NBD and multiplicity fluctuation in CGC

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RHIC-AGS users meet, BNL

- Particle production in CGC
- Single inclusive distributions.
- Two particle and three particle correlation
- n-particle correlation
- Multiplicity distribution, origin of NBD
- Phenomenology, probability distribution in p+p and A+A
- Numerical simulations and NBD parameters
- Summary

Some features of multiplicity distribution

Probability distribution of multiplicity $P(n)$ contains the information of correlated n-particle production.

One can define a generating function

$$F(z) \equiv \sum_{n=0}^{\infty} z^n P_n$$

The fundamental properties of P_n are reflected in the factorial moments and cumulants. The qth factorial moment

$$f_q = \langle n(n-1)\cdots(n-q+1)\rangle = \left.\frac{\mathrm{d}^q F(z)}{\mathrm{d}z^q}\right|_{z=1}$$

The qth factorial cumulant

$$m_q = \left.\frac{\mathrm{d}^q \ln F(z)}{\mathrm{d}z^q}\right|_{z=1} = f_q \pm \text{lower cumulants}$$

Some features of multiplicity distribution

- Color Glass Condensate $\rightarrow$ *ab into* framework to study correlated multi-particle production.
- We need to study the relation between higher and lower cumulants.

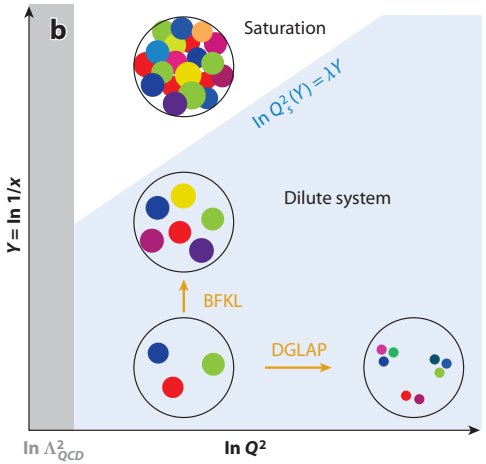
$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_1\,\mathrm{d}^2\mathbf{p}_{\perp 1}\ldots\,\mathrm{d}y_q\,\mathrm{d}^2\mathbf{p}_{\perp q}} \right\rangle \Longleftrightarrow \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_1\,\mathrm{d}^2\mathbf{p}_{\perp 1}} \right\rangle \ldots \left\langle \frac{\mathrm{d}N}{\mathrm{d}y_q\,\mathrm{d}^2\mathbf{p}_{\perp q}} \right\rangle$$

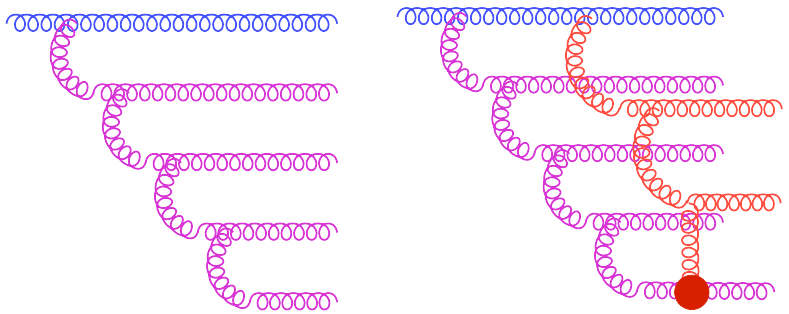
$$\begin{aligned} \langle n^2 \rangle &= \langle n \rangle^2 + \langle n \rangle \rightarrow \text{Poisson} \\ \langle n^2 \rangle &= 2\langle n \rangle^2 + \langle n \rangle \rightarrow \text{Geometric} \end{aligned}$$

Gluons saturation

Gribov, Levin & Ryskin 1983

At High energies gluon density inside hadrons/nuclei saturate by two competing process:



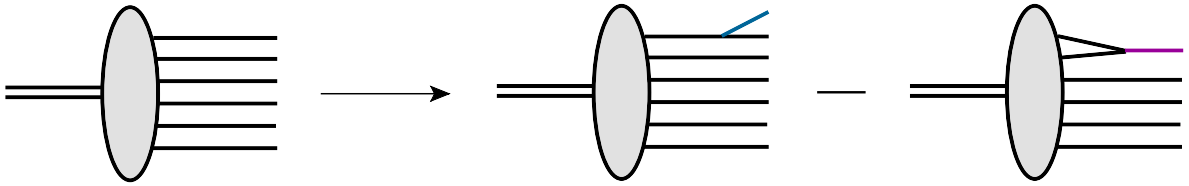


Bremstrahlung

$\propto$ No. of gluons

Recombination

$\propto$ (No. of gluons)^2



$$\frac{\partial \phi(x, k_\perp)}{\partial \log(x_0/x)} \approx \mathcal{K} \otimes \phi(x, k_\perp) - \phi(x, k_\perp)^2 \quad \textit{BK/JIMWLK equation}$$

Non-linear equation gives rise a scale, $Q_s^2(x) \rightarrow$ saturation scale.

High energy Nuclei/hadrons $\rightarrow$ large parton density $\rightarrow$ classical approx.

Color Glass Condensate: MV model

- Color: QCD (gluons carry color charge)
- Glass: Stochastic interactions, dynamics on very long time scales (time dilation).
- Condensate: Fields with large occupation # $\sim 1/\alpha_S$ with mom. peaked at $k_T \approx Q_S$

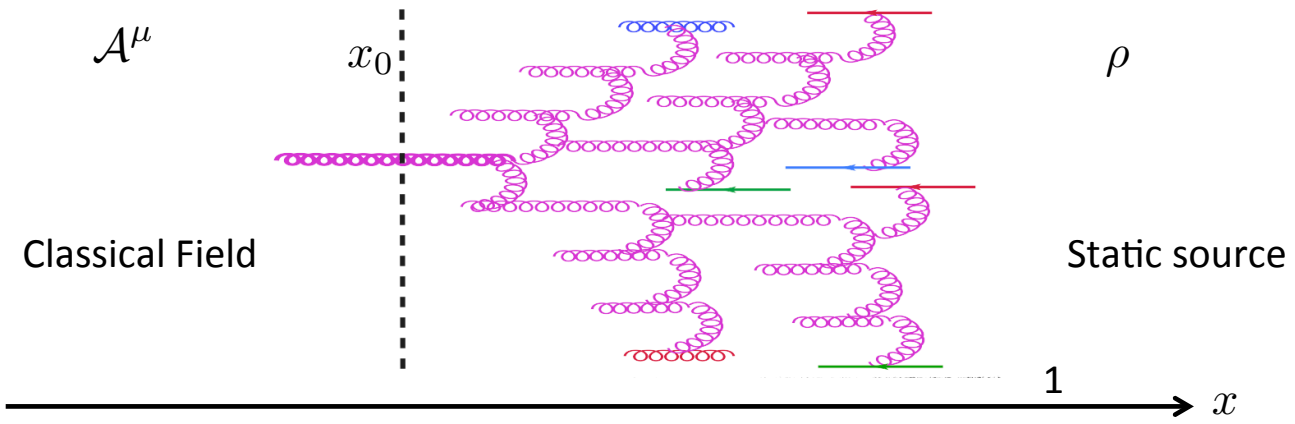


figure : Albacete QM12

Unknown color charge distribution (weight functional):

$$W[\rho] = \exp\left[-\frac{\rho^2(x_\perp)}{2\mu^2(x_\perp)}\right], \; g^2\mu(x_\perp) \sim Q_S(x, x_\perp)$$

McLerran , Venugopalan 1994 (hep-ph/9309289)

Color Glass Condensate: MV model

- Solve classical Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

for color current due to colliding sources (ρ_1 and ρ_2)

$$J_a^\nu = g\delta^{\nu+}\delta(x^-)\rho_{1,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{2,a}(\mathbf{x}_\perp)$$

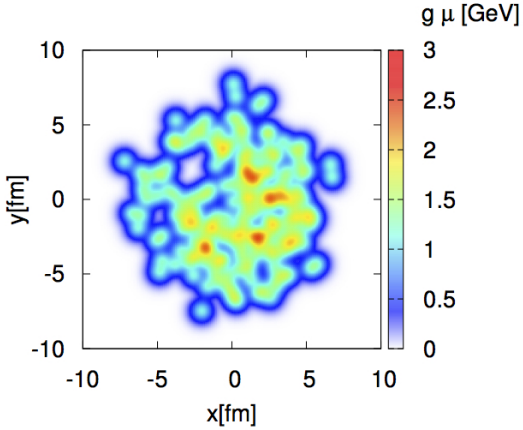
- Extract the gauge field after collision for given color charge configuration.
- Analytical calculation possible for lowest order of sources.
- Final observables $\mathcal{O}(\rho_1, \rho_2)$ should be averaged over color charge configuration

$$\langle \mathcal{O} \rangle = \int [\,\mathrm{d}\rho_1][\,\mathrm{d}\rho_2] W_y[\rho_1] W_y[\rho_2] \mathcal{O}(\rho_1, \rho_2).$$

Color correlation in particle production

Averaging $\langle \mathcal{O} \rangle \Rightarrow$ connection between sources $\Rightarrow$ color correlation.
In the MV model

$$W[\rho] \equiv \exp\left(- \int d^2\mathbf{x}_\perp \frac{\rho^a(\mathbf{x}_\perp)\rho^a(\mathbf{x}_\perp)}{2\,\mu_A^2} \right)$$



$\rho^a \rightarrow$ random sources distributed from local Gaussian.

fields $\mathcal{A}(\mathbf{x}_\perp) \sim -\rho(\mathbf{x}_\perp)/\nabla_\perp^2 \implies \mathcal{A}(\mathbf{k}_\perp) \sim -\rho(\mathbf{k}_\perp)/\mathbf{k}_\perp{}^2$

Yang-Mills introduces non-local correlation over length scale $1/Q_s$
$\rightarrow$ Glasma flux tube picture.

Correlated production $\rightarrow$ Gaussian correlations in momentum space,

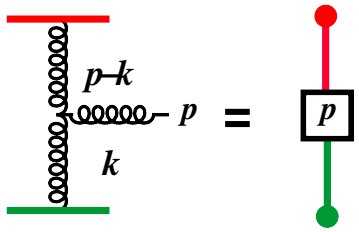
$$\left\langle \tilde{\rho}^{*a}(\mathbf{k}_\perp)\tilde{\rho}^b(\mathbf{k}'_\perp) \right\rangle = (2\pi)^2 \mu_A^2\, \delta^{ab} \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp)$$

Further discussion $\rightarrow$ MV model and LO particle production.

Single inclusive distribution

The one gluon production amplitude for fixed color charges ρ_1, ρ_2

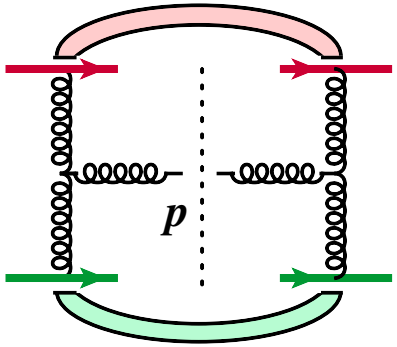
$$\mathcal{M} \sim \frac{\rho_1(\mathbf{k}_\perp)}{\mathbf{k}_\perp{}^2} \frac{\rho_2(\mathbf{p}_\perp - \mathbf{k}_\perp)}{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2} L^\gamma(\mathbf{p}, \mathbf{k}_\perp) \,,$$



$L^\gamma(\mathbf{p}, \mathbf{k}_\perp) \to$ Lipatov vertex.

inclusive gluon distribution $\left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle \sim \langle |\mathcal{M}|^2 \rangle \sim \langle \rho_1^* \rho_1 \rho_2^* \rho_2 \rangle$

$\to$ contracted by Gaussian correlator.



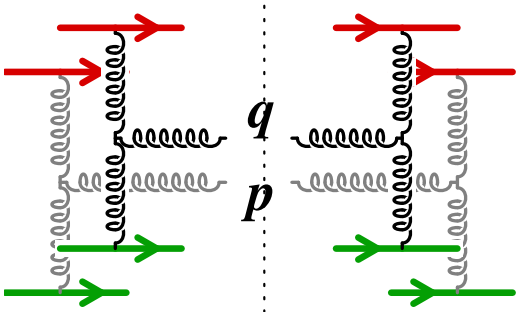
$$\left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle = \frac{S_\perp}{8\pi^4} \frac{(g^2\mu_A)^4}{g^2} \frac{N_c(N_c^2-1)}{p_\perp^4} \ln\left(\frac{p_\perp}{Q_s}\right) \,, \; p_\perp \gg Q_s$$

IR divergence ($\sim \int_{\mathbf{k}_\perp} \mathrm{d}^2\mathbf{k}_\perp / \mathbf{k}_\perp{}^4$) $\to$ regulated over $1/Q_s$.
$S_\perp \to$ transverse overlap area of collision.

Two particle correlation

Dumitru, Gelis, McLerran, Venugopalan 0804.3858

The dominant contribution comes from disconnected diagrams connected by color averaging.



The two gluon production amplitude for fixed color charges ρ_1, ρ_2

$$\mathcal{M} \sim \frac{\rho_1(k_{1\perp})}{k_{1\perp}^2} \frac{\rho_1(k_{2\perp})}{k_{2\perp}^2} \frac{\rho_2(\mathbf{p}_\perp - k_{1\perp})}{(\mathbf{p}_\perp - k_{1\perp})^2} \frac{\rho_2(\mathbf{q}_\perp - k_{2\perp})}{(\mathbf{q}_\perp - k_{2\perp})^2} L^\mu(\mathbf{p}, k_{1\perp}) L^\nu(\mathbf{q}, k_{2\perp}) \,,$$

$\langle |\mathcal{M}|^2 \rangle \to \langle \rho_1^* \rho_1^* \rho_1 \rho_1 \rho_2^* \rho_2^* \rho_2 \rho_2 \rangle \Rightarrow$ 9 possible ways to contract.

$\langle |\mathcal{M}|^2 \rangle \to$ 8 connected & 1 disconnected diagram

Two particle correlation

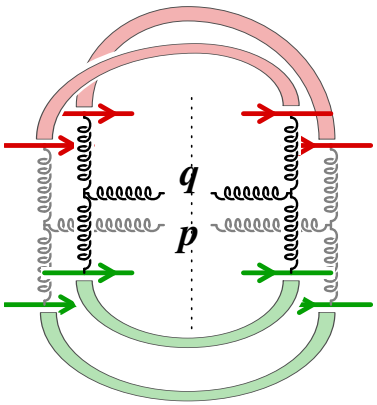
Dumitru, Gelis, McLerran, Venugopalan 0804.3858

Correlated and non-correlated contribution

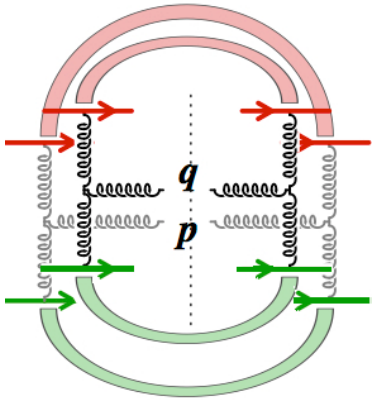
$$C_2(\mathbf{p},\mathbf{q}) \equiv \left\langle \frac{dN_2}{dy_p d^2\mathbf{p}_\perp dy_q d^2\mathbf{q}_\perp} \right\rangle - \left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q d^2\mathbf{q}_\perp} \right\rangle ,$$

$\downarrow$ connected diagrams

$\downarrow$ disconnected diagrams



8 topologies



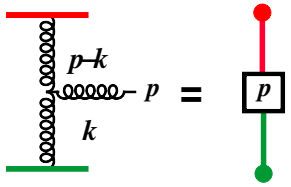
1 topology

It can be shown

$$C_2(\mathbf{p},\mathbf{q}) = \frac{\kappa_2}{S_\perp Q_s^2} \left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q d^2\mathbf{q}_\perp} \right\rangle ,$$

$\kappa_2 \rightarrow$ non-perturbative constant.

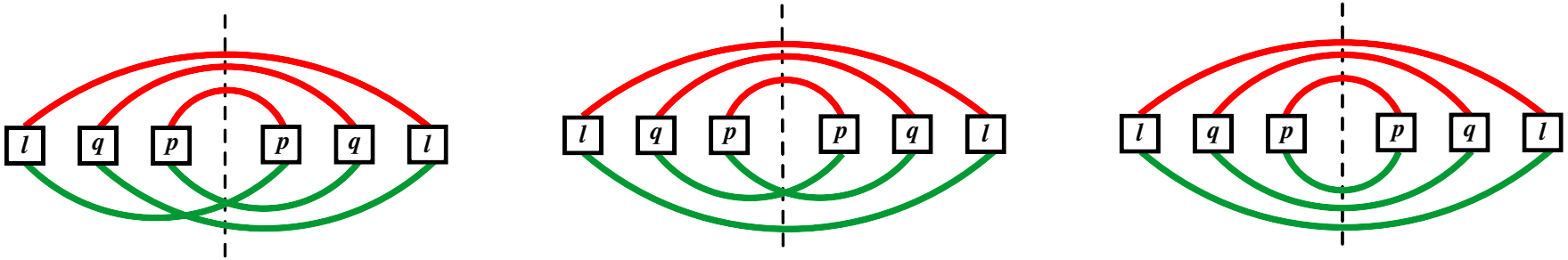
Three particle correlation



The different terms of $C_3(\mathbf{p},\mathbf{q},\mathbf{l})$

$$\left\langle \frac{dN_3}{dy_p d^2\mathbf{p}_\perp dy_q d^2\mathbf{q}_\perp dy_l d^2\mathbf{l}_\perp} \right\rangle \qquad \left\langle \frac{dN_2}{dy_p d^2\mathbf{p}_\perp dy_q d^2\mathbf{q}_\perp} \right\rangle \left\langle \frac{dN}{dy_l d^2\mathbf{l}_\perp} \right\rangle \qquad \left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q d^2\mathbf{q}_\perp} \right\rangle \left\langle \frac{dN}{dy_p d^2\mathbf{l}_\perp} \right\rangle$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$



Dusling, Fernandez-Fraile, Venugopalan 0902.4435

A total 255 topologies $\rightarrow$ 16 contribute to the correlation term.
For $Q_S \ll p_\perp, q_\perp, l_\perp$

$$C_3(\mathbf{p},\mathbf{q},\mathbf{l}) = \frac{\kappa_3}{S_\perp^2 Q_S^4} \left\langle \frac{dN}{dy_p d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q d^2\mathbf{q}_\perp} \right\rangle \left\langle \frac{dN}{dy_l d^2\mathbf{l}_\perp} \right\rangle$$

n-particle correlation: factorial moments

LO Two-particle and three-particle correlation results indicates

$$\begin{aligned} \left\langle n^2 \right\rangle &- \langle n \rangle^2 \propto \langle n \rangle^2 \\ \left\langle n^3 \right\rangle &- 3 \langle n \rangle^2 \langle n \rangle + 2 \langle n \rangle^3 \propto \langle n \rangle^3 \end{aligned}$$

and in Glasma $\langle n \rangle \sim 1/g^2 \sim 1/\alpha_s$

The qth factorial moment

$$\begin{aligned} f_q &= \langle n(n-1)\cdots(n-q+1) \rangle = \left. \frac{\mathrm{d}^q F(z)}{\mathrm{d}z^q} \right|_{z=1} \\ f_q &= \langle n^q \rangle + \text{lower order terms} \\ f_q &= \mathcal{O}\left(\frac{1}{\alpha_s^{2q}}\right) + \text{lower order in } \left(\frac{1}{\alpha_s}\right) \end{aligned}$$

The LO contribution $\mathcal{O}\left(\frac{1}{\alpha_s^q}\right) \implies f_q \approx \langle n^q \rangle$

factorial moments in LO $\rightarrow$ from connected diagrams.

factorial cumulants

$$m_q = \left.\frac{\mathrm{d}^q \ln F(z)}{\mathrm{d}z^q}\right|_{z=1}$$

$$\begin{aligned} m_2 &= \langle n(n-1)\rangle - \langle n\rangle^2 \\ m_3 &= \langle n(n-1)(n-2)\rangle - 3\left\langle n^2\right\rangle\langle n\rangle + 2\langle n\rangle^3 \\ \Longrightarrow\ m_q &= f_q \pm \text{lower cumulants} \end{aligned}$$

In the Glasma for LO in α_s ($\equiv \mathcal{O}(1/\alpha_s^q)$)

$$m_q = \langle n^q\rangle - \text{disconnected diagrams}$$

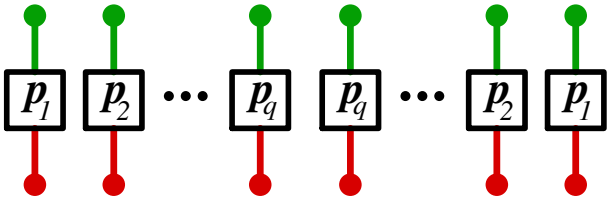
Two-particle and three particle correlation results show

$$m_{q=2,3} = \langle n^q\rangle - \text{disconnected diagrams} = \langle n\rangle^q$$

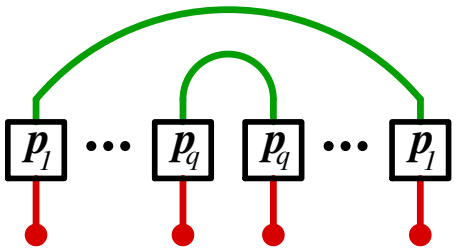
multi-particle production topologies

Gelis, Lappi, McLerran 0905.3234

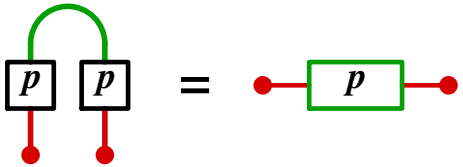
$\langle n^q \rangle \to q$ ladders to be contracted.



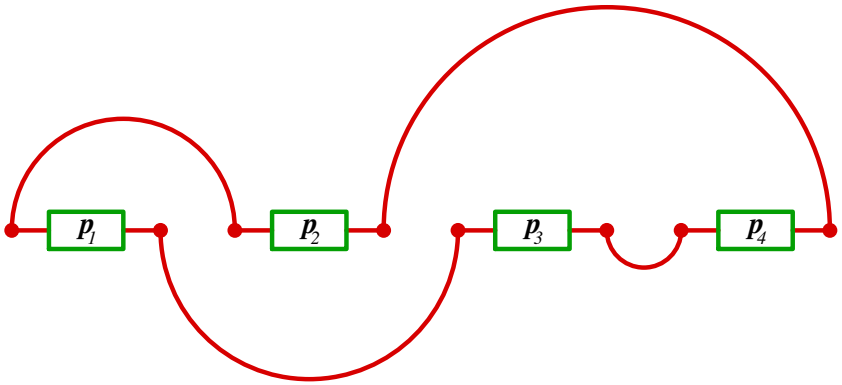
- Step 1: Connect $2q$ building blocks



define



- Step 2: combine loose ends to form connected loops



$\to 2^q(q-1)!$ topologies

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_1\,\mathrm{d}^2\mathbf{p}_{\perp 1}\ldots\,\mathrm{d}y_q\,\mathrm{d}^2\mathbf{p}_{\perp q}} \right\rangle_{\text{conn.}} =$$

$$(q-1)!\,\frac{({N_\mathrm{c}}^2-1)\kappa {Q_\mathrm{s}}^2 S_\perp}{2\pi}\,\frac{\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_1\,\mathrm{d}^2\mathbf{p}_{\perp 1}}\right\rangle\cdots\left\langle \frac{\mathrm{d}N}{\mathrm{d}y_q\,\mathrm{d}^2\mathbf{p}_{\perp q}}\right\rangle}{\left(({N_\mathrm{c}}^2-1)\kappa {Q_\mathrm{s}}^2 S_\perp/(2\pi)\right)^q}$$

$$\Rightarrow \quad m_q = (q-1)!\,k\left(\frac{\bar{n}}{k}\right)^q$$

with

$$k = \kappa\frac{({N_\mathrm{c}}^2-1){Q_\mathrm{s}}^2 S_\perp}{2\pi}$$

This form of m_q defines the Negative Binomial distribution

$$P_n^{\mathrm{NB}} = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)}\,\frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

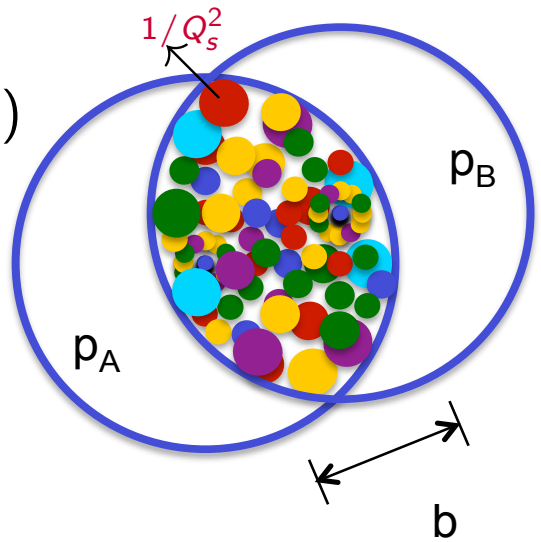
With the generating function

$$F_{k,\bar{n}}(z) \equiv \sum_{n=0}^{\infty} z^n\,P_n \implies m_q \equiv \frac{\mathrm{d}^q}{\mathrm{d}z^q}\ln F_{k,\bar{n}}(z)\bigg|_{z=1} = (q-1)!\,k\left(\frac{\bar{n}}{k}\right)^q$$

Negative Binomial distributions

NBD comes naturally due to

- Gaussian combinatorics of classical sources
- Correlated production over length scale $\mathcal{O}(1/Q_s)$



The parameter of NBD ($\bar{n}$, k)

$$k = \kappa \frac{(N_c{}^2 - 1) Q_s{}^2 S_\perp}{2\pi}$$

$\kappa \rightarrow$ includes the details of IR diverges regulated at the scale Q_s.
$Q_s{}^2 S_\perp \rightarrow$ no. of flux tubes (size $1/Q_s^2$) in the transverse area $S_\perp$.
$(N_c^2 - 1)$ no. of gluon colors emitted from each flux tube.

For $k = 1$, NBD $\rightarrow$ Bose-Einstein (BE) distribution.

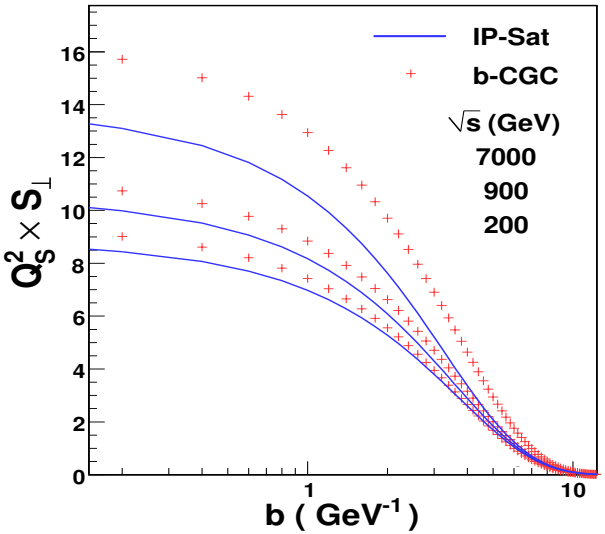
Single color gluon emitted from single flux tube $\rightarrow$ BE distribution.

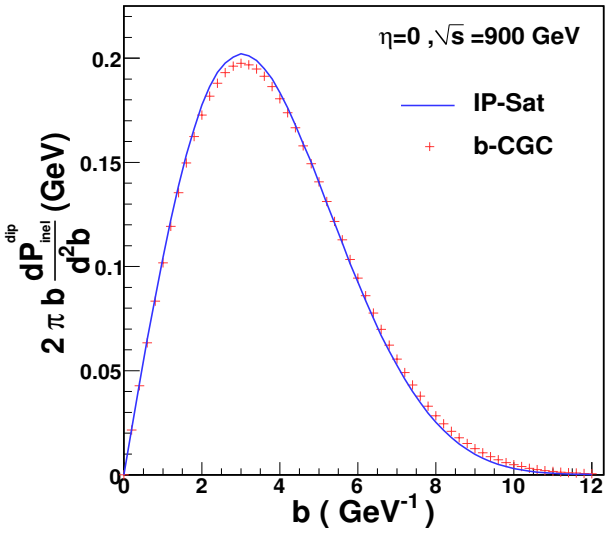
Many sources of color $\rightarrow$ entropy maximization.

Modelling multiplicity distribution in p+p collisions

Using IP-Sat model + $\mathbf{k}_\perp$−factorization approach.

- $\bar{n}(b) \rightarrow k_\perp$-factorization
- $k(b) \propto Q_s^2 S_\perp(b) \rightarrow$ IP-Sat parametrization of Q_s
- κ (non-perturbative constant) $\rightarrow$ unknown.
- convolution of multiple NBDs with probability distributions of impact parameter $dP/d^2b_\perp \rightarrow k_\perp$-factorization.



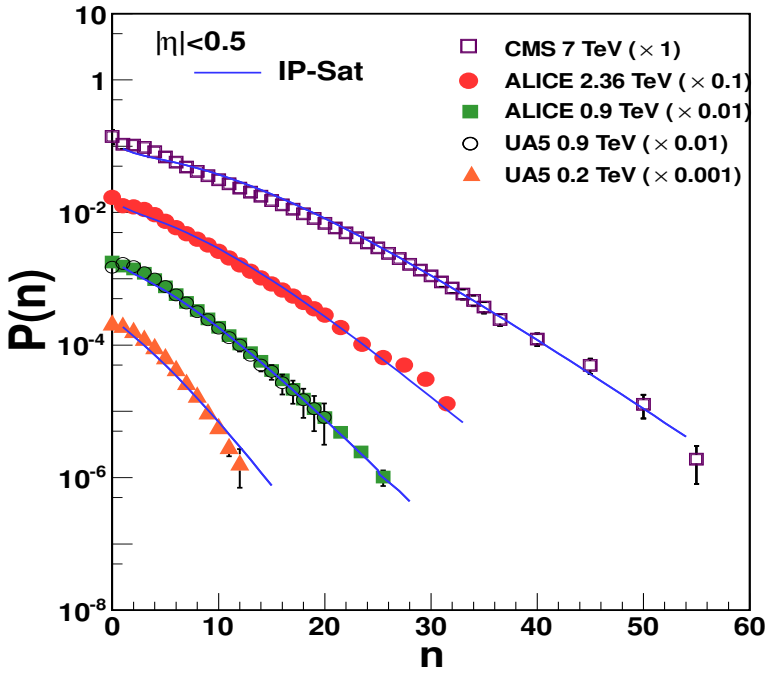


Tribedy, Venugopalan 1011.1895

Modelling multiplicity distribution in p+p collisions

$$P(n) = \int d^2 b_\perp \frac{dP_{\rm inel.}}{d^2 b_\perp} P_n^{\rm NBD}(\bar{n}(b_\perp), k(b_\perp))$$

$\rightarrow$ good agreement over wide range of energy.



The non-perturbative constant κ extracted from fits $\sim 1/6$

Tribedy, Venugopalan 1011.1895 , 1112.2445
see also Dumitru *et al*, arXiv:1111.3031

E-by-E 2+1D CYM simulation : IP-Glasma

IP-Glasma (Impact parameter dependent Glasma) model: Even-by-event solves

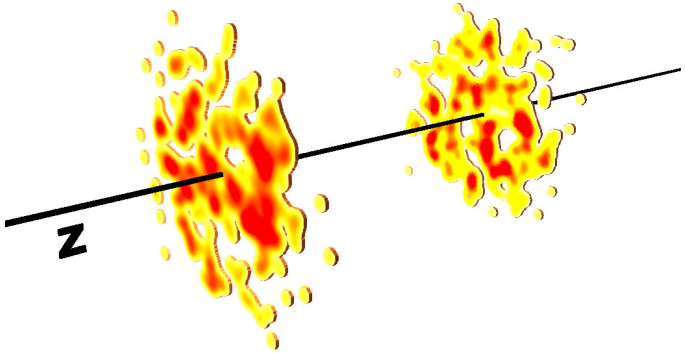
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

In the presence of color current

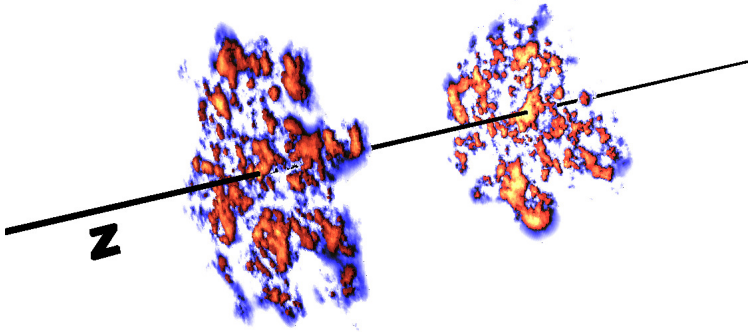
$$J^\nu_{A,B} \approx \delta(x^\mp)\rho(x_\perp)\delta^{\mu\pm}$$

Nuclear color charge distribution $W[\rho] \rightarrow$ from IP-Sat model

Nucleons sampled from Woods-Saxon distribution.



Color charge density for one A+A collision



Two point correlator for one A+A collision

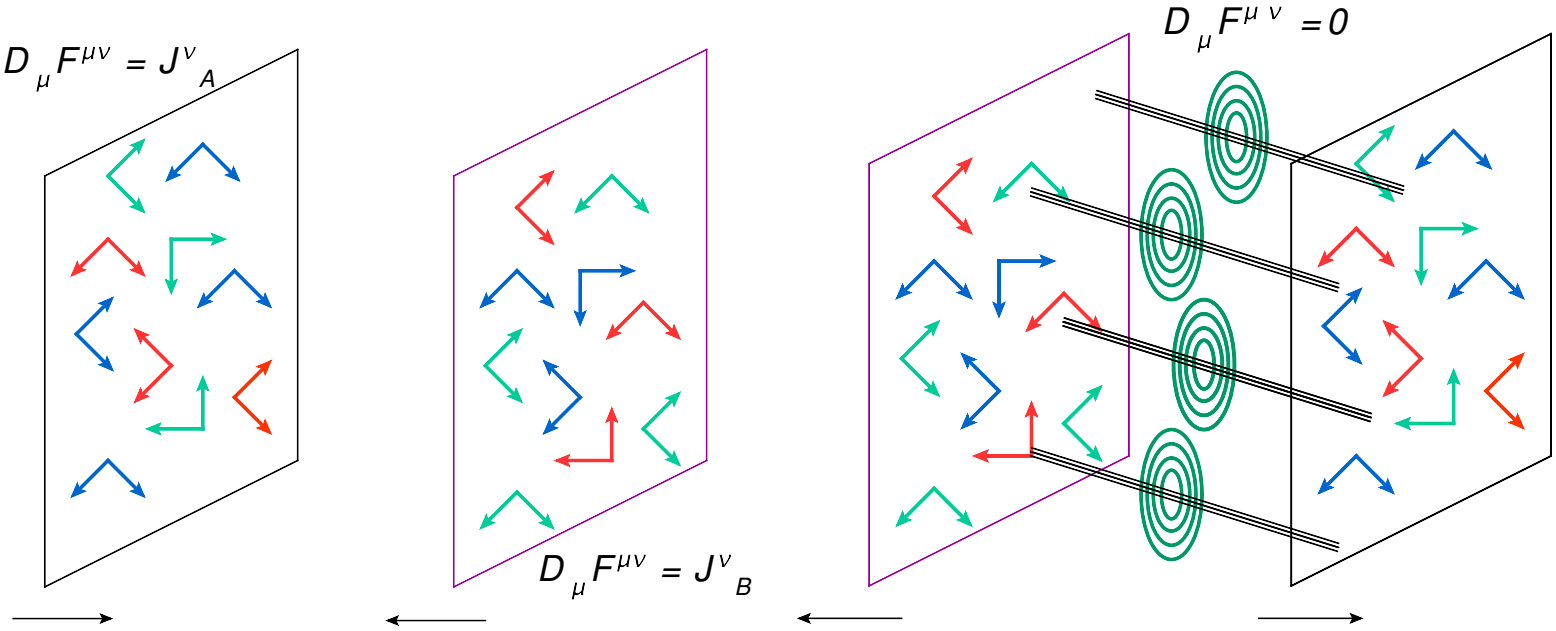
Schenke, Tribedy, Venugopalan PRL 108(2012)

Glasma : CYM evolution after collision

The field after collision at $\tau = 0$ has simple relation

$$A^i = A^i_{(A)} + A^i_{(B)} \ , \ A^\eta = \frac{ig}{2}\left[A^i_{(A)}, A^i_{(B)}\right]$$

The fields are evolved at $\tau > 0$ according to $[D_\mu, F^{\mu\nu}] = 0$



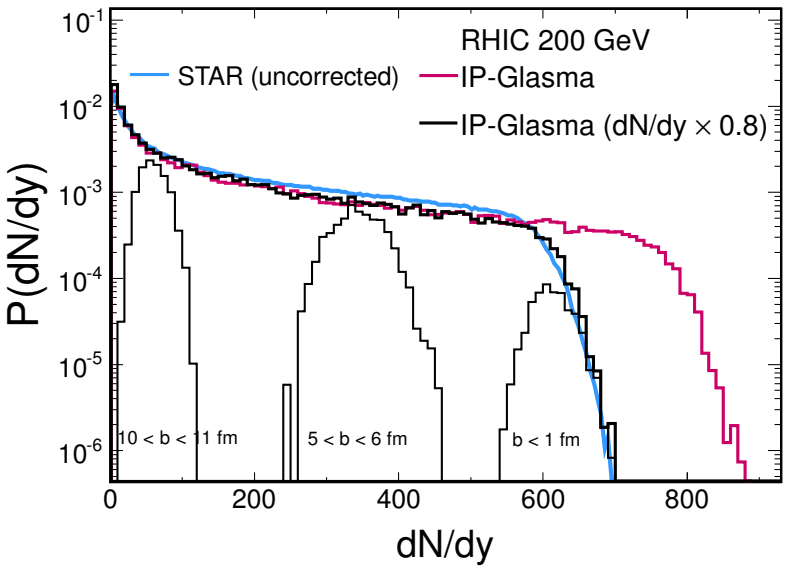
Multiplicity calculation done at $\tau \sim 1/Q_s$

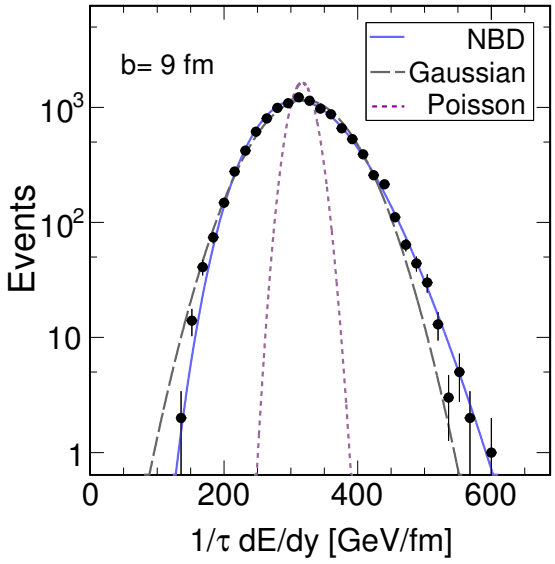
Schenke, Tribedy, Venugopalan PRC 86(2012)

Multiplicity and Energy density fluctuation for A+A collisions

Schenke, Tribedy, Venugopalan PRC 86(2012)

IP-Glasma naturally produces negative-binomial multiplicity distribution.



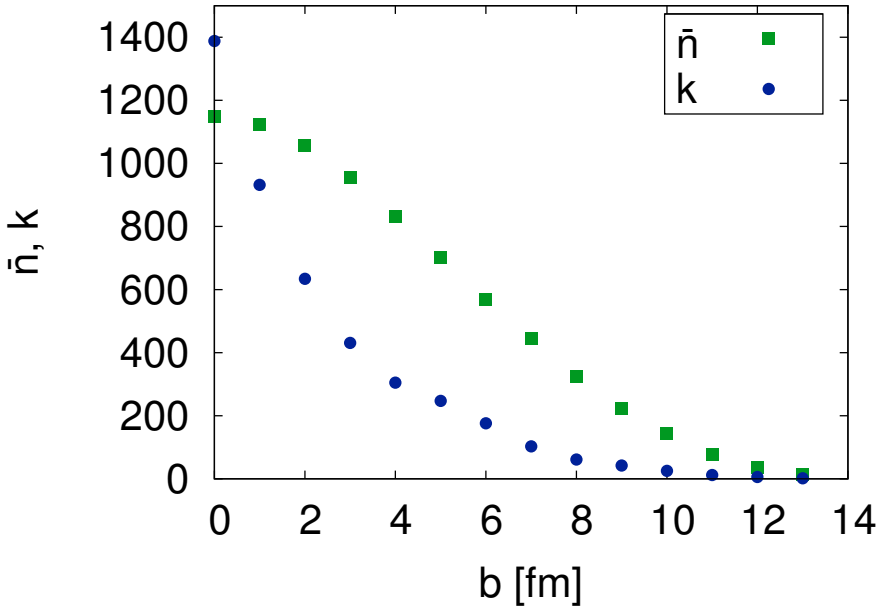


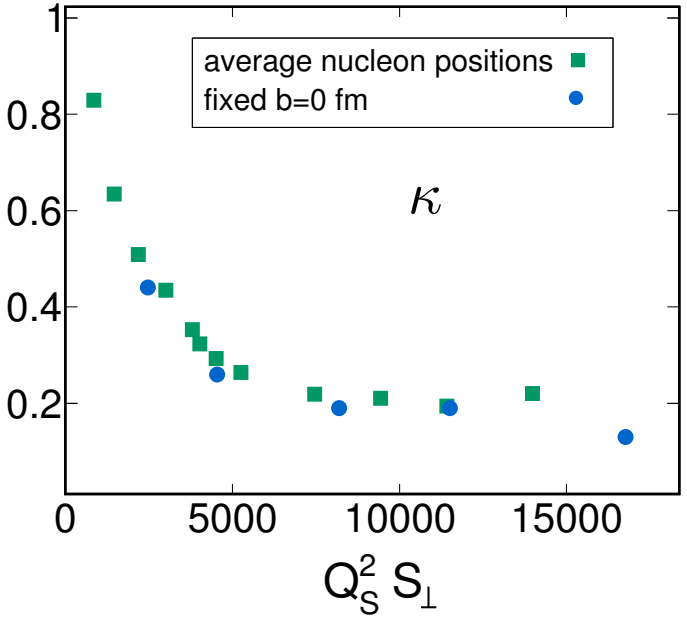
- NBD multiplicity distribution obtained for a given configuration of color charge distribution inside colliding nucleus.
- Color charge distribution changes with fluctuating nucleon positions and impact parameter.
- Min-bias distribution → convolution of many NBDs.

NBD parameters from IP-Glasma

Schenke, Tribedy, Venugopalan PRC 86(2012)

Parameters of NBD in A+A $\rightarrow$ consistent with perturbative approach.





At large $Q_s^2 S_\perp$ the result approaches to Glasma flux tube model for n-gluon correlations.
Glasma flux tube picture $\rightarrow$ non-perturbatively consistent.

Summary

- Glasma flux tube picture describes microscopic origin of NBD.
- Saturation model based calculation gives good description of experimental multiplicity distribution in p+p collisions.
- Event-by-event solutions of CYM naturally produces NBD.
- Distribution of transverse energy also follows NBD.
- IP-Glasma model validates the perturbative picture at large gluon densities.