Collins and Sivers effects and Drell-Yan production

The origin (A_N, $p\,p \to \pi\,X$)

more theory and data: SIDIS and TMDs

TMDs and the nucleon structure

TMDs and QCD: factorization, universality, back to A_N

Future experiments (D-Y)

M. Anselmino, Torino University & INFN, June 25, 2013

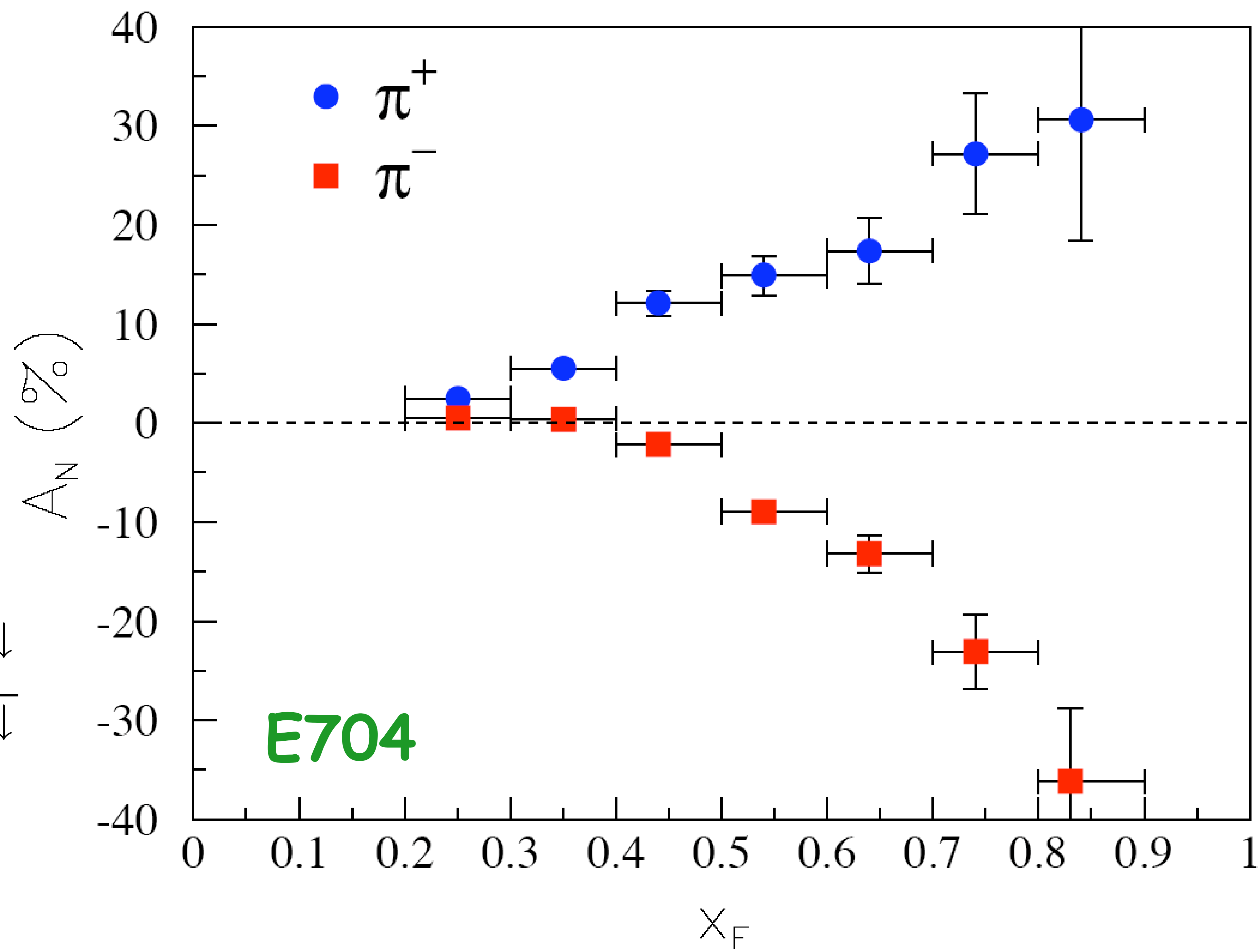
where it all started from ... (1991)

large P_T

$$p^{\uparrow}\, p \to \pi\, X$$

Single Spin Asymmetry

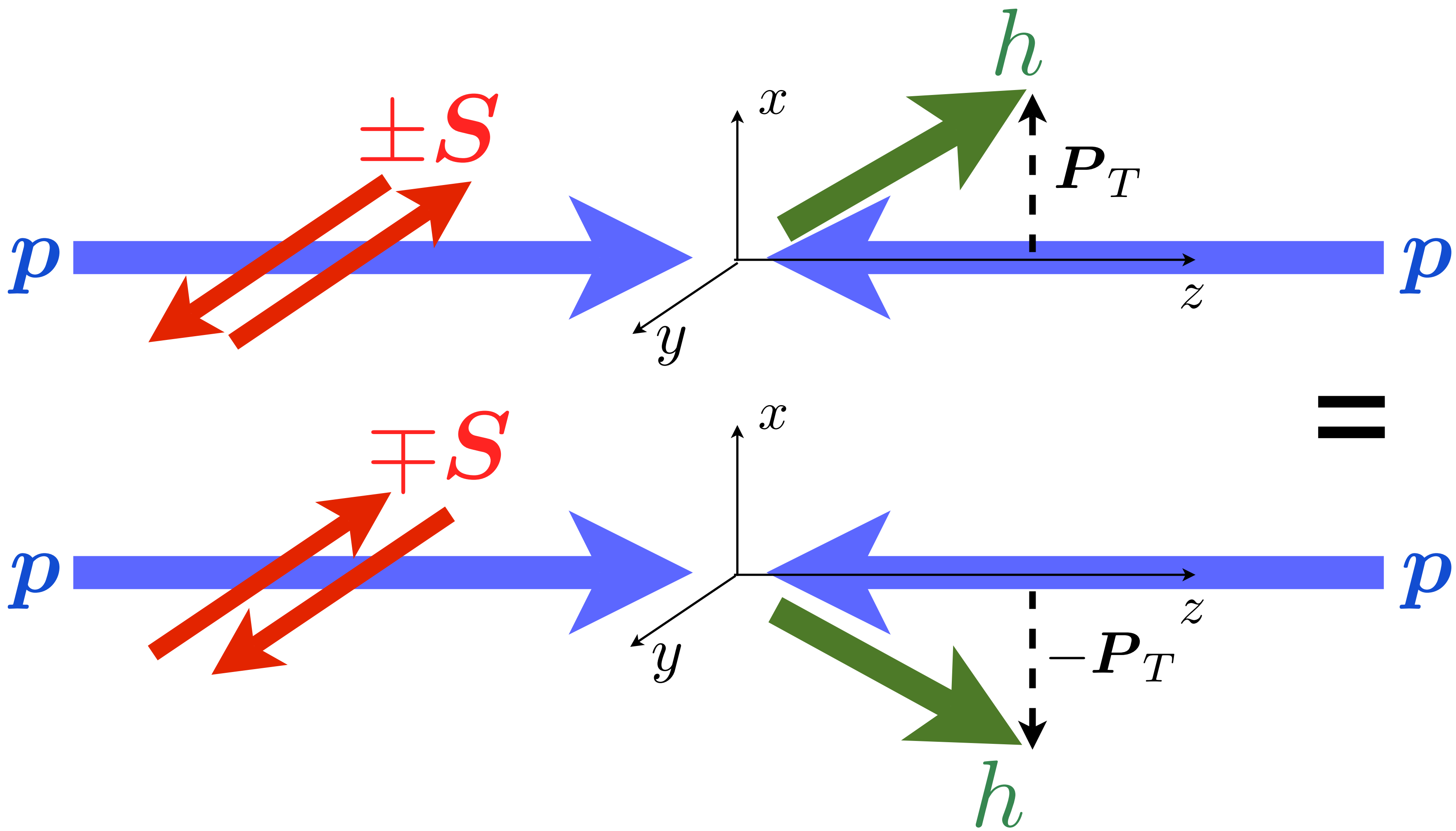
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



E704 $\sqrt{s}$ = 20 GeV 0.7 < p_T < 2.0

(some earlier measurements at smaller energy and p_T)

A_N = simple left-right asymmetry



$$A_N = \frac{d\sigma^{\uparrow}(\boldsymbol{P}_T) - d\sigma^{\downarrow}(\boldsymbol{P}_T)}{d\sigma^{\uparrow}(\boldsymbol{P}_T) + d\sigma^{\downarrow}(\boldsymbol{P}_T)} = \frac{d\sigma^{\uparrow}(\boldsymbol{P}_T) - d\sigma^{\uparrow}(-\boldsymbol{P}_T)}{2\, d\sigma^{\mathrm{unp}}(P_T)}$$

$$A_N \equiv \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{P}_T) \propto \sin\theta$$

transverse Single Spin Asymmetry (SSA) $\pm \boldsymbol{S} = \uparrow, \downarrow$

The birth of TMDs: D. Sivers

PRD 41 (1990) 83

$$G_{a/p}(x;\mu^2) \to G_{a/p}(x,\boldsymbol{k}_T;\mu^2)$$

The relevance of the transverse momentum for the asymmetry can be seen from the venerable Chou-Yang[1] model of the constituent structure of a transversely polarized proton. If we assume a correlation between the spin of the proton and the orbital motion of its constituents, Chou and Yang showed the existence of a nontrivial A_N in elastic scattering. *The coherent dynamics which correlates the spin of the proton with the orbital angular momentum of the quarks and gluons can also produce a constituent-level asymmetry in transverse momentum:*

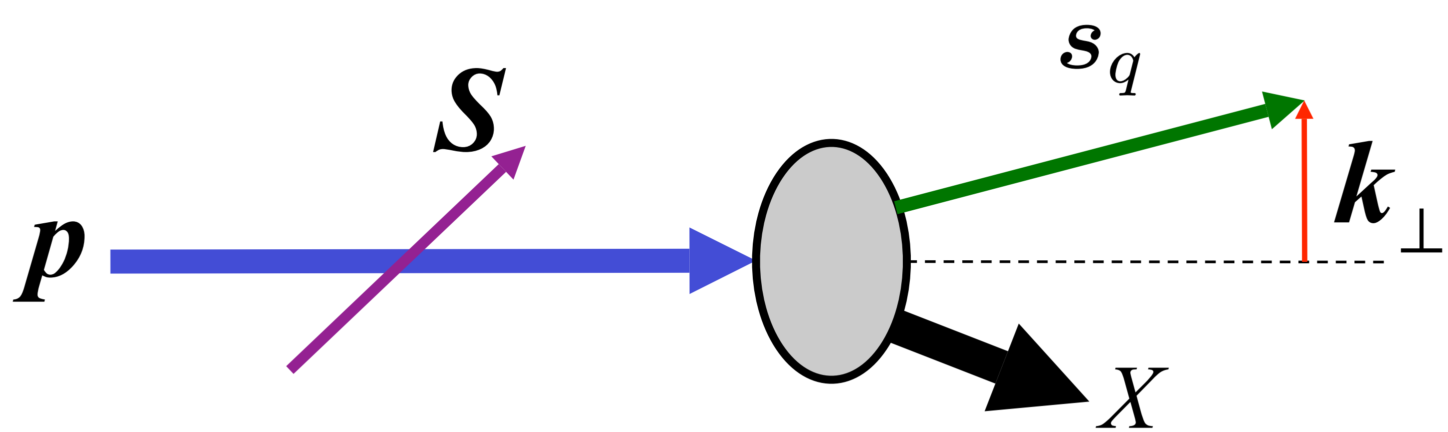
$$\Delta^N G_{a/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) = \sum_h \left[G_{a(h)/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) - G_{a(h)/p(\downarrow)}(x,\boldsymbol{k}_T;\mu^2)\right]$$

$$= \sum_h \left[G_{a(h)/p(\uparrow)}(x,\boldsymbol{k}_T;\mu^2) - G_{a(h)/p(\uparrow)}(x,-\boldsymbol{k}_T;\mu^2)\right]$$

1 T. T. Chou and C. N. Yang, Nucl. Phys. B107, 1 (1976)

$$A_N\left[E\frac{d^3\sigma}{d^3p}(pp_\uparrow\to mX)\right]\simeq\sum_{ab\to cd}\int d^2\boldsymbol{k}_T^a\,dx_a\int d^2\boldsymbol{k}_T^b\,dx_b\int d^2\boldsymbol{k}_{TC}\,\frac{dx_c}{x_c^2}\Delta^N G_{a/p_\uparrow}(x_a,k_T^a;\mu^2)$$
$$\times\,G_{b/p}(x_b,k_T^b;\mu^2)\,D_{m/c}(x_c,k_T^c:\mu^2)\times\tilde{s}\,\frac{d\sigma}{d\tilde{t}}(ab\to cd)\,\delta(\tilde{s}+\tilde{t}+\tilde{u})$$

.... this equation corresponds to a probabilistic formula in the original spirit of the parton model where the unknown soft nonperturbative dynamics have been absorbed into the specification of the density $\Delta^N G$ ...

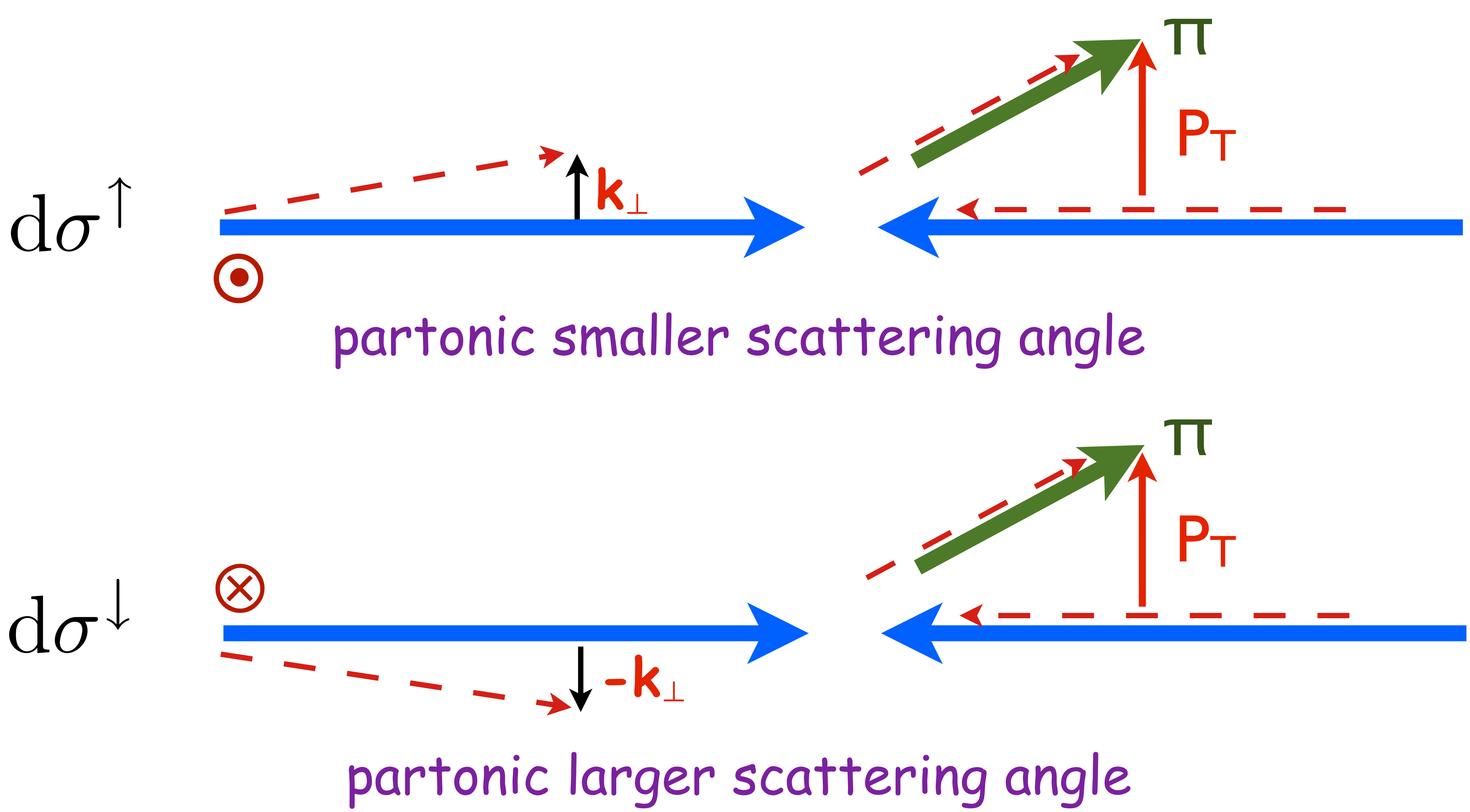


Sivers function

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp)=f_{q/p}(x,k_\perp)+\frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$=f_{q/p}(x,k_\perp)-\frac{k_\perp}{M}f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

simple physical picture for Sivers effect

(correlation between **S** and $\mathbf{k}_\perp$)

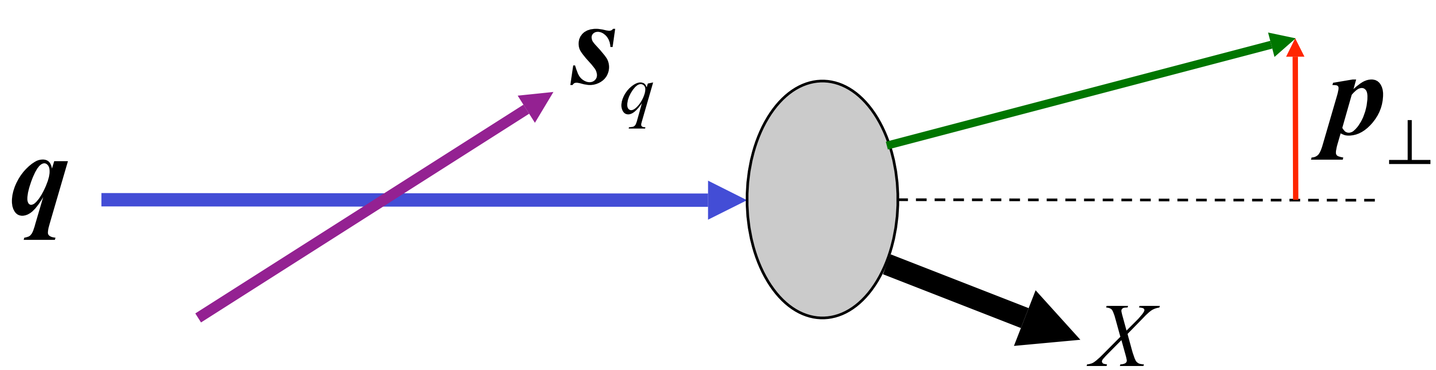


$$d\sigma^\uparrow \neq d\sigma^\downarrow$$

Collins fragmentation function

Nucl. Phys. B396 (1993) 161

It is shown that the azimuthal dependence of the distribution of hadrons in a quark jet is a probe of the transverse spin of the quark initiating the jet. This results in a new spin-dependent fragmentation function that acts at the twist-2 level.



Collins function

$$
\begin{aligned}
D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) &= D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp) \\
&= D_{h/q}(z,p_\perp) + \frac{p_\perp}{zM_h}H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)
\end{aligned}
$$

Collins, Nucl. Phys. B396 (1993) 161

It follows from the parity and time-reversal invariance of QCD that the number density of quarks is independent of the spin state of the initial hadron, so that we have

$$\hat{f}_{a/A}(x, |k_\perp|) \equiv \int \frac{\mathrm{d}y^- \, \mathrm{d}^2 y_\perp}{(2\pi)^3} \mathrm{e}^{-ixp^+y^- + ik_\perp \cdot y_\perp} \langle p| \, \bar{\psi}_i(0, y^-, y_\perp) \, \frac{\gamma^+}{2} \, \psi_i(0) \, |p\rangle$$

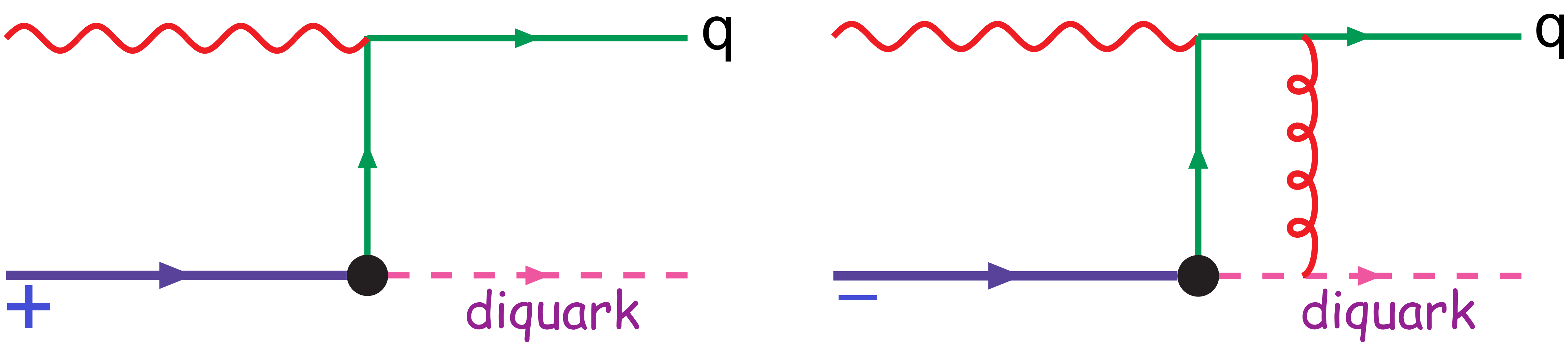
We have ignored here the subtleties needed to make this a gauge invariant definition: an appropriate path ordered exponential of the gluon field is needed [18].

Sivers suggested that the $k_\perp$ distribution of the quark could have an azimuthal asymmetry when the initial hadron has transverse polarization. However, such an asymmetry is prohibited because QCD is time-reversal invariant....

premature death of Sivers effect?

gauge links have physical consequences; quark models for non vanishing Sivers function,

SIDIS final state interactions



Brodsky, Hwang, Schmidt, PL B530 (2002) 99 - Collins, PL B536 (2002) 43

An earlier proof that the Sivers asymmetry vanishes because of time-reversal invariance is invalidated by the path-ordered exponential of the gluon field in the operator definition of parton densities. Instead, the time-reversal argument shows that the Sivers asymmetry is reversed in sign in hadron-induced hard processes (e.g., Drell-Yan), thereby violating naive universality of parton densities. Previous phenomenology with time-reversal-odd parton densities is therefore validated.

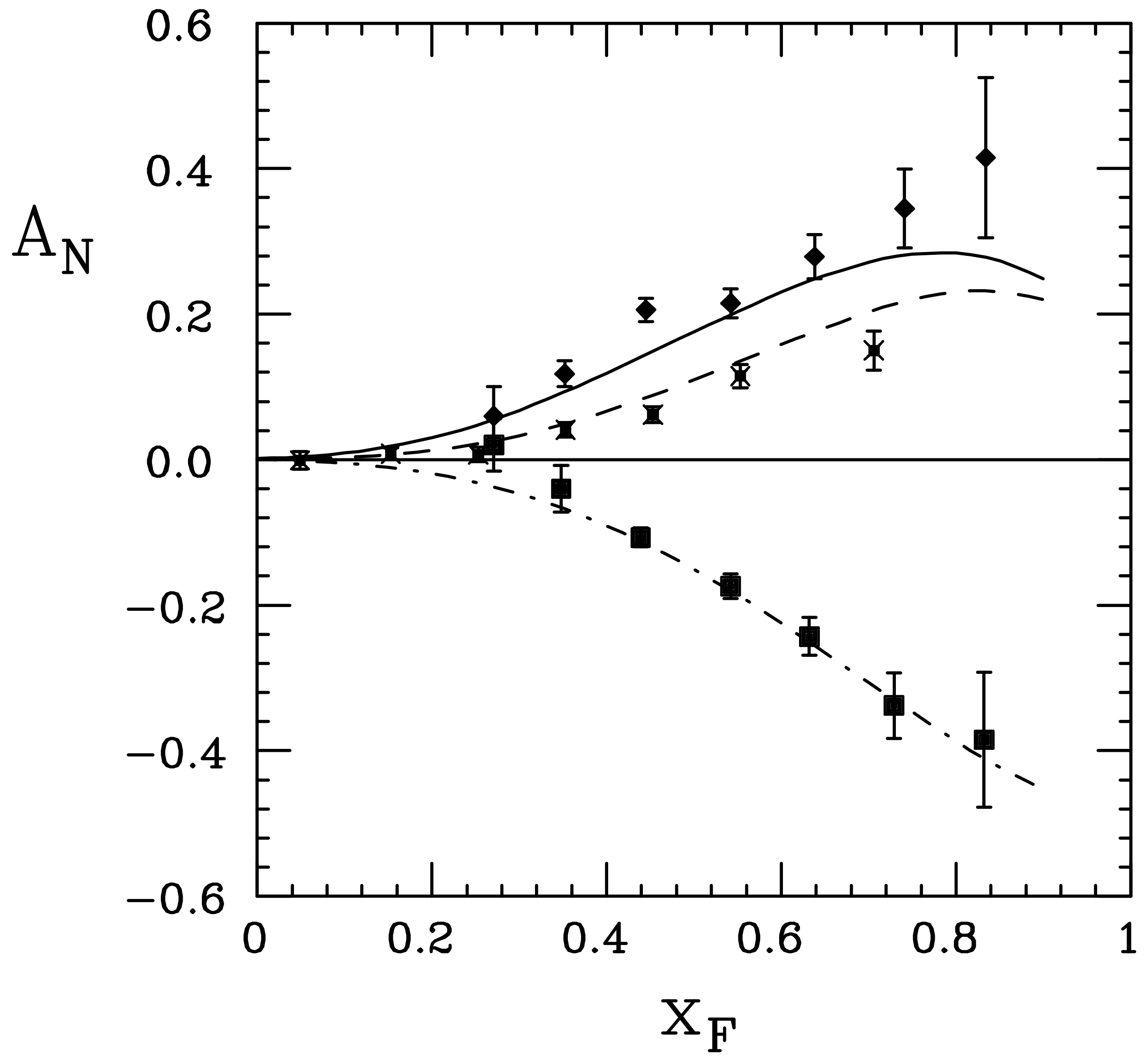
$$[f_{1T}^{q\perp}]_{\mathrm{SIDIS}} = -[f_{1T}^{q\perp}]_{\mathrm{DY}}$$

early A_N phenomenology with Sivers function

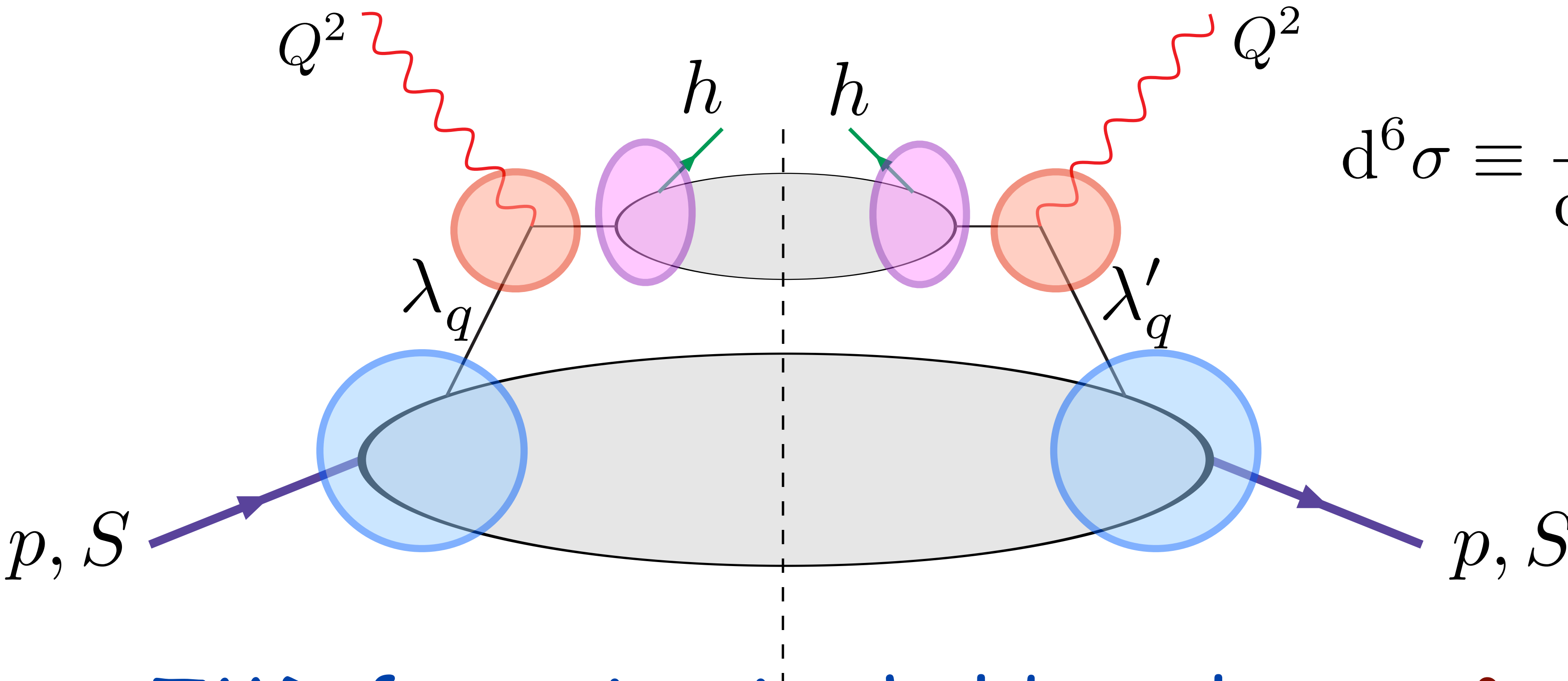
(M.A., M. Boglione and F. Murgia, PL B 362 (1995) 164)

$$\frac{E_\pi \, d\sigma^{p^\uparrow p \to \pi X}}{d^3\boldsymbol{p}_\pi} \sim \frac{1}{2} \sum_{a,b,c,d} \sum_{\lambda_a,\lambda'_a;\lambda_b;\lambda_c,\lambda'_c;\lambda_d} \int dx_a \, dx_b \, \frac{1}{z}$$

$$\rho^{a/p^\uparrow}_{\lambda_a,\lambda'_a} \, f_{a/p^\uparrow}(x_a) \, f_{b/p}(x_b) \, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda_b} \, D^{\lambda_c,\lambda'_c}_{\pi/c}(z)$$



theory: SSAs and TMDs in SIDIS



$$d^6\sigma \equiv \frac{d^6\sigma^{\ell p^\uparrow \to \ell h X}}{dx_B \, dQ^2 \, dz_h \, d^2\boldsymbol{P}_T \, d\phi_S}$$

$$\boldsymbol{p}_\perp \simeq \boldsymbol{P}_T - z_h \, \boldsymbol{k}_\perp$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

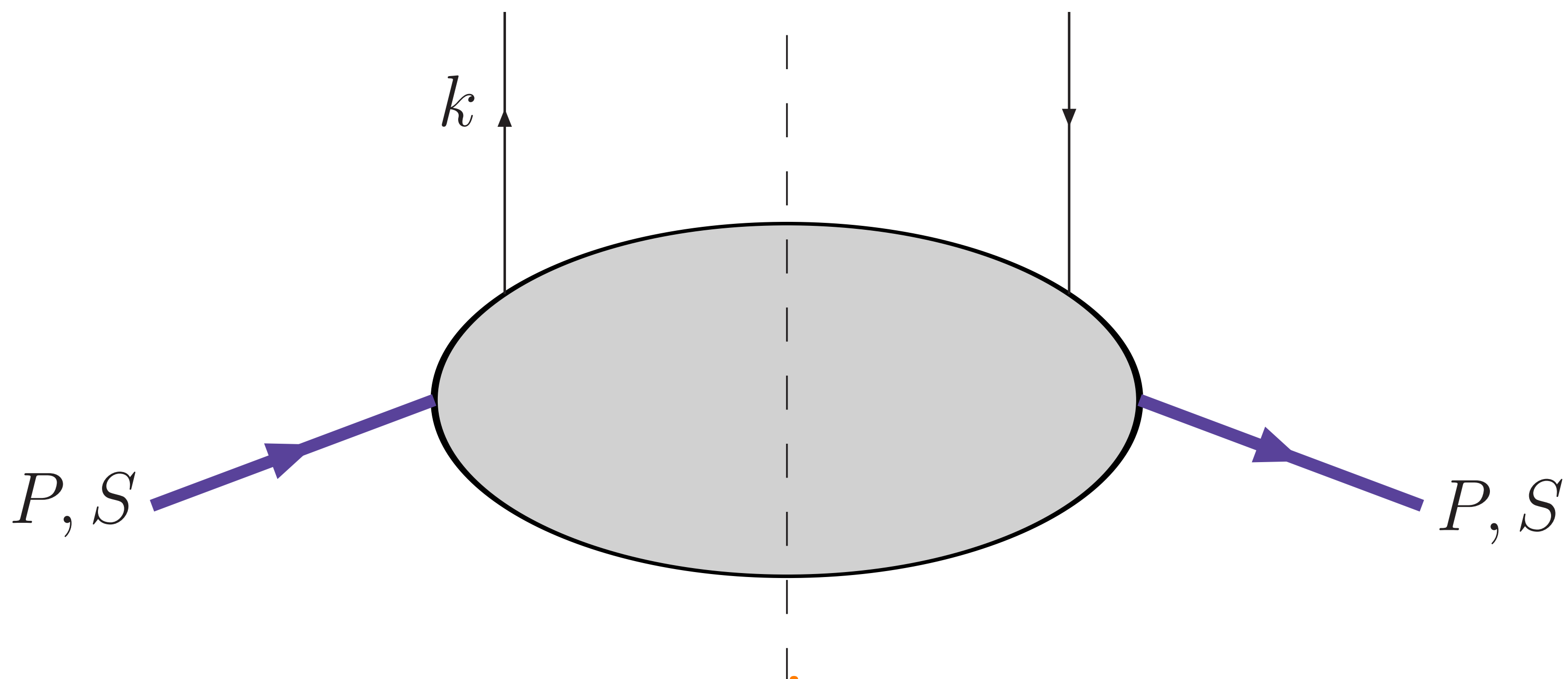
Two scales: $P_T \ll Q^2$

TMD-PDFs hard scattering TMD-FFs

$$d\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

The nucleon, as probed in DIS, in collinear configuration: 3 distribution functions



Correlator:

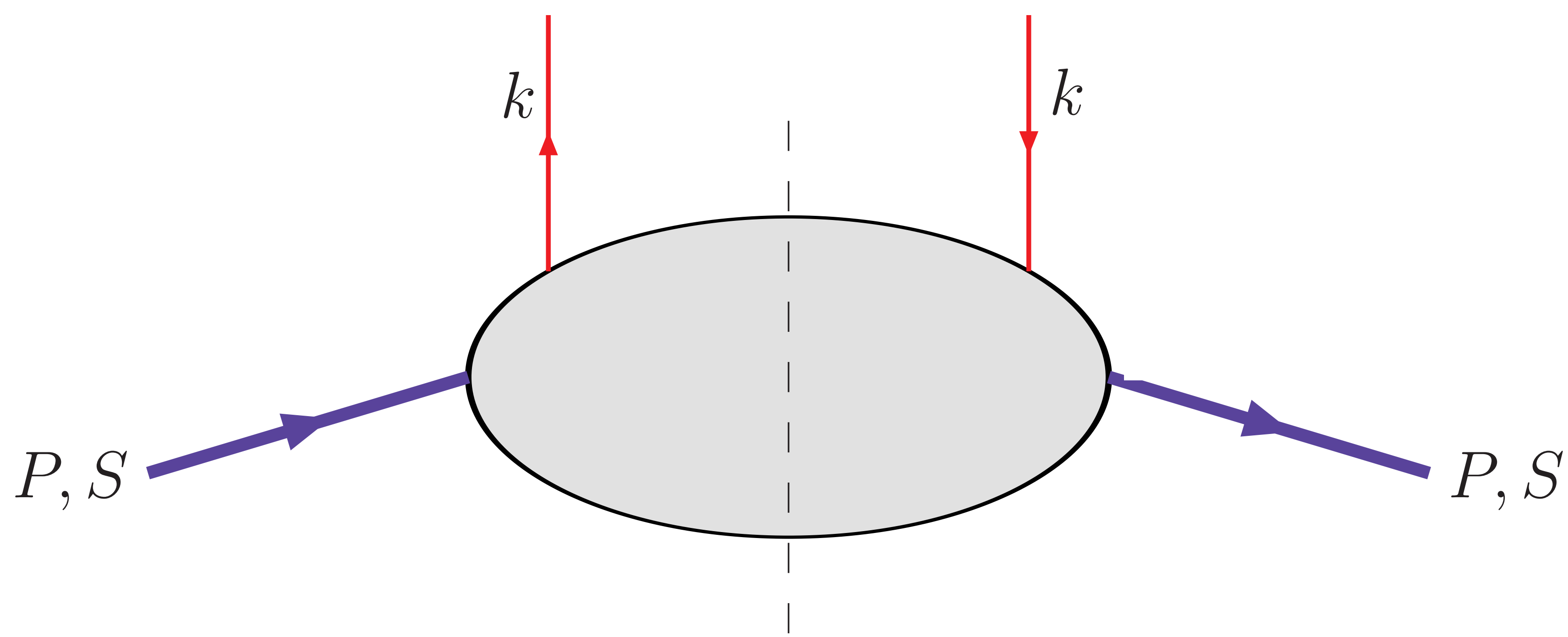
$$\Phi_{ij}(k;P,S) = \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X}\, (2\pi)^4\, \delta^4(P-k-P_X) \langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle$$

$$= \int \mathrm{d}^4\, \xi\, e^{ik\cdot\xi} \langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle$$

$$\Phi(x,S) = \frac{1}{2}\left[\underbrace{f_1(x)}_{q} \not{n}_+ + S_L \underbrace{g_{1L}(x)}_{\Delta q} \gamma^5 \not{n}_+ + \underbrace{h_{1T}}_{\Delta_T q} i\sigma_{\mu\nu}\gamma^5 n_+^{\mu} S_T^{\nu} \right]$$

TMD-PDFs: the leading-twist correlator, with intrinsic k⊥, contains 8 independent functions

$$\begin{aligned}\Phi(x,\boldsymbol{k}_\perp) \;=\;& \frac{1}{2}\Bigg[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\ &+ h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\ &+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Bigg]\end{aligned}$$



with partonic interpretation

not the complete story yet....

Sivers function universality - gauge links

$$\Phi_{ij}^{[U]}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P|\overline{\psi}_j(0)\,U_{[0,\xi]}\psi_i(\xi)|P\rangle\Big|_{\xi\cdot n=0}$$

$$\begin{aligned}\Phi^{[U]}(x,p_T;n) \;=\; & \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 \\ & + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + ih_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2}\end{aligned}$$

$$g_{1s}^{[U]}(x,p_T) = S_L g_{1L}^{[U]}(x,p_T^2) - \frac{p_T\cdot S_T}{M}g_{1T}^{[U]}(x,p_T^2)$$

factorization of TMD moments in:
(universal TMDs) x (process dependent factors)

$$f_{1T}^{\perp(1)[U]}(x) = C_G^{[U]}\,f_{1T}^{\perp(1)}(x)$$

Buffing, Mukherjee, Mulders, PRD 86 (2012) 074030

there are 8 independent TMD-PDFs

(partonic structure of the nucleon in momentum space)

$f_1^q(x, \boldsymbol{k}_\perp^2)$ unpolarized quarks in unpolarized protons
unintegrated unpolarized distribution

$g_{1L}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_L of quark with S_L of proton
unintegrated helicity distribution

$h_{1T}^q(x, \boldsymbol{k}_\perp^2)$ correlate s_T of quark with S_T of proton
unintegrated transversity distribution

only these survive in the collinear limit

$f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ of quark with S_T of proton (Sivers)

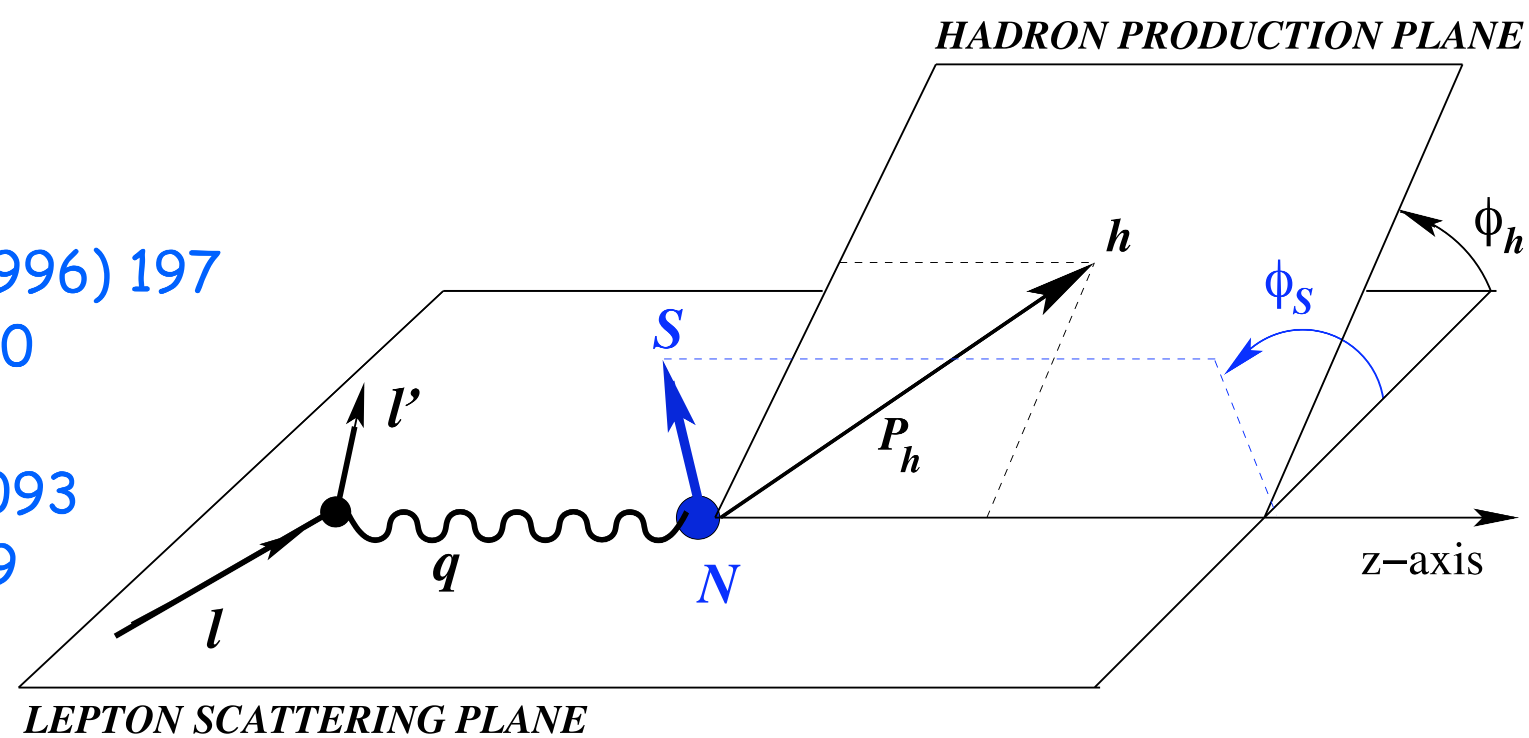
$h_1^{\perp q}(x, \boldsymbol{k}_\perp^2)$ correlate $k_\perp$ and s_T of quark (Boer-Mulders)

$g_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ $\quad h_{1L}^{\perp q}(x, \boldsymbol{k}_\perp^2)$ $\quad h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$

different double-spin correlations

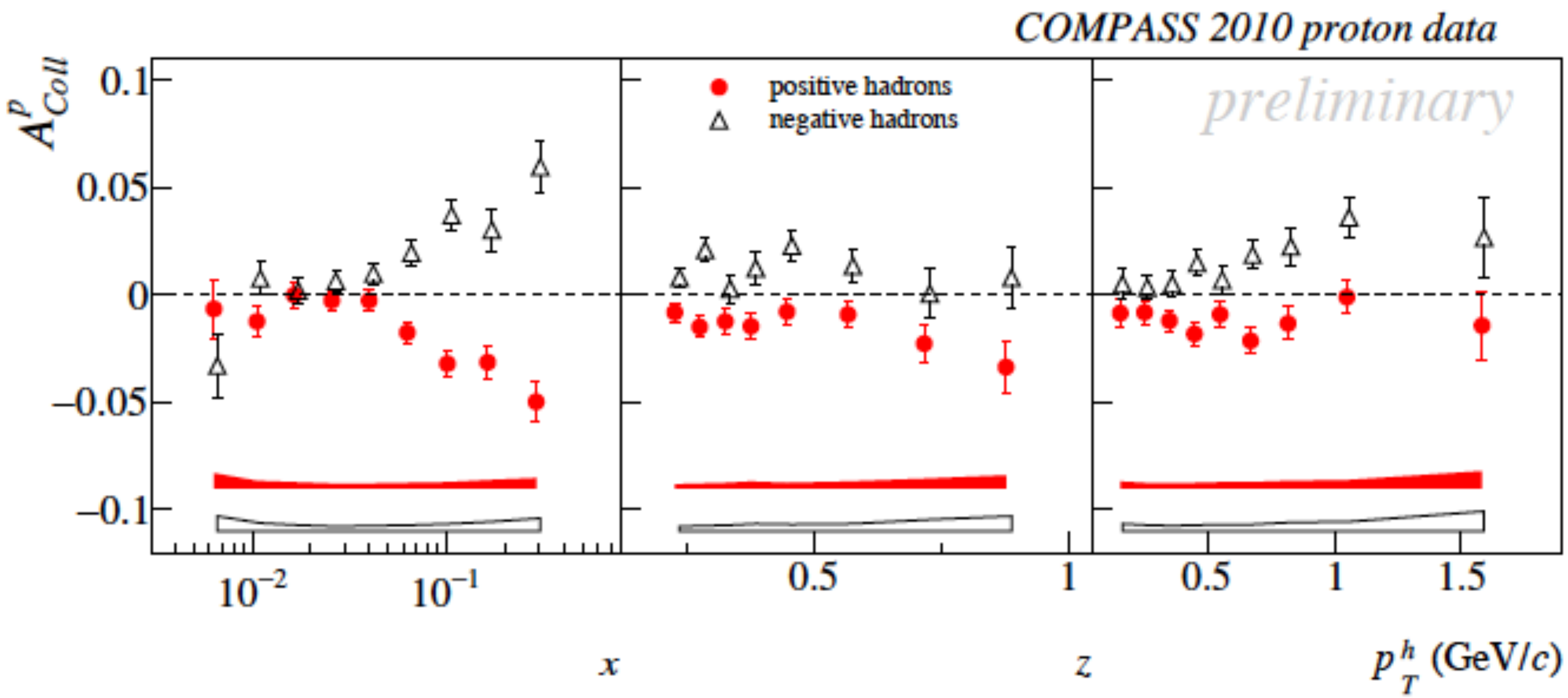
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = {} & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
& + S_T \Big\{ \underbrace{\sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)}}_{\text{Sivers}} + \underbrace{\sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)}}_{\text{Collins}} + \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \frac{1}{Q} \left[\sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
& + \lambda \left[\cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

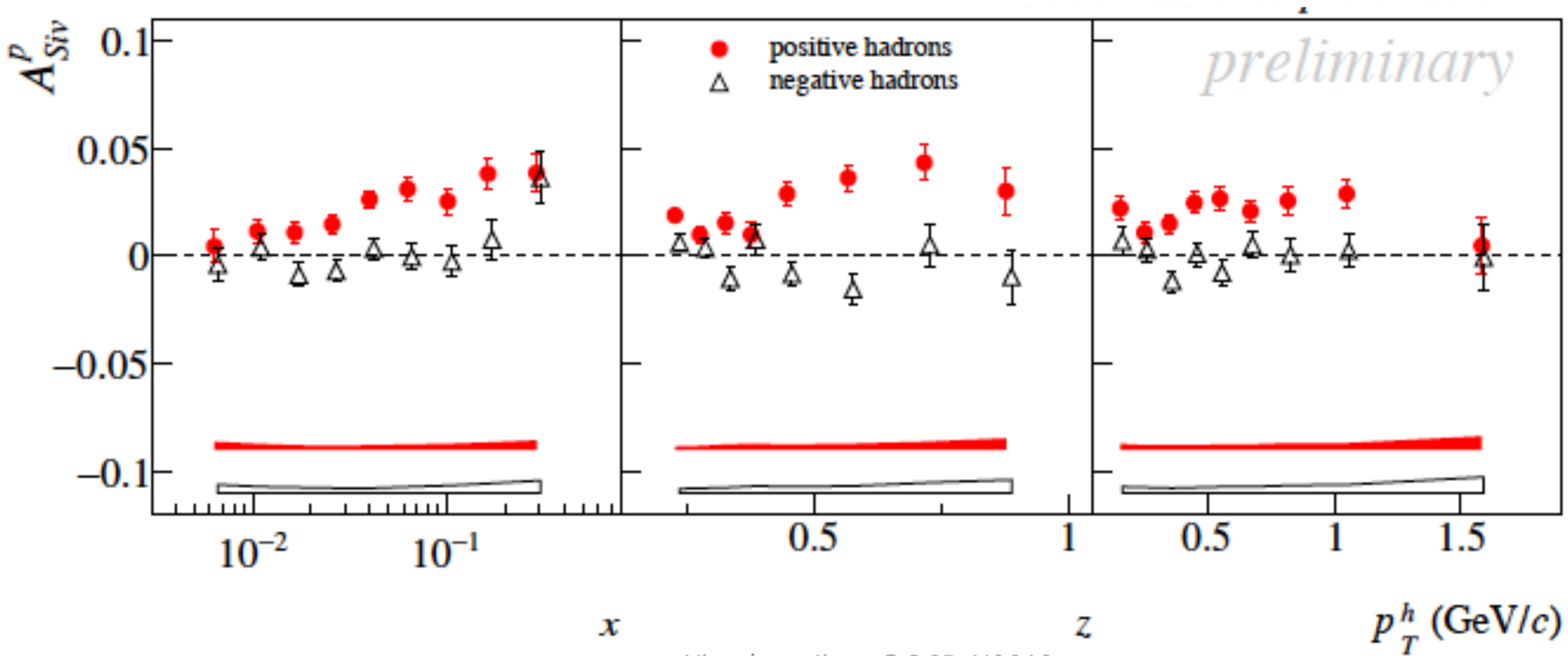
Kotzinian, **NP B441** (1995) 234
Mulders and Tangermann, **NP B461** (1996) 197
Boer and Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093
Anselmino et al., **PR D83** (2011) 114019



the $F^{(\cdots)}_{S_B\,S_T}$ contain the TMDs; plenty of Spin Asymmetries

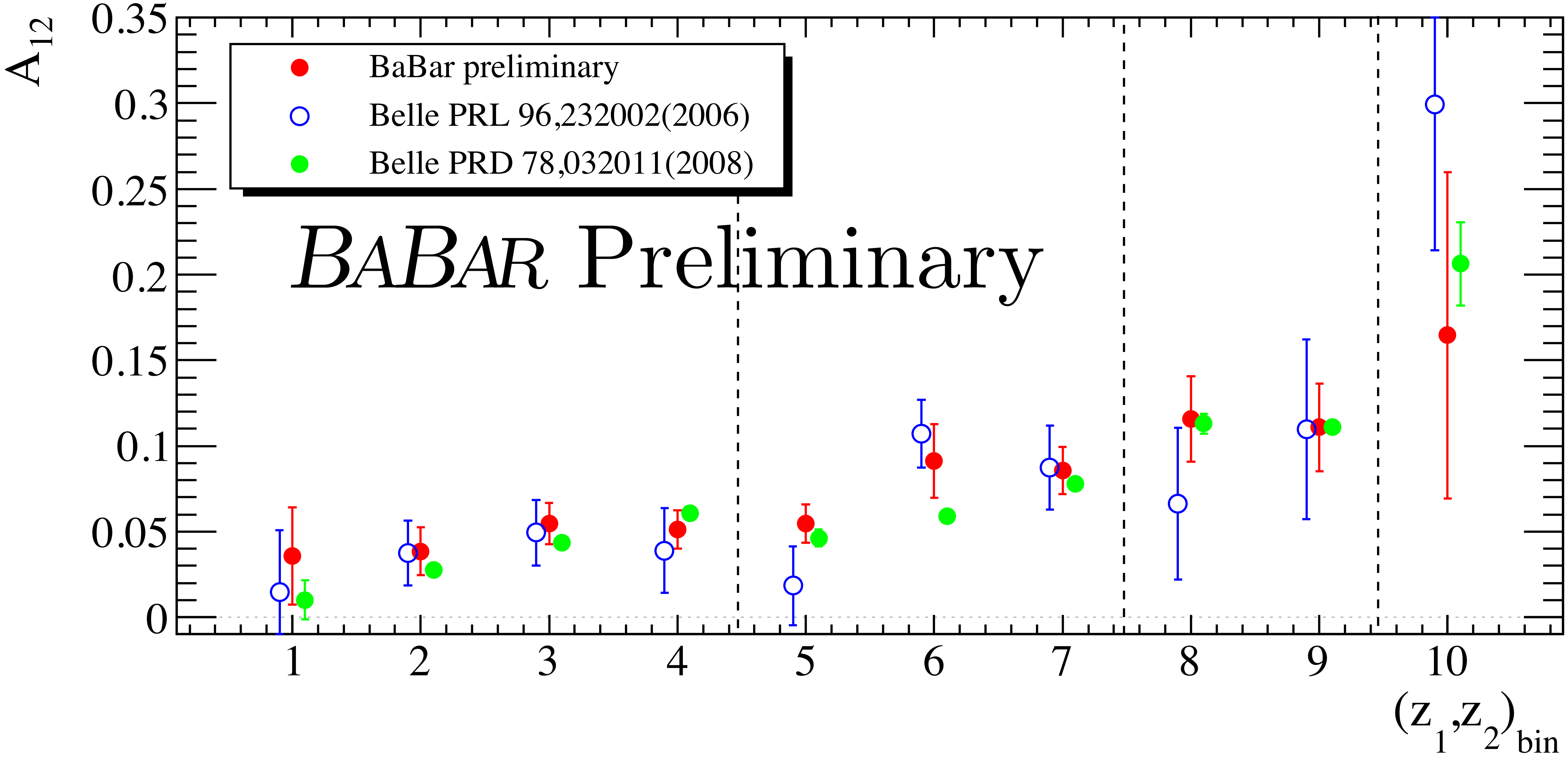
Clear evidence for Sivers and Collins effects from SIDIS data





independent evidence for Collins effect from e^+e^- data at Belle and BaBar

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^\uparrow}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$$

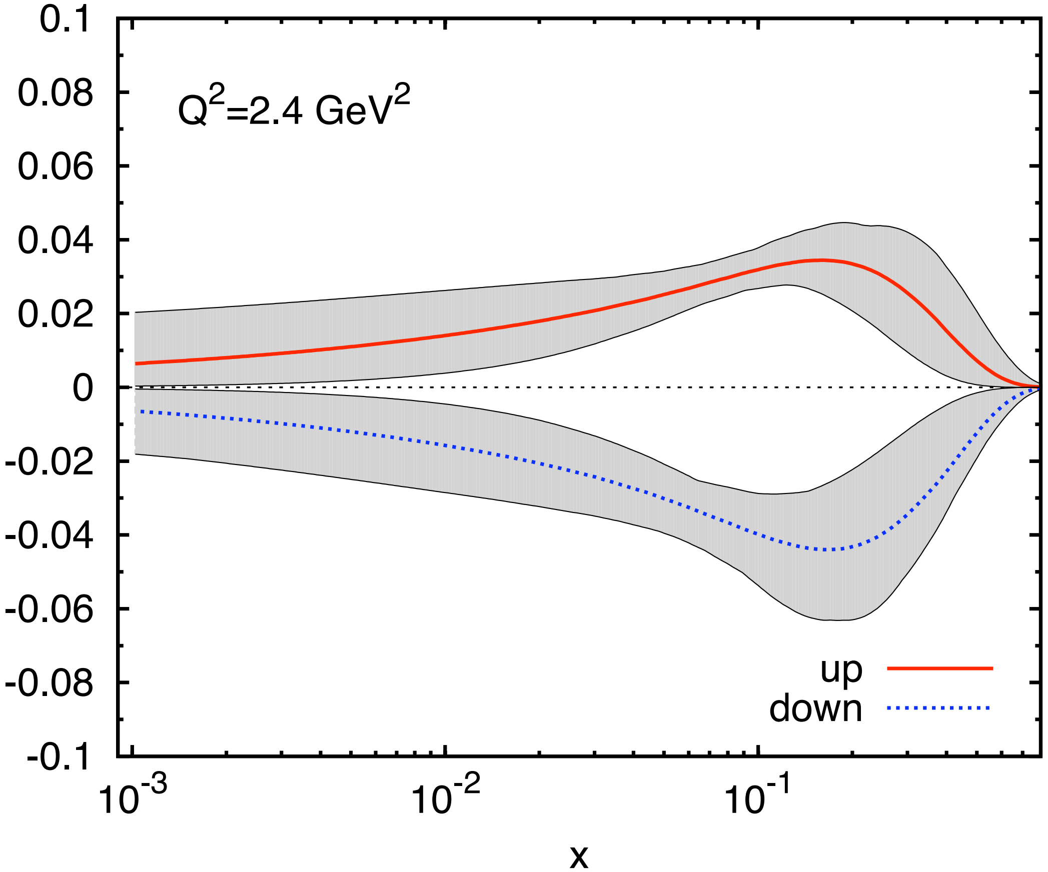


I. Garzia, arXiv:1201.4678

extraction of u and d Sivers functions - first phase

M.A, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin (agreement with several other groups)

$$x\,\Delta^N f_q^{(1)}(x,Q)$$



$$\Delta^N f_q^{(1)}(x,Q) = \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4M_p}\, \Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = -f_{1T}^{\perp(1)q}(x,Q)$$

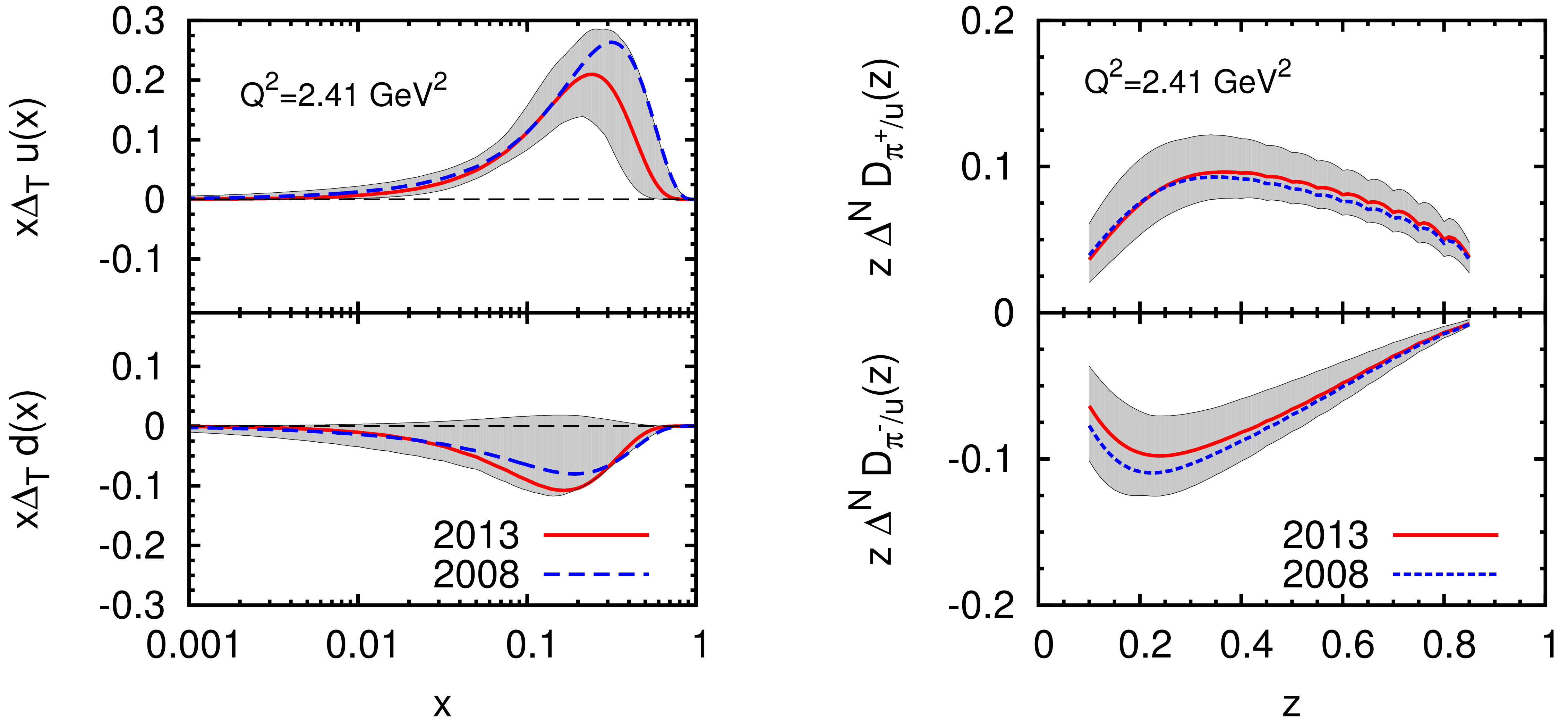
parameterization of the Sivers function:

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = 2\,\mathcal{N}(x)\,h(k_\perp)\,f_q(x,Q)\,\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Q^2 evolution only taken into account in the collinear part (usual PDF)

extraction of transversity and Collins functions

M. A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PRD 87 (2013) 094019



$$\Delta_T q(x,k_\perp) \;=\; \frac{1}{2}\,\mathcal{N}_q^T\,(x)\,[f_{q/p}(x)+\Delta q(x)]\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

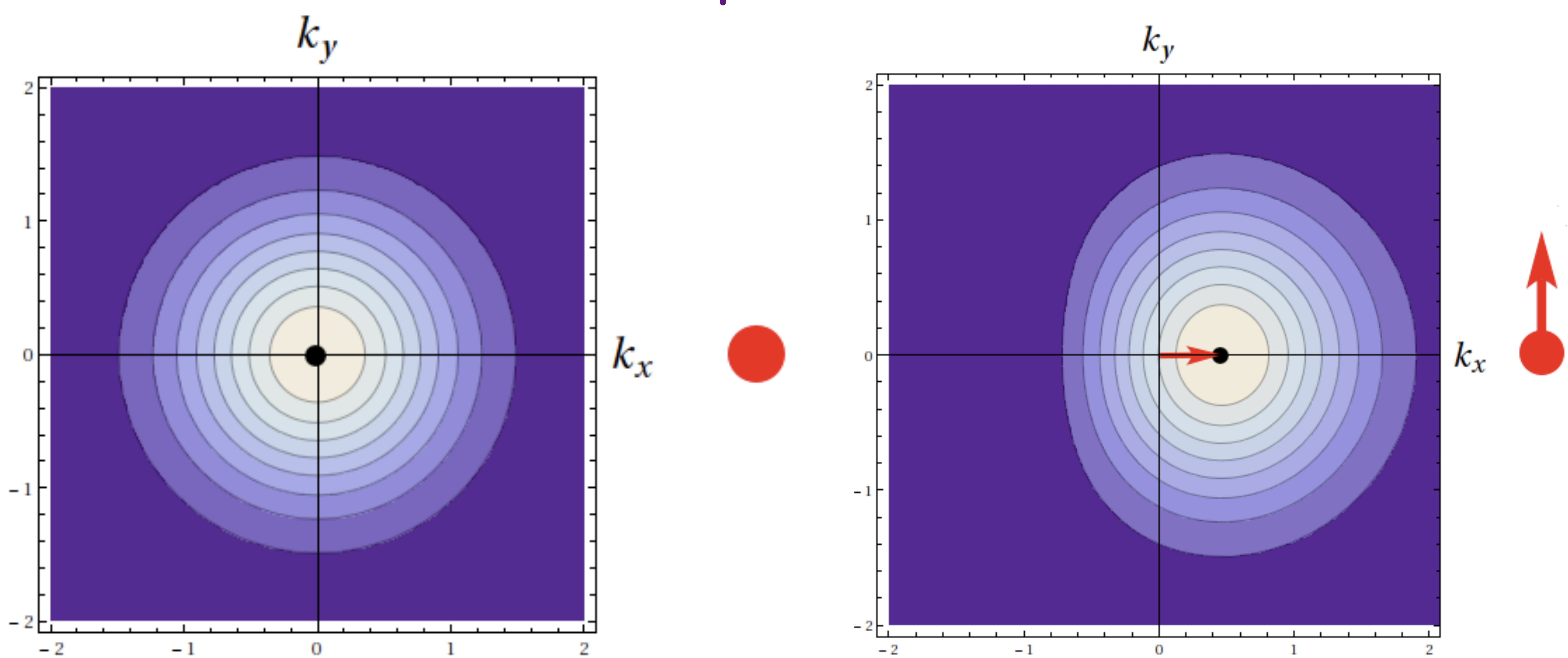
$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) \;=\; 2\,\mathcal{N}_q^C\,(z)\,D_{h/q}(z)\,h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

simple parameterization, no TMD evolution

what do we learn from the Sivers function?
dipole deformation

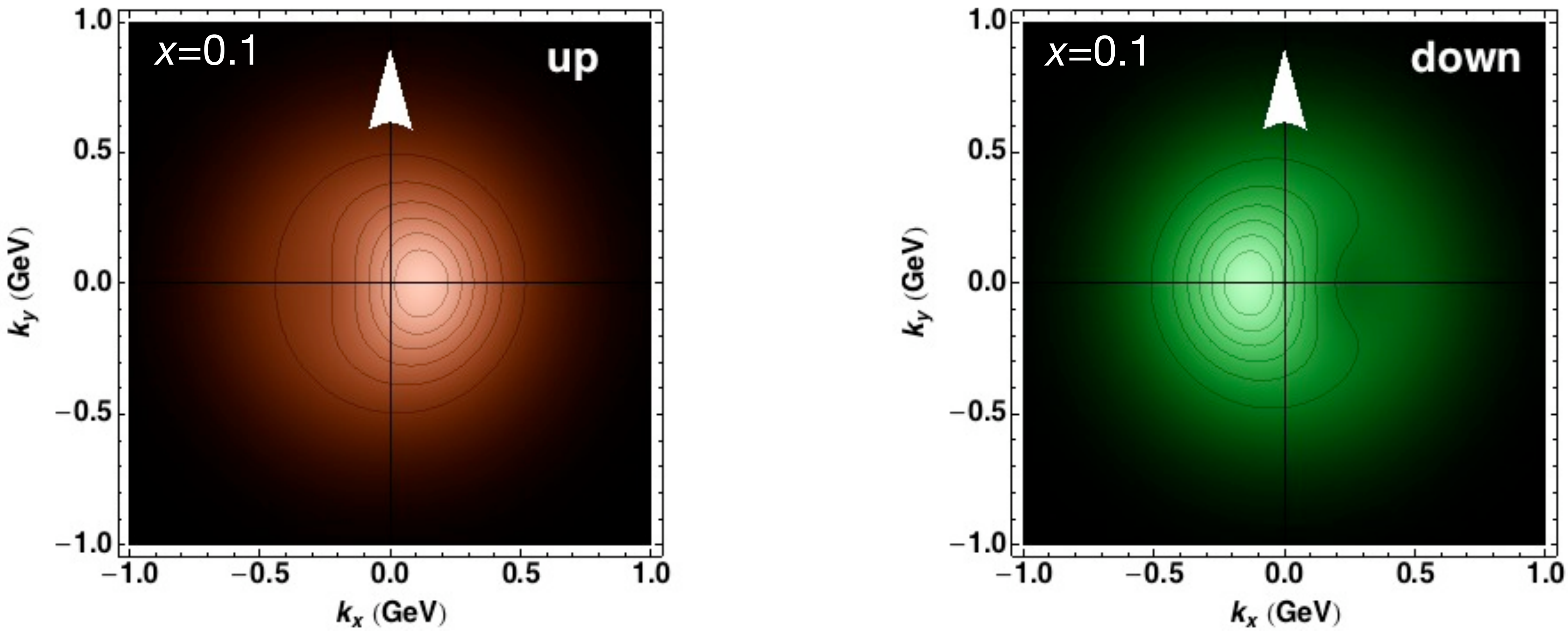
$$\widehat{f}_{q/p^\uparrow}(x, \boldsymbol{k}_\perp, S\,\hat{\boldsymbol{j}}; Q) = \widehat{f}_{q/p}(x, k_\perp; Q) - \widehat{f}_{1T}^{\perp q}(x, k_\perp; Q)\,\frac{k_\perp^x}{M_p}$$

$\boldsymbol{S} = 0$ u quark $\boldsymbol{S} = S\,\hat{\boldsymbol{j}}$



courtesy of Alexei Prokudin

transverse momentum distortion



courtesy of Alessandro Bacchetta

Sivers function and orbital angular momentum

Ji's sum rule

forward limit of GPDs

$$J^q = \frac{1}{2}\int_0^1 dx\, x\, [H^q(x,0,0) + E^q(x,0,0)]$$

usual PDF $q(x)$

cannot be measured directly

anomalous magnetic moments

$$\kappa^p = \int_0^1 \frac{dx}{3}\,\left[2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$\kappa^n = \int_0^1 \frac{dx}{3}\,\left[2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right]$$

$$(E^{q_v} = E^q - E^{\bar{q}})$$

Sivers function and orbital angular momentum

assume

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)E^a(x,0,0;Q_L^2)$$

$$f_{1T}^{\perp(0)a}(x,Q) = \int d^2\boldsymbol{k}_\perp\, \widehat{f}_{1T}^{\perp a}(x,k_\perp;Q)$$

L(x) = lensing function
(unknown, can be computed in models)

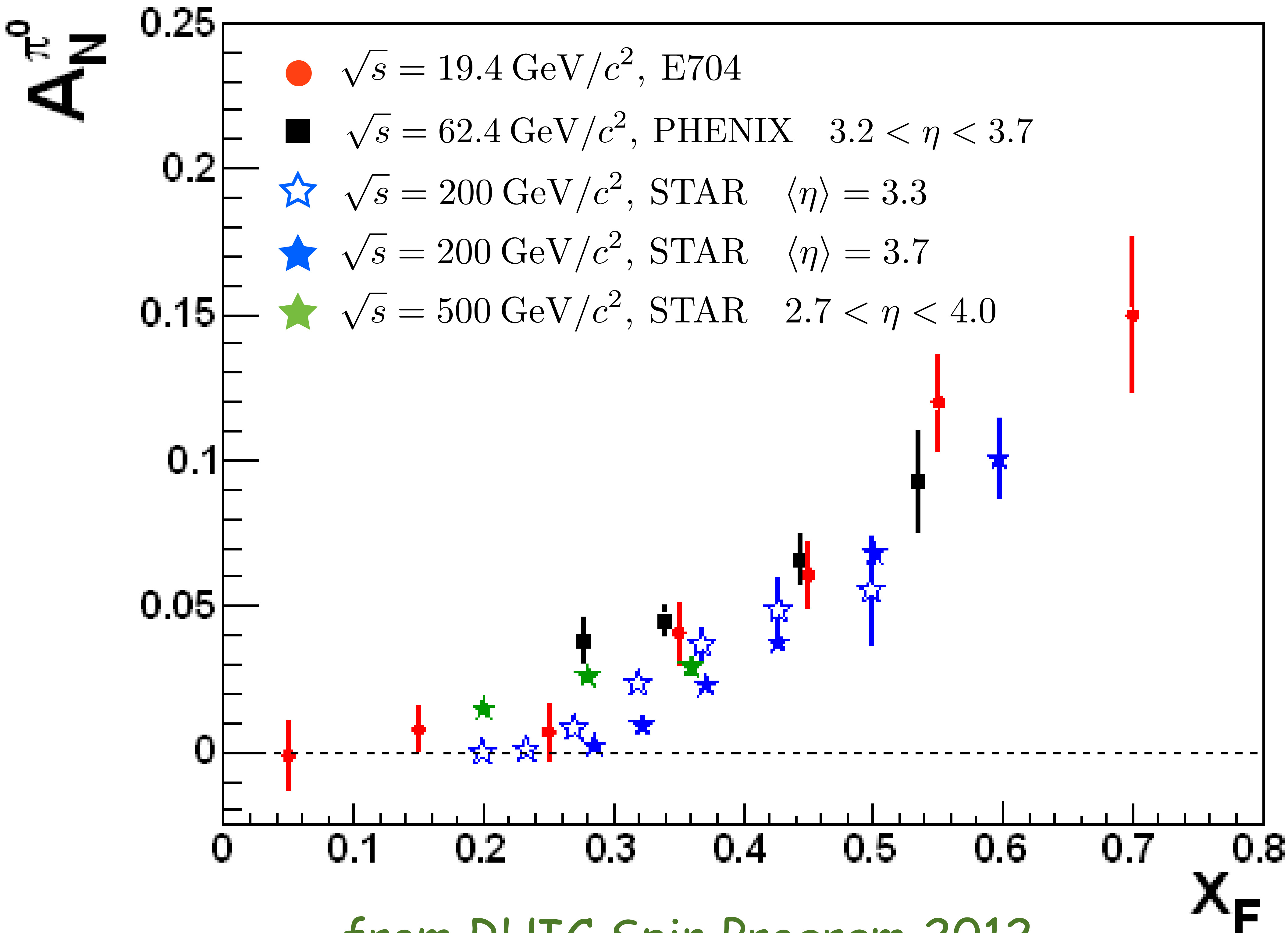
parameterize Sivers and lensing functions

fit SIDIS and magnetic moment data

obtain E^q and estimate orbital angular momentum

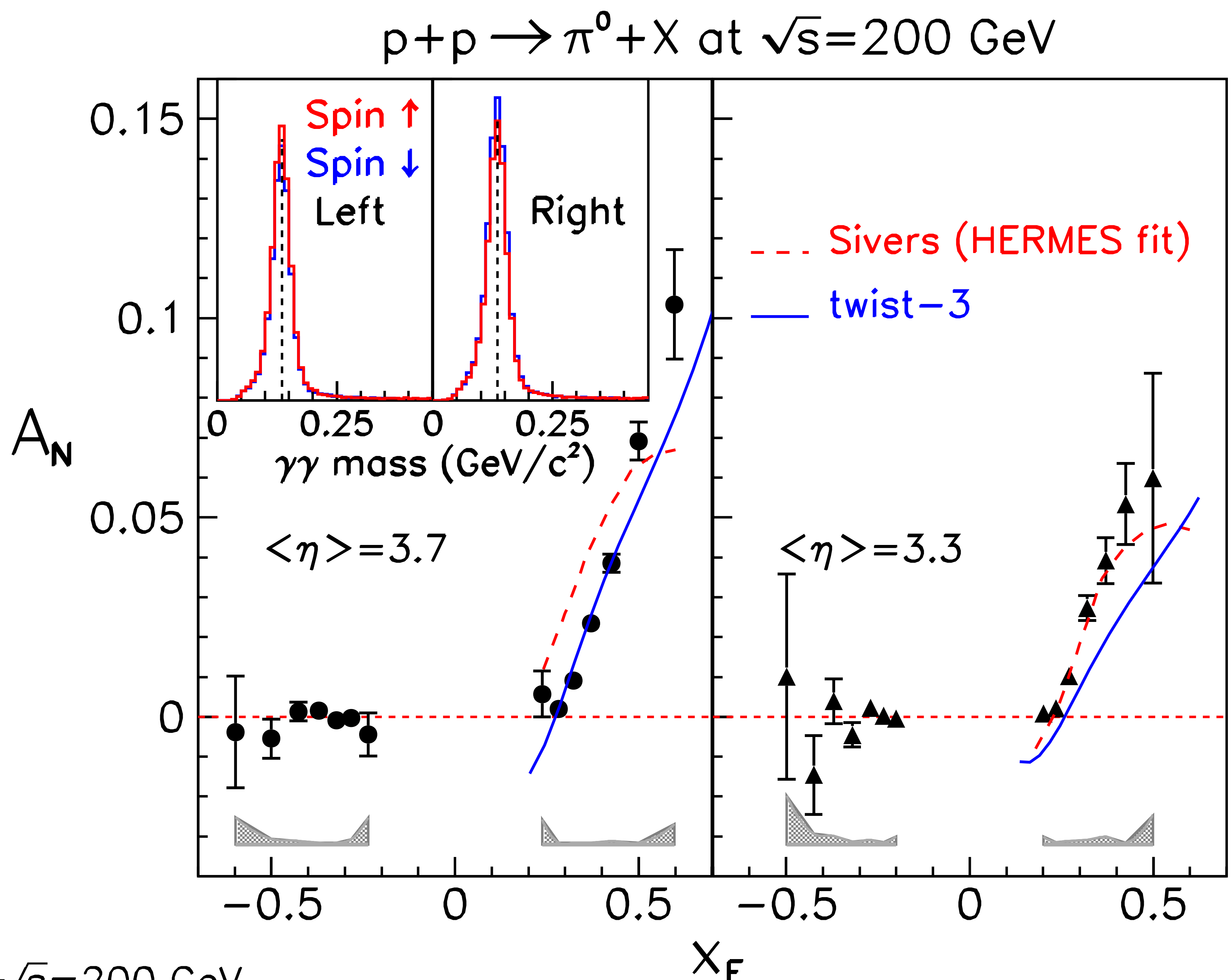
results at $Q^2 = 4$ GeV^2: $J^u \approx 0.23$, $J^{q\neq u} \approx 0$

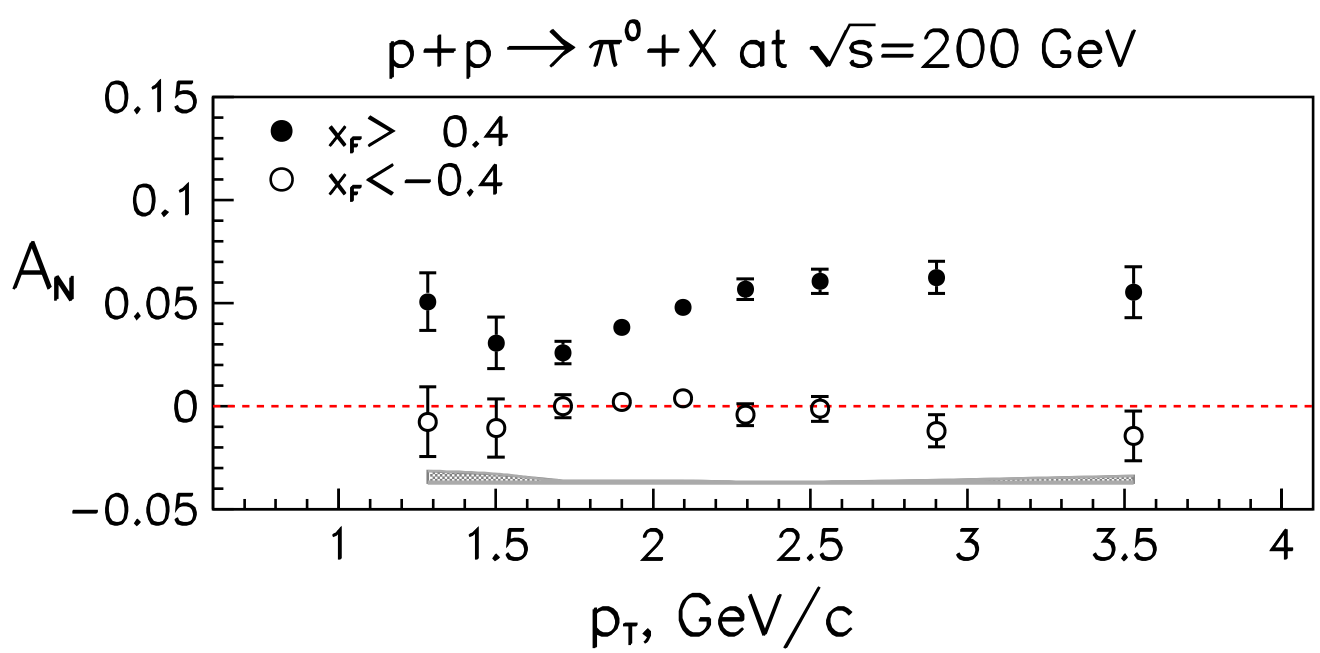
Bacchetta, Radici, PRL 107 (2011) 212001 meanwhile, what happened to A_N? it remained, of course ....



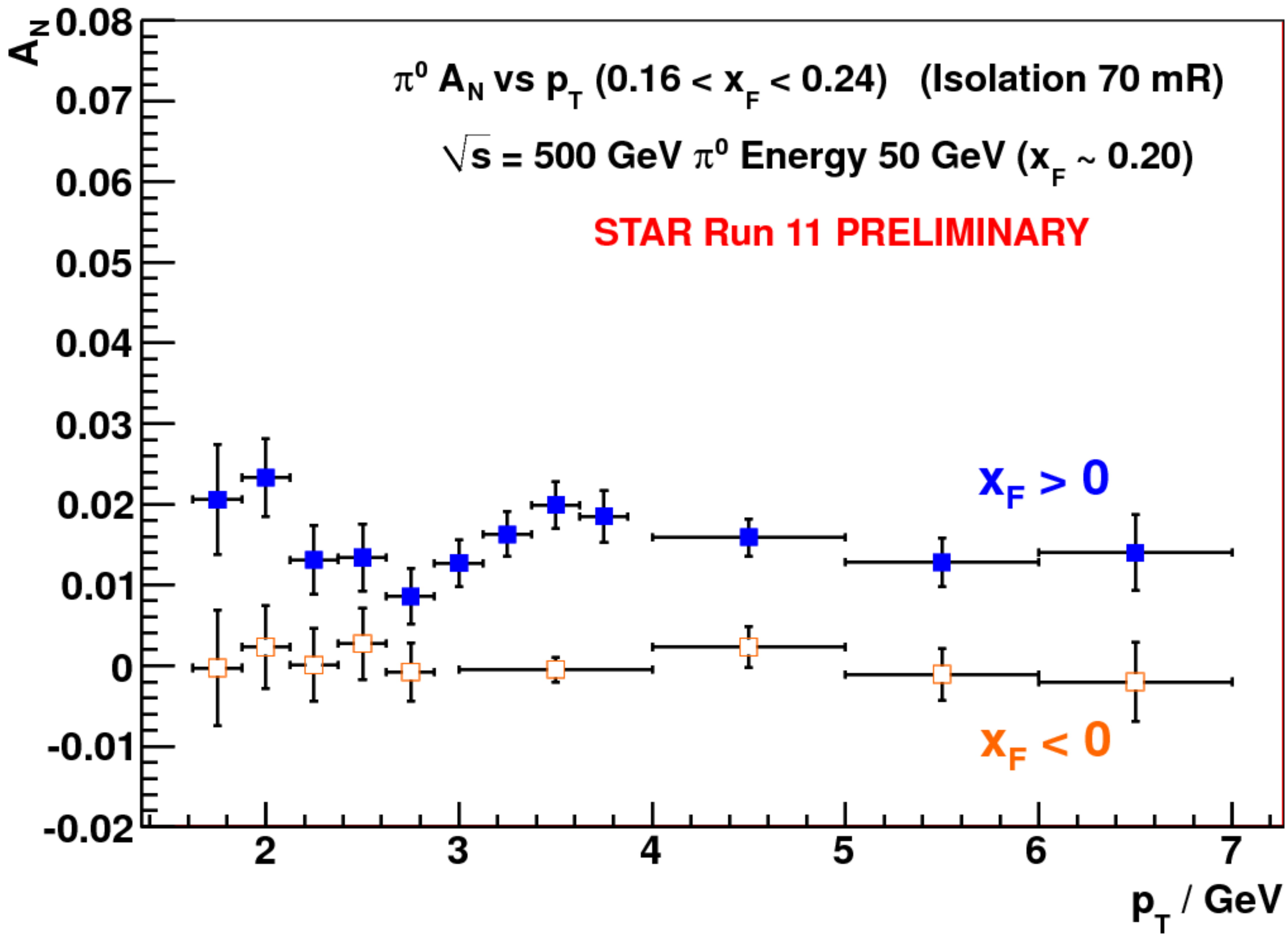
from RHIC Spin Program 2012

STAR data





PRL 101, 222001 (2008)

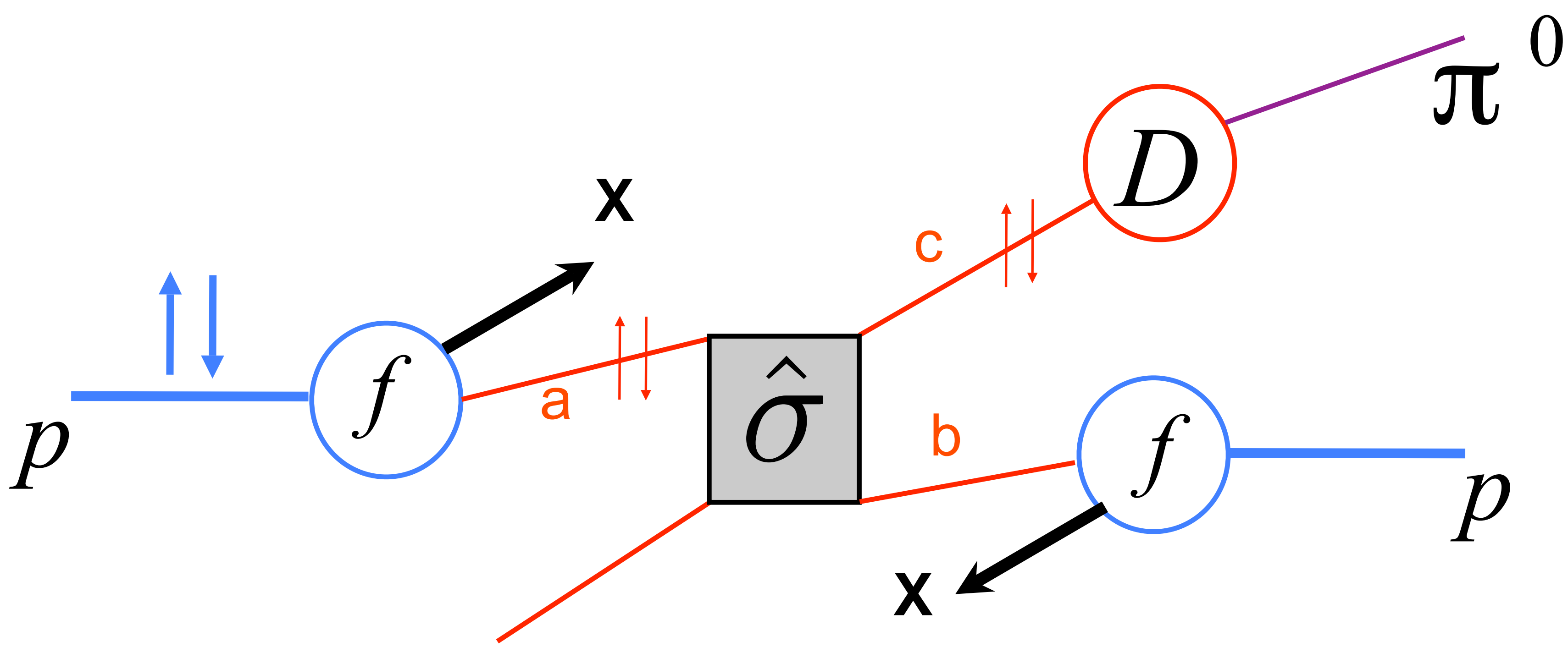


π^0 A_N vs p_T ($0.16 < x_F < 0.24$) (Isolation 70 mR)
$\sqrt{s}$ = 500 GeV π^0 Energy 50 GeV ($x_F \sim 0.20$)
STAR Run 11 PRELIMINARY
$x_F > 0$
$x_F < 0$
A_N
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
2
3
4
5
6
7
p_T / GeV

SSA in hadronic processes: TMDs, higher-twist correlations?

Two main different (?) approaches

1. Generalization of collinear scheme (assuming factorization)



$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes d\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

single spin effects in TMDs

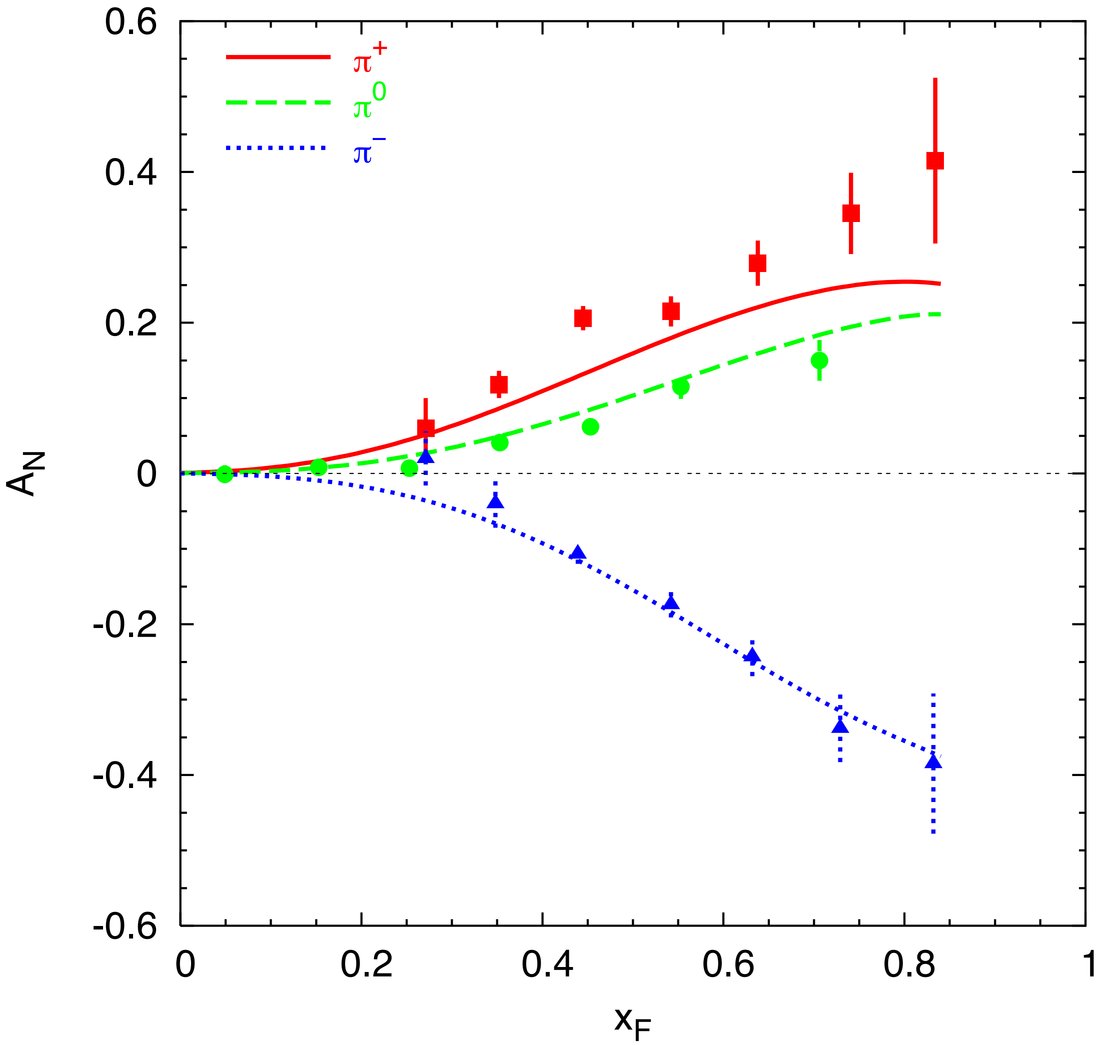
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...

Field-Feynman

generalized TMD factorization at work

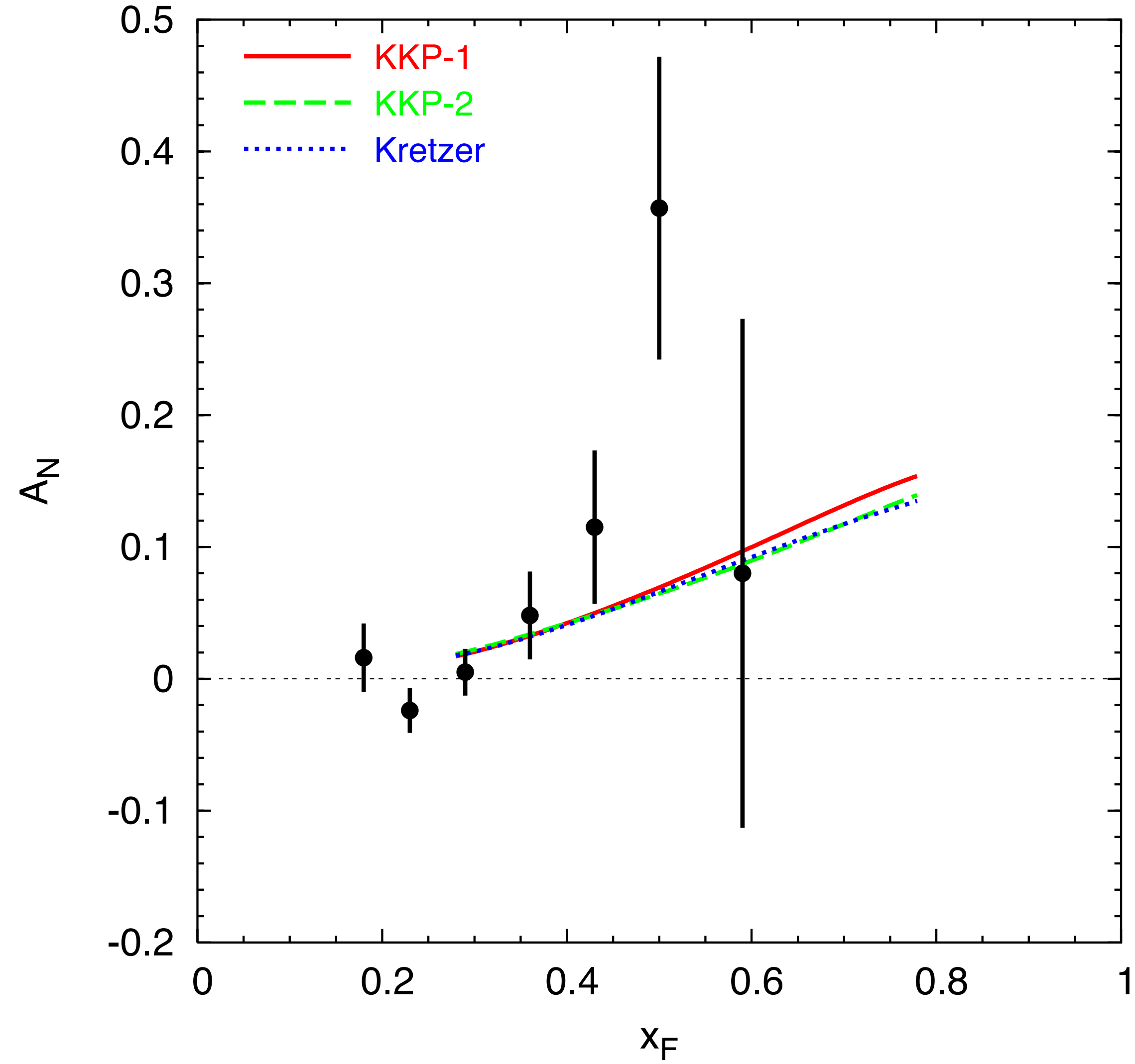
U. D'Alesio, F. Murgia, PR D70 (2004) 074009

E704 data



fit

STAR data



prediction

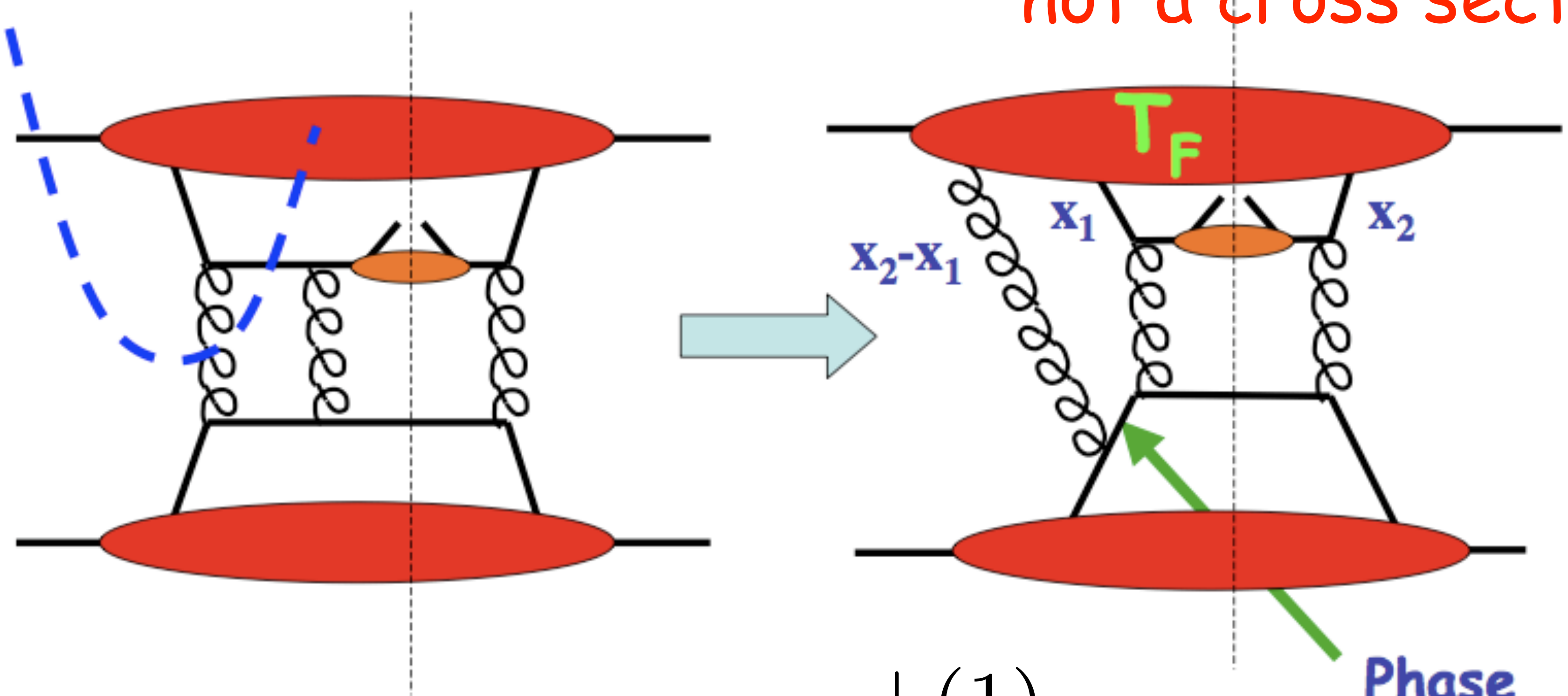
Sivers effect $p\,p \to \pi\,X$

2. Higher-twist partonic correlations

(Efremov, Teryaev, Ratcliffe; Qiu, Sterman; Kouvaris, Vogelsang, Yuan; Bacchetta, Bomhof, Mulders, Pijlman; Koike; Gamberg, Kang...)

higher-twist partonic correlations - factorization OK

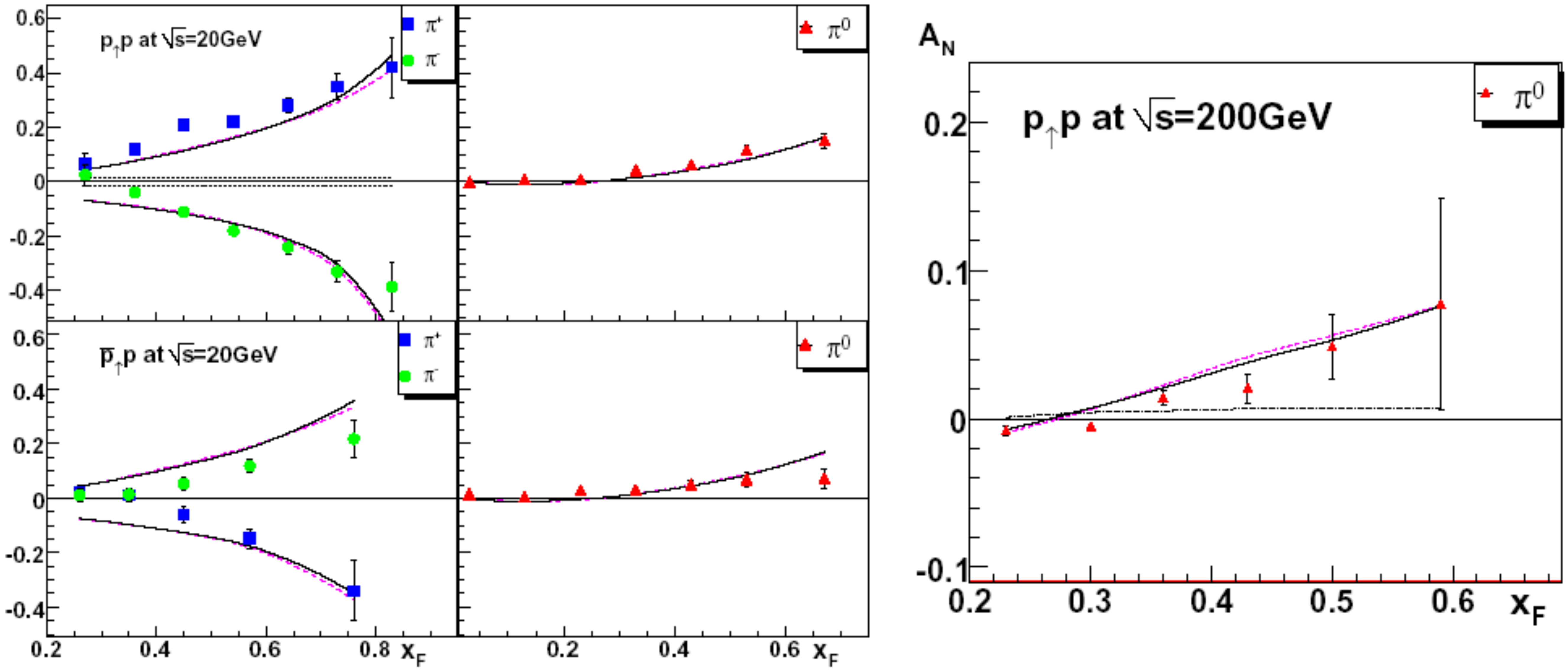
$$\mathrm{d}\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, \boldsymbol{S}_\perp)}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\to c}(k_1, k_2)}_{\text{hard interaction, not a cross section}} \otimes D_{h/c}(z)$$



$$(T_a \propto f_{1T}^{\perp(1)})$$

possible project: compute T_a using SIDIS extracted Sivers functions

(courtesy of W. Vogelsang)



fits of E704 and STAR data

Kouvaris, Qiu, Vogelsang, Yuan

sign mismatch

(Kang, Qiu, Vogelsang, Yuan, PR D83 (2011) 094001)

compare

$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\rm SIDIS}$$

as extracted from fitting A_N data, with that obtained by inserting in the above relation the SIDIS extracted Sivers functions

similar magnitude, but opposite sign!

the same mismatch does not occur adopting TMD factorization; the reason is that the hard scattering part in higher-twist factorization is negative

node in the Sivers function (Boer, Kang, Prokudin...)?
Study it at large x values

generalized TMD factorization: role of (SIDIS extracted) Collins and Sivers functions

(M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin)

A phenomenological study

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

main contribution from Sivers and Collins effects, negligible contributions from other TMDs

$$d\sigma^\uparrow - d\sigma^\downarrow \equiv \frac{E_\pi \, d\sigma^{\,p\to\pi\, X}}{d^3\boldsymbol{p}_\pi} - \frac{E_\pi \, d\sigma^{\,p\to\pi\, X}}{d^3\boldsymbol{p}_\pi} = [d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Sivers}} + [d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Collins}}$$

$$\begin{aligned}
[d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Sivers}} \quad = \quad & \sum_{q_a, b, q_c, d} \int \frac{dx_a\, dx_b\, dz}{16\,\pi^2\, x_a\, x_b\, z^2 s}\; d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{p}_\perp\, \delta(\boldsymbol{p}_\perp \cdot \hat{\boldsymbol{p}}_c)\, J(p_\perp)\, \delta(\hat{s}+\hat{t}+\hat{u}) \\
\times \quad & \Delta^N f_{a/}(x_a, k_{\perp a})\, \cos\phi_a \longrightarrow \text{Sivers function + phase} \\
\times \quad & f_{b/p}(x_b, k_{\perp b})\, \frac{1}{2}\left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2\right]_{ab\to cd} \, D_{\pi/c}(z, p_\perp)
\end{aligned}$$

unpolarized cross section

TMD-FF

$$
\begin{aligned}
[d\sigma^{\uparrow} - d\sigma^{\downarrow}]_{\rm Collins} \;&=\; \sum_{q_a, b, q_c, d} \int \frac{dx_a\, dx_b\, dz}{16\,\pi^2\, x_a\, x_b\, z^2 s}\; d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{p}_{\perp}\, \delta(\boldsymbol{p}_{\perp}\cdot \hat{\boldsymbol{p}}_c)\, J(p_\perp)\, \delta(\hat s + \hat t + \hat u) \\
&\times\; \Delta_T q_a(x_a, k_{\perp a}) \cos(\phi_a + \varphi_1 - \varphi_2 + \phi_\pi^H) \\
&\times\; f_{b/p}(x_b, k_{\perp b}) \left[\hat M_1^0\, \hat M_2^0\right]_{q_a b \to q_c d} \Delta^N D_{\pi/q_c}(z, p_\perp)
\end{aligned}
$$

transversity + phases

spin transfer
cross section

Collins function

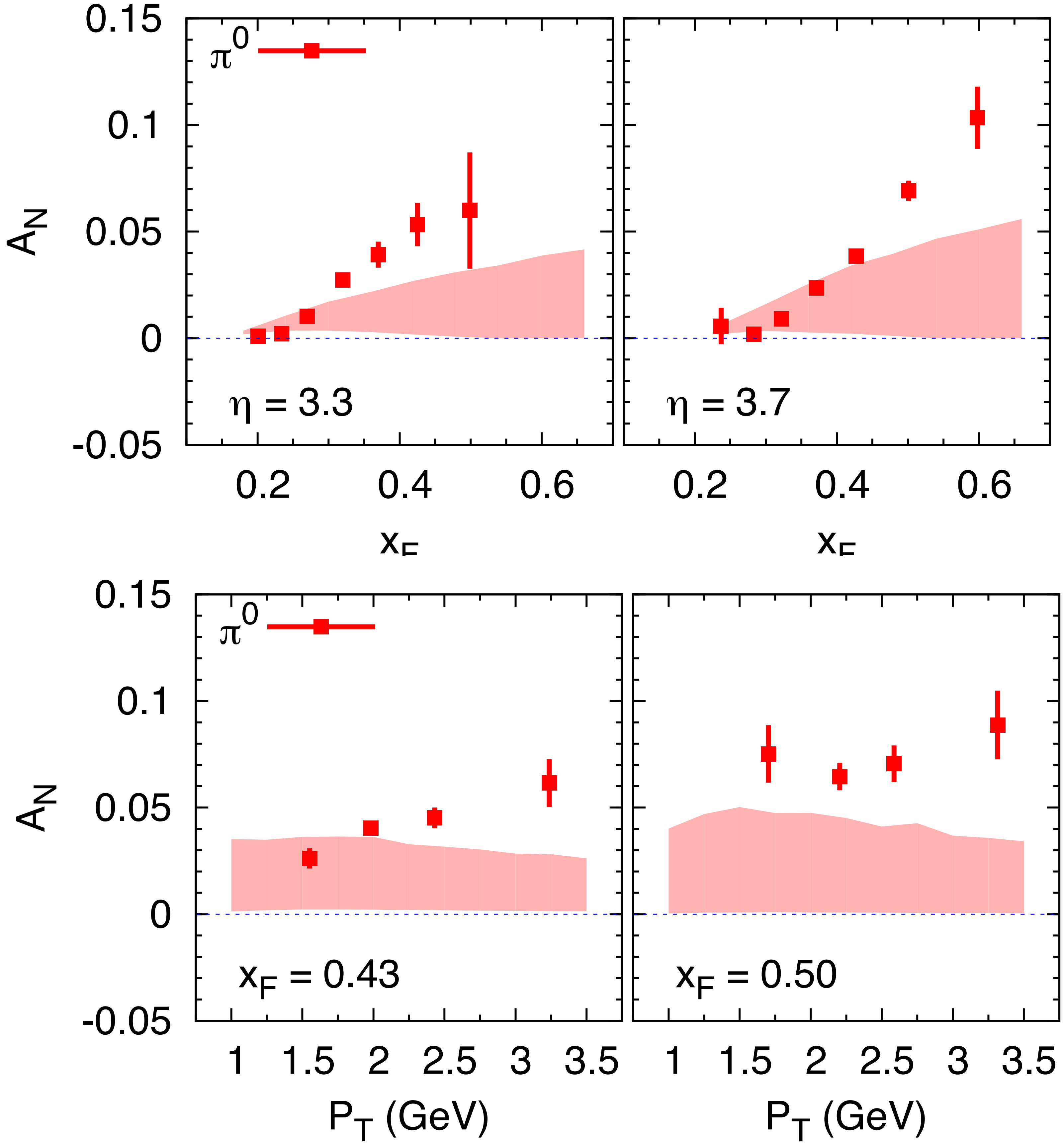
SIDIS data do not fix the large x behaviour of the transversity and Sivers distributions

$$
\Delta_T q(x, k_\perp) = \frac{1}{2}\, \mathcal{N}_q^T(x)\, \left[\, f_{q/p}(x) + \Delta q(x)\,\right] \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi \langle k_\perp^2\rangle_T}
$$

$$
\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2\, \mathcal{N}_q^S(x)\, f_{q/p}(x)\, h(k_\perp)\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi \langle k_\perp^2\rangle}
$$

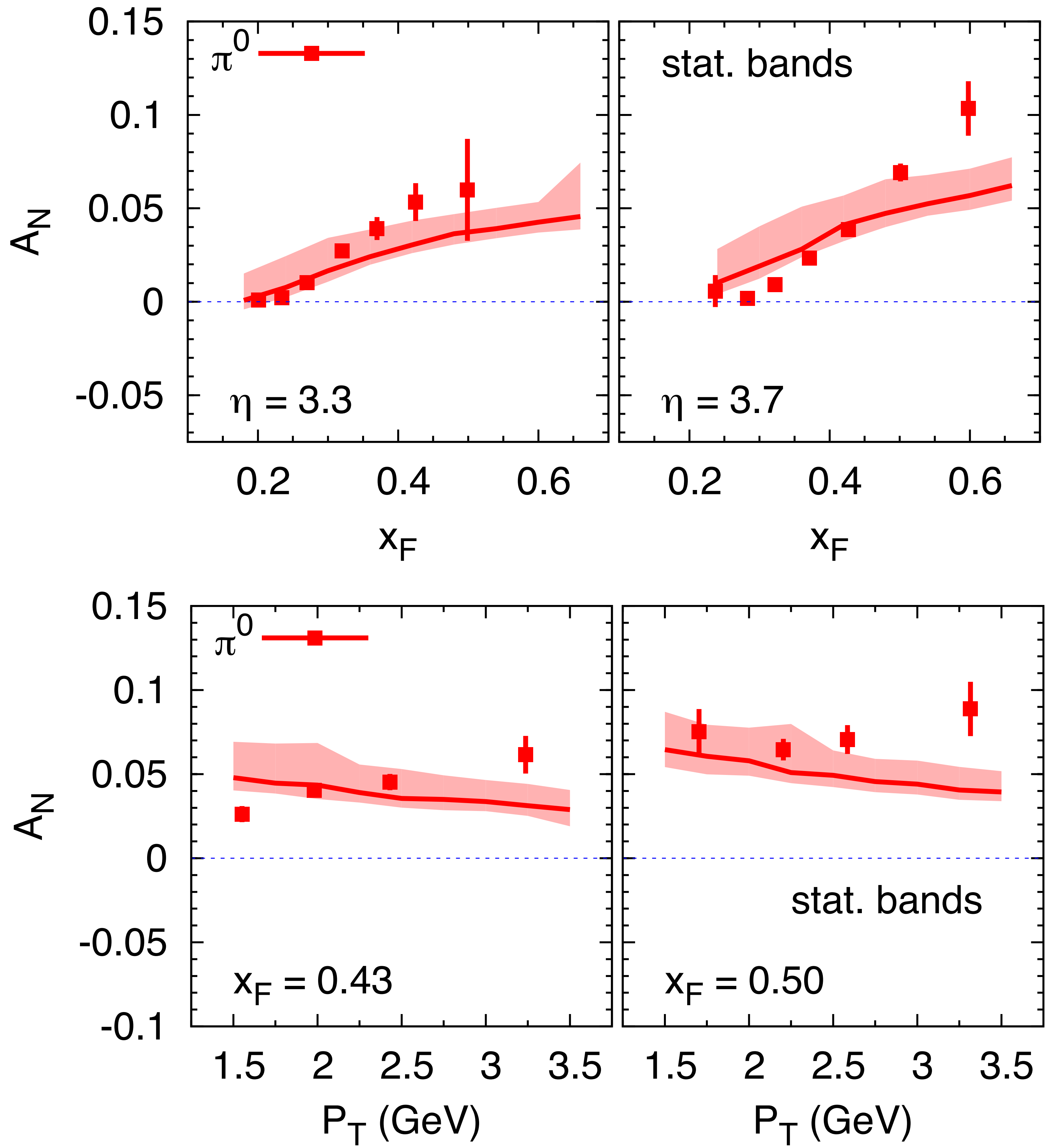
$$
\mathcal{N}(x) \sim x^\alpha (1-x)^\beta
$$

A_N sensitive to large x values: let β parameters vary



Collins effect STAR data, problems at large x_F:
Collins effect alone is not enough

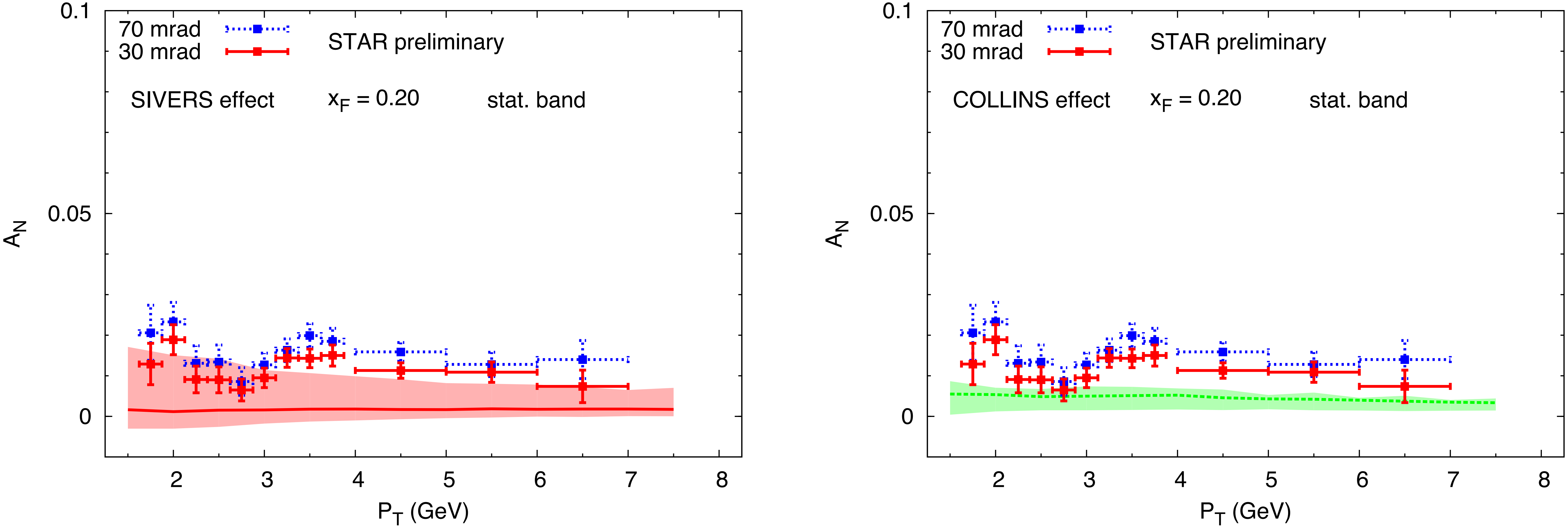
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, PR D86 (2012) 074032



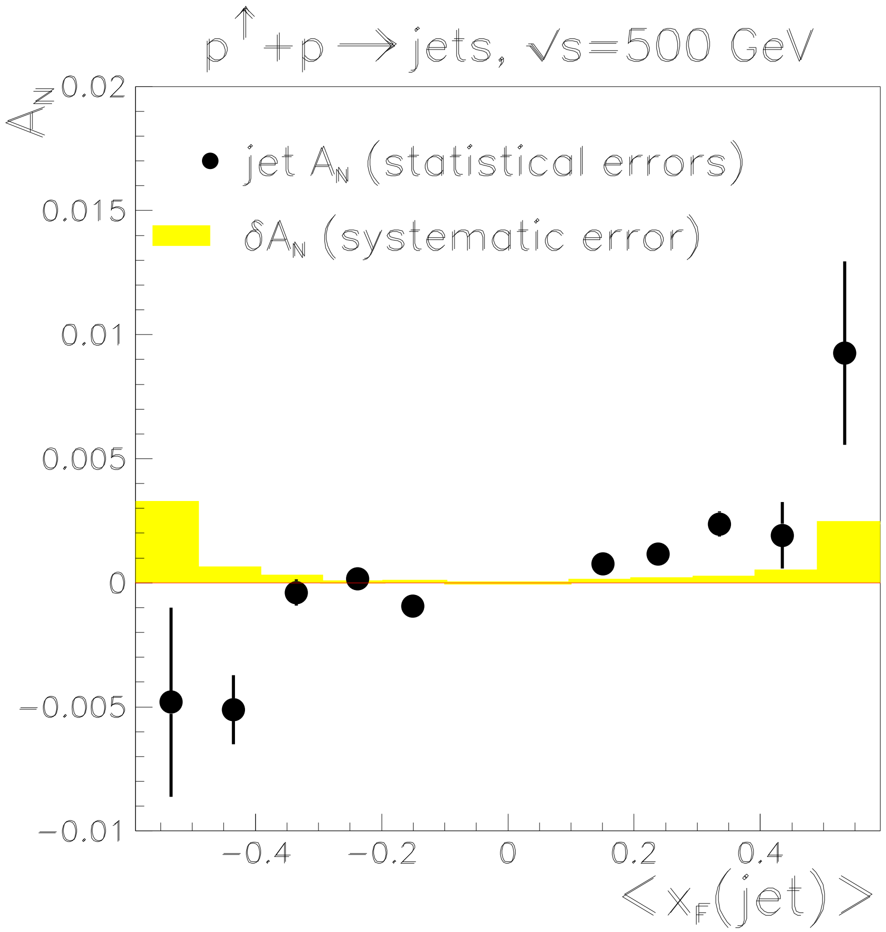
Sivers effect STAR data, one particular set of parameters with statistical band:

Sivers effect alone can explain most data

M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, arXiv:1304.7691



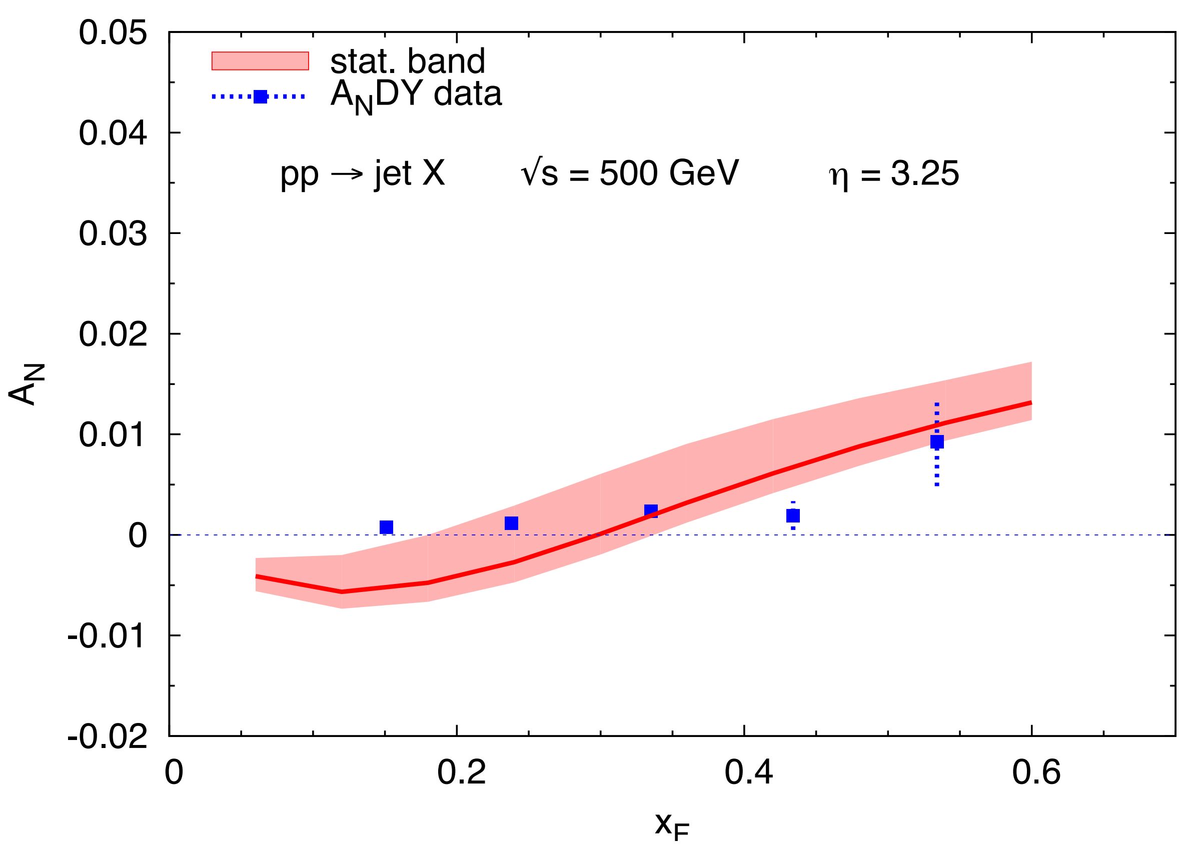
Sivers + Collins effects might explain also the 500 GeV, large P_T STAR data

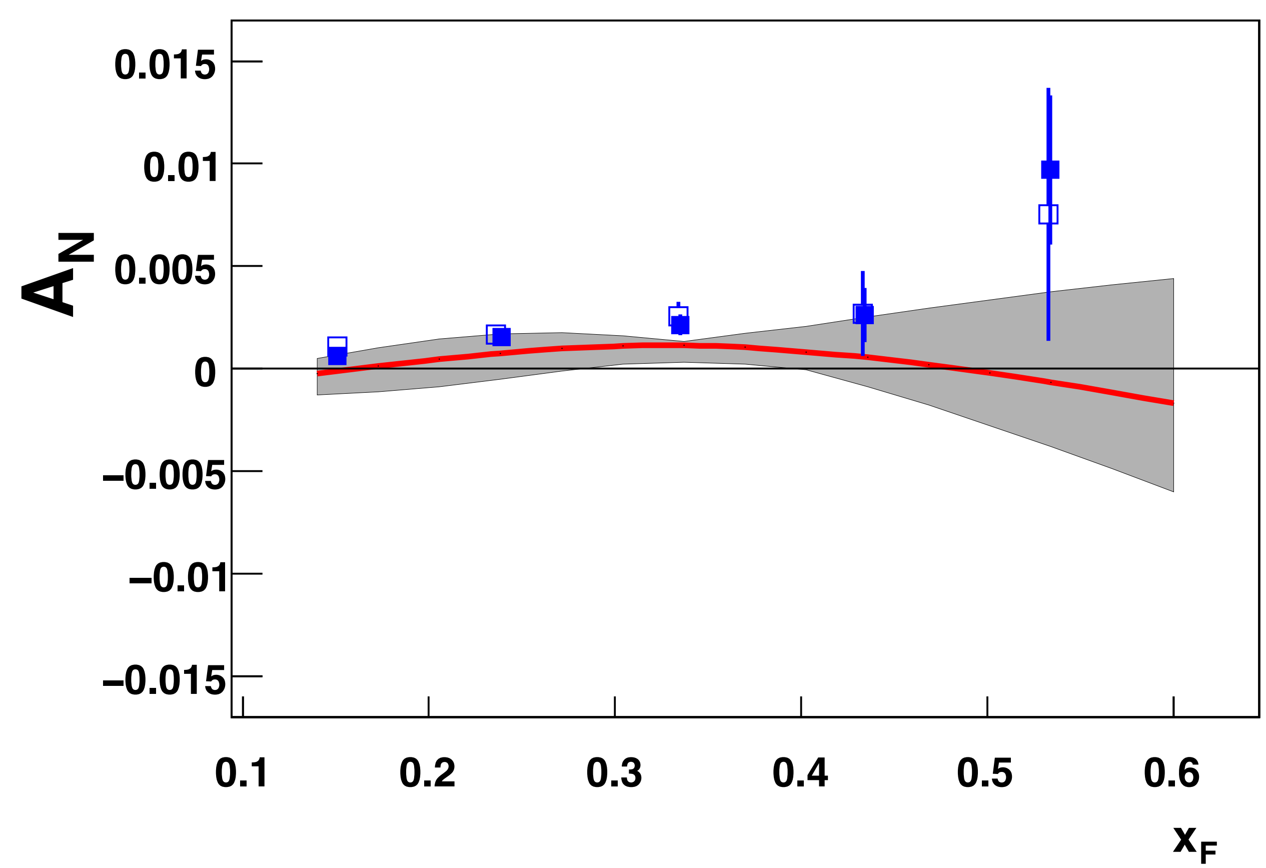


A_N for jet production at A_NDY

lower left plot: A_N assuming TMD factorization

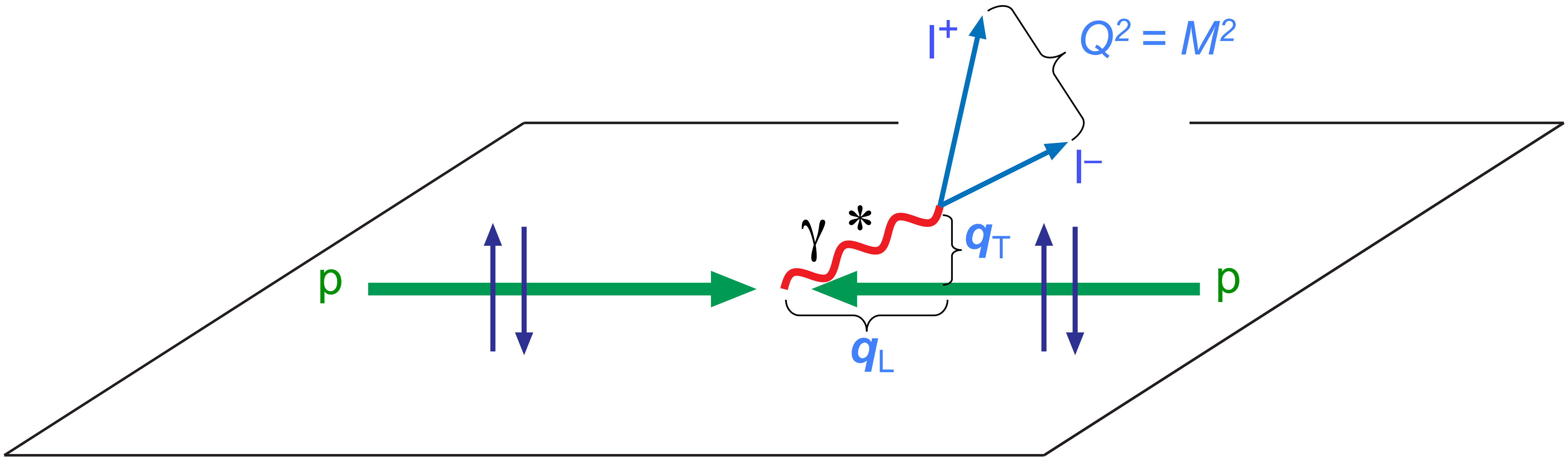
lower right plot: A_N with twist-3 correlation function (Gamberg, Prokudin)





Future: TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M^2, and $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, d\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs

no fragmentation process

cross-section: most general pp leading-twist expression

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times$$

S. Arnold, A. Metz and M. Schlegel, PR D79 (2009) 034005

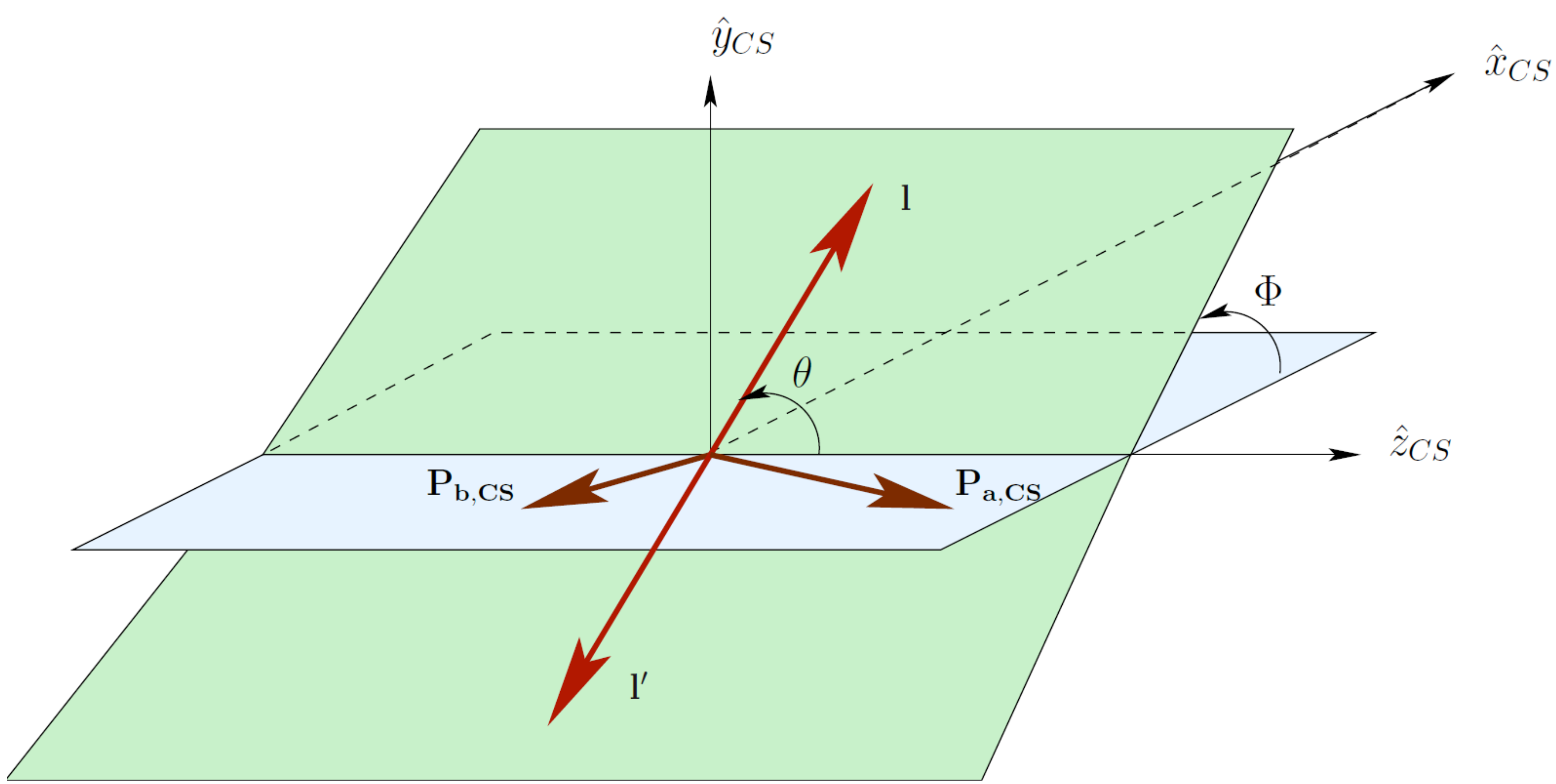
$$\begin{aligned}
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big)\\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big)\\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big)\\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big]\\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big)\\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big)\\
&\qquad\quad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}
\end{aligned}$$

Drell-Yan

48 structure functions ...

Case of one polarized nucleon only

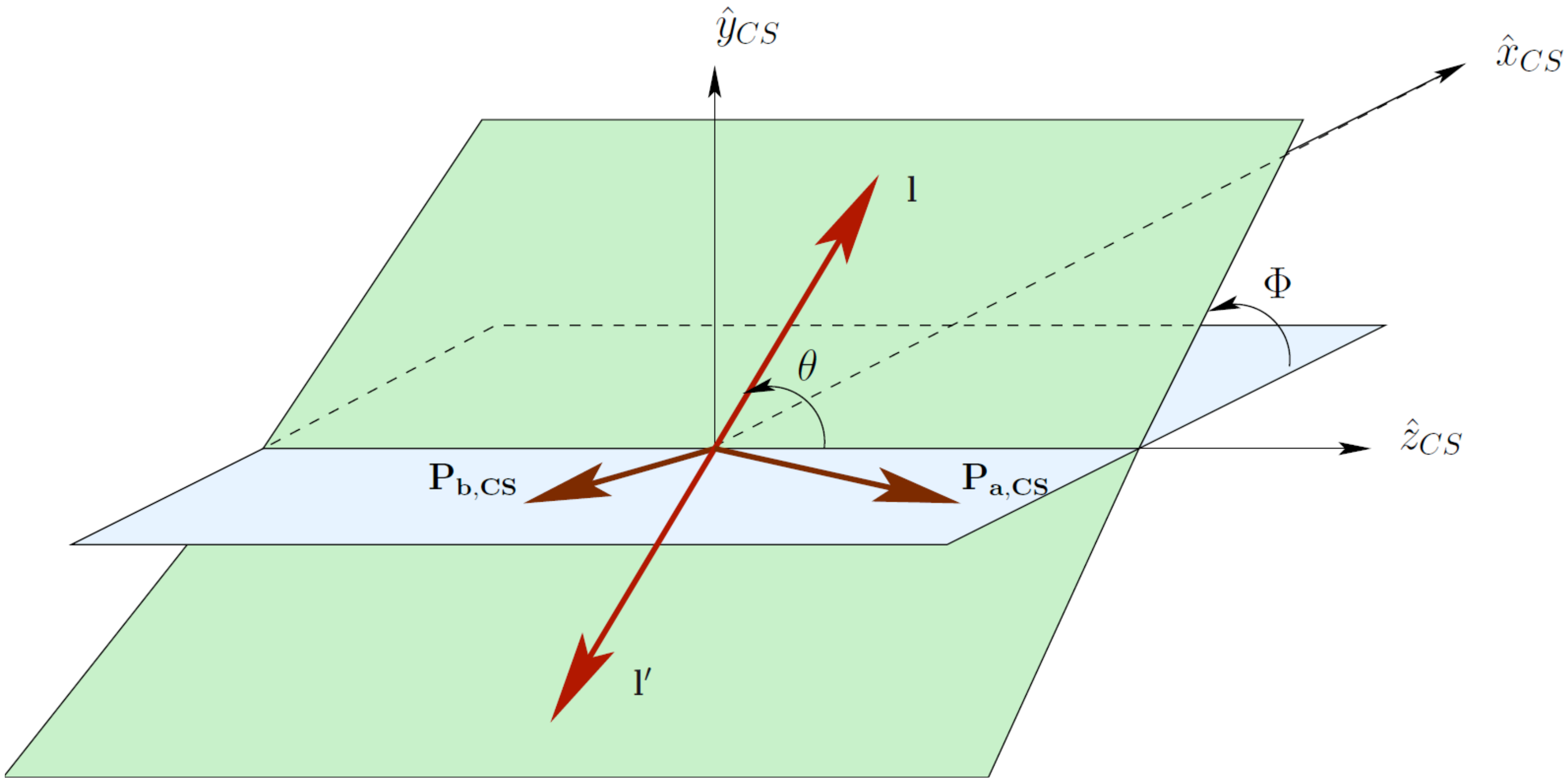
$$
\begin{aligned}
\frac{d\sigma}{d^4q\, d\Omega} \;=\; & \frac{\alpha^2}{\Phi\, q^2}\Big\{(1+\cos^2\theta)\, F_U^1 + (1-\cos^2\theta)\, F_U^2 + \sin 2\theta \cos\phi\, F_U^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_U^{\cos 2\phi} \\
& + S_L \left(\sin 2\theta \sin\phi\, F_L^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_L^{\sin 2\phi}\right) \\
& + S_T \Big[\left(F_T^{\sin\phi_S} + cos^2\theta\, \tilde{F}_T^{\sin\phi_S}\right) \sin\phi_S + \sin 2\theta \Big(\sin(\phi+\phi_S)\, F_T^{\sin(\phi+\phi_S)} \\
& \qquad + \sin(\phi-\phi_S)\, F_T^{\sin(\phi-\phi_S)}\Big) \\
& + \sin^2\theta \left(\sin(2\phi+\phi_S)\, F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\, F_T^{\sin(2\phi-\phi_S)}\right)\Big]\Big\}
\end{aligned}
$$



Collins-Soper frame

Unpolarized cross section already very interesting

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\,\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$



Collins-Soper frame

naive collinear parton model: $\lambda = 1 \quad \mu = \nu = 0$

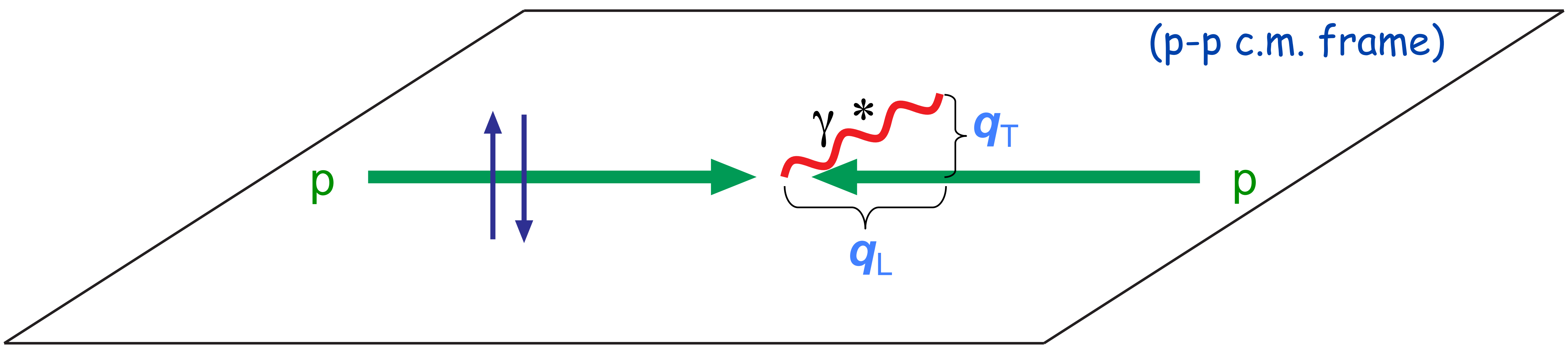
Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_{\perp 1}) \otimes f_{\bar{q}/p}(x_2, k_{\perp 2}) \otimes \mathrm{d}\hat{\sigma}$$

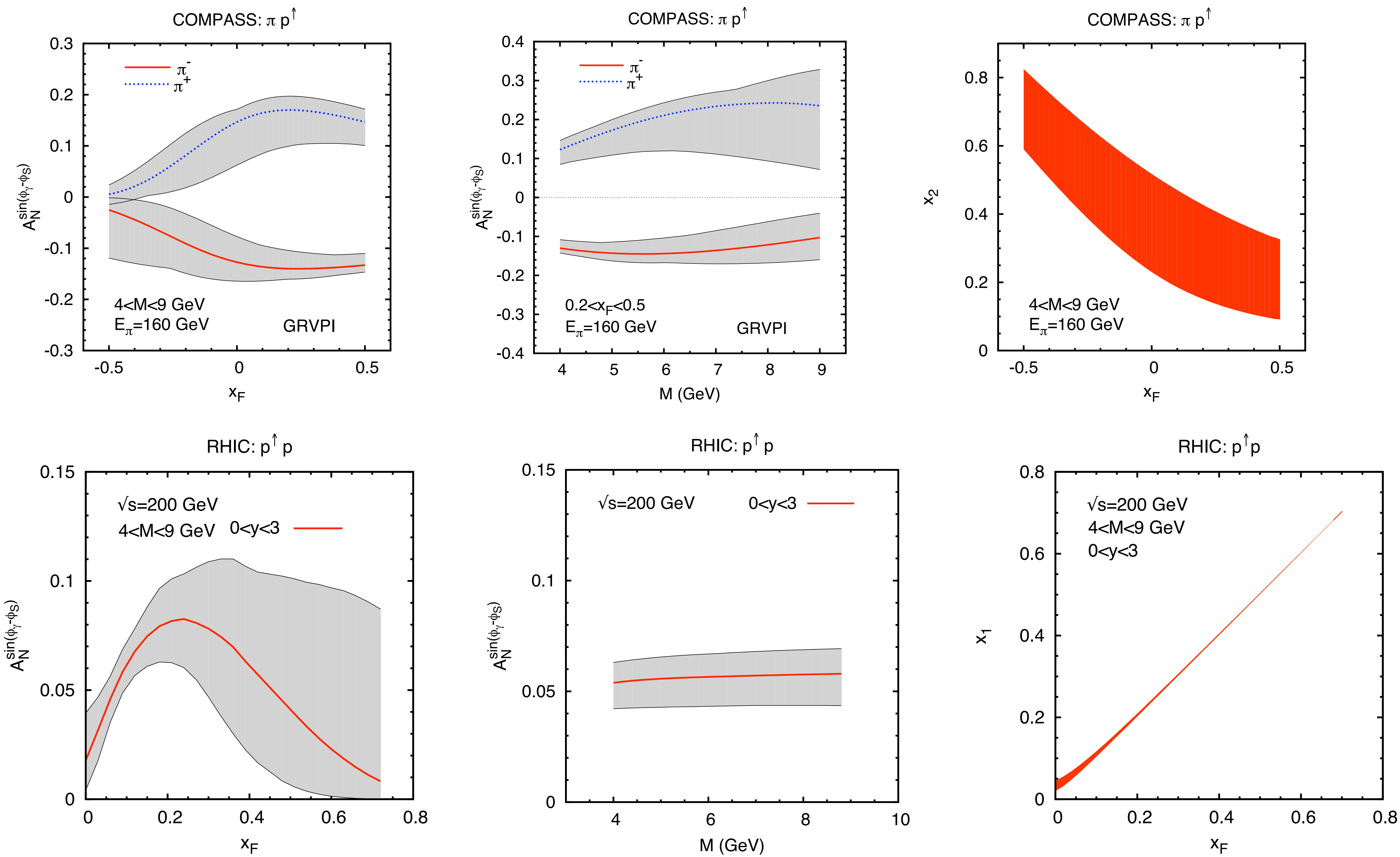
$$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow] \, \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$



Predictions for A_N - no TMD evolution

Sivers functions as extracted from SIDIS data, **with opposite sign**



M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PR D79 (2009) 054010

Conclusions

SSAs in pp interactions keep being surprising and very interesting...

are they related to intrinsic nucleon properties? quark angular momentum...?

is there a common origin for SSAs observed in different processes? TMDs?

sign mismatch? sign change between SIDIS and D-Y Sivers function?

A_N for pions in jets (separation of Collins and Sivers effects); transversity via di-hadron fragmentation functions; ...

thank you