Thermal dilepton rate, electrical conductivity & heavy quark diffusion from lattice QCD

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based on work with

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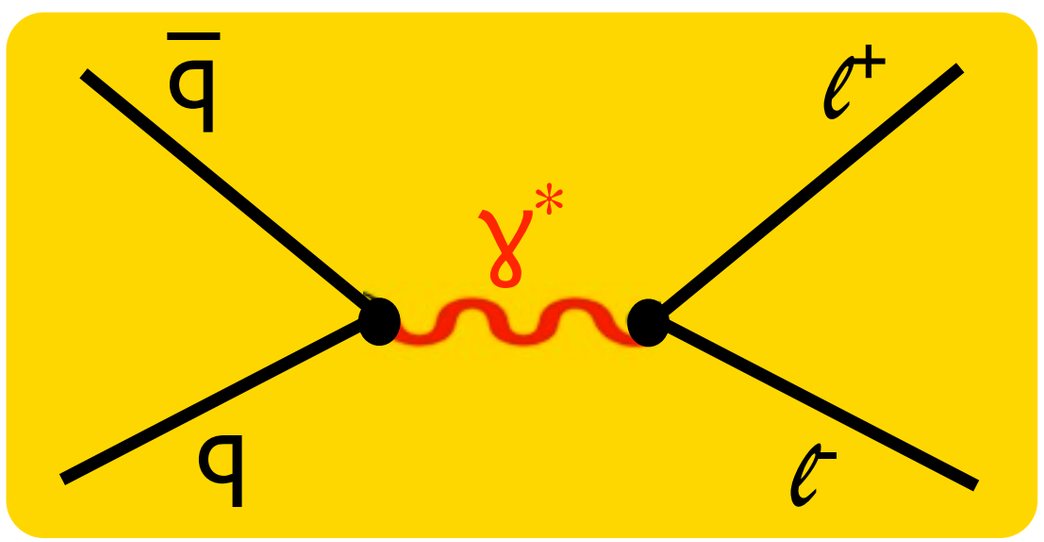
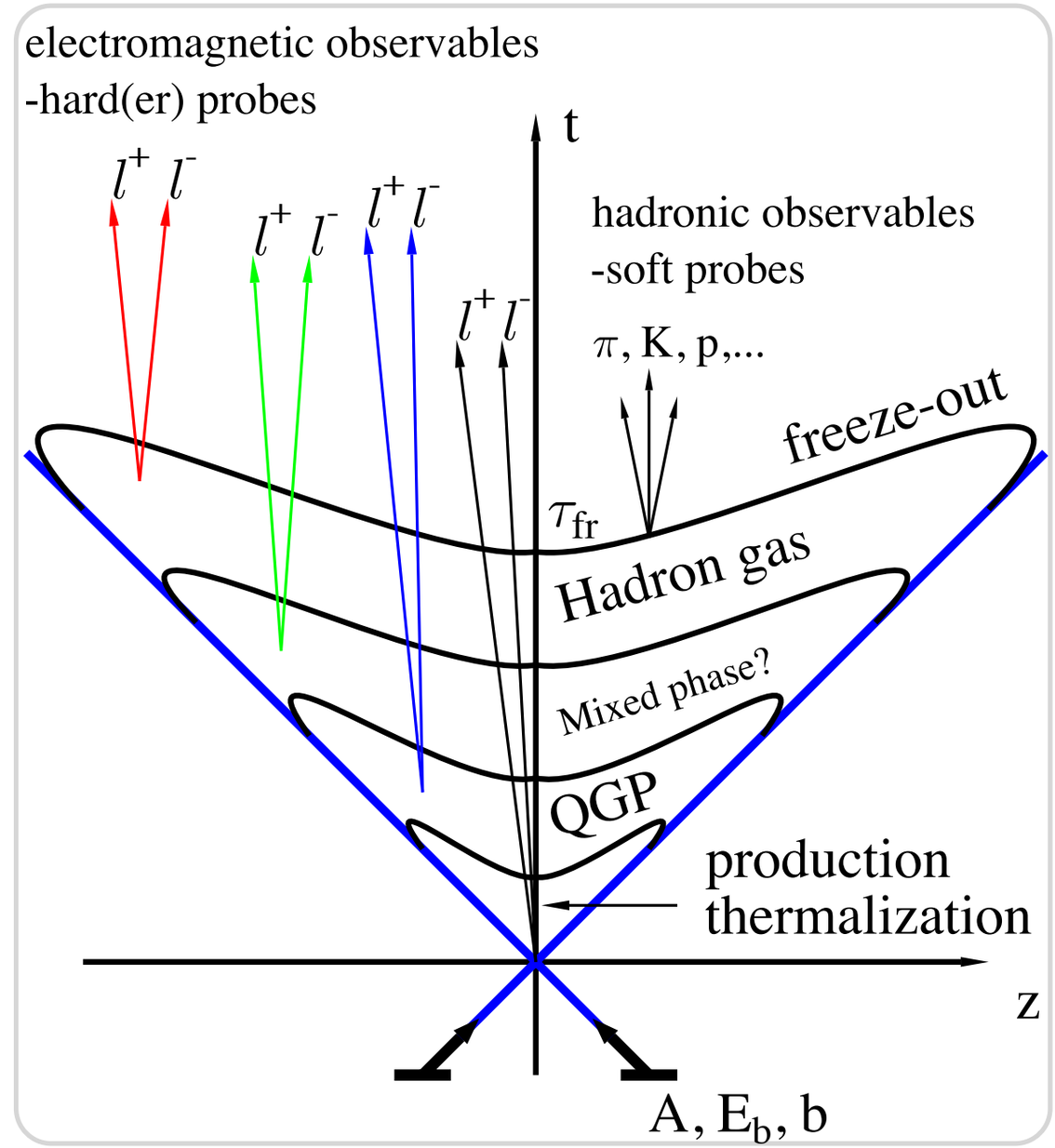
Phys.Rev.D83(2011)034504, arXiv:1204.4945

dileptons in heavy ion collisions

- Dileptons: no strong interactions
- produced in all stages of collisions
- In medium & vacuum production of dilepton needs to be understood

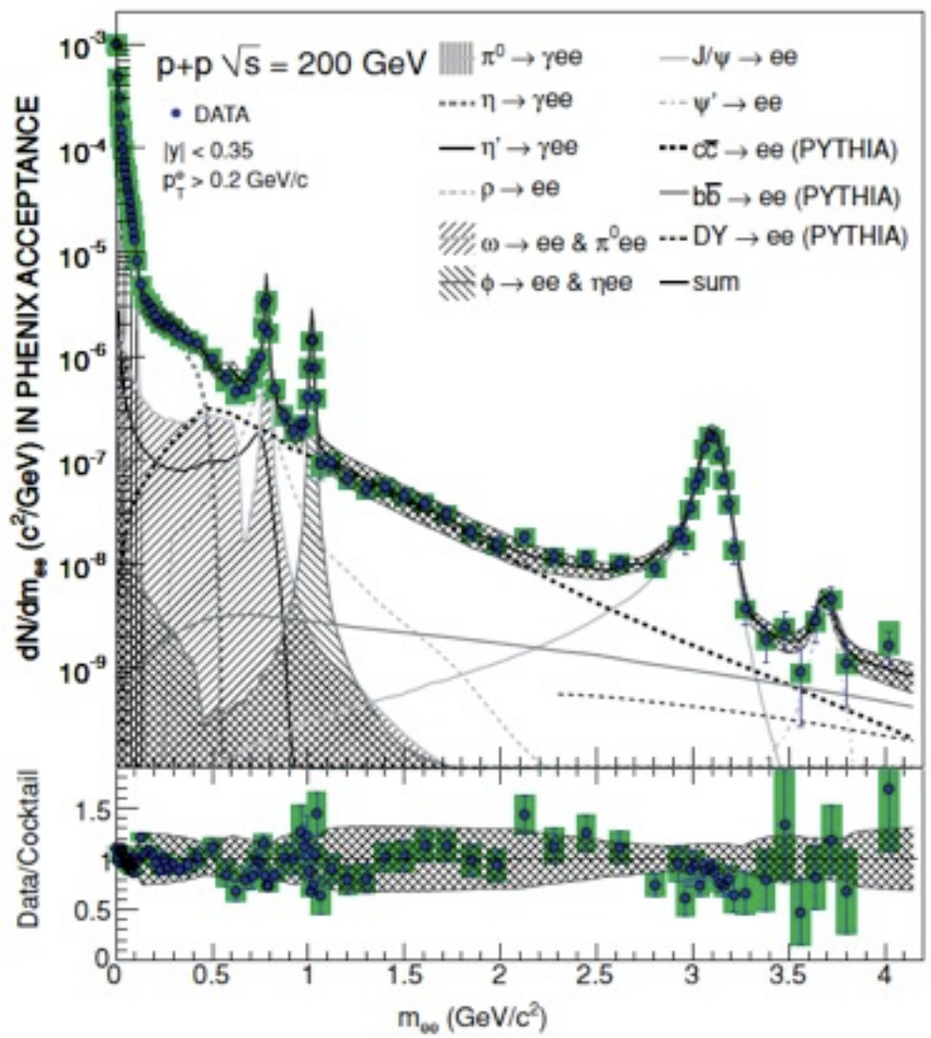


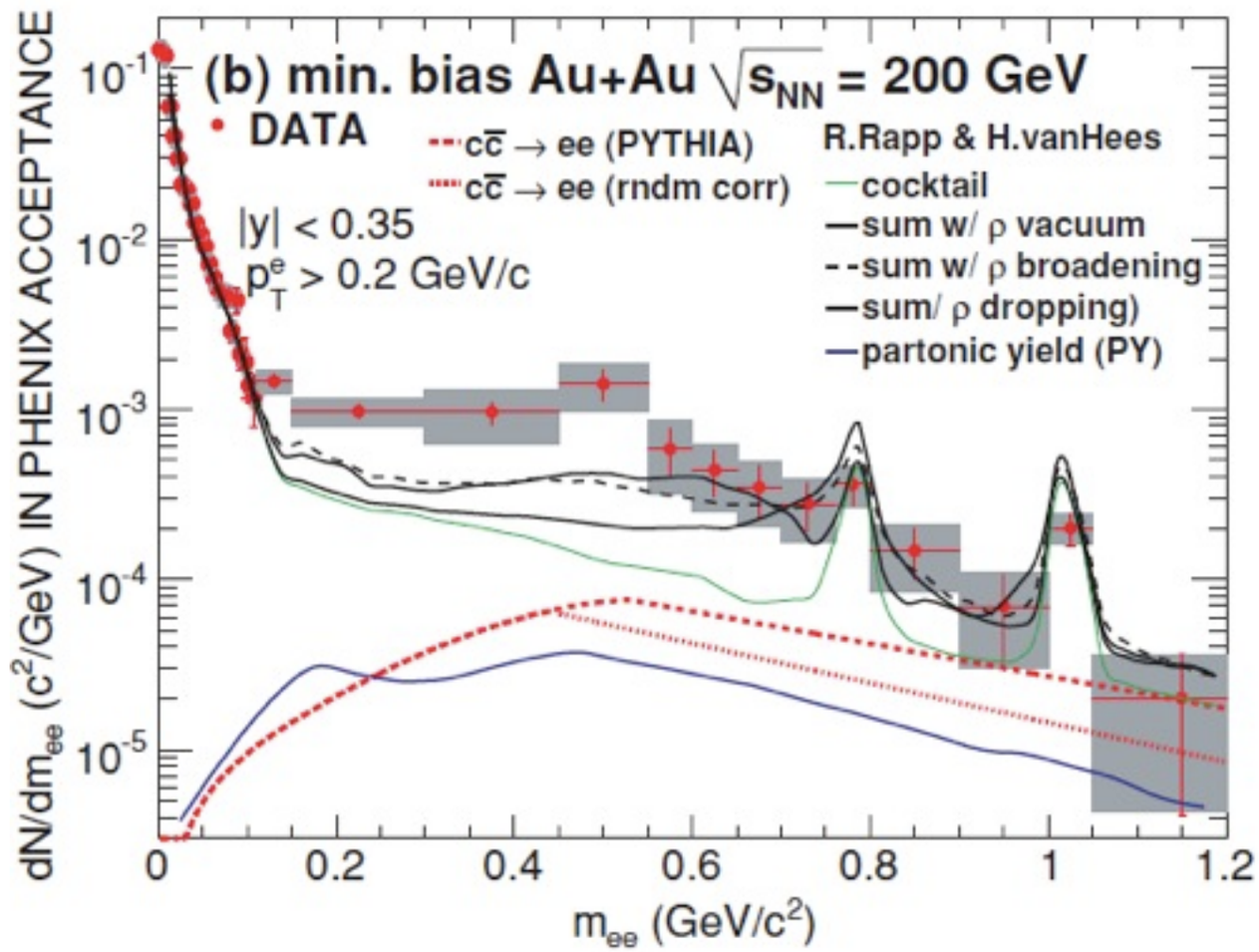


- Thermal dilepton production: annihilation of thermal qqbar, proportional to the self energy of a virtual photon

Experimental results on dilepton rates

- dilepton rates: $$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)} \qquad C_{em} = e^2\sum_{f=1}^{n_f} Q_f^2$$





- pp data well described by cocktails
- enhancement in the low mass region in AuAu data
- the low mass, the low pt PHENIX puzzle (c.f. Bratkovskaya et al, Dusling et al)

Response of the QGP to electromagnetic fields

- Maxwell equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \frac{1}{\mu}\nabla \times \mathbf{B} = \epsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$$

- Ohm's law: $\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$

μ: permeability of QGP
ϵ: permittivity of QGP
J: electrical current
σ: electrical conductivity
v: flow velocity
R: nuclear radius

- Evolution of magnetic field

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B_m} \frac{R^2}{R^2 + 4(t - t_0)/\sigma} \exp\left[-\frac{l^2}{R^2 + 4(t - t_0)/\sigma}\right]$$

Tuchin, PRC82(2010)034904

Relative strength of R^2 and $4(t-t_0)/\sigma$: Whether B is static or not ?

- Electrical conductivity σ:

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

- The emission rate of soft photons:

$$\lim_{\omega \to 0} \omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = \lim_{\omega \to 0} C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1} = \frac{3}{2\pi^2} \sigma(T) T \alpha_{em}$$

Meson correlation & spectral functions

Meson properties are all enclosed in Spectral functions:

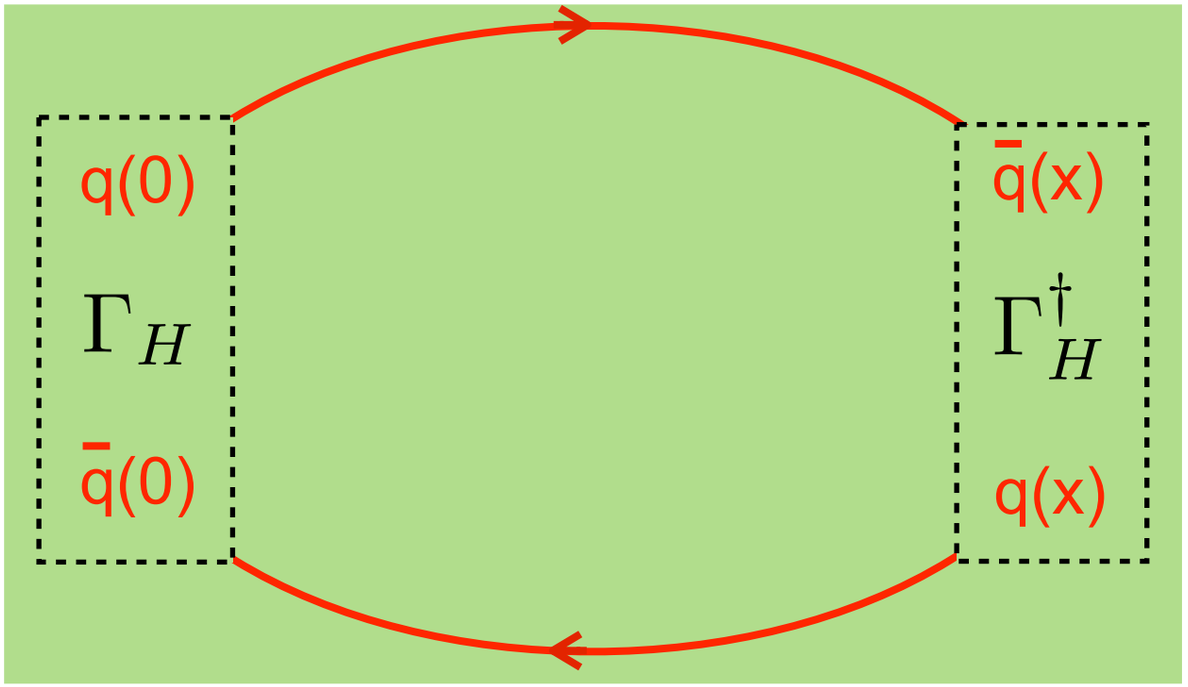
$$\rho(\omega,\vec{p}) = D^{+}(\omega,\vec{p}) - D^{-}(\omega,\vec{p}) = 2\,\mathrm{Im}D_R(\omega,\vec{p})$$

Euclidean correlation function

$$G_H(\tau,T) = \sum_{\vec{x}} \left\langle J_H(0,\vec{0}) J_H^{\dagger}(\tau,\vec{x}) \right\rangle$$

$$J_H(\tau,\vec{x}) = \bar{q}(\tau,\vec{x})\,\Gamma_H\, q(\tau,\vec{x})$$

channel	Γ	$^{2S+1}L_J$	J^{PC}	$u\bar{u}$	$c\bar{c}$
pseudo-scalar	γ_5	$^1S_0$	0^{-+}	π	η_c
vector	γ_μ	$^3S_1$	1^{--}	ρ	J/ψ
scalar	1	$^3P_0$	0^{++}	a_0	χ_{c0}
axial-vector	$\gamma_5\gamma_\mu$	$^3P_1$	1^{++}	a_1	χ_{c1}



Spectral representation

$$G(\tau,\vec{p}) = \int \mathrm{d}^3x\, e^{-i\vec{p}\cdot\vec{x}}\, D^{+}(-i\tau,\vec{x})\,, \qquad D^{+}(t,\vec{x}) = D^{-}(t+i\beta,\vec{x})$$

$$G_H(\tau,\vec{p},T) = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \rho_H(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \quad H = 00,\ ii,\ V\ .$$

Vector correlation function

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\,\rho_H(\omega,\vec{p},T)\,\frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \quad H = 00,\ ii,\ V\ .$$

p=0 in this work

time like correlator G_{00} and space like correlator G_{ii}

$$G_V(\tau,\vec{p},T) = G_{ii}(\tau,\vec{p},T) + G_{00}(\tau,\vec{p},T)$$

conserved current, J_0, gives τ-independent correlator G_{00}

$$G_{00}(T) \equiv -\chi_q T + \mathcal{O}(a^2)$$

the local, non-conserved current needs to be renormalized

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\gamma_\mu\psi(\tau,\vec{x})$$

avoid ambiguities of renormalization

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} \quad ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)G_V^{free}(\tau T)}$$

Prior information on spectral functions

- free vector spectral function (in the infinite temperature limit)

$$\rho_{00}^{free}(\omega) = -2\pi T^2 \omega \delta(\omega)$$

$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$$

  - δ-functions cancel in $\rho_V(\omega) \equiv \rho_{00}(\omega) + \rho_{ii}(\omega)$

- vector spectral function at T<∞
  - δ-function in ρ_{00} is protected

$$\rho_{00}(\omega, T) = -2\pi \chi_q \omega \delta(\omega)$$

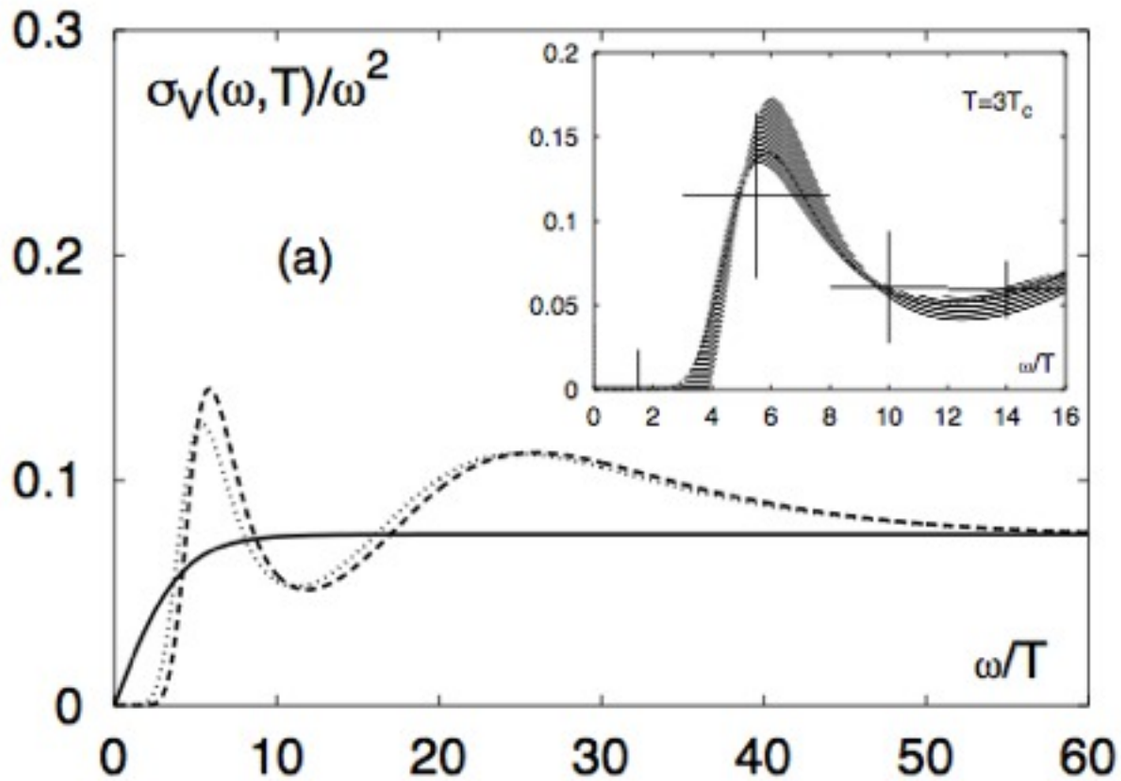
  - δ-function in ρ_{ii} is smeared out

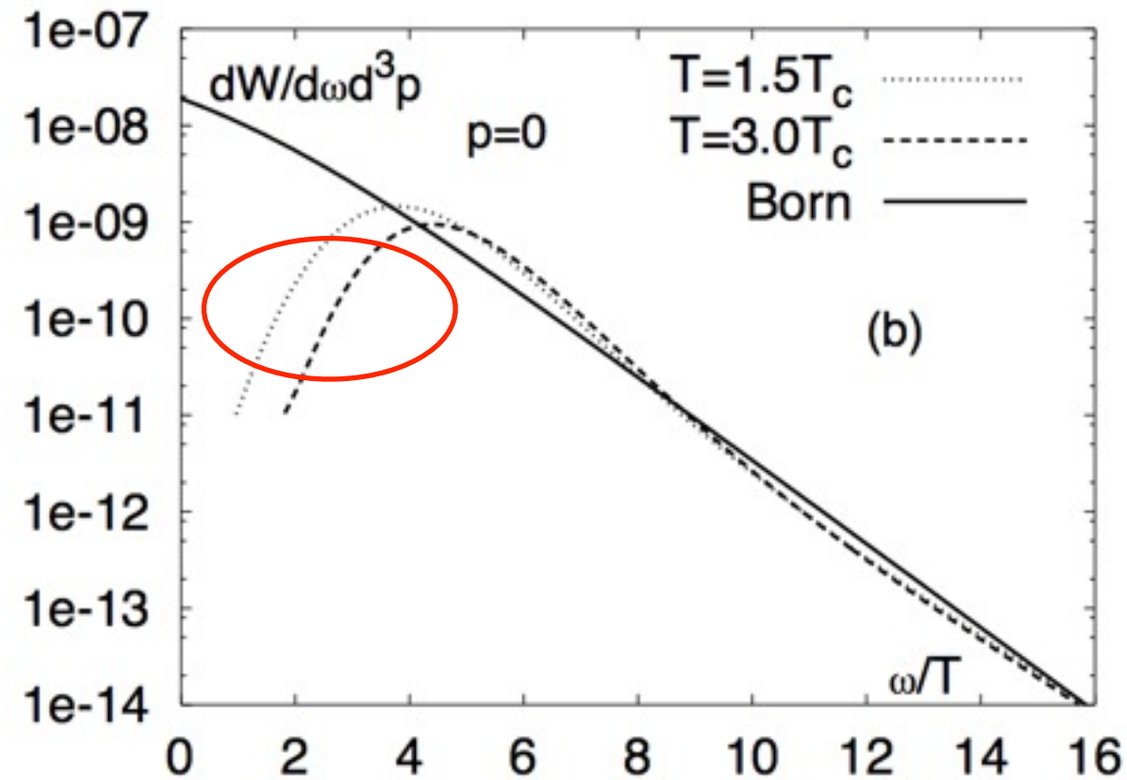
possible form: Breit-Wigner (BW) form + modified continuum

$$\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}\left(1 + \frac{\alpha_s}{\pi}\right) \omega^2 \tanh(\frac{\omega}{4T})$$

3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \alpha_s$

Previous lattice results on thermal dilepton rate





Wilson fermions

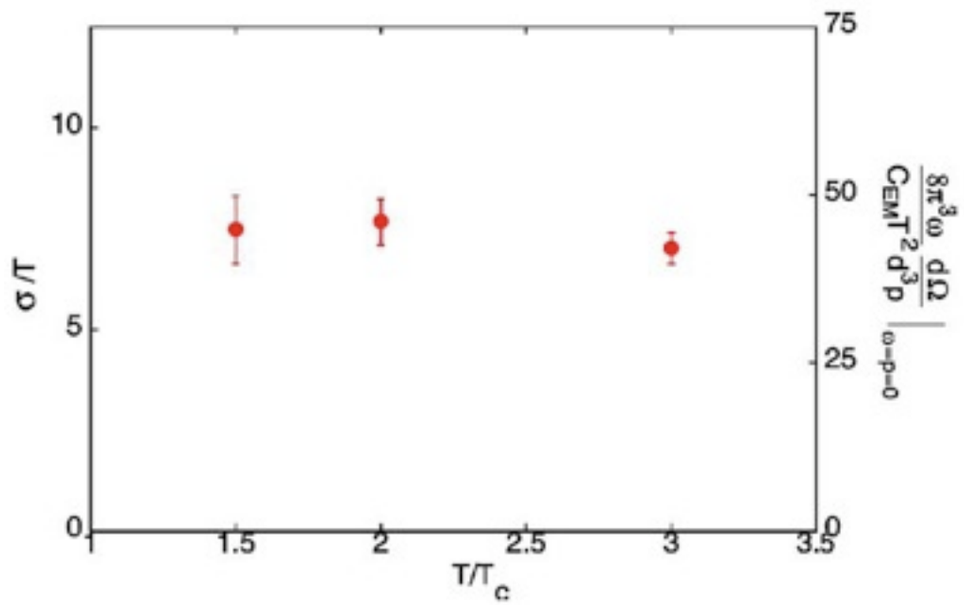
F. Karsch, E. Laermann, P. Petreczky, S. Stickan and I. Wetzorke, Phys.Lett. B530 (2002) 147-152

- 64^3x16 lattices, finite volume & lattice cutoff effects?
- N_τ not sufficiently large to extract spectral function
- $\rho(\omega)$ should be linear in ω at small ω, not captured by the MEM analysis
- Integral Kernel needs to be redefined in MEM to explore low frequency region G. Aarts et al, PRL '07

Previous lattice results on electrical conductivity

Quite different results from previous lattice calculations:

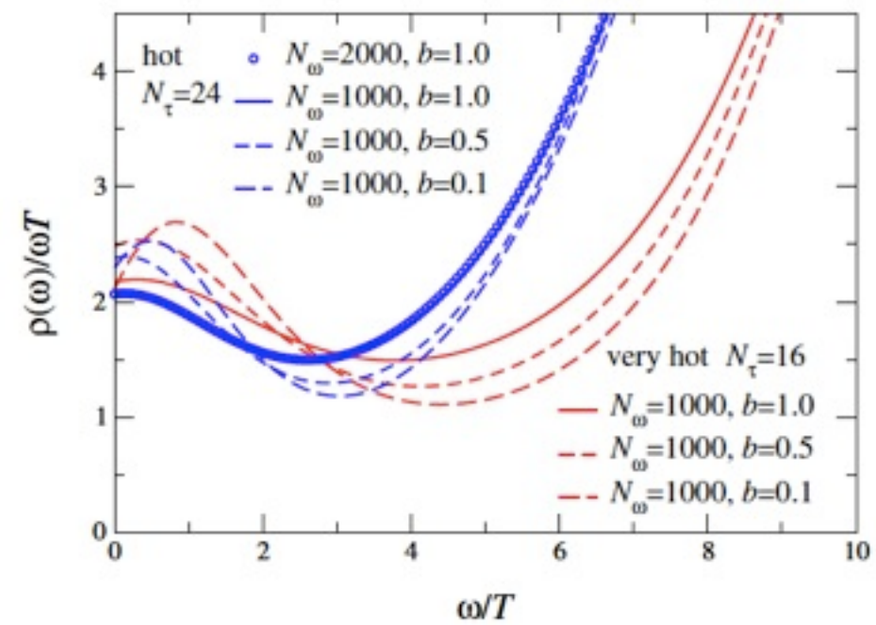
$\sigma/T \simeq 7C_{em}$



S. Gupta PLB 597(2004)57

$N_\tau = 8 - 14,\ N_\sigma \leq 44$

$\sigma/T \simeq (0.4 \pm 0.1)C_{em}$



G. Aarts et al., PRL 99(2007)

$N_\tau = 16, 24,\ N_\sigma = 64$

- Staggered fermions used, ρ_{even} and ρ_{odd} need to be distinguished
- Unrenormalized currents are used

Vector correlation functions on large & fine lattices

- SU(3) gauge configurations at $T/T_c \simeq 1.45$
- lattice size $N_\sigma^3 \times N_\tau$ with N_σ = 32-128 & N_τ=16,24,32,48
- Non-perturbatively clover O(a) improved Wilson fermions
- Quark masses close to chiral limit $\kappa \simeq \kappa_c$

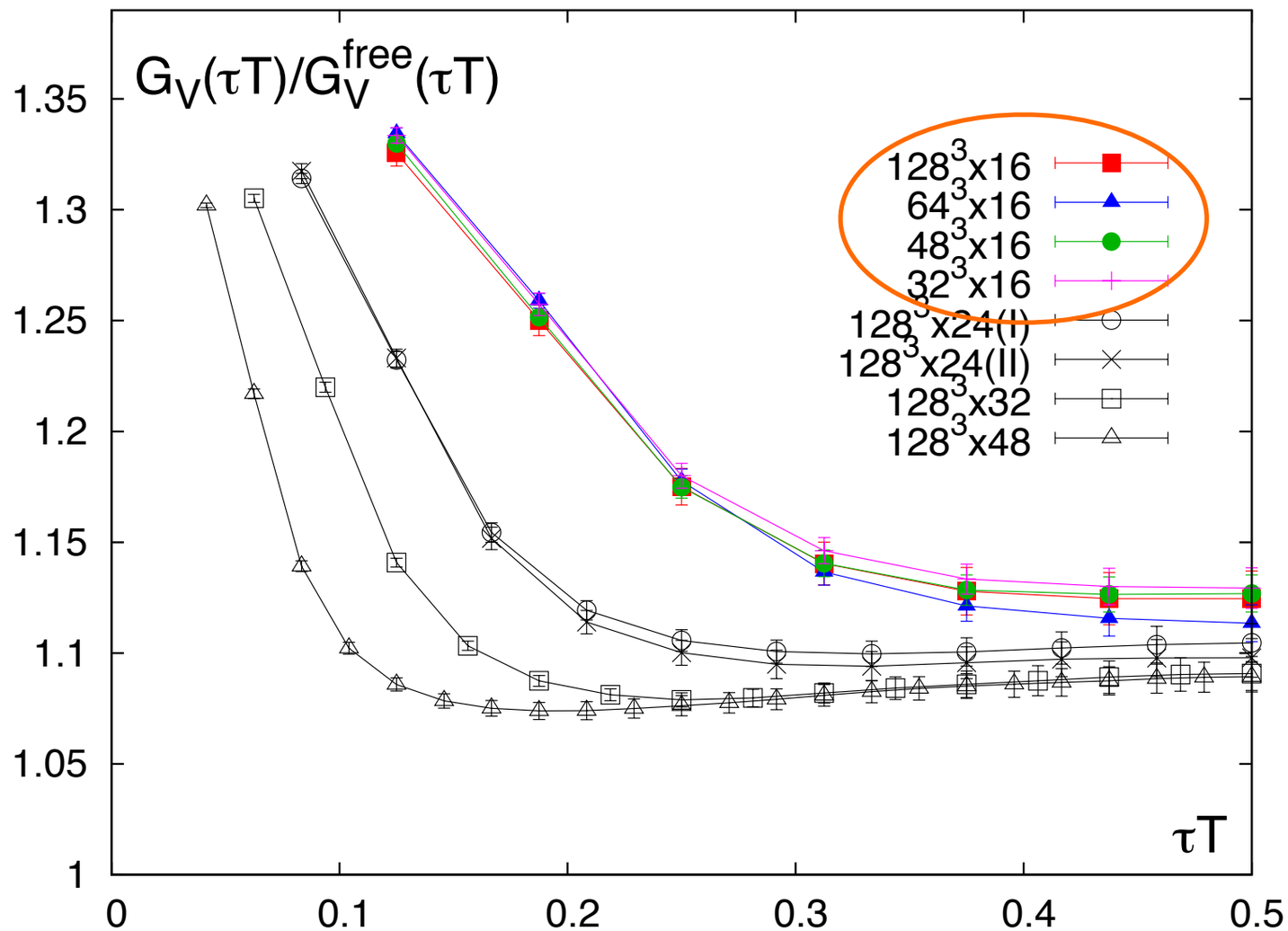
volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	a^{-1}[GeV]	a[fm]	#conf
16	32	6.872	1.4125	0.13495	0.829	6.43	0.031	251
16	48	6.872	1.4125	0.13495	0.829	6.43	0.031	229
16	64	6.872	1.4125	0.13495	0.829	6.43	0.031	191
16	128	6.872	1.4125	0.13495	0.829	6.43	0.031	191
24	128	7.192	1.3673	0.13431	0.842	9.65	0.020	340
	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	18.97	0.010	451

cut-off dep. & continuum extrapolation

close to continuum

Volume & cut-off dep. of vector corr. function



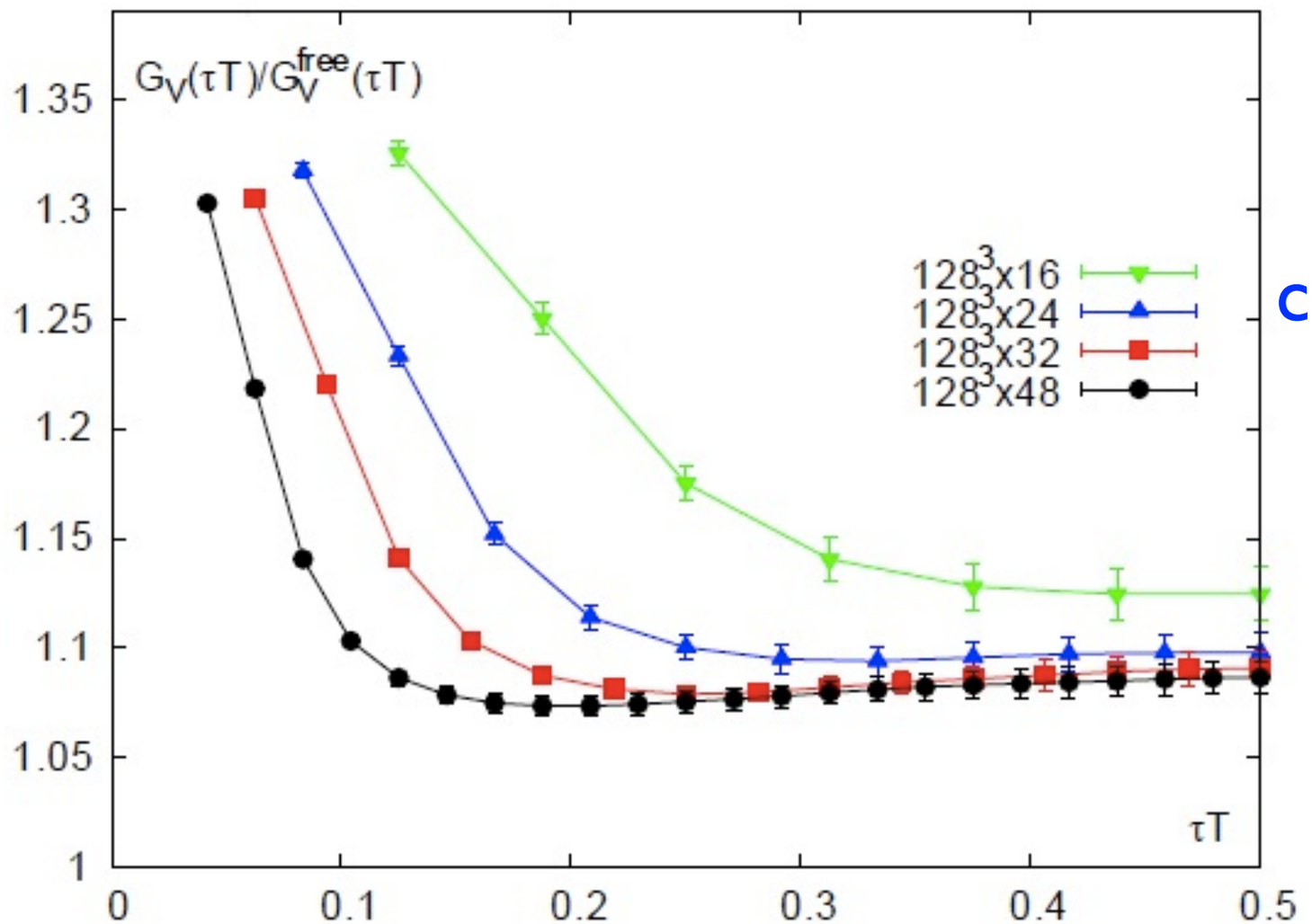
small volume dep.

weak τT dep. near 1/2

Normalized by free correlators in the continuum $G_V^{free}(\tau T)$

$$G_V^{free}(\tau T) = 6T^3 \left(\pi \, (1 - 2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2 \, \frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

Volume & cut-off dep. of vector corr. function



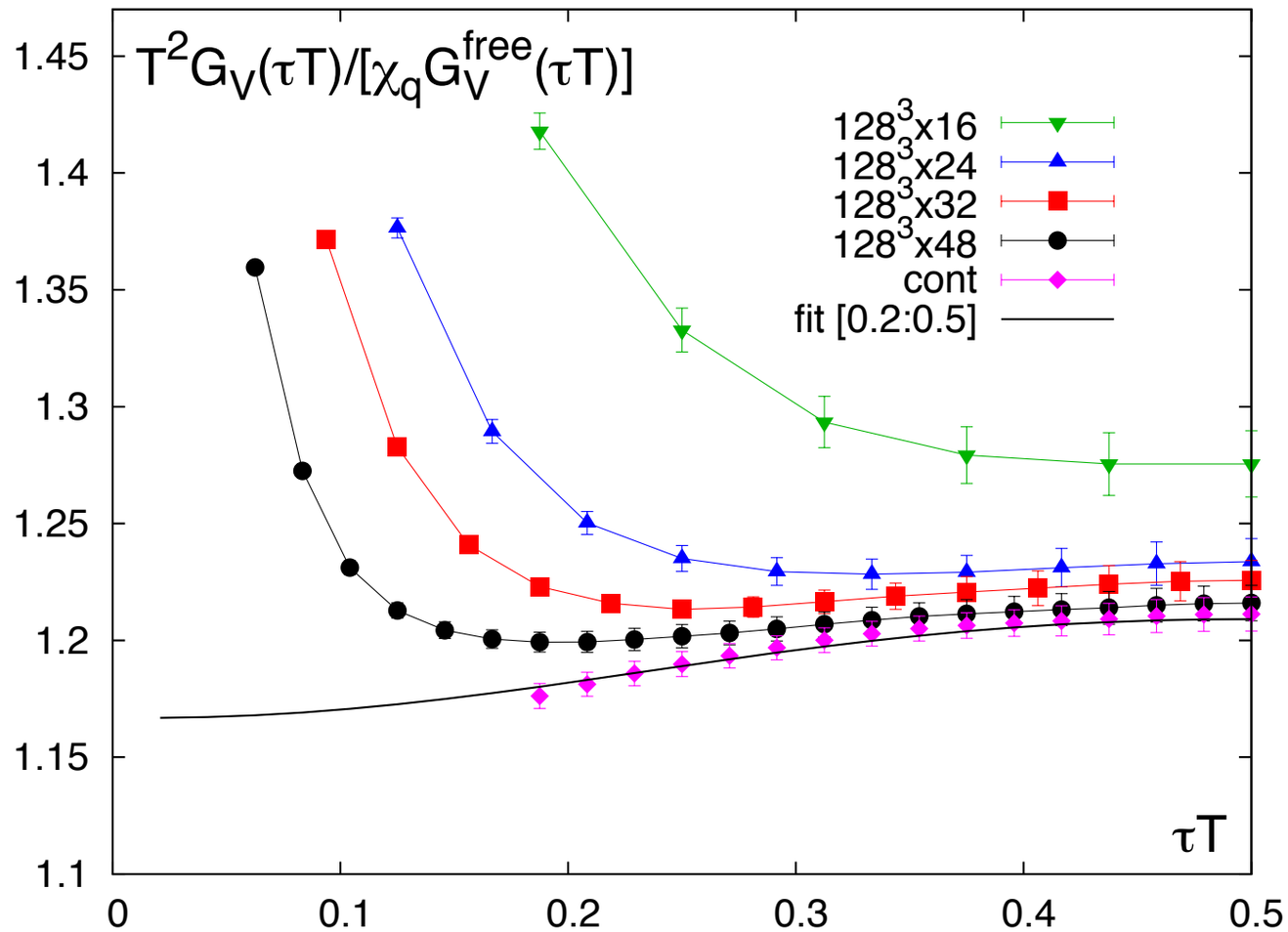
cut-off effects are more server than finite volume effects

large N_τ needed to perform continuum extrapolation

$G_V(\tau T)$ is close to the free case at large τT

incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$?

Continuum extrapolation



$$\frac{G_V(1/2)}{G_V^{free}(1/2)} = 1.086 \pm 0.008 \,,$$

$$\frac{G_V(1/4)}{G_V^{free}(1/4)} = (0.982 \pm 0.005)\frac{G_V(1/2)}{G_V^{free}(1/2)}$$

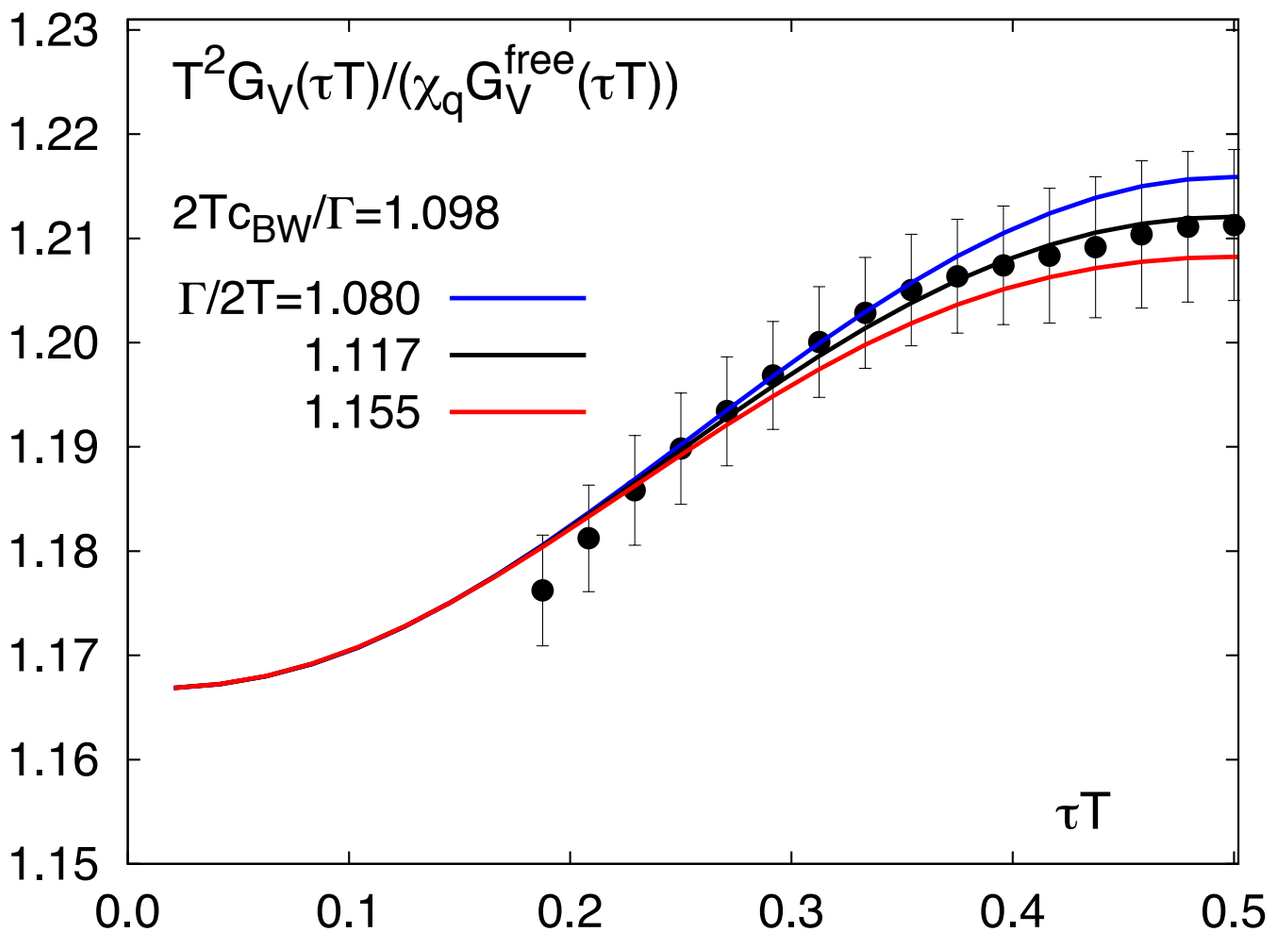
- Increase of $G_V(\tau T)/G_V^{free}(\tau T)$ with τT is obvious
- The rise with τT indicates that vector spectral function in the low frequency region is different from the free case
- Motivation for the Breit-Wigner type ansatz fitting

Fit to vector correlation functions

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$$k = 0.0465(30)\ ,\ \widetilde{\Gamma} = 2.235(75)\ ,\ 2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$$

→ vary width Γ with the other two parameters fixed

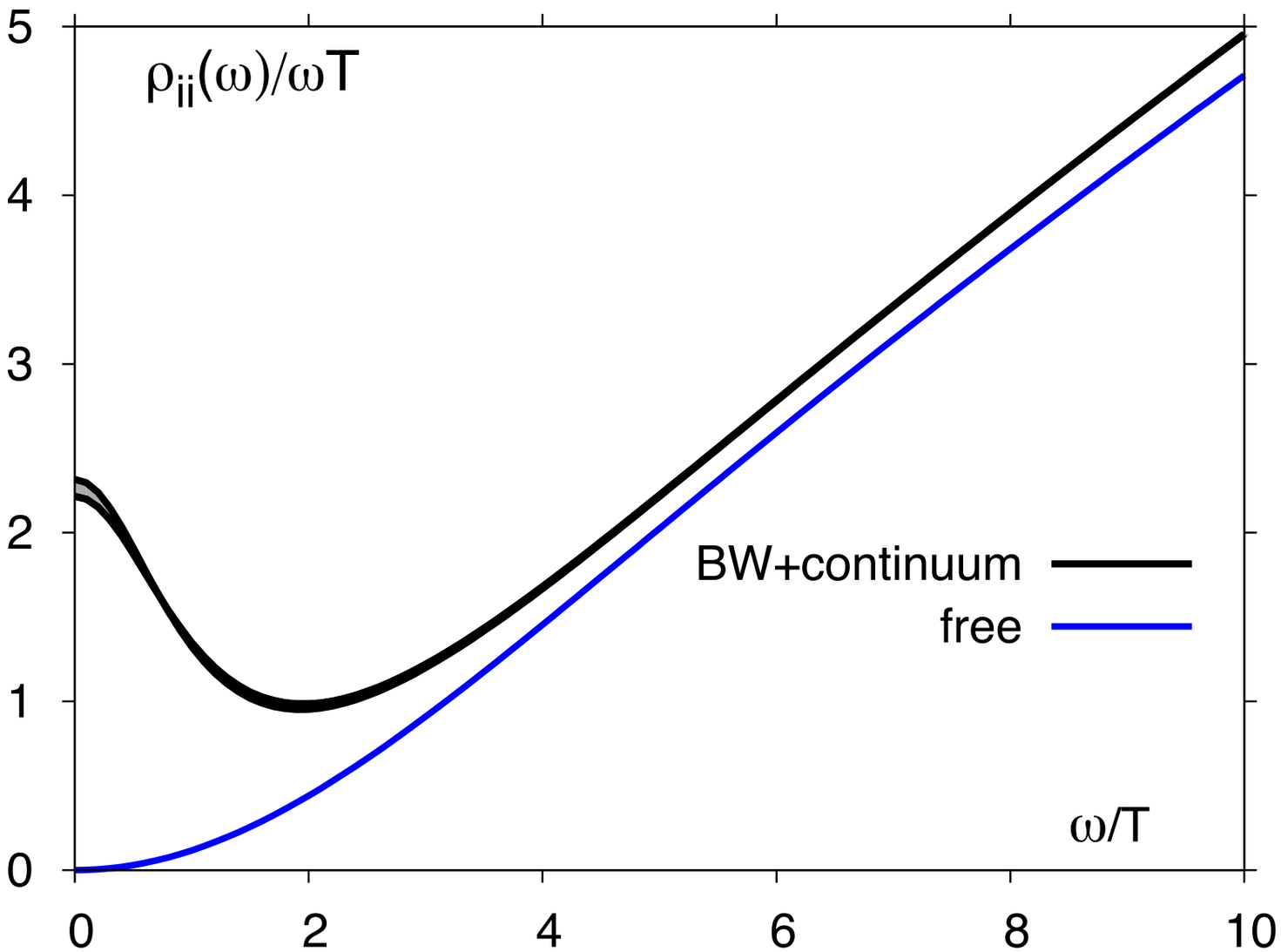


- vector correlation function is sensitive to the low energy, Breit-Wigner contribution only for distance $\tau T \gtrsim 0.25$

Estimate of electrical conductivity

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$k = 0.0465(30)$, $\widetilde{\Gamma} = 2.235(75)$, $2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$



$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$
$$= \frac{C_{em}}{3} \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}}$$
$$= (0.37 \pm 0.01) C_{em}$$

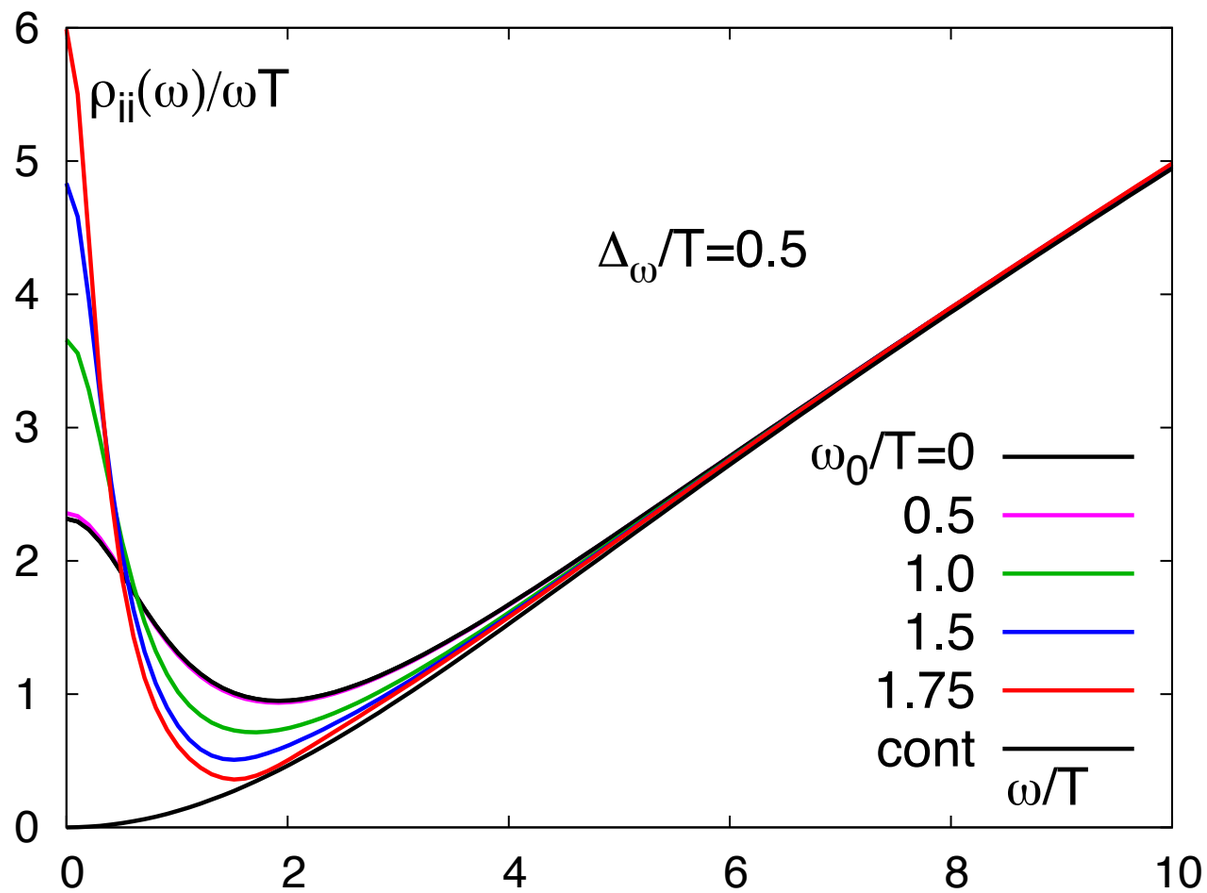
(accidentally) close to Aarts' result!

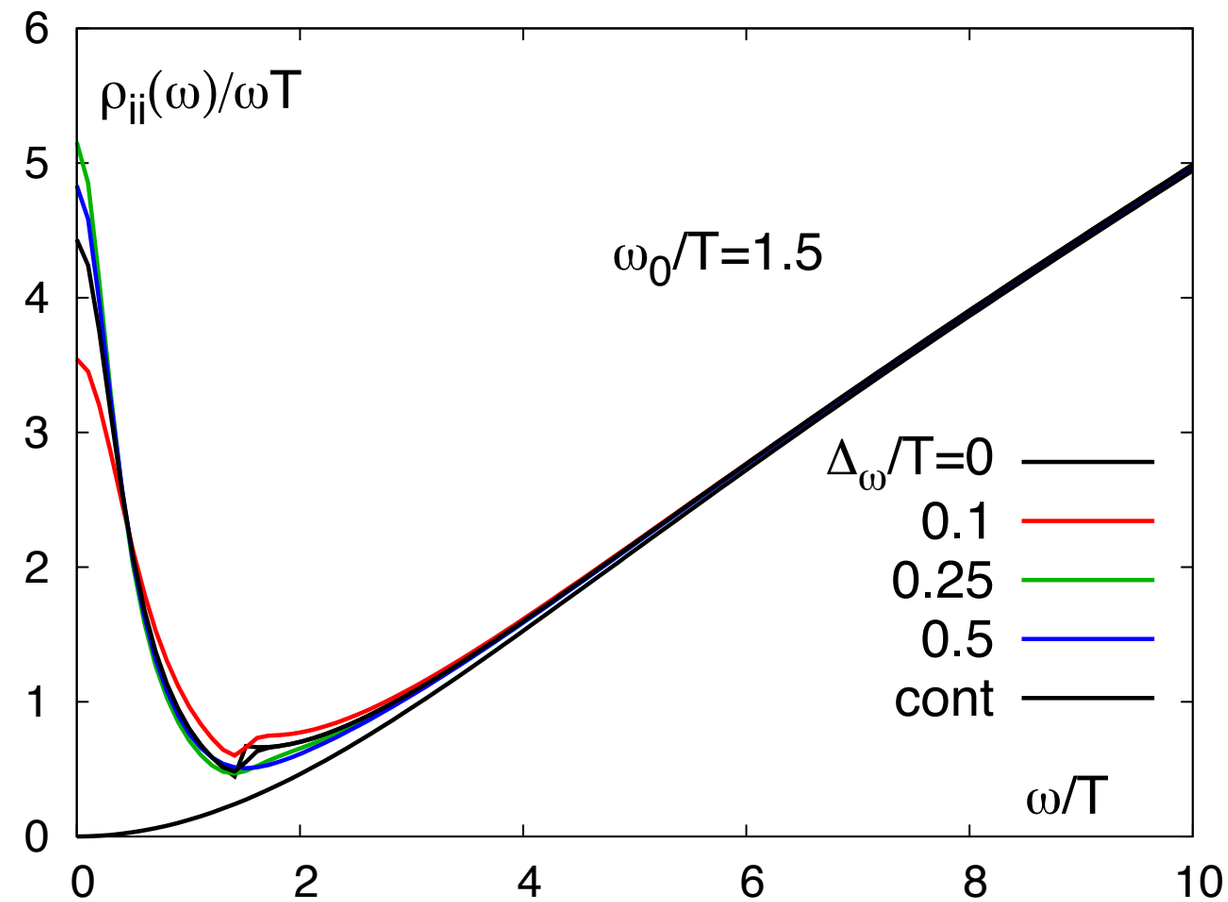
Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\ \Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + \mathrm{e}^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

delay the onset (ω_0) of the continuum part

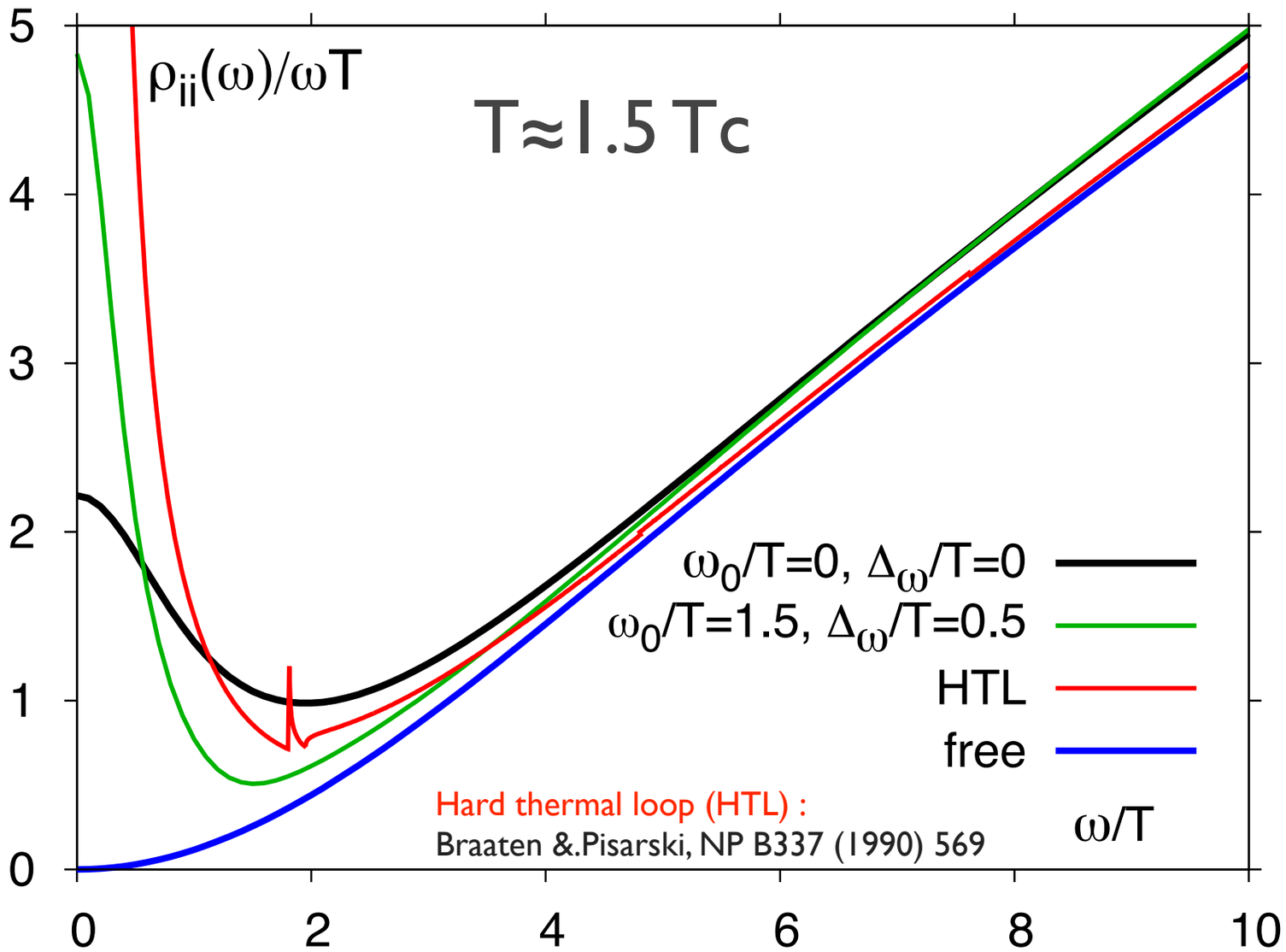




- Rise of BW peaks compensate for the cut from continuum parts
- Fits become worse with increasing ω_0 and/or increasing Δ_ω

Electrical conductivity

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\ \Theta(\omega_0, \Delta_\omega)$$



$$\Theta(\omega_0, \Delta_\omega) = \left(1 + \mathrm{e}^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

electrical conductivity

$$1/3 \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 1$$

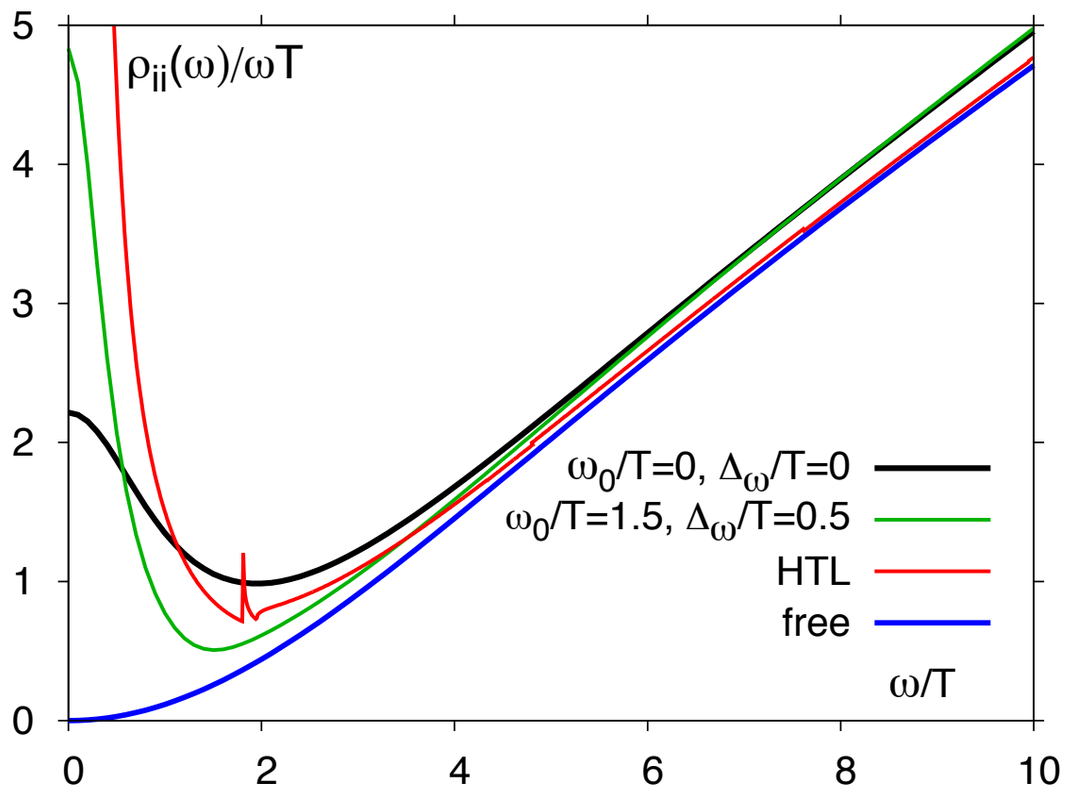
Soft photon emission rate

$$\lim_{\omega\to 0} \omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = (0.0004 - 0.0013)\, T_c^2$$

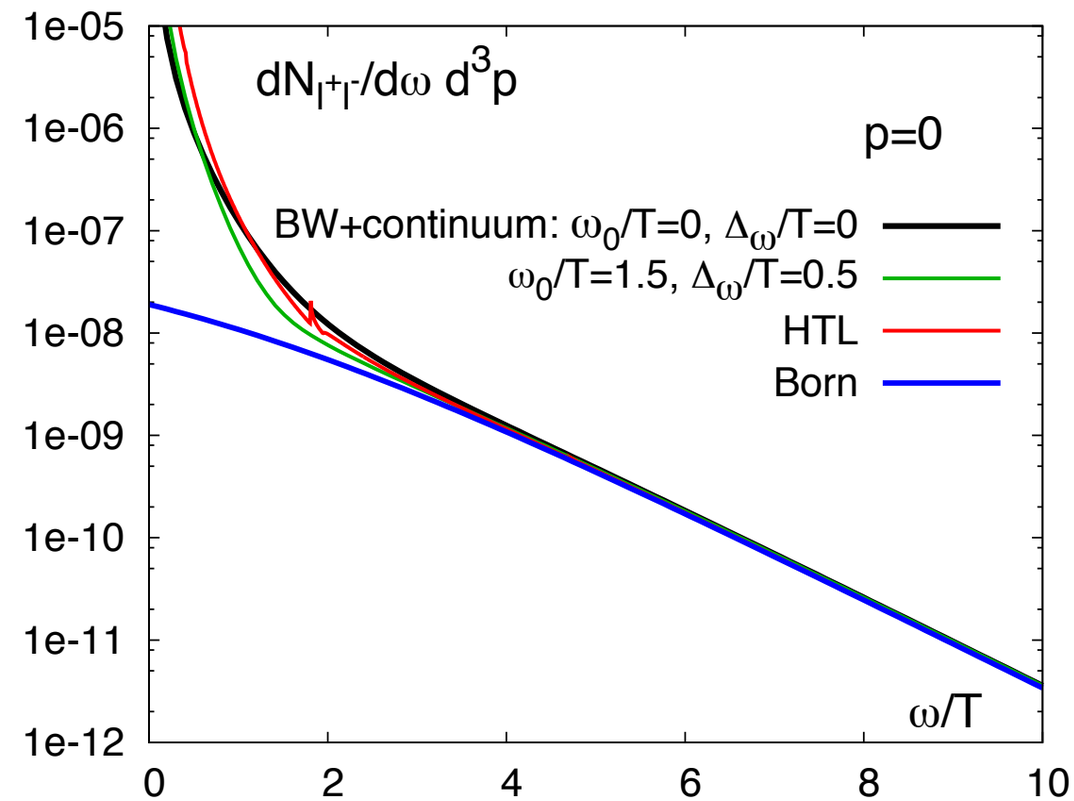
HTL conductivity is divergent at w~0

Thermal dilepton rates

$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)}$$



Hard thermal loop (HTL): Braaten &.Pisarski, NP B337 (1990) 569



HTD, Francis, Kaczmarek,Karsch, Laermann, Soeldner, Phys.Rev. D83 (2011) 034504

- thermal dilepton rate approaches leading order Born rate at w/T ≳ 4
- enhancement at small w/T

Heavy quark diffusion

- Langevin Equation

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{p^i}{M},$$

$$\frac{\mathrm{d}p^i}{\mathrm{d}t} = -\eta_D\, p^i + \xi^i(t)$$

drag — random force

- Fluctuation-dissipation relation

$$\eta_D = \frac{\kappa}{2MT},$$

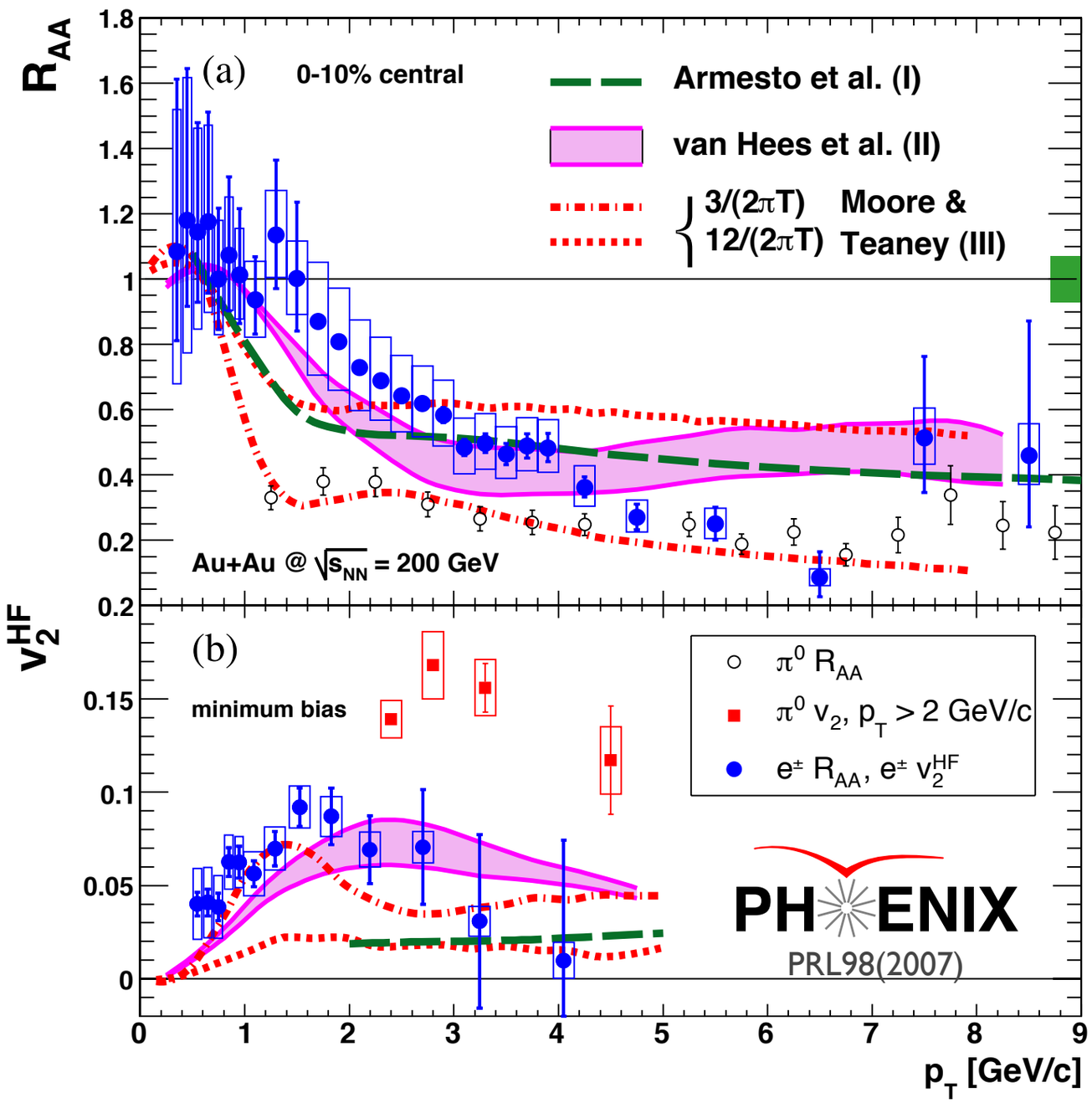
$$\langle \xi^i(t)\xi^j(t')\rangle = \kappa\, \delta^{ij}\, \delta(t-t')$$

- Fick's law of diffusion

$$\partial_t N + D\, \nabla^2 N = 0$$

- Einstein relation

$$D = \frac{T}{M\,\eta_D} = \frac{2T^2}{\kappa}$$

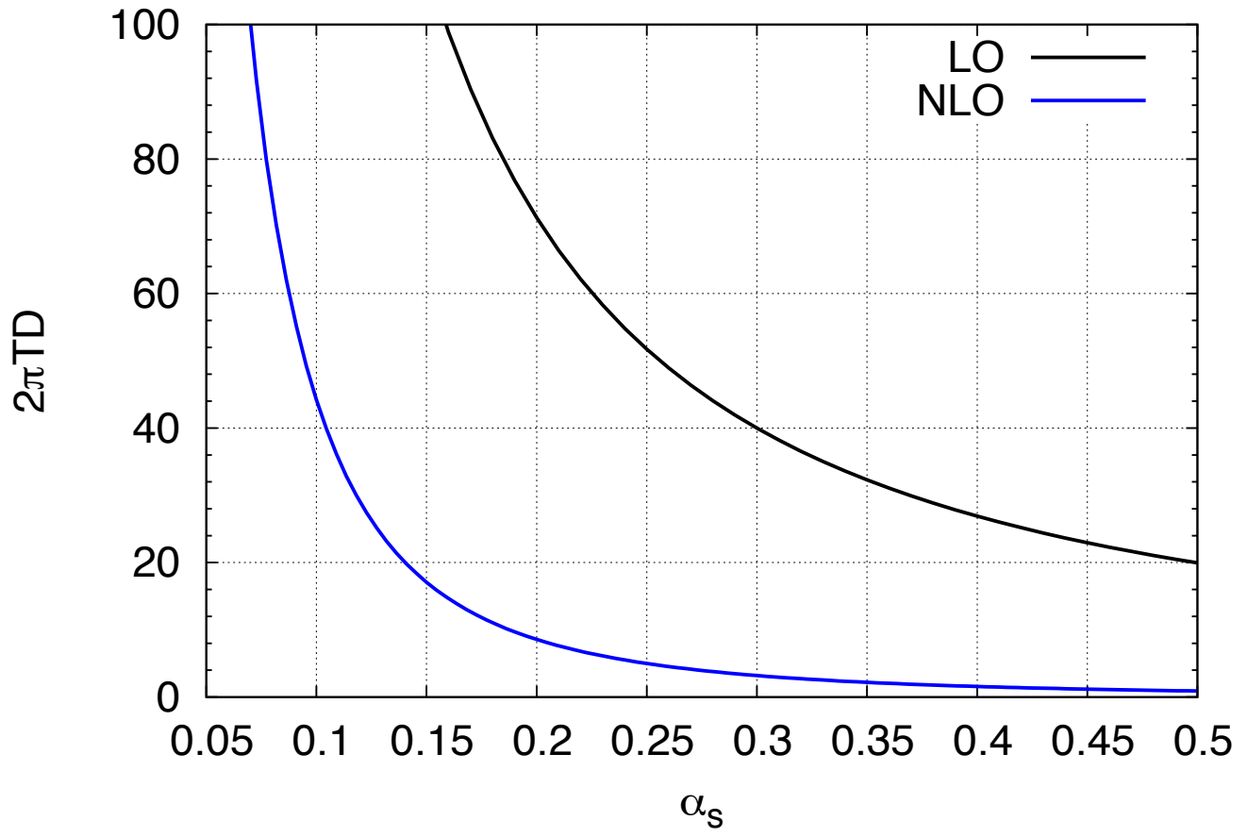


D: diffusion coefficient

3κ: mean squared momentum transfer per time

Heavy quark diffusion

- pQCD calculations



$\alpha_s \sim 0.2, \; g \sim 1.6$

LO: $2\pi TD \approx 71.2$
NLO: $2\pi TD \approx 8.4$

Moore & Teaney, PRD71(2005)064904
Caron-Huot & Moore, PRL 100(2008)052301

★ Compute heavy quark diffusion coefficient on the lattice

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \lim_{\vec{p}\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}(\omega,\vec{p})}{\omega}$$

Kubo formula: heavy quark diffusion constant ~ intercept of σ(ω,0)/ω at ω=0

$$\rho_{ii}(\omega,\vec{p}) = \int \mathrm{d}^4x \, e^{i\omega t - i\vec{p}\cdot\vec{x}} \left\langle \left[j_i(t,\vec{x}), j_i(0,\vec{0}) \right] \right\rangle$$

EM current: $j_i = \bar{\psi}\gamma_i\psi$

Lattice setup for charmonium simulation

- ★ non-perturbatively clover improved Wilson fermions
- ★ isotropic quenched lattice
- ★ simulation parameters tuned to reproduce nearly physical J/ψ mass

lattice cutoff dep. | Volume dep.

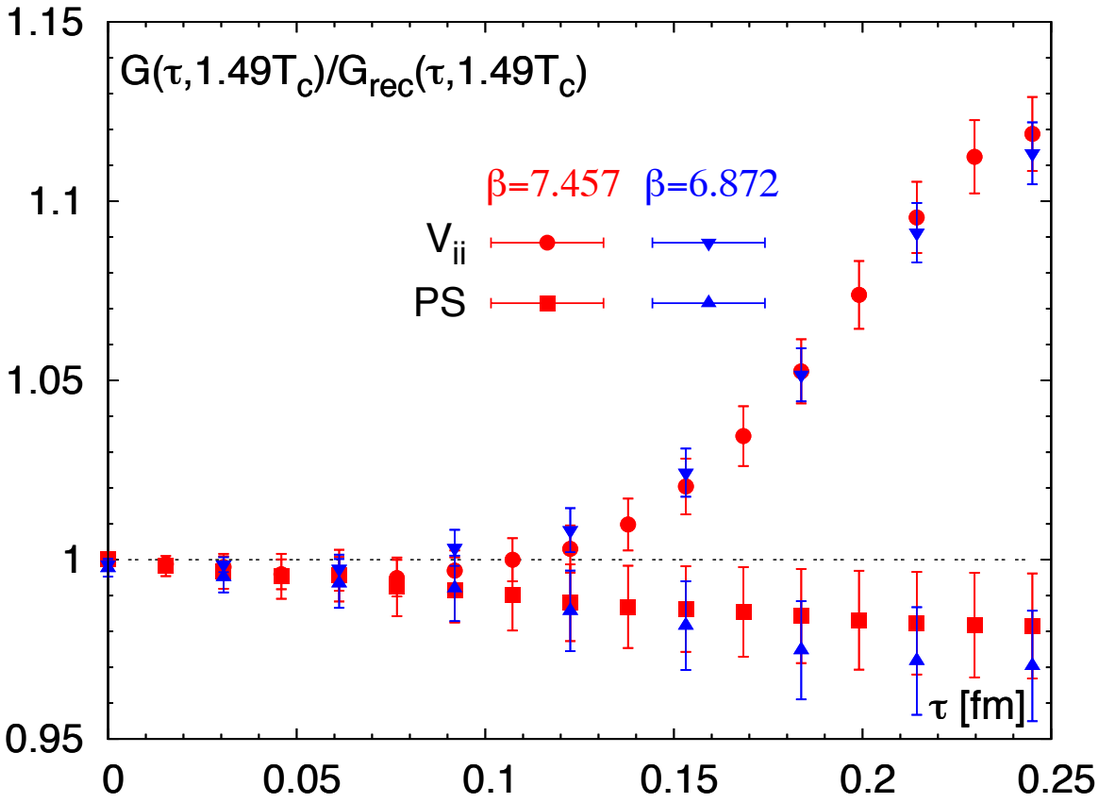
β	a[fm]	a^{-1}[GeV]	L_σ[fm]	c_{SW}	κ	$N_\sigma^3 \times N_\tau$	T/T$_c$	N_{conf}
6.872	0.031	6.432	3.93	1.412488	0.13035	$128^3 \times 32$	0.74	126
						$128^3 \times 16$	1.49	198
7.457	0.015	12.864	1.96	1.338927	0.13179	$128^3 \times 64$	0.74	179
						$128^3 \times 32$	1.49	250
7.793	0.010	18.974	1.33	1.310381	0.13200	$128^3 \times 96$	0.73	234
						$128^3 \times 48$	1.46	461
						$128^3 \times 32$	2.20	105
						$128^3 \times 24$	2.93	81

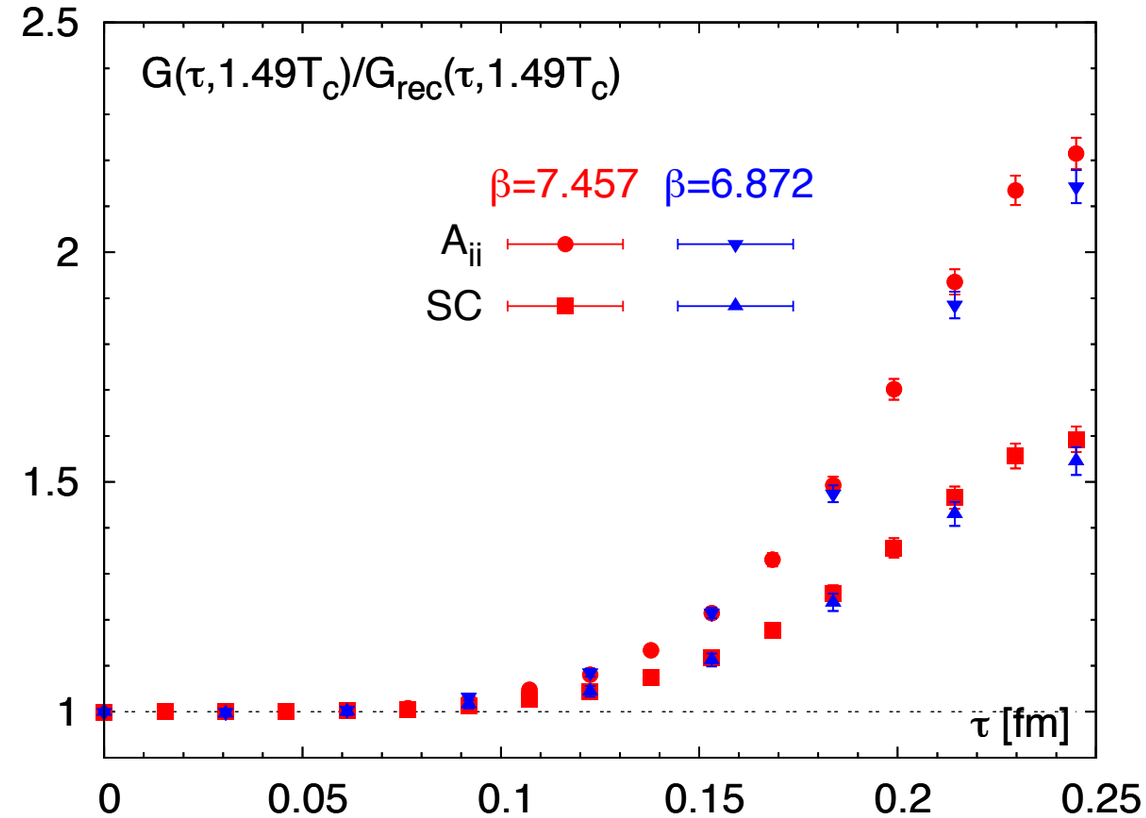
close to continuum

T dep.

- ★ large N_T makes the extraction of spf more reliable

Lattice cutoff/volume effects





$$G(\tau, T) = \int \mathrm{d}\omega\, \rho(\omega, T)\, K(\tau, T)$$

$$G_{\mathrm{rec}}(\tau, T) = \int \mathrm{d}\omega\, \rho(\omega, T')\, K(\tau, T)$$

$$T' < T_c < T$$

- small volume and lattice spacing dependences

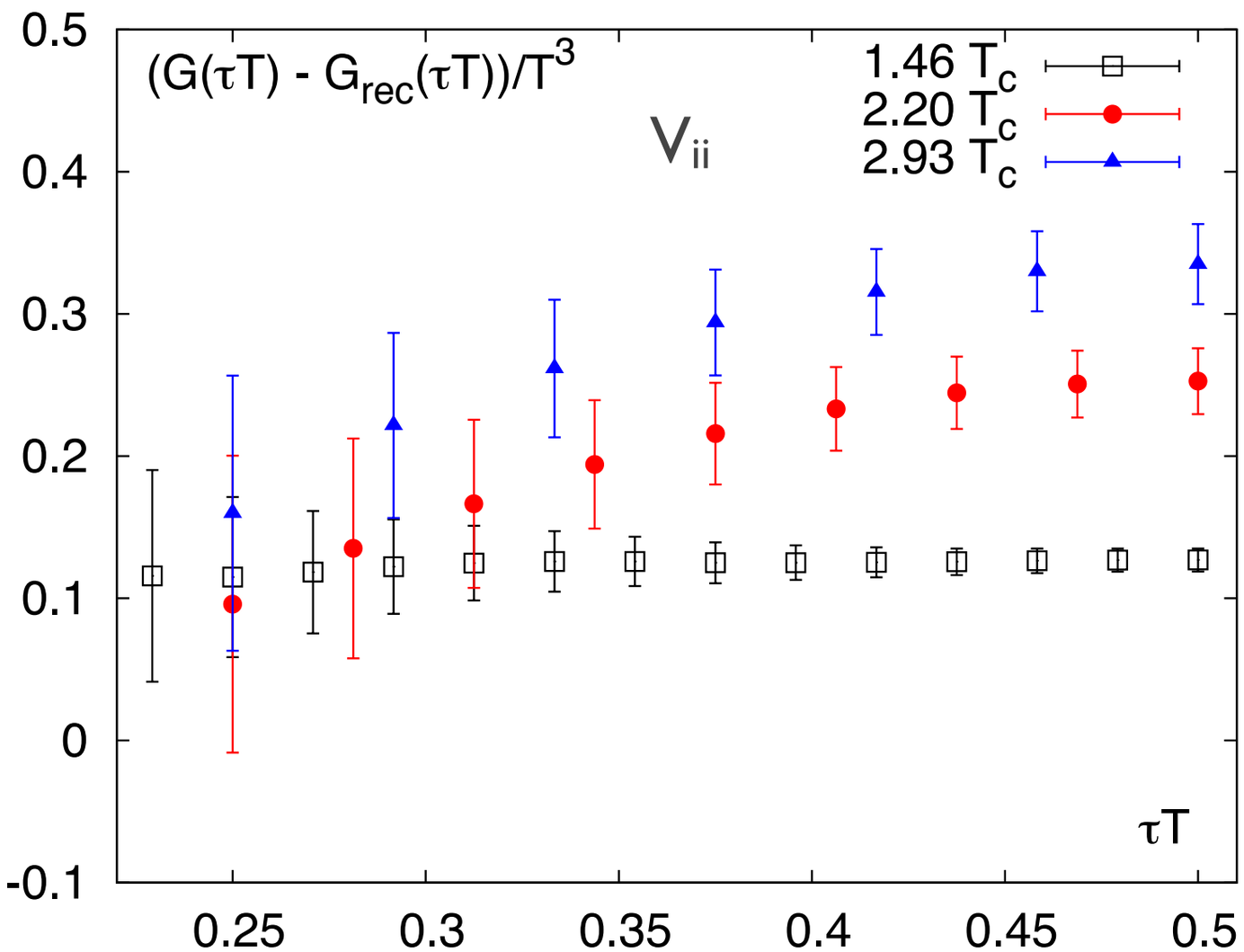
Differences of vector spfs at $T>T_c$ from at $T<T_c$

reconstructed correlator:

$$G_{rec}(\tau,T) = \int \frac{d\omega}{2\pi}\, \rho(\omega, 0.73T_c)\; K(\omega,\tau,T)$$ calculated from $G(\tau,0.73T_c)$ directly

differences of the correlation functions

$$G(\tau,T) - G_{rec}(\tau,T) = \int \frac{d\omega}{2\pi}\, \Delta\rho(\omega)\; K(\omega,\tau,T), \quad \Delta\rho(\omega) = \rho(\omega,T) - \rho(\omega,0.73T_c)$$



- At $T>1.46\,T_c$, $G(\tau T)-G_{rec}(\tau T)$ is monotonically increasing with τT
- At $1.46\,T_c$, $G(\tau T)-G_{rec}(\tau T)$ is almost independent of τT at $\tau T>0.35$

Estimation of charm quark diffusion at 1.46 T_c

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \frac{\rho^V(\omega, T)}{\omega}$$

Assume: from T=0.73 T_c to T=1.46 T_c, only the very low frequency part (ω<T) of the vector spectral function changes

Fit to the value of $G(\tau T=1/2)-G_{rec}(\tau T=1/2)$ at 1.46 T_c

Ansatz of the very low frequency part of spectral function:

I $$\rho(\omega \ll T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}, \quad \eta = \frac{T}{MD}$$

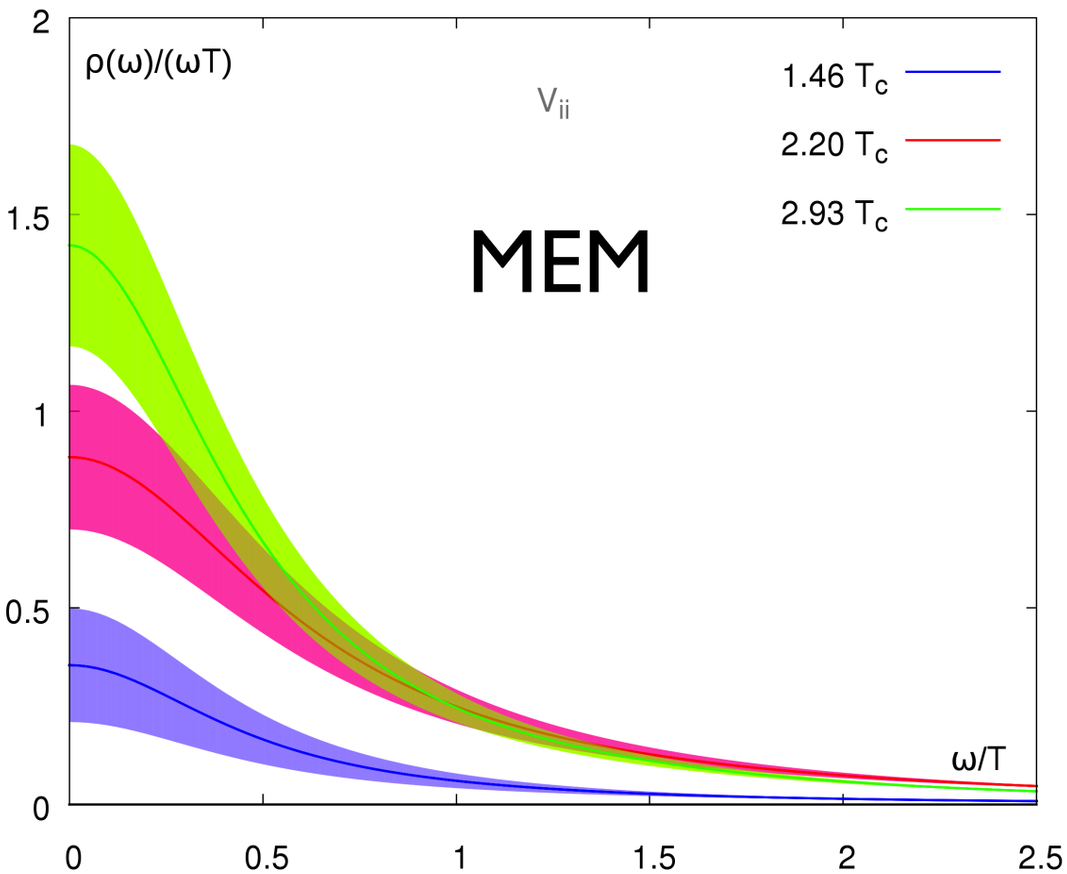
M=1.0GeV	➜	$2\pi TD \approx 0.6$
M=1.8GeV	➜	$2\pi TD \approx 3.6$

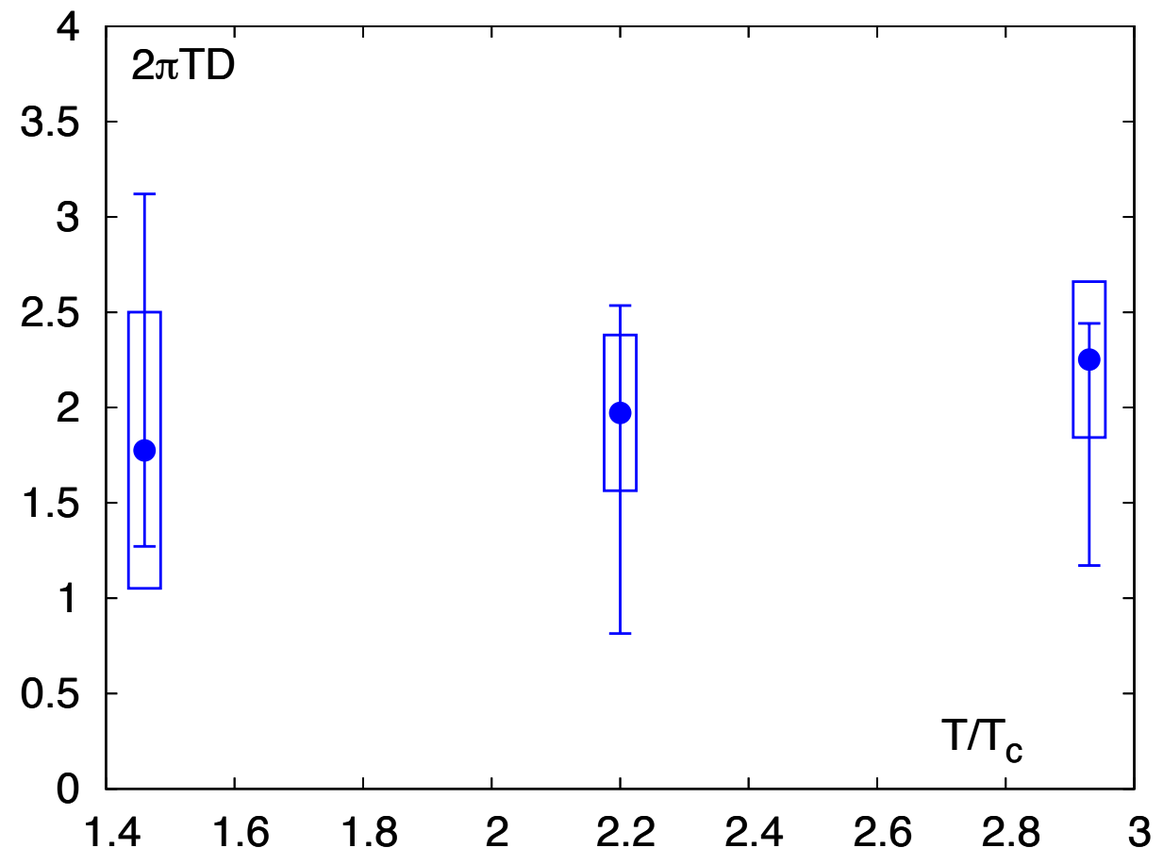
II $$\rho(\omega \ll T) = b\,\omega$$

$$2\pi TD = \frac{\pi T}{3\chi_{00}} b \approx 2$$

Charm diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \frac{\rho^V(\omega, T)}{\omega}$$





- Strong coupling limit: $2\pi TD=1$ — Kovtun, Son & Starinets, JHEP 0310(2003)064
- pQCD: $\alpha_s \sim 0.2$, LO: $2\pi TD \approx 71$, NLO: $2\pi TD \approx 8$ — Moore & Teaney, PRD71(2005)064904; Caron-Huot & Moore, PRL 100(2008)052301
- T-Matrix (U-pot.): $T/T_c \approx 1.5$, $2\pi TD \approx 8$ — M. He, R. Rapp, arXiv: 1204.4442
- HQET(lQCD): $T_c < T < 2T_c$ $2\pi TD \approx 6$ — D. Banerjee, S. Datta, R. Gavai & P. Majumdar, PRD 85(2012)014510; A. Francis, O. Kaczmarek, M. Laine & J. Langelage, PoSLAT(2011)202

Conclusion & Outlook

- We calculated the vector correlation function at $T \simeq 1.45 T_c$ in quenched lattice QCD and performed a continuum extrapolation

- $G_V(\tau T)$ is well reproduced using a Breit-Wigner plus continuum ansatz for the vector spectral function

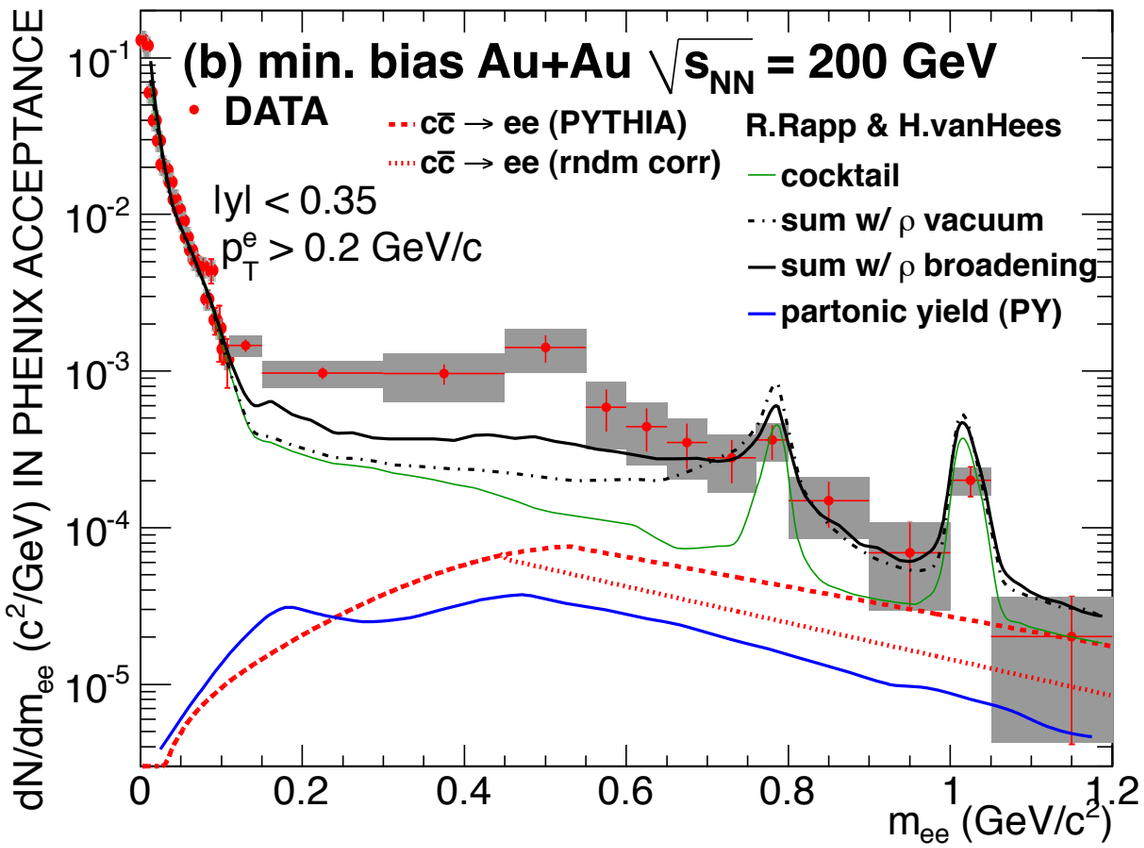
- Electrical conductivity $1/3 \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 1 \quad \text{at} \quad T \simeq 1.45\, T_c$

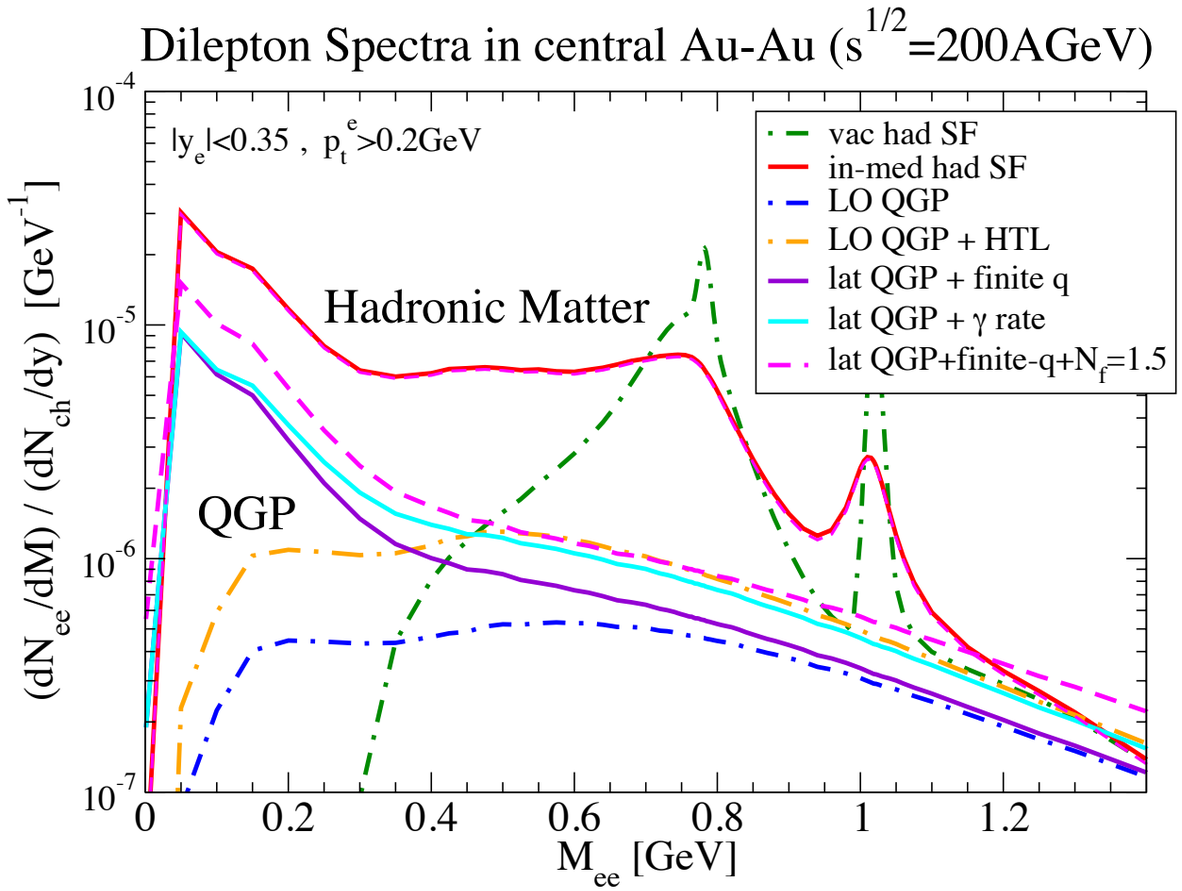
- Dilepton rate approaches leading order Born rate at $\omega/T \gtrsim 4$

- Charm diffusion coefficients are estimated to be approximately $1/\pi T$ in the region of $1.46 T_c \ldots 2.93\, T_c$

▸ Vector correlation functions at other temperatures and non-zero momenta

Dilepton rates

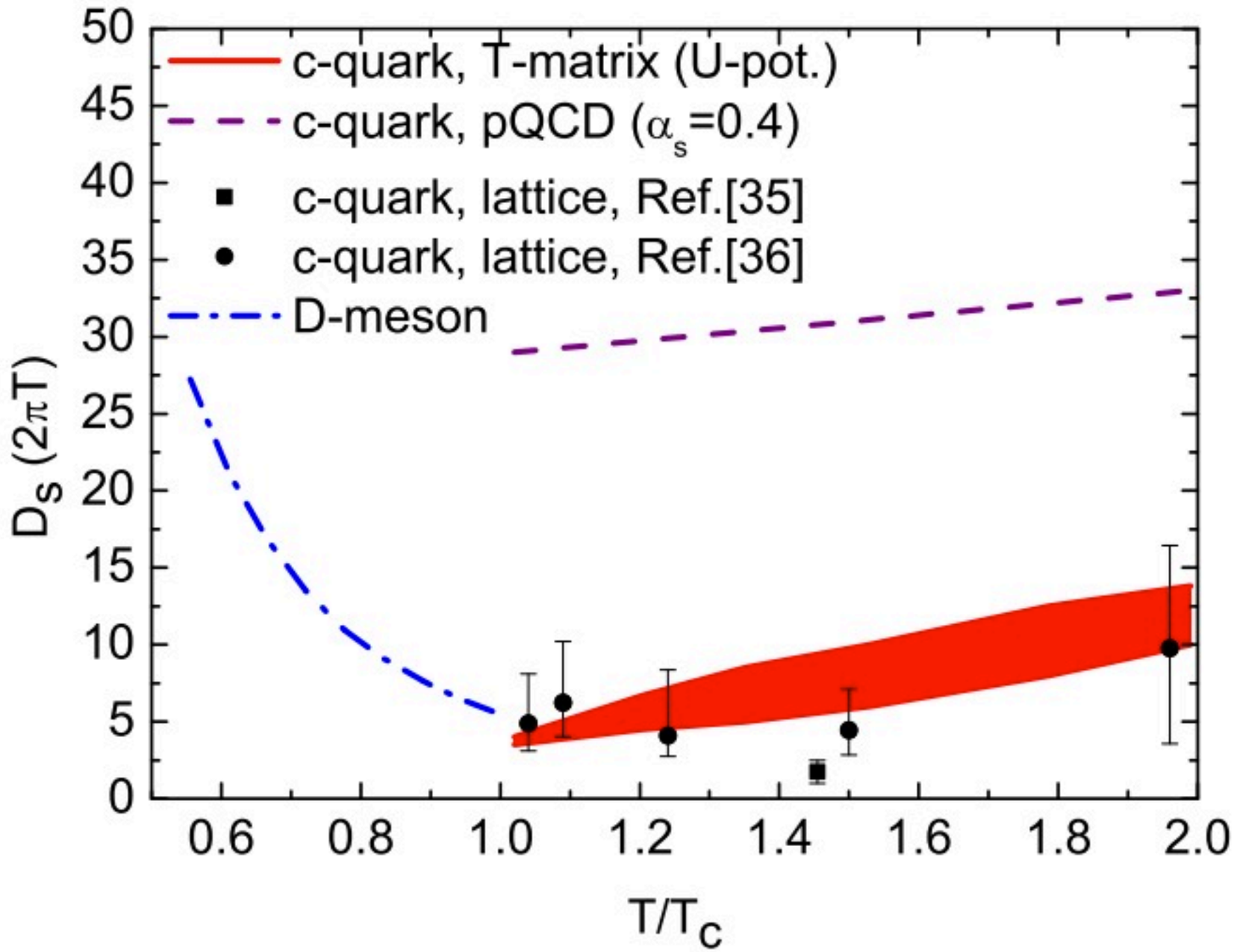




Rapp, arXiv:1010.1719

- lattice QCD results have larger contributions than LO pQCD
- the QGP contribution from several attempts is smaller than the hadronic contribution
- Note: thermal dilepton rates from lQCD are calculated only at ~1.5 Tc

Heavy quark diffusion coefficients



M. He, R. Rapp, arXiv: 1204.4442

Maximum Entropy Method [Asakawa, Hatsuda & Nakahara, '01]

- Hard to extract spectral function (spf)

$$G(\tau,T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, K(\tau,\omega,T)\, \rho(\omega,T)$$

Discretized $\mathcal{O}(10)$ — Continuous $\mathcal{O}(10^3)$

χ^2 fitting inconclusive

- Maximum Entropy Method (MEM) ⟵ Bayesian theorem
  - A method to obtain the most probable image from insufficient data
  - Ingredients of MEM: $P[\sigma|GH] \propto P[G|\sigma H]\; P[\sigma|H]$

$P[G|\sigma H] \propto \exp(-\chi^2/2)$: likelihood function

$P[\sigma|H] \propto \exp(\alpha S)$: prior probability

ρ: spectral function
G: lattice data
H: prior information on ρ

Information entropy: $S = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$

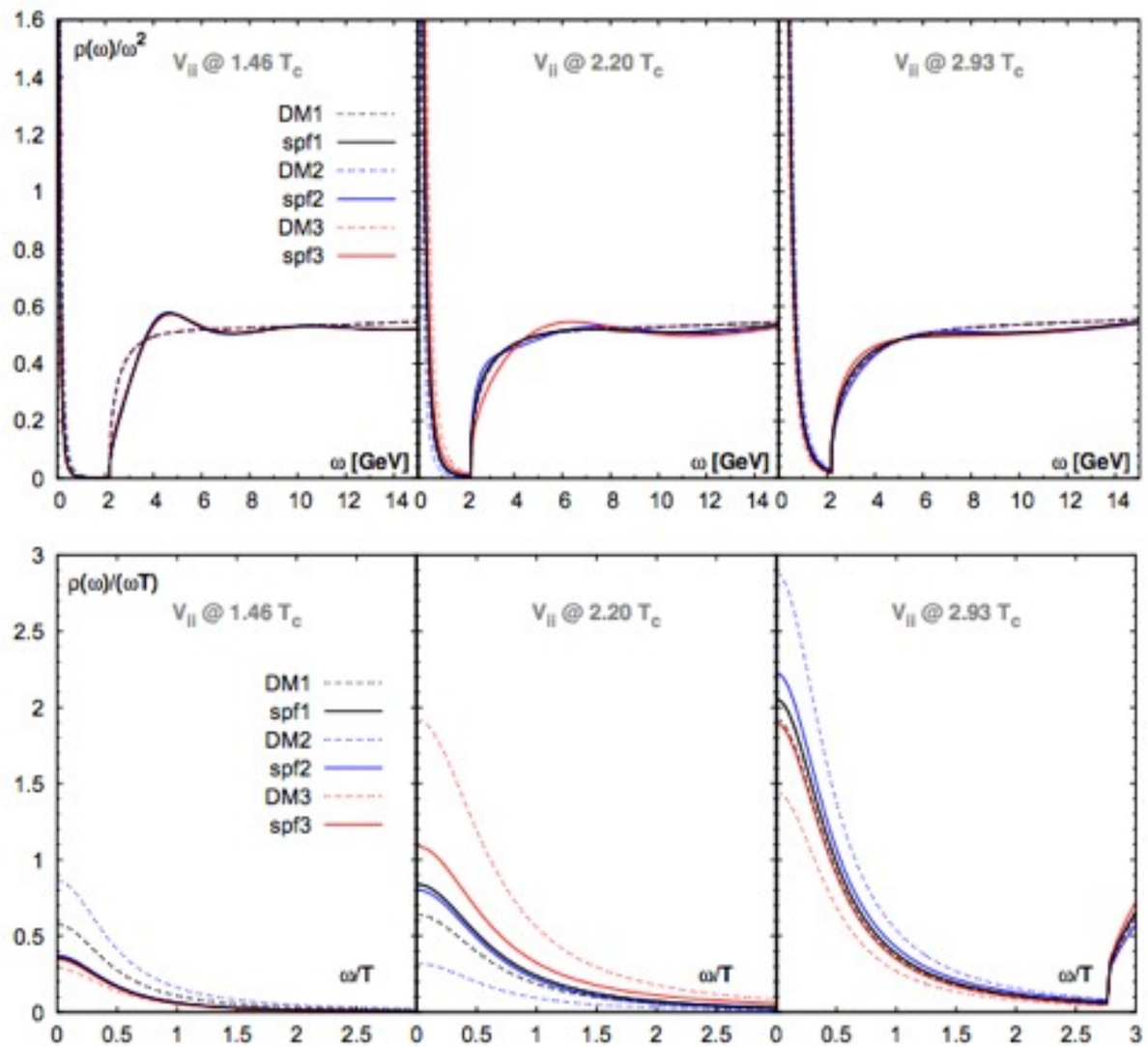
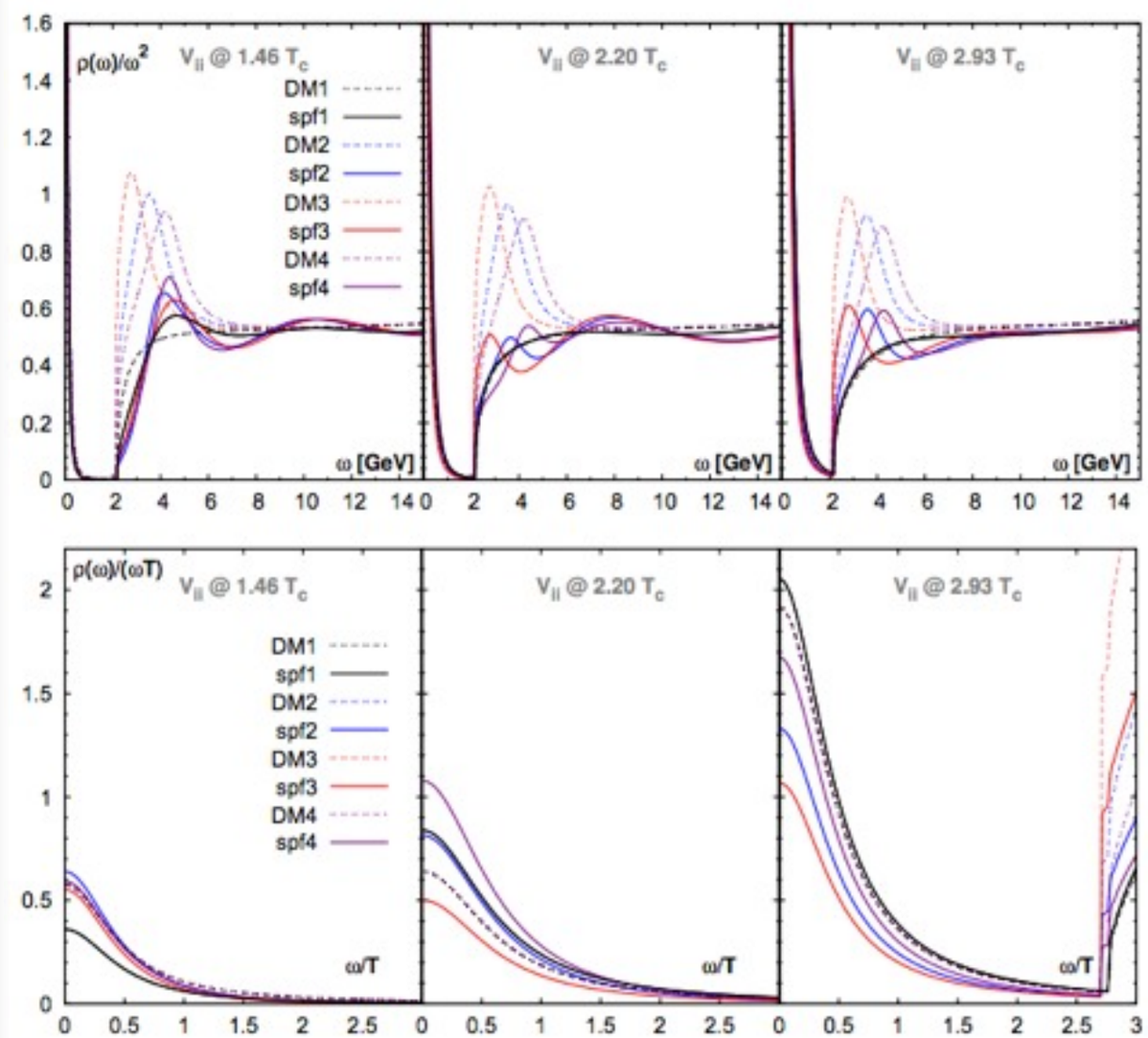
Default Model (DM): m(ω), includes the prior information on ρ, e.g. ρ is positive-definite

DM is the only input parameter in the MEM analysis

- Important to check the dependence of output spf on DMs

Default model dependences of charmonium spf

variations on transport peaks



variations on resonance peaks