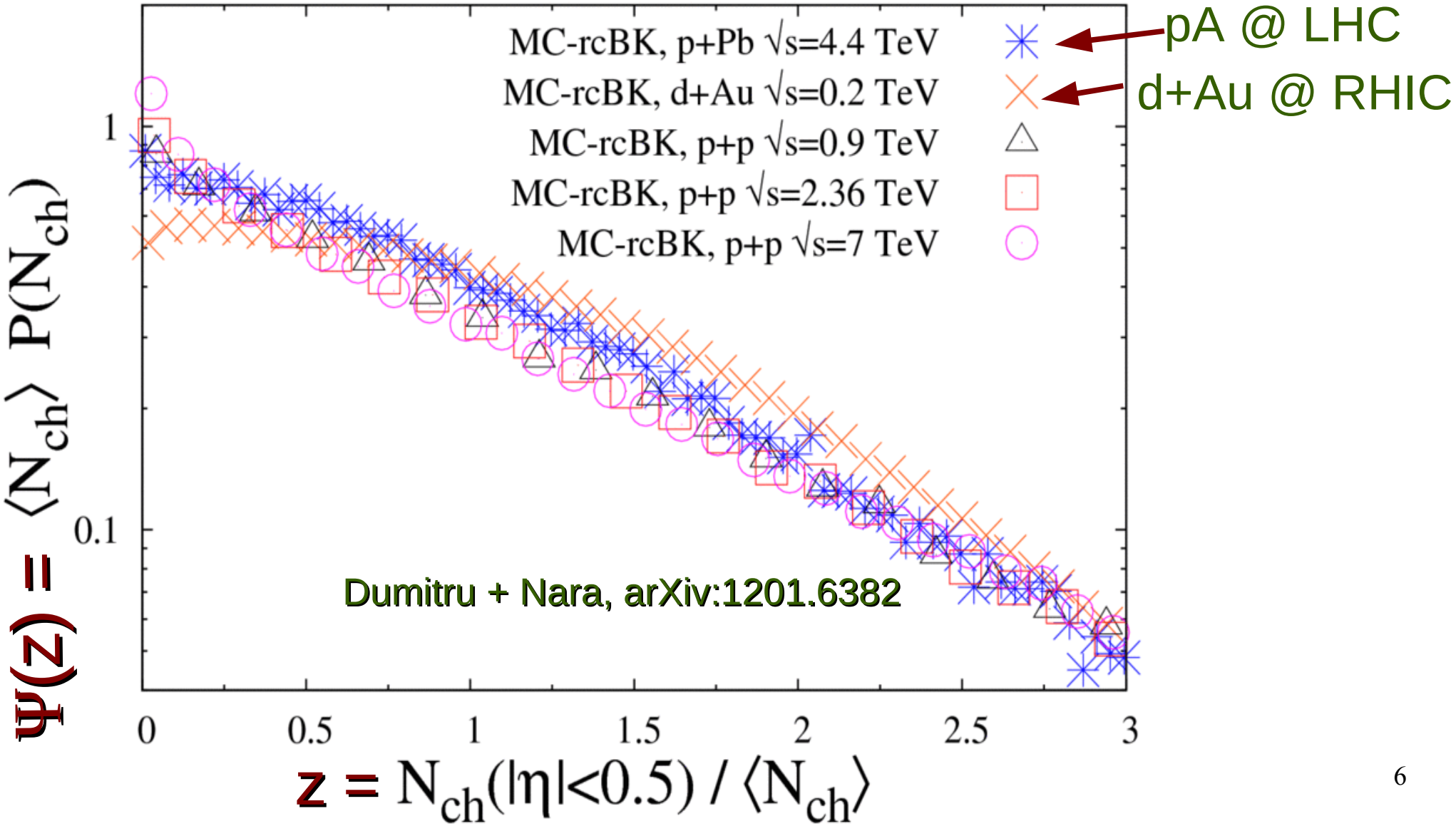
$$N_{\mathrm{part,A}}(\vec{b}) = \sum_{i=1\cdots A} \Theta\left(P(\vec{b} - \vec{r}_i) - \nu_i\right) \ .$$

$$P(b) = 1 - \exp[-\sigma_g T_{pp}(b)], \qquad T_{pp}(b) = \int d^2s\, T_p(s)\, T_p(s-b)$$

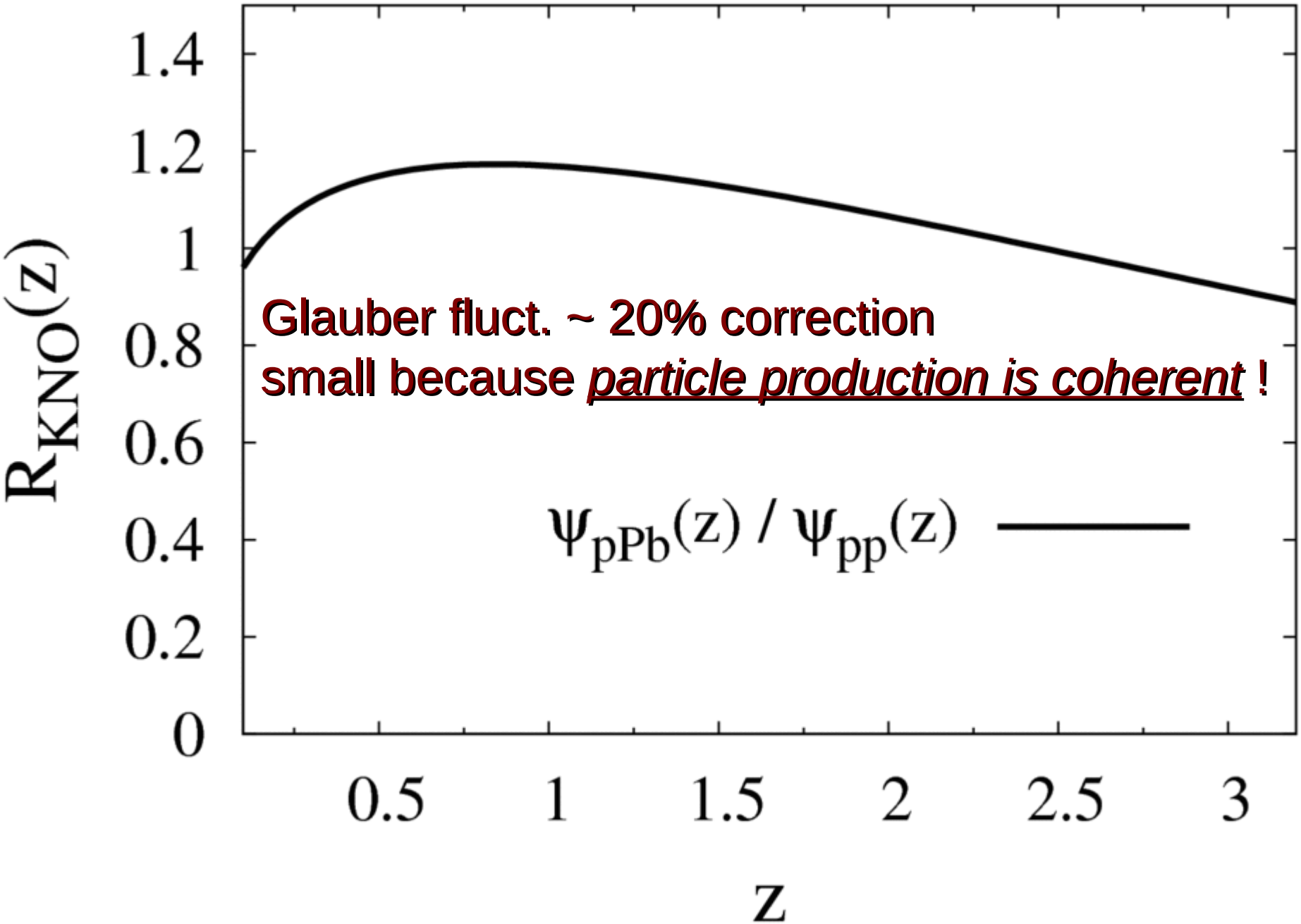
$$T_p(r) = \frac{1}{2\pi B} \exp[-r^2/(2B)] \qquad \sigma_{NN}(\sqrt{s}) = \int d^2b\, (1 - \exp[-\sigma_g T_{pp}(b)])$$

KNO scaling (even p+Pb approx.; prediction)

for A+B : $k_{AB} \sim k_{pp} \min(T_A, T_B)$

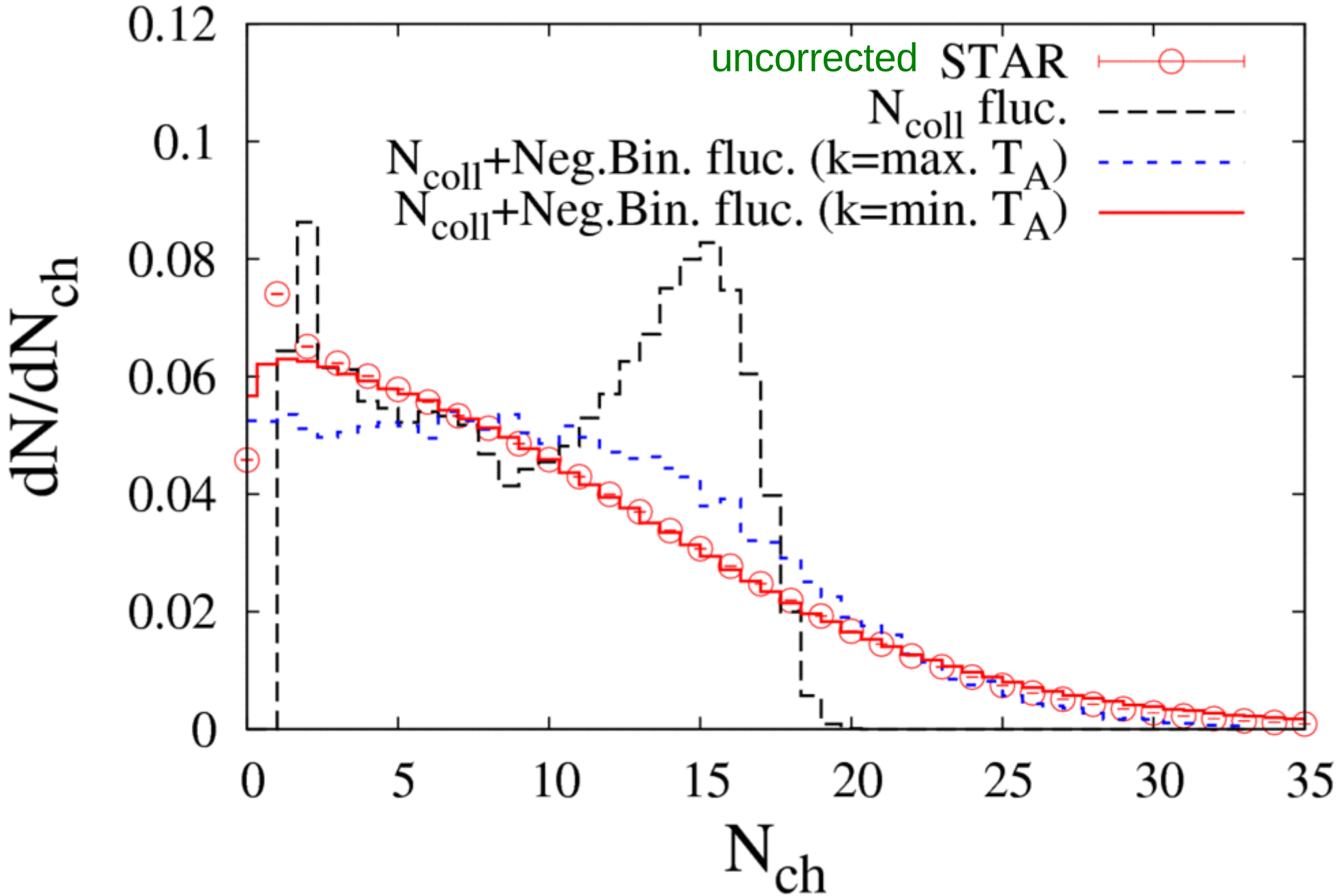


KNO scaling in p+Pb @ 4.4TeV on linear scale



Glauber fluct. ~ 20% correction
small because *particle production is coherent* !

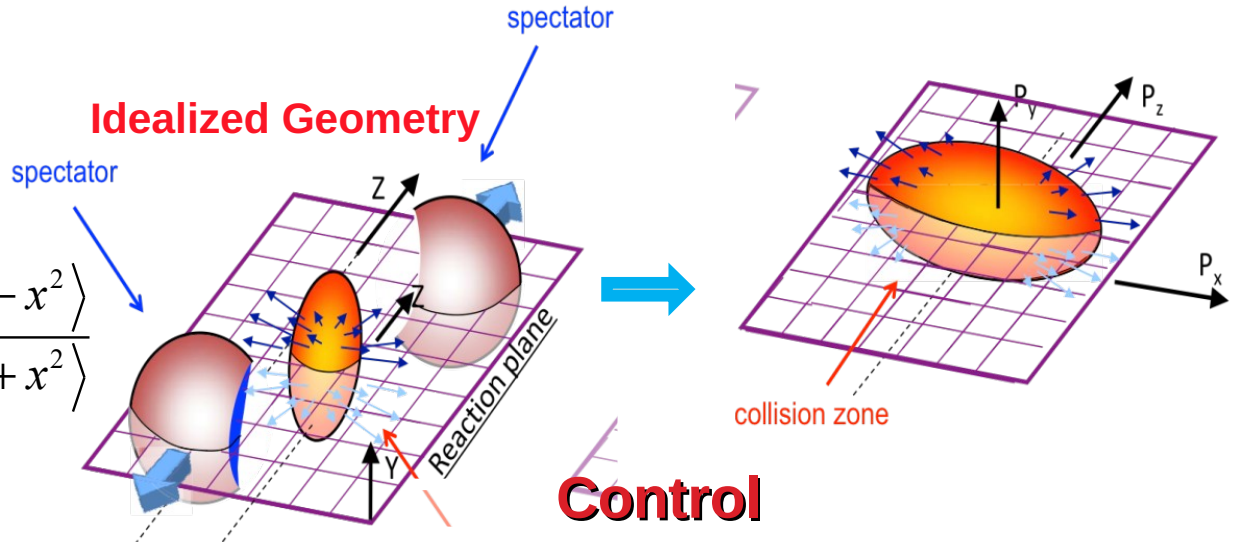
need KNO flucs also for d+Au@RHIC



The Flow Probe

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

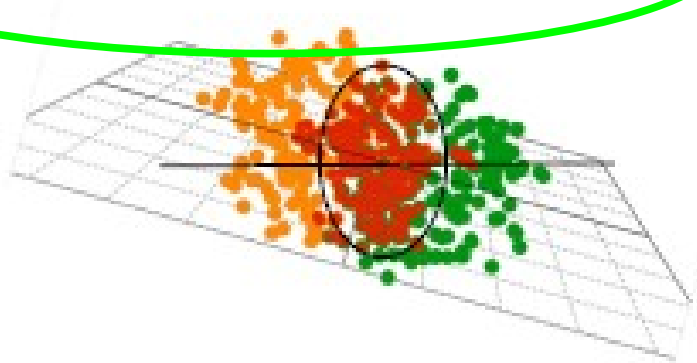
Idealized Geometry

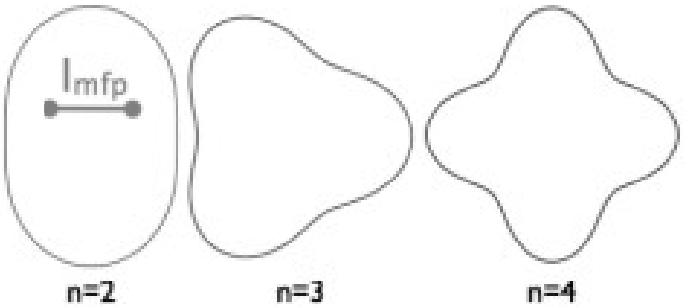


Control parameters

Initial Geometry characterized by many harmonics

Actual collision profiles are not smooth, due to fluctuations!

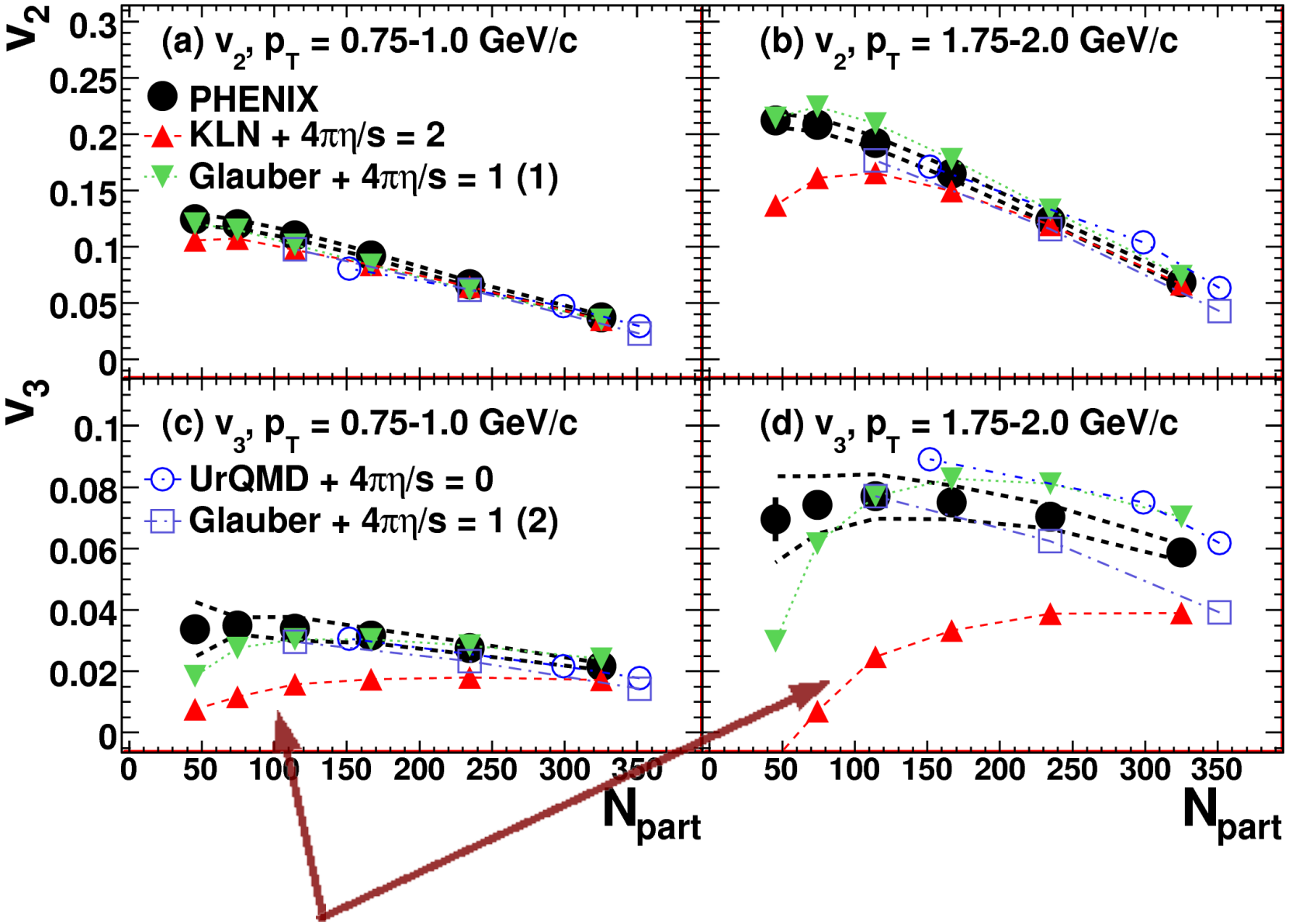




$$\varepsilon_n = \frac{\left\langle r^n \cos\left(n\varphi_{part}\right)\right\rangle^2 + \left\langle r^n \sin\left(n\varphi_{part}\right)\right\rangle^2}{\left\langle r^n \right\rangle^2}$$

Initial eccentricity (and its attendant fluctuations) ε_n drive momentum anisotropy v_n with specific scaling properties

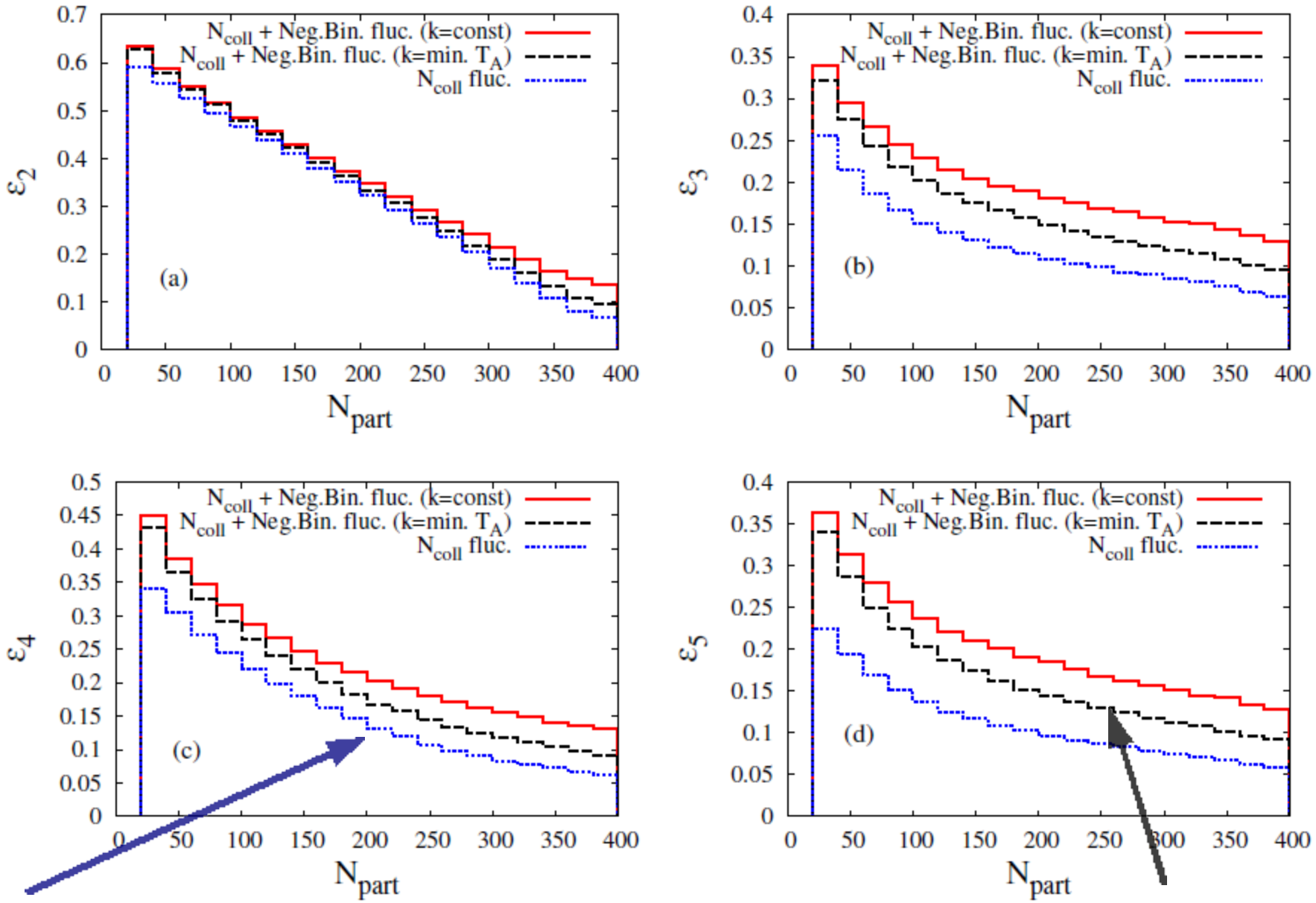
Phenix Collab., arXiv:1105.3928



- MC-KLN initial condition (k_T factorization with Glauber fluct. only) leads to underestimate of v_3 !

Eccentricities ε_n in Au+Au

Dumitru + Nara, arXiv:1201.6382



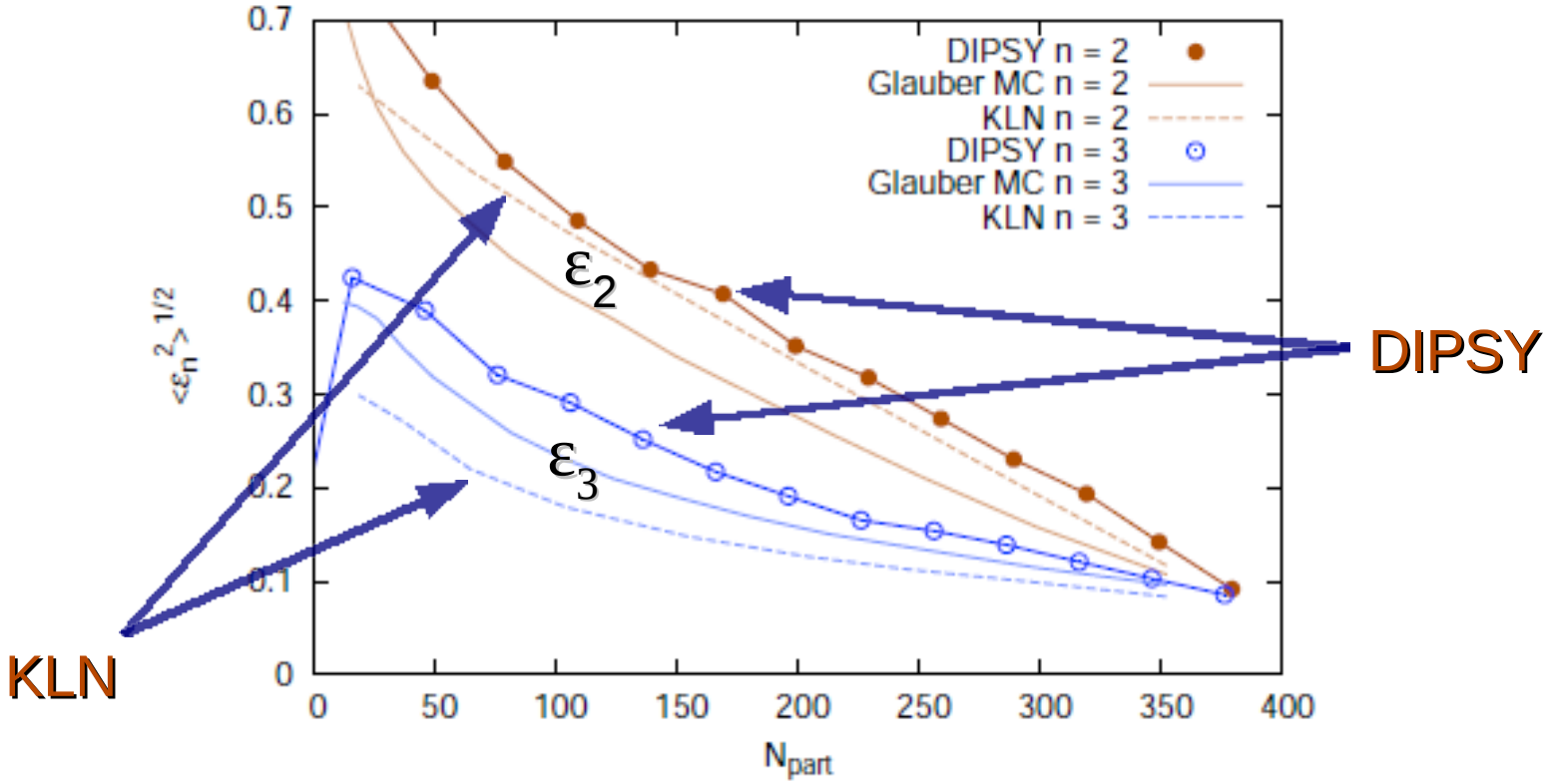
Glauber fluc only

Glauber + NBD
$k \sim \min(T_A, T_B)$

DIPSY MC (Lund) w. fluctuations in BFKL ladders

Results: $\varepsilon_2, \varepsilon_3$

C. Flensburg: ISMD 2011, Hiroshima

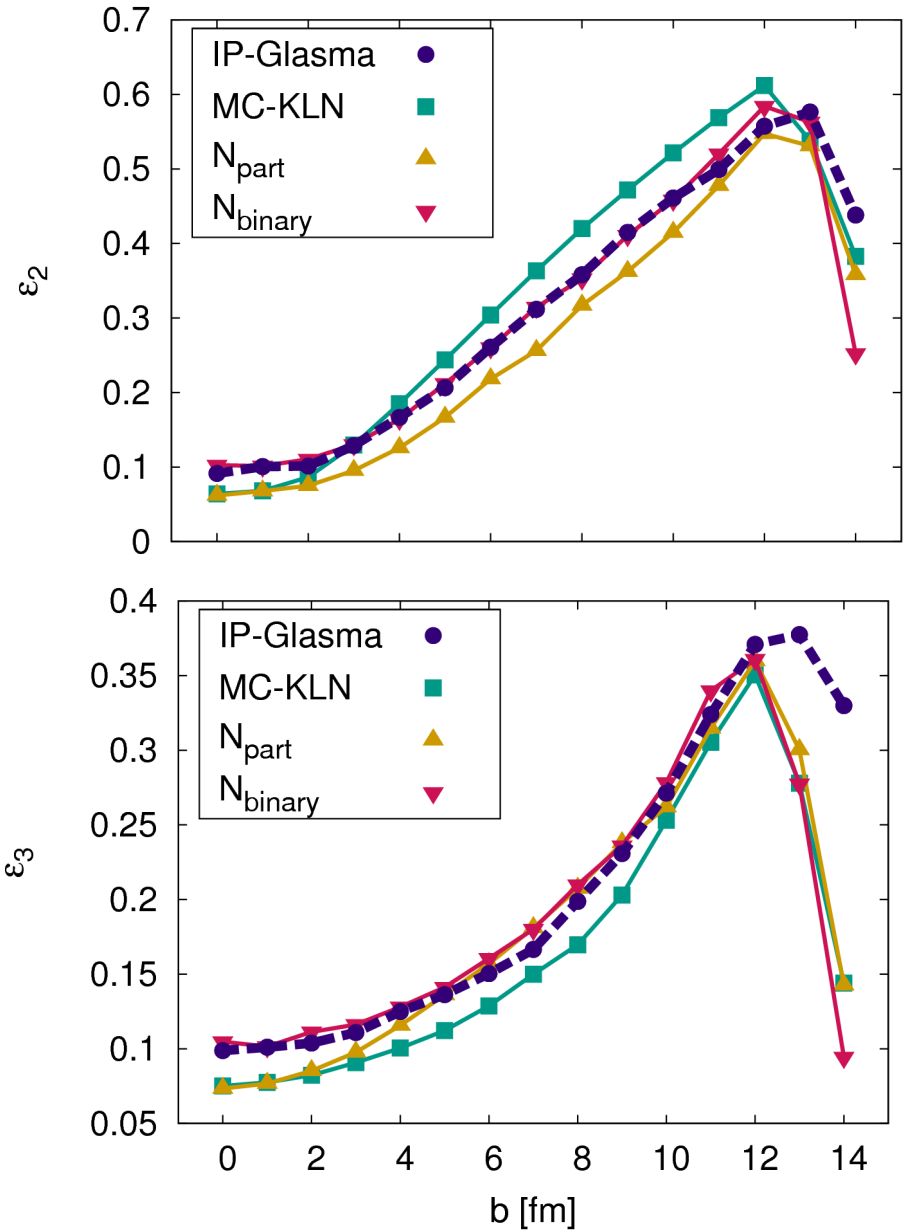


compare DIPSY <-----> MC-KLN:
ε_2 similar, ε_3 larger

- does DIPSY reproduce KNO in pp, pA ?

Classical YM solutions w/ fluctuating initial conditions (incl. NBD !)

Schenke, Tribedy, Venugopalan, arXiv:1202.6646



- lower ε_2 than MC-KLN
- higher ε_3

KNO scaling:

Koba, Nielsen, Olesen, NPB 40 (1972) 317

$\bar{n}\, P(n) \equiv \psi(z)$ is universal (independent of energy); $z \equiv n/\bar{n}$

Note that if $k \ll \bar{n}$, NBD can be written as

$$\bar{n}\, P(n)\, dz \sim z^{k-1}\, e^{-kz}\, dz \qquad , \quad z \equiv n/\bar{n}$$

So, if k≈const, this leads to KNO scaling !

fit to pp @ LHC: $k / \bar{n} \sim 0.16$ at 2360 GeV

but why is

i) P(n) a NBD ?

ii) $k \ll \bar{n}$?

NBD from MV model

Gelis, Lappi, McLerran: arXiv:0905.3234

for large nucleus, $A^{1/3} \to \infty$

$$S_{\rm MV} = \int d^2x_\perp \frac{1}{2\mu^2}\rho^a\rho^a$$

+ soft YM fields +

coupling of soft ↔ hard

$$\left\langle \frac{dN}{dy_1\cdots dy_n}\right\rangle_{\rm conn.} = \beta_n \left\langle \frac{dN}{dy_1}\right\rangle \cdots \left\langle \frac{dN}{dy_n}\right\rangle$$

$$\beta_n = (n-1)!\ \ k^{1-n}$$

→ NBD

$$\bar{n} = \# \frac{N_c(N_c^2-1)}{\alpha_s} Q_s^2 \pi R^2$$

$$k = \# \frac{N_c^2-1}{2\pi} Q_s^2 \pi R^2 \quad \sim T_A$$

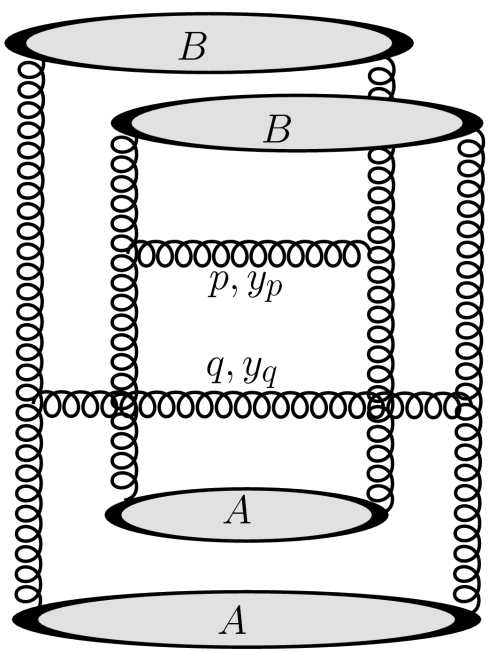
So, why KNO then ?

$$\bar{n} = \# \frac{N_c(N_c^2-1)}{\alpha_s} Q_s^2 \pi R^2$$

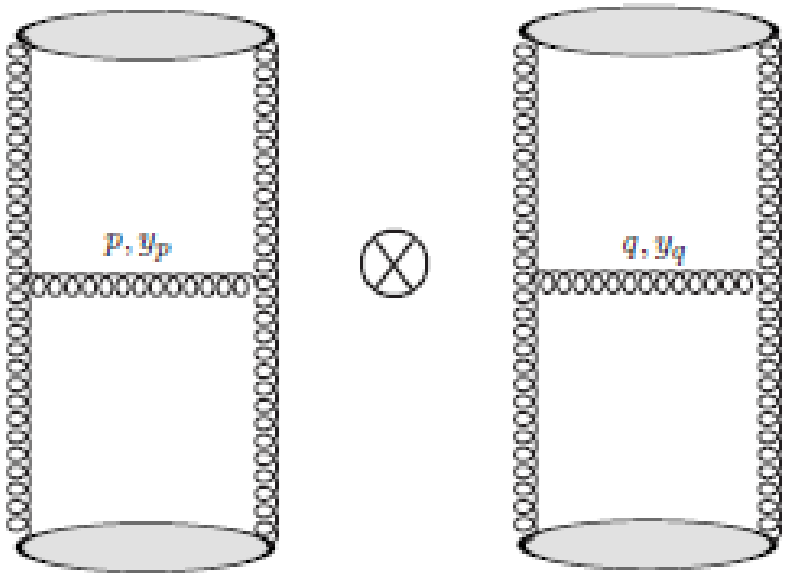
$$k = \# \frac{N_c^2-1}{2\pi} Q_s^2 \pi R^2$$

$$\boxed{\frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \gg 1}$$

Why is k = O(α_s^0) ?



same order in α_s



KNO scaling emerges if

i) Gaussian action

ii) high occupation number

How about

i) quantum evolution

ii) corrections to Gaussian action ?

Beyond MV action...

Dumitru, Jalilian-Marian, E.P.
PRD 2011

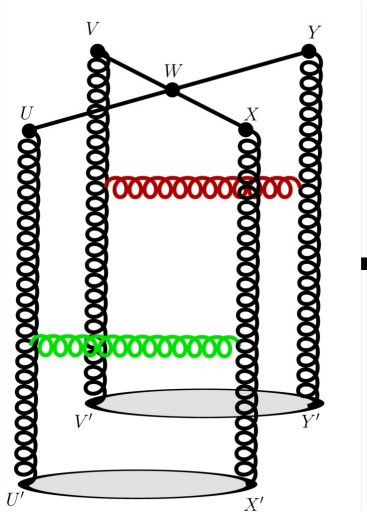
$$S = \int d^2x_\perp \left\{ \frac{1}{2\mu^2}\rho^a\rho^a + \frac{1}{\kappa_4}\left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right)\rho^a\rho^b\rho^c\rho^d \right\}$$

+ soft YM fields + coupling of soft $\leftrightarrow$ hard

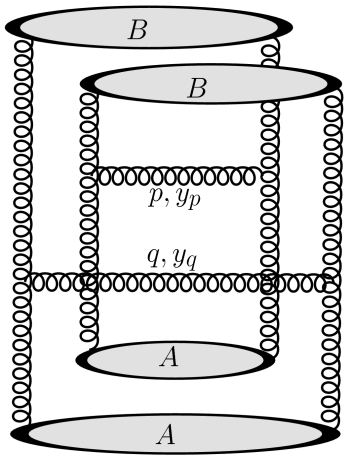
- $\mu^2 \sim g^2 A^{1/3}$; $\kappa_4 \sim g^4 A$

Recalculate width k^{-1} of mult. distribution

$$\left\langle \frac{dN_2}{dy_1 dy_2} \right\rangle_{\text{conn.}} = \frac{1}{k} \left\langle \frac{dN}{dy_1} \right\rangle \left\langle \frac{dN}{dy_2} \right\rangle$$

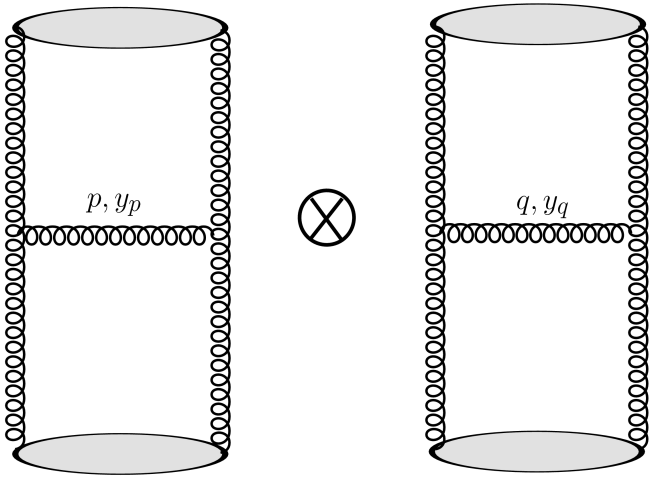


+

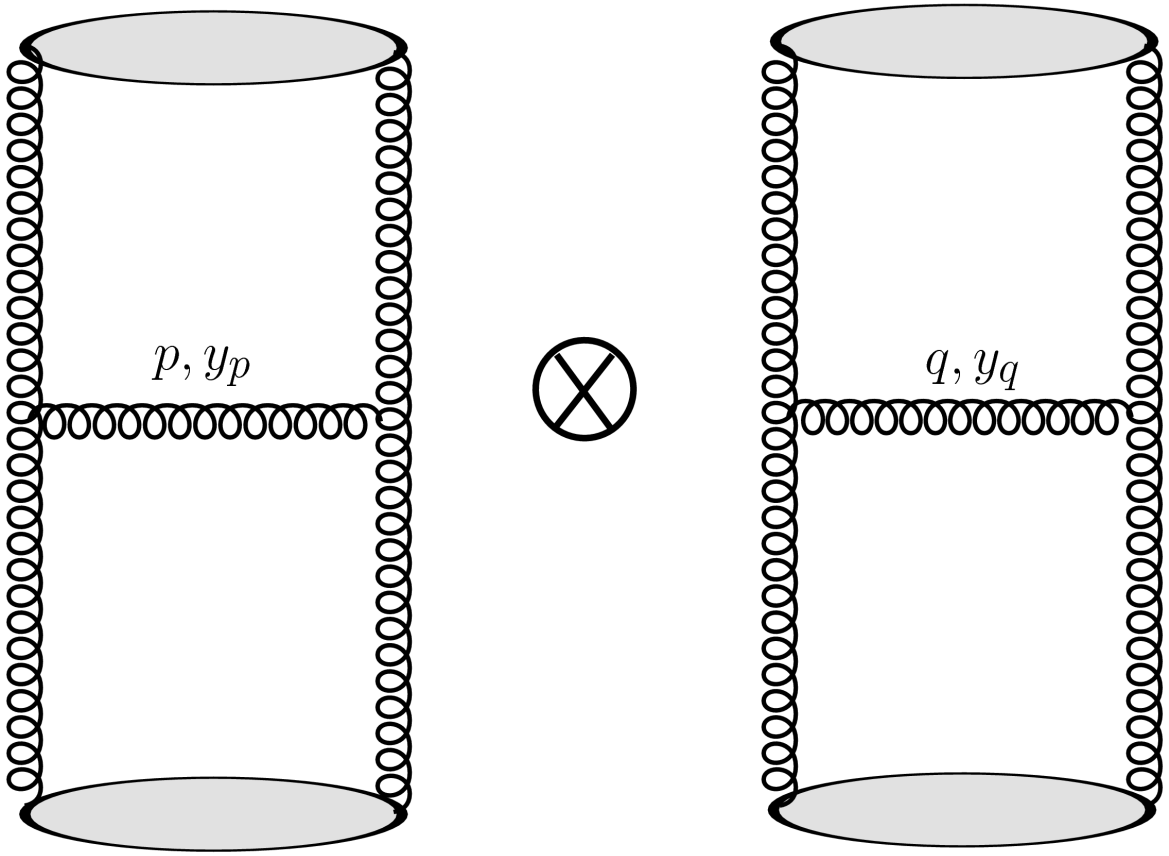


+ ... =

k^{-1} x



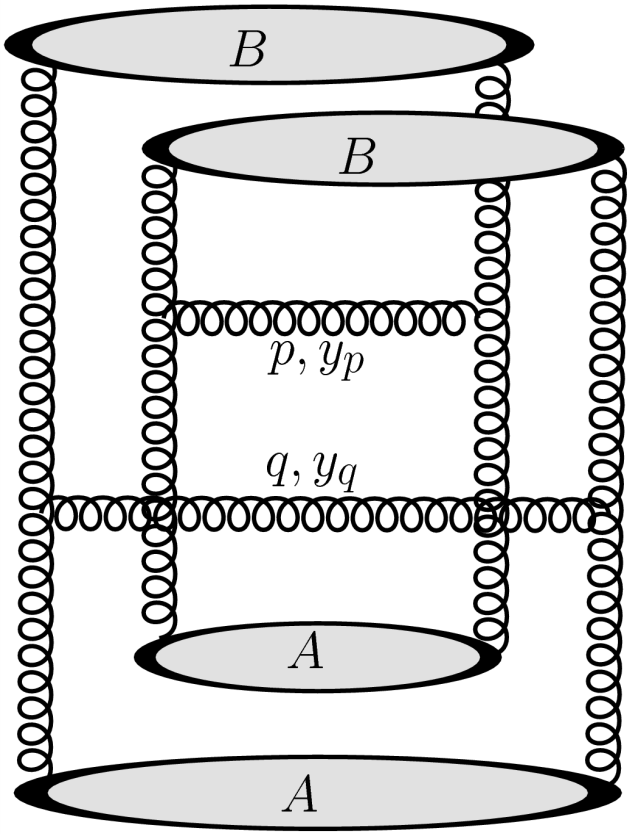
independent 2-gluon production with MV action



$$\sim \quad N_c^2(N_c^2-1)^2\,(\pi R^2)^2\,\frac{(\mu^2)^4}{p^2\,q^2}\int\frac{dk^2}{k^2}\frac{1}{(p-k)^2}\int\frac{dk'^{\,2}}{k'^{\,2}}\frac{1}{(q-k')^2}$$

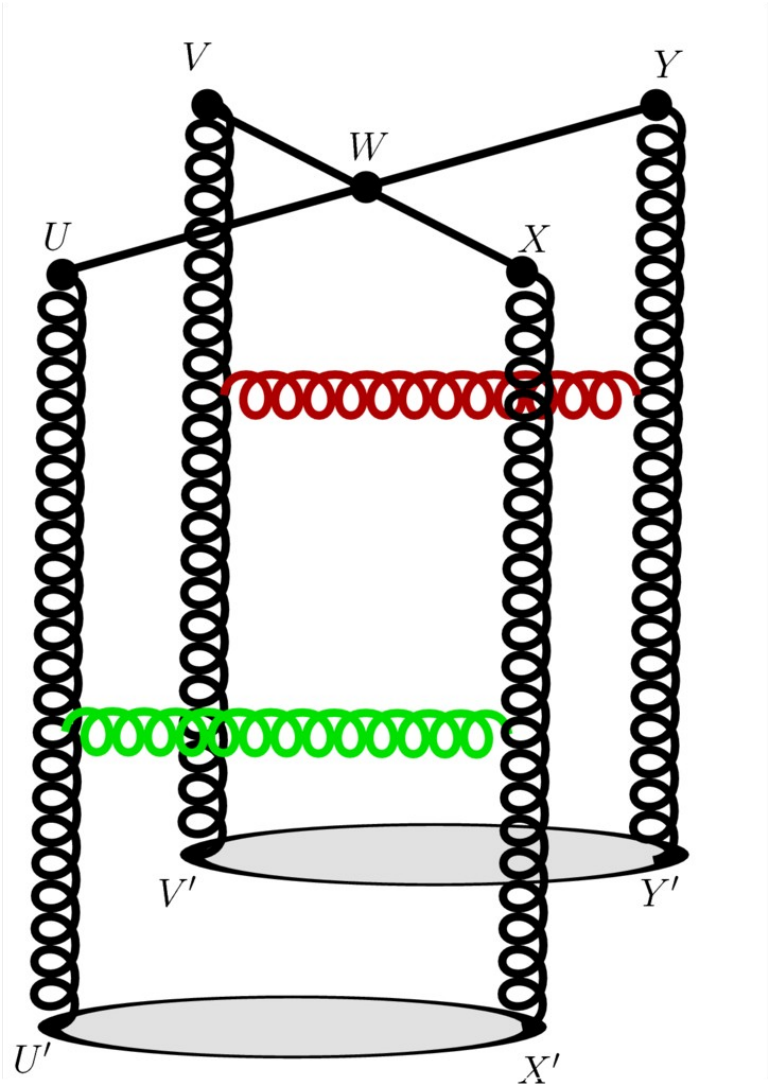
$$\sim \quad \frac{N_c^2(N_c^2-1)^2}{p^4q^4}\,(\mu^2)^4\,(\pi R^2)^2\,\log\frac{p^2}{Q_s^2}\,\log\frac{q^2}{Q_s^2}$$

double gluon production with MV action



$$\sim \; N_c^2(N_c^2-1)\,\pi R^2\,\frac{(\mu^2)^4}{p^2q^2}\int\frac{dk^2}{k^4}\,\frac{1}{(p-k)^2}\,\frac{1}{(q-k)^2}$$

$$\sim \; \frac{N_c^2(N_c^2-1)}{p^4q^4}\,(\mu^2)^4\,\pi R^2\,Q_s^{-2}$$



$$\sim \quad -N_c^2(N_c^2-1)^2\,\frac{(\mu^2)^2\,(\mu^4)^2}{\kappa_4}\,\frac{\pi R^2}{p^2\,q^2}\int\frac{dk^2}{k^2}\,\frac{1}{(p-k)^2}\int\frac{dk'^{\,2}}{k'^{\,2}}\,\frac{1}{(q-k')^2}$$

$$\sim \quad -\frac{N_c^2(N_c^2-1)^2}{p^4q^4}\,\frac{(\mu^2)^2\,(\mu^4)^2}{\kappa_4}\,\pi R^2\log\frac{p^2}{Q_s^2}\,\log\frac{q^2}{Q_s^2}$$

compute k^{-1} from 2-particle connected diagrams:

Dumitru + E.P. 2012

$$\frac{Q_s^2 S_\perp}{2\pi}\,\frac{1}{k} \simeq \frac{1}{N_c^2-1} - 3\,\frac{N_c^2+1}{N_c^2-1}\,\beta + \cdots$$

MV $\qquad\qquad$ ρ^4

$\beta>0$ makes k bigger,
should be sufficiently small so as not to ruin KNO !

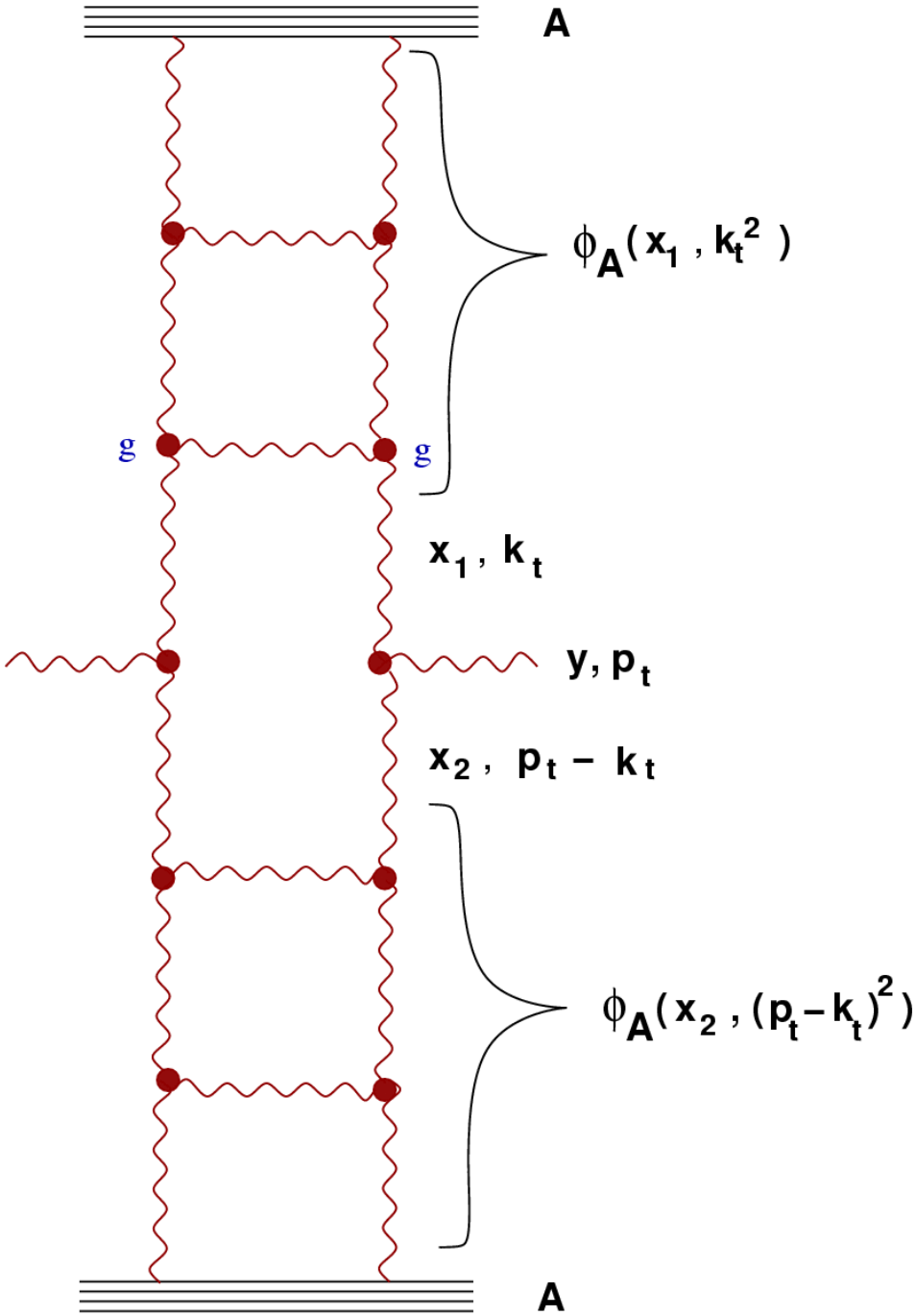
$$\beta \equiv \frac{C_F^2}{6\pi^3}\,\frac{g^8}{Q_s^2\,\kappa_4}\left[\int_{-\infty}^{\infty} dz^-\mu^4(z^-)\right]^2$$

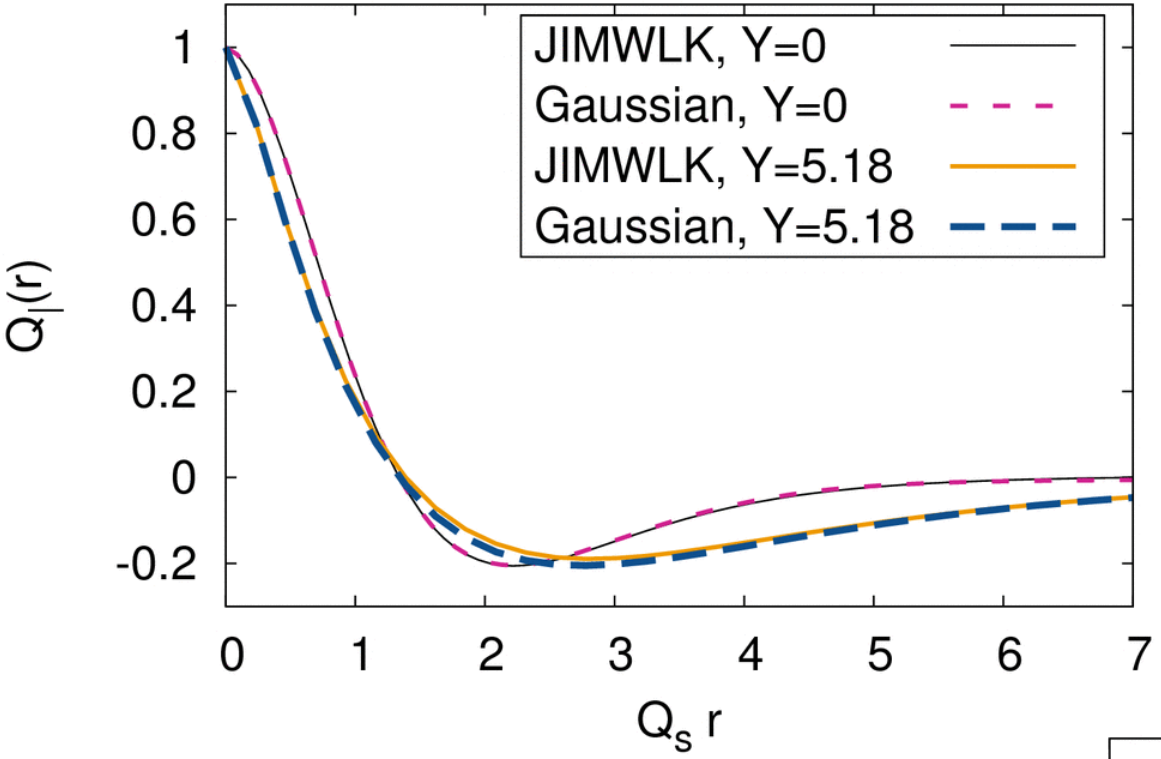
$\approx 0.01\ A^{-2/3}$ (from fit to AAMQS dipole)

Evolution with energy

Fluctuations in evolution ladders:

do (rapidity-enhanced) quantum fluctuations satisfy KNO scaling ?

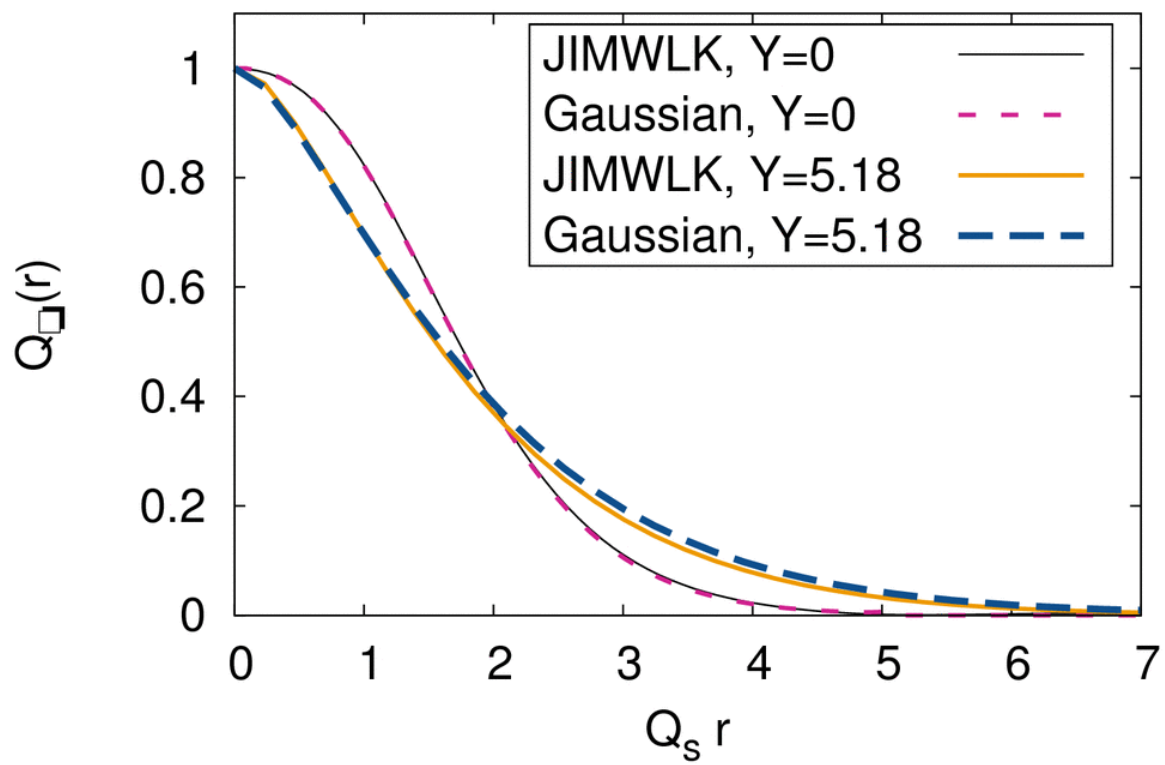




JIMWLK evolution appears to preserve Gaussianity of initial MV action ! (more to come)

Quadrupole evolution

$$Q \equiv \frac{1}{N_c}\langle \mathrm{tr}\ V_x V_y^\dagger V_u V_w^\dagger \rangle$$



A.D., Jalilian-Marian, Lappi, Schenke, Venugopalan: arXiv:1108.4764

Summary

- multiplicity distributions in pp @ LHC exhibit KNO scaling (η=0, n/n <~ 3)
- can be described by NBD with k « $\bar{n}$
- approx. KNO scaling predicted for p+Pb @ LHC
  (slight distortion of KNO due to Glauber flucs)
- higher-order eccentricities ε_3 etc. in HIC increase
- theoretical studies of fluctuations:
  - constrain magnitude of higher ρ^n operators
  - evolution with energy to test validity of Gaussian approximation from JIMWLK is in progress with B. Schenke, R. Venugopalan

Backup Slides

$k_\perp$-factorization, multiplicity in A+B --> g+X

(generalized) unintegrated gluon distribution:

$$\varphi(k, Y; b, A) = \frac{C_F\, k^2}{\alpha_s(k)} \int \frac{d^2\mathbf{r}}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{r}} \mathcal{N}_A(r, Y; b, A)$$

multiplicity: (Kharzeev, Levin, Nardi ansatz)

$$\frac{dN^{A+B\to g}}{dy\, d^2b} = K\, \frac{1}{2C_F} \int \frac{d^2p_t}{p_t^2} \int^{p_t} d^2k_t \; \alpha_s(Q)\, \varphi\left(\frac{|p_t + k_t|}{2}, x_1\right) \varphi\left(\frac{|p_t - k_t|}{2}, x_2\right)$$

- finite at $p_t \to 0$ if UGD does not blow up
- $x_{1,2} = (p_t/\sqrt{s}) \exp(\pm y)$; $Y_{1,2} = \log(x_0/x_{1,2})$
  where x_0=0.01 is assumed onset of rcBK evol.

rcBK evolution:

basic "degrees of freedom": dipole scattering amplitude in fund. rep. (2-point fct)

$$\mathcal{N}_F(r, Y; b, A) \equiv \frac{1}{N_c} \, \mathrm{tr} \, \langle 1 - V^\dagger(y) V(z) \rangle_Y$$

$$\mathbf{r} = \mathbf{y} - \mathbf{z}$$

BK equation (incl. non-linear terms → saturation of scattering amplitude!)

$$\frac{\partial \mathcal{N}(r, Y)}{\partial Y} = \int d^2 r_1 \, K(r, r_1, r_2) \left[\mathcal{N}(r_1, Y) + \mathcal{N}(r_2, Y) - \mathcal{N}(r, Y) - \mathcal{N}(r_1, Y) \mathcal{N}(r_2, Y) \right]$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r_2}$$

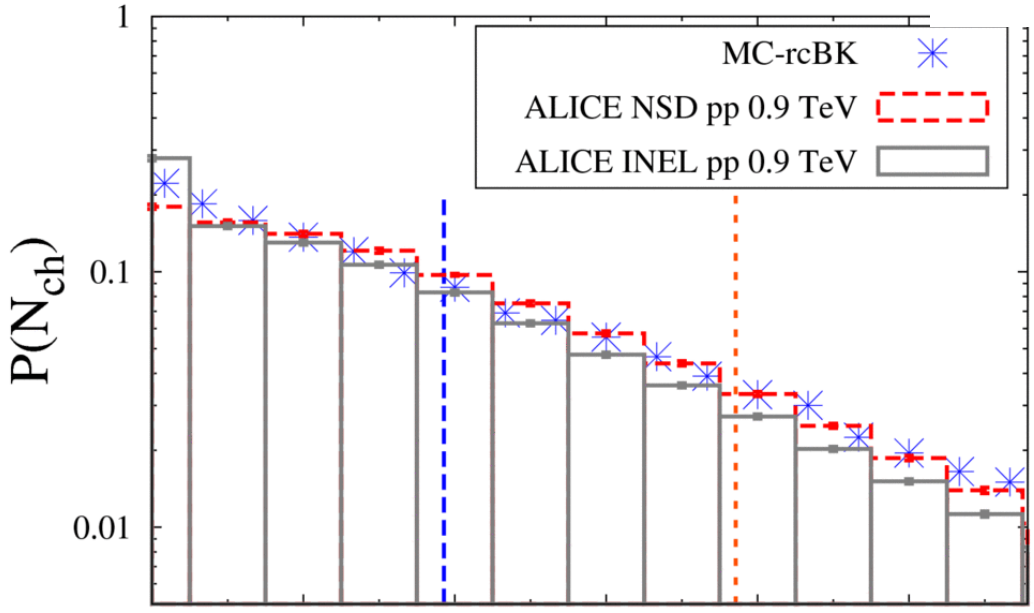
running-coupling kernel (Balitsky prescription)

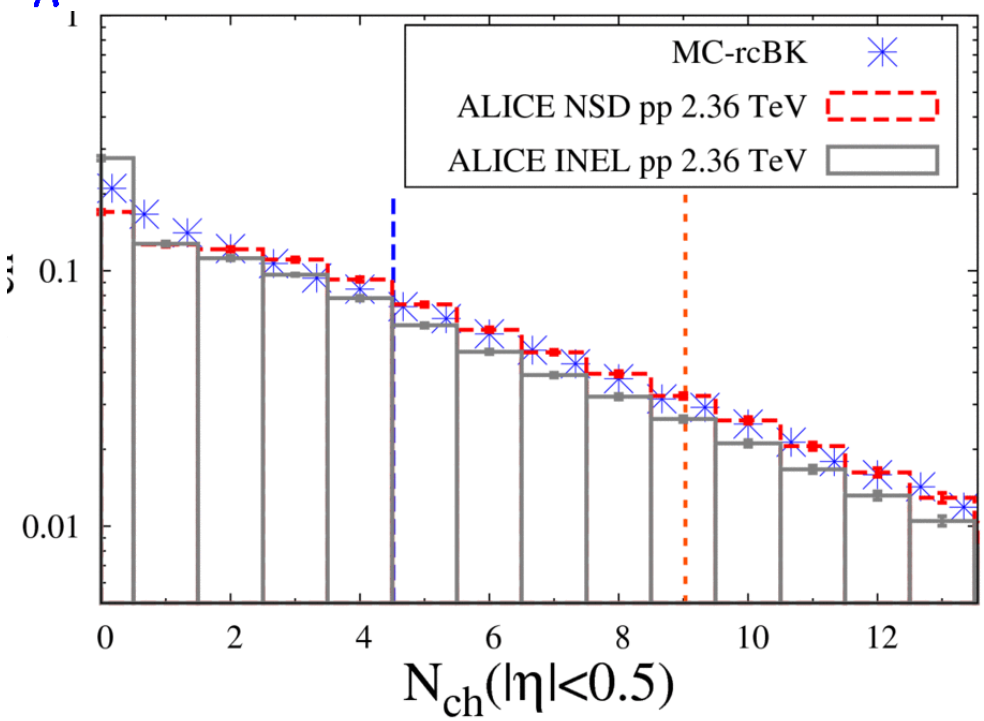
$$K(\mathbf{r}, \mathbf{r_1}, \mathbf{r_2}) = \frac{N_c \, \alpha_s(r^2)}{2\pi^2} \left[\frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2 \, r_2^2} + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

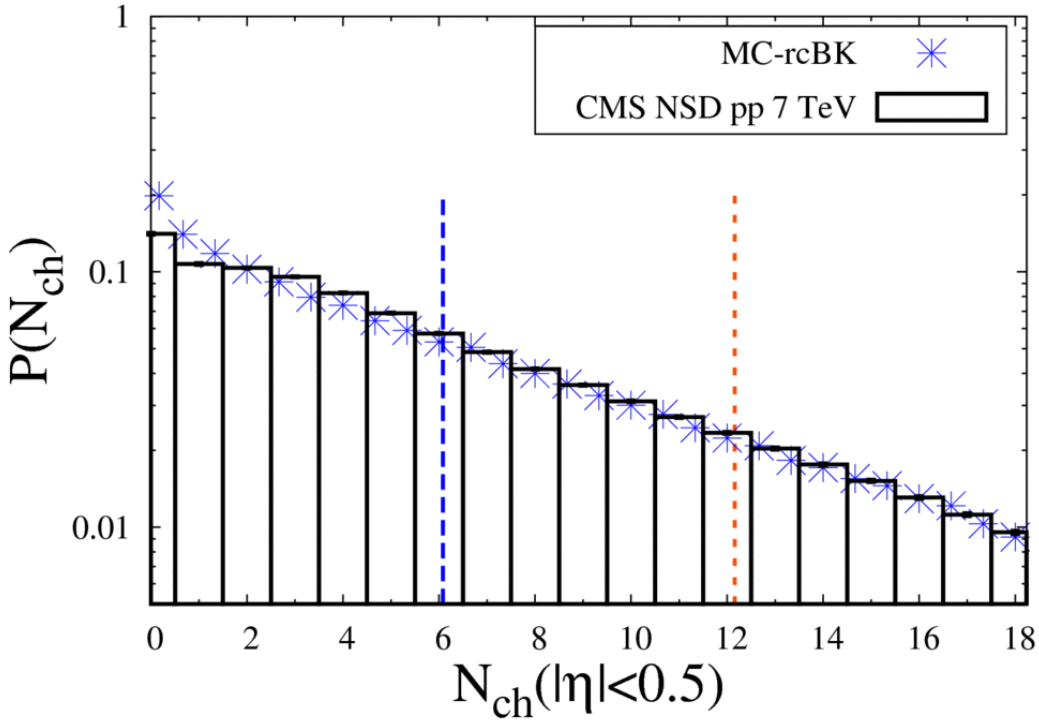
dipole scattering amplitude in adj. rep.

$$\mathcal{N}_A = 2 \, \mathcal{N}_F - \mathcal{N}_F^2$$

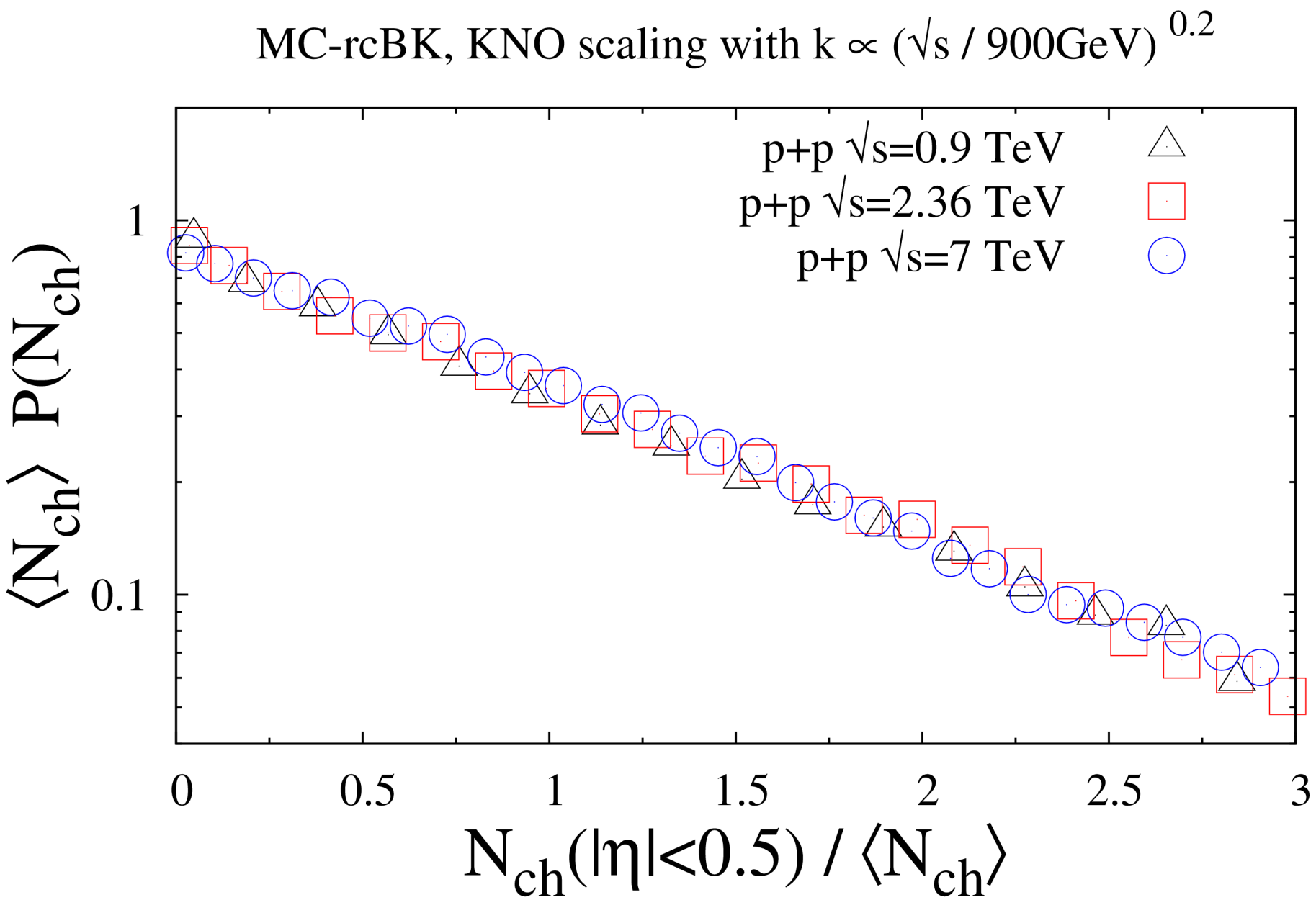
result for constant $k = \frac{1}{\pi} \Delta x_\perp^2 \, \Delta\eta \, \Lambda_{\rm QCD}^2$







energy dependent $k \sim E_{CM}^{0.2}$



Non-Gaussian initial conditions for high-energy evolution

- Odderon operator $-d^{abc}\rho^a\rho^b\rho^c/\kappa_3$ S. Jeon and R. Venugopalan, Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- Effective action for a system of $k \sim N_c A^{1/3} \gg 1$ valence quarks in SU(3);
- Random walk of SU(3) color charges in the space of representations (m,n);

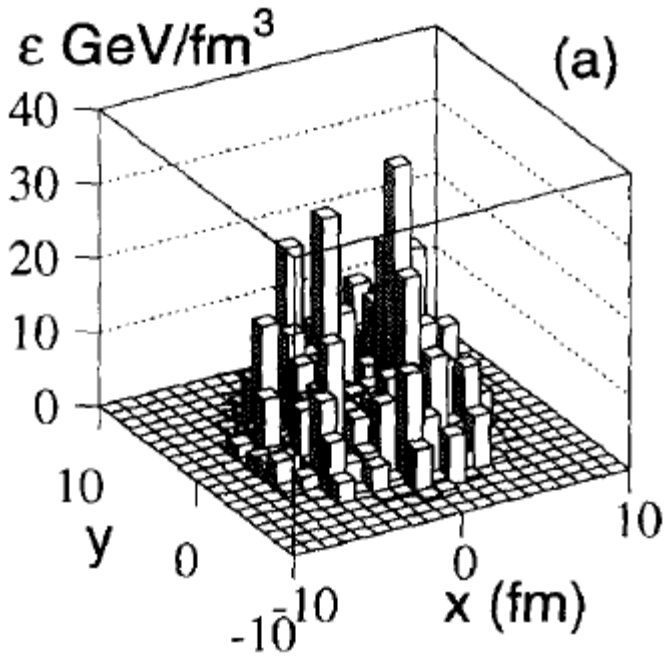
- Probability $P(m,n) = e^{-S(m,n)}$

$$S(m,n;k) \simeq \frac{N_c}{k} C_2(m,n) - \frac{1}{3}\left(\frac{N_c}{k}\right)^2 C_3(m,n) + \frac{1}{6}\left(\frac{N_c}{k}\right)^3 C_4(m,n)$$

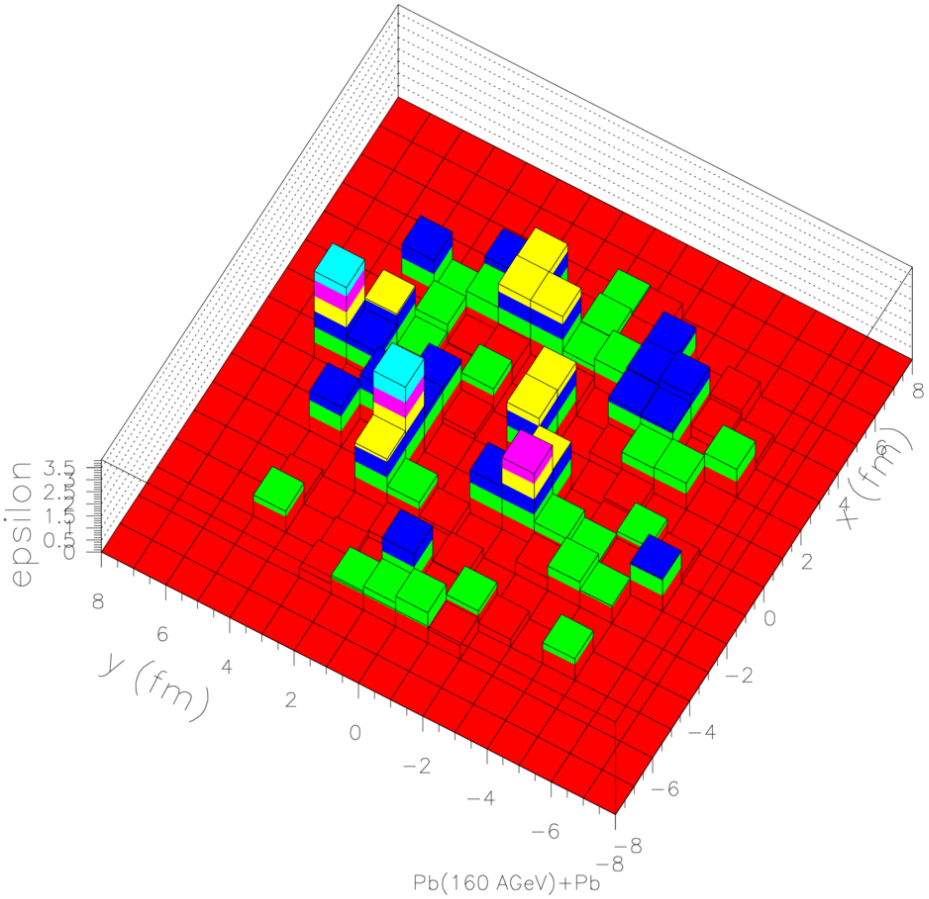
- Casimir operator $C_2,\ C_3,\ C_4$ ntation (m,n)

- Define color charge per unit area $\rho^a \equiv g\, Q^a/\Delta^2 x$
where $|Q| = \sqrt{Q^a Q^a} \equiv \sqrt{C_2}$

Lumpy initial conditions in AA collisions

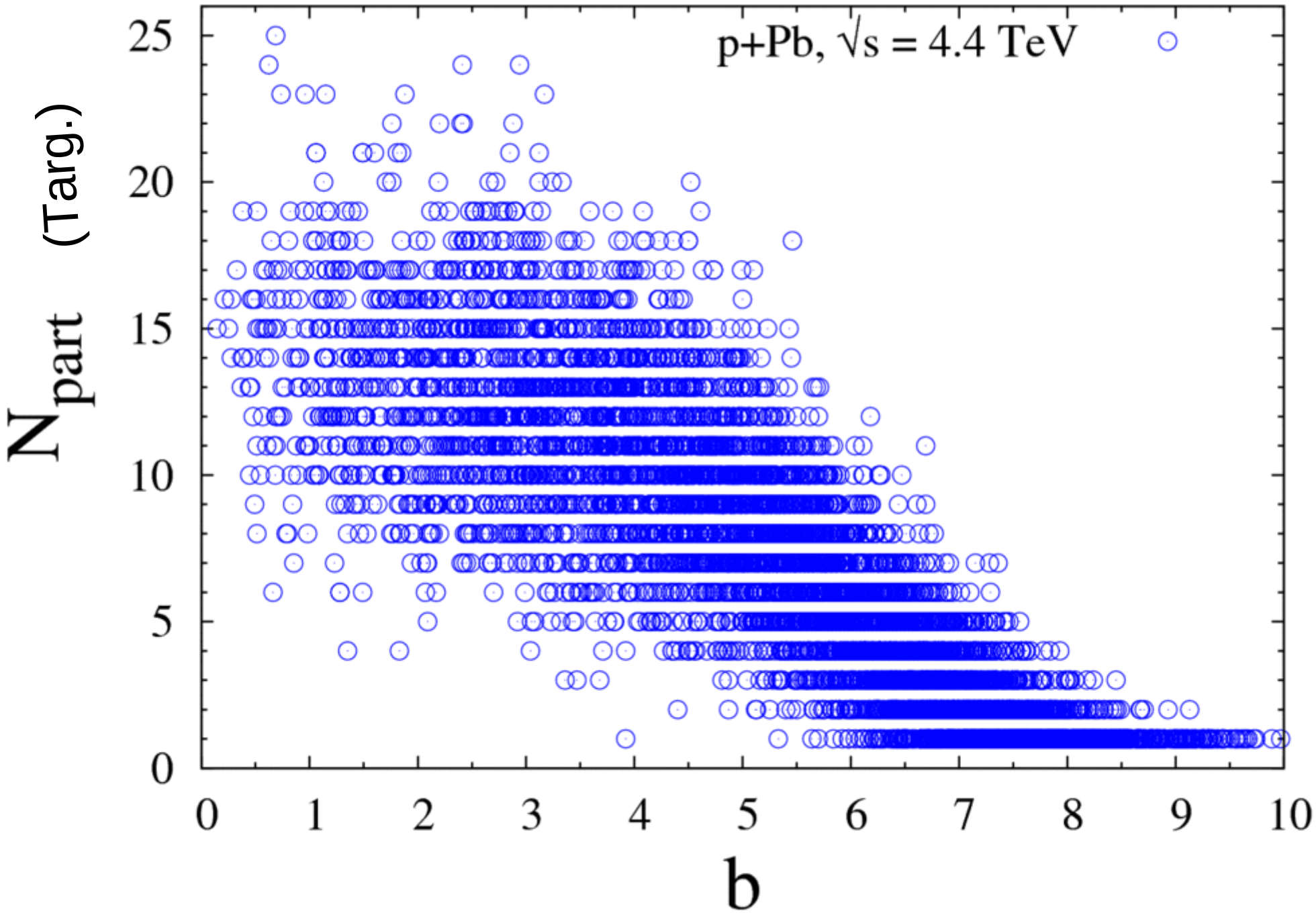


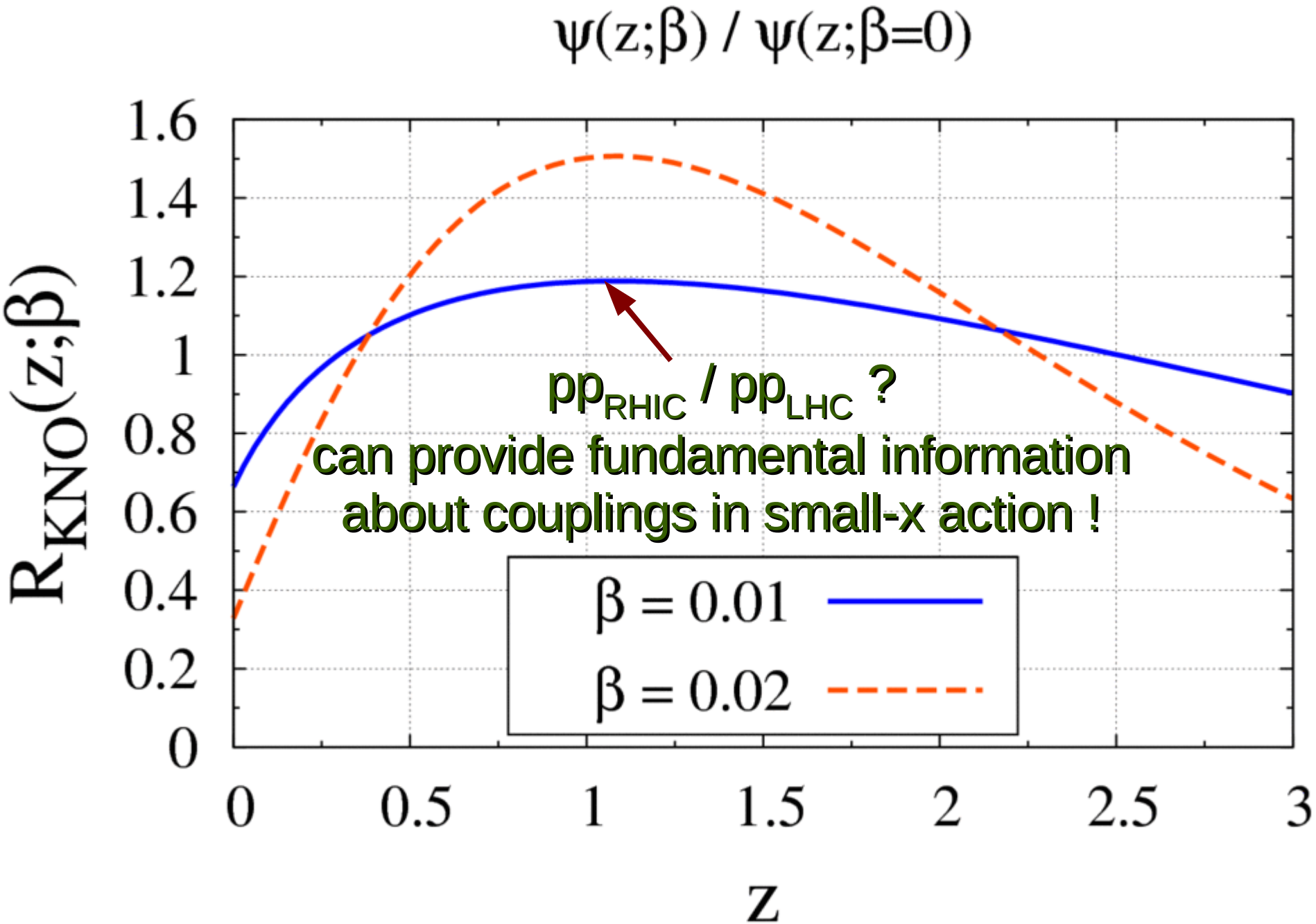
Gyulassy, Rischke, Zhang:
NPA 613 (1997)



M. Bleicher *et al.:* QM97 (Tsukuba)
NPA 638 (1998) p.391

N_{part} fluctuations in p+Pb:




ψ(z;β) / ψ(z;β=0)
$R_{KNO}(z;\beta)$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
z
pp_{RHIC} / pp_{LHC} ?
can provide fundamental information
about couplings in small-x action !
β = 0.01
β = 0.02