Initial state geometry and fluctuations in Cu+Au and U+U collisions

Prithwish Tribedy

Variable Energy Cyclotron Center, Kolkata, India

in collaboration with Björn Schenke (BNL) and Raju Venugopalan (BNL)

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Introduction

Aim:

- An *ab initio* description of initial stages of heavy ion collisions.
- Study the sensitivity of initial geometry of collision in various systems.

Approach:
Color Glass Condensate (CGC) framework combining,

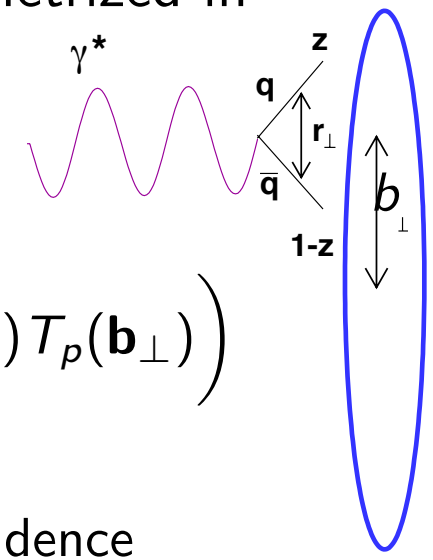
- Saturation model of HERA DIS to construct nuclear wave-function (IP-Sat model).
- Classical Yang-Mills description to calculate gluon field after collisions (Glasma description).

Saturation models of HERA DIS

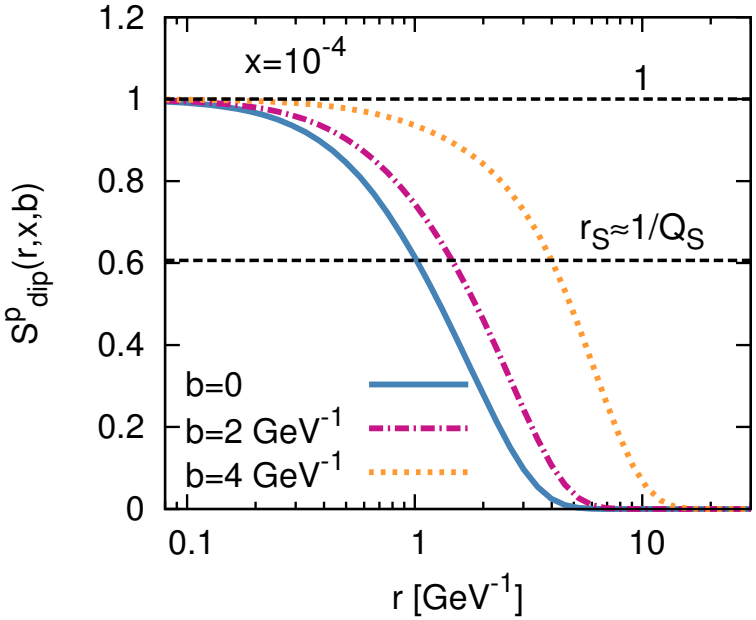
Kowalski, Teaney

IP-Sat $\rightarrow$ (Impact parameter dependent dipole model)

The dipole cross section in $e+p$ collisions is parametrized in IP-Sat model.
The dipole scattering matrix for proton is



$$S^p_{\rm dip}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = \exp\left(-\frac{\pi^2}{2N_c}\mathbf{r}_\perp{}^2\alpha_S(\tilde{\mu}^2)xg(x,\tilde{\mu}^2)\,T_p(\mathbf{b}_\perp)\right)$$



Impact parameter dependence through thickness profile

$$T_p(\mathbf{b}_\perp) = \frac{1}{2\pi B_G}\exp\left(\frac{-\mathbf{b}_\perp^2}{2B_G}\right)$$

$\langle b^2 \rangle = 2B_G \rightarrow$ gluonic radius of proton

Color charge distribution inside Nuclei

The nuclear scattering matrix is obtained as

$$S^A_{\rm dip}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = \prod_{i=0}^{A} S^p_{\rm dip}(\mathbf{r}_\perp, x, \mathbf{b}_\perp)$$

$i \to$ nucleons are distributed according to Fermi distribution.

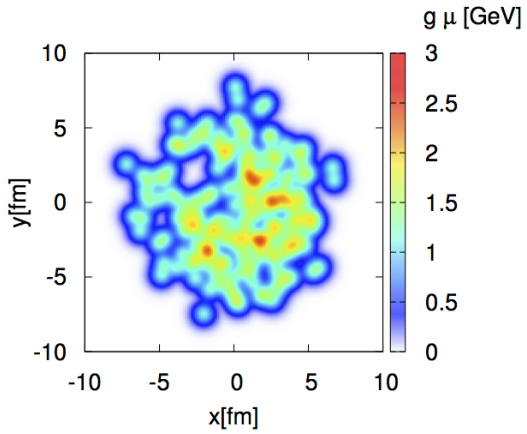
$S^A_{\rm dip}$ gives the distribution of nuclear saturation solving

$$S^A_{\rm dip}(\mathbf{r}_\perp = r_S, x, \mathbf{b}_\perp) = \exp(-1/2) \implies Q_s^2 = \frac{2}{r_S^2}$$

Iteratively solving

$$x = \frac{Q_s(\mathbf{b}_\perp, x)}{\sqrt{s}}$$

gives nuclear Q_s in transverse plane at given energy
Color charge density distribution $g^2\mu \sim Q_s$



Lappi, arXiv:0711.3039, 1104.3725

Color Glass Condensate

- Color: QCD (gluons carry color charge)
- Glass: Stochastic interactions, dynamics on very long time scales (time dilation).
- Condensate: Fields with large occupation $\# \sim 1/\alpha_S$ with mom. peaked at $k_T \approx Q_S$

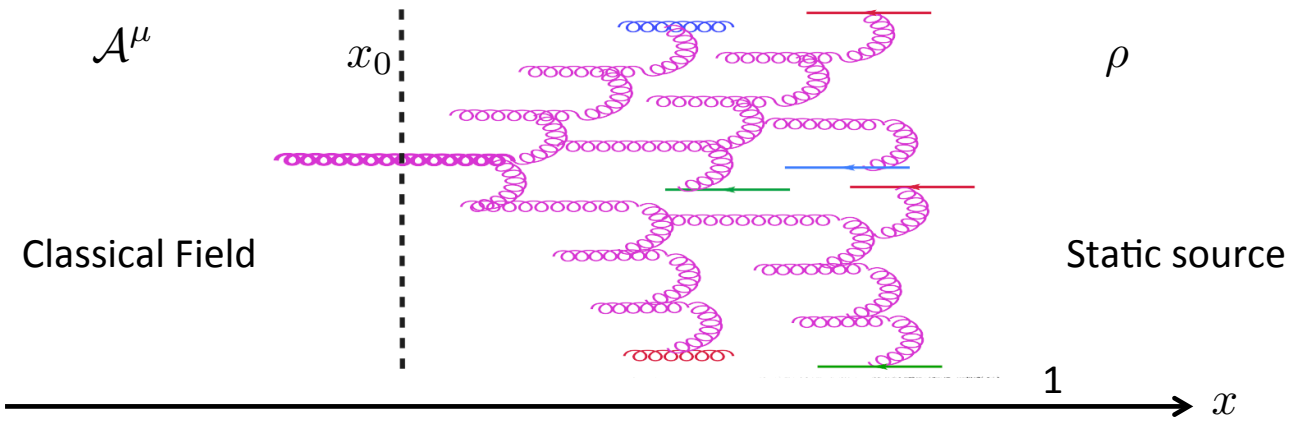


figure :Albacete

Unknown color charge distribution (weight functional):

$$W[\rho] = \exp\left[-\frac{\rho^2(\mathbf{x}_\perp)}{2\mu^2(\mathbf{x}_\perp)}\right], \; g^2\mu(\mathbf{x}_\perp) \sim Q_S(x, \mathbf{x}_\perp)$$

McLerran , Venugopalan 1994 (hep-ph/9309289)

IP-Glasma : Classical Yang-Mills approach on 2+1D lattice

Schenke, Tribedy, Venugopalan PRL 108(2012)

The effective lagrangian for a colliding nucleus,

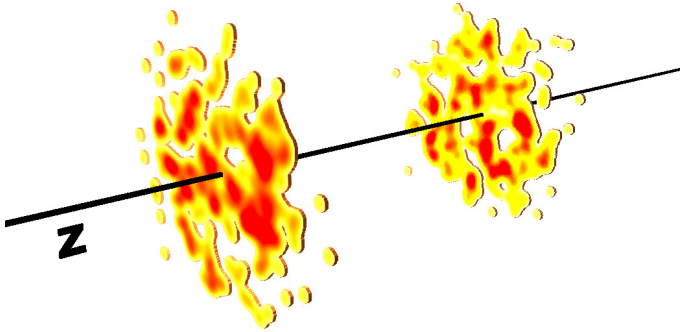
$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu$$

The corresponding eq. of motion

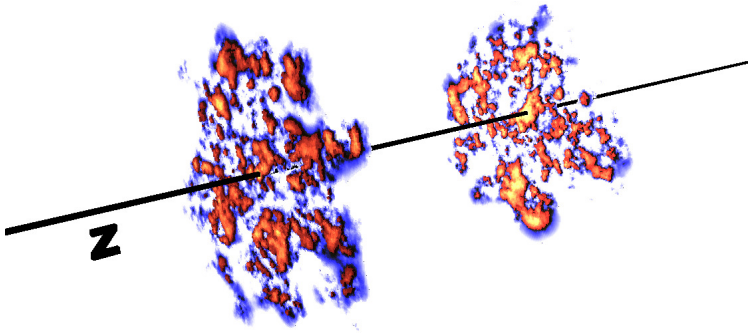
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

In the presence of color current

$$J^\nu_{A,B} \approx \delta(x^\mp)\rho(\mathbf{x}_\perp)\delta^{\mu\pm}$$



Color charge density for one A+A collision



Two point correlator for one A+A collision

$\rho(\mathbf{x}_\perp)$ sampled from local Gaussian distribution $W[\rho]$

$$\left\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\right\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

lattice implementation Krasnitz, Venugopalan, hep-ph/9809433 Lappi, hep-ph/0303076

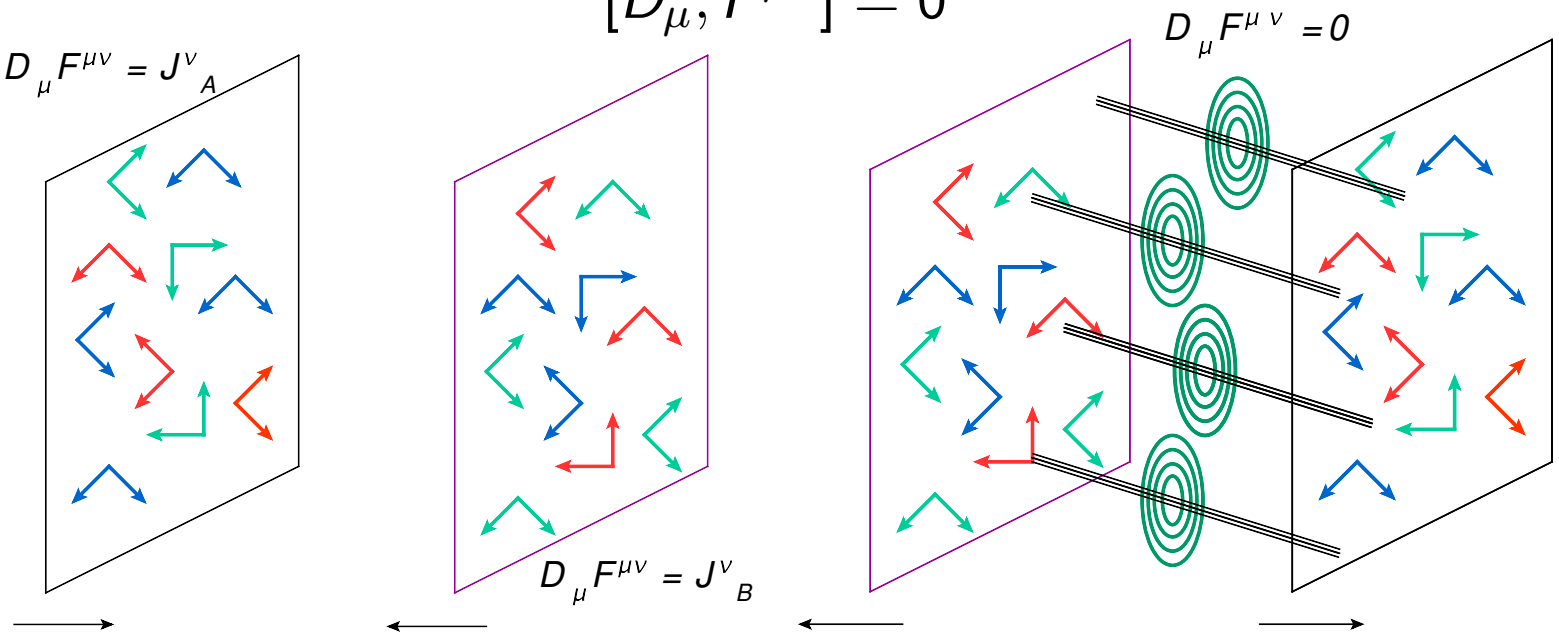
IP-Glasma : CYM evolution after collision

The field after collision at $\tau = 0$ has simple relation

$$A^i = A^i_{(A)} + A^i_{(B)} \ , \ A^\eta = \frac{ig}{2}\left[A^i_{(A)}, A^i_{(B)}\right]$$

The fields are evolved at $\tau > 0$ according to

$$[D_\mu, F^{\mu\nu}] = 0$$



Schenke, Tribedy, Venugopalan PRC 86(2012)

IP-Glasma : Multiplicity and Energy density

E-by-E soln. of CYM equation on 2+1D lattice $\rightarrow F^{\mu\nu}(\tau, \mathbf{x}_\perp, \eta)$.

- Multiplicity (n): $F^{\mu\nu}(\tau, \mathbf{x}_\perp, \eta) \rightarrow \mathcal{H}(\mathbf{x}_\perp)$ (Hamiltonian density)
  Fourier transform $\mathcal{H}(\mathbf{k}_\perp) \rightarrow$ number density $n(\mathbf{k}_\perp)$ of gluon,
  $\mathcal{H}(\mathbf{k}_\perp) \sim n(\mathbf{k}_\perp)\omega(\mathbf{k}_\perp)$, assuming dispersion relation, $\omega(k) = k$.

- Energy density (ϵ): $F^{\mu\nu} \rightarrow T^{\mu\nu}$ (stress energy tensor).

$$T^{\mu\nu} = -g^{\gamma\delta} F^{\mu}_{\ \gamma} F^{\nu}_{\ \delta} + \frac{1}{4} g^{\mu\nu} F^{\gamma}_{\ \beta} F_{\gamma}^{\ \delta}$$

solving eigen value eq. $u_\mu T^{\mu\nu} = \epsilon u^\nu$ gives ϵ and flow u^ν
$T^{\mu\nu}_{CYM}$ can be Landau matched with viscous hydro.

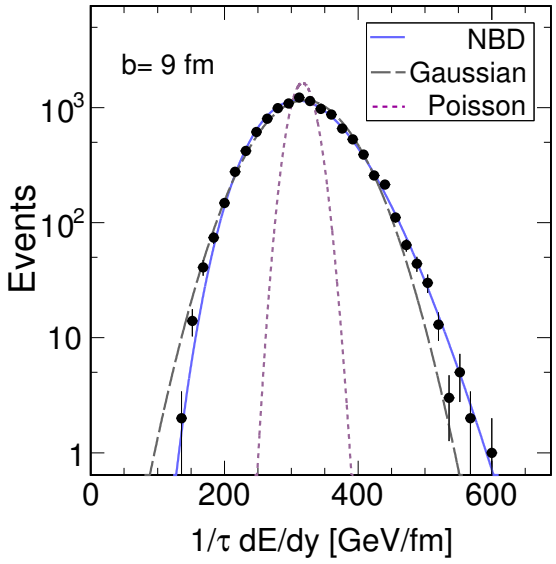
Gale, Jeon, Schenke, Tribedy, Venugopalan

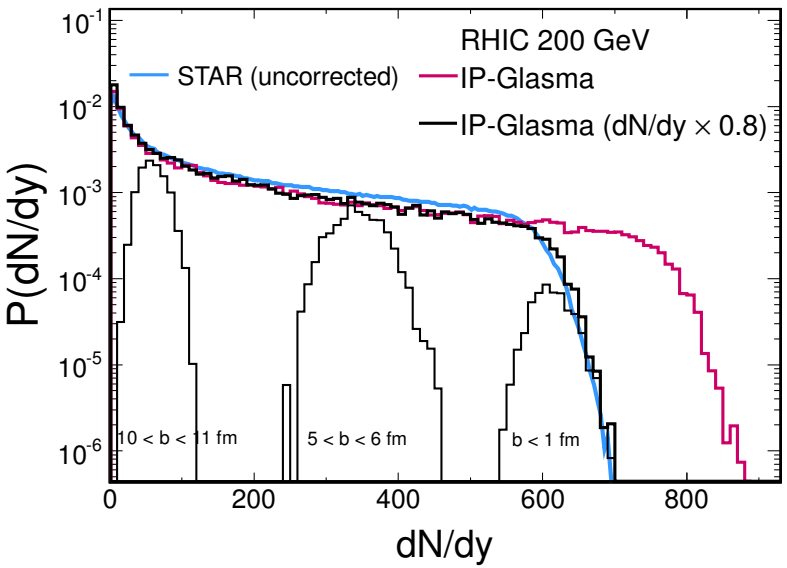
Negative binomial fluctuation in IP-Glasma model

IP-Glasma includes :

- Initial color charge fluctuation at sub-nucleonic length scale $1/Q_S$.
- E-by-E fluctuation of nucleon position and impact parameter.

$\rightarrow$ IP-Glasma Naturally produces negative-binomial transverse energy and multiplicity fluctuation (no need to put by hand).





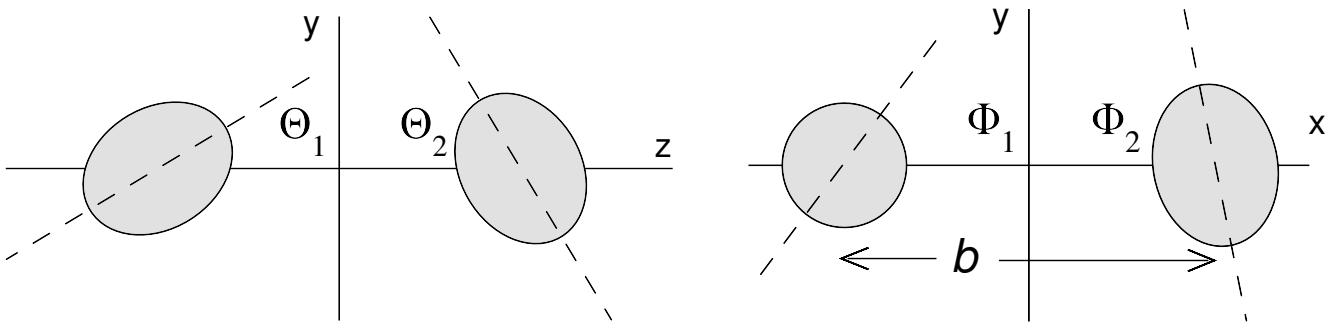
Schenke, PT, Venugopalan PRC 86(2012)

Sampling uranium collisions at RHIC

The Woods-Saxon distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp\left([r - R']/a\right)}$$
$$R' = R\left[1 + \beta_2 Y_2^0(\theta) + \beta_4 Y_4^0(\theta)\right]$$

$R = 6.81$ fm, a=0.55, β_2=0.28, β_4=0.093.

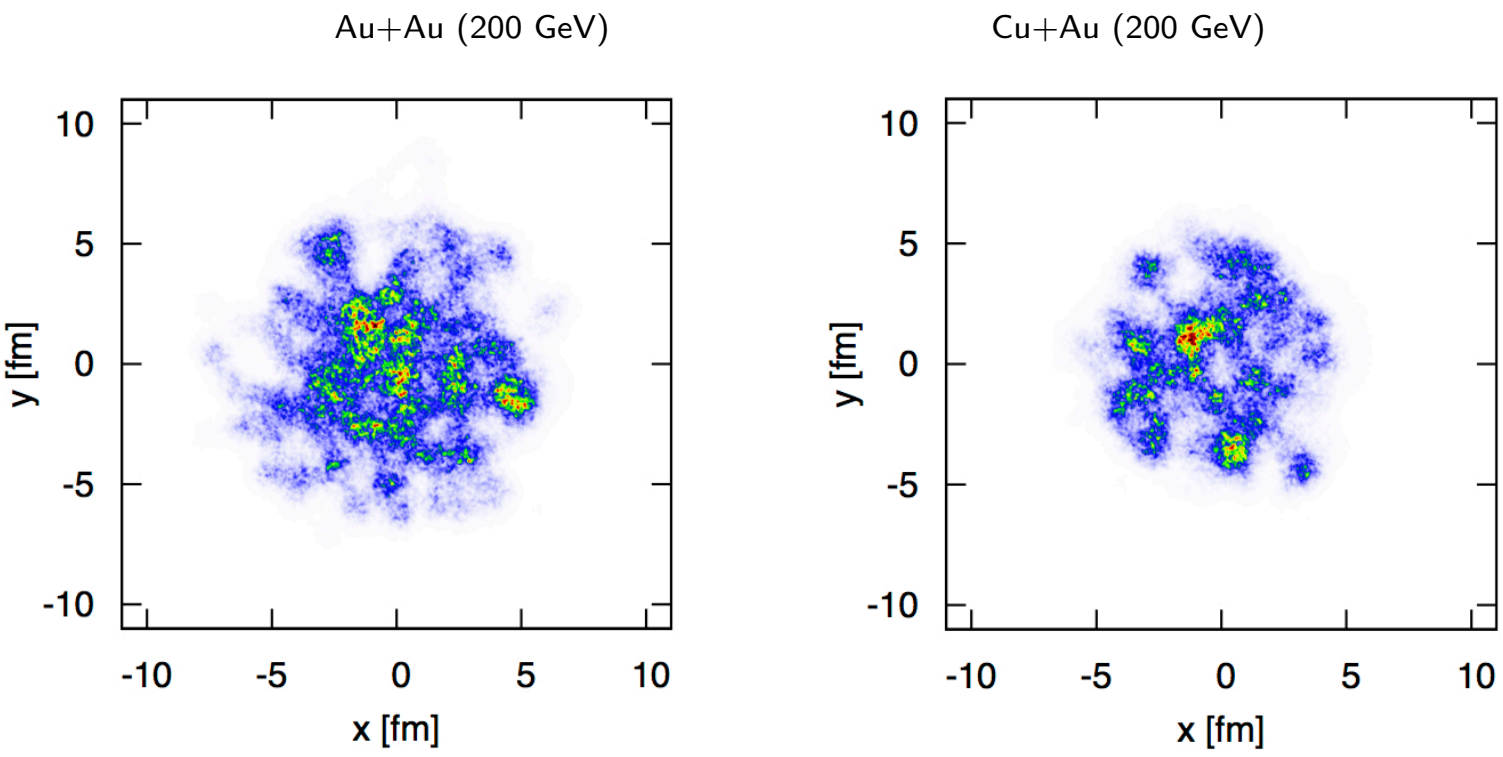


4 more d.o.f in addition to impact parameter.

- $b \rightarrow$ linearly
- $\Theta_{1,2} \rightarrow$ according to $\sin(\Theta_{1,2})$
- $\Phi_{1,2} \rightarrow$ uniformly

Initial energy density for different systems

Energy density (ϵ) from IP-Glasma model (at $\tau = 0$)



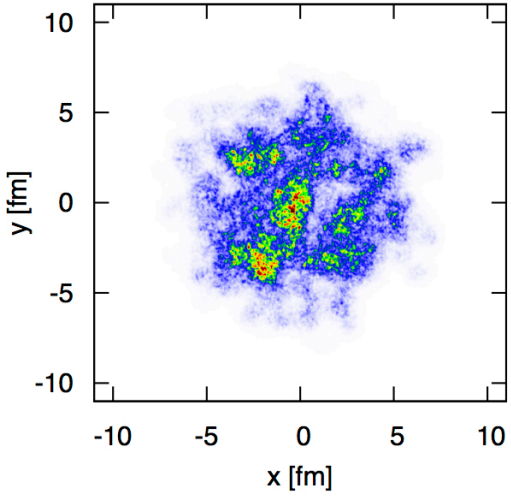
- The length scale of fluctuation is $1/Q_s <$ nucleonic size.
- For Cu+Au system size is determined by Cu nucleus.

Initial energy density for different systems

$\epsilon(\mathbf{x}_\perp)$ for different configurations of U+U collisions ($\tau = 0, b = 0$)

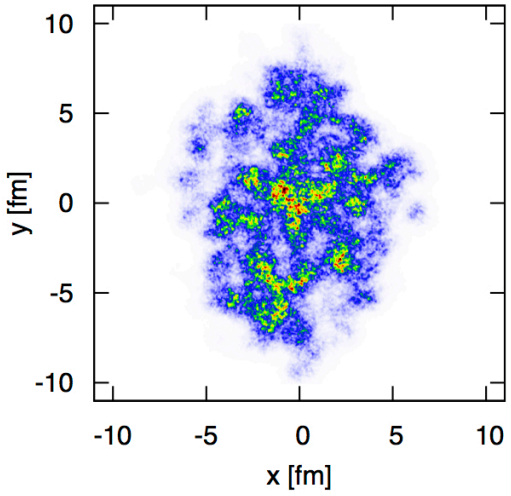
U+U (Tip-Tip)

$\Phi_{1,2} = 0, \Theta_{1,2} = 0$



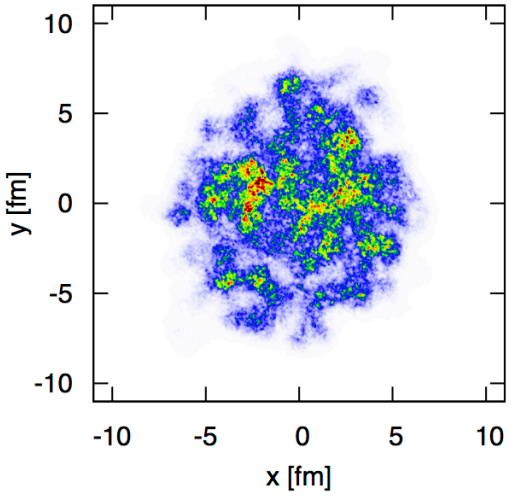
U+U (Side-Side)

$\Phi_{1,2} = \pi/2, \Theta_{1,2} = \pi/2$



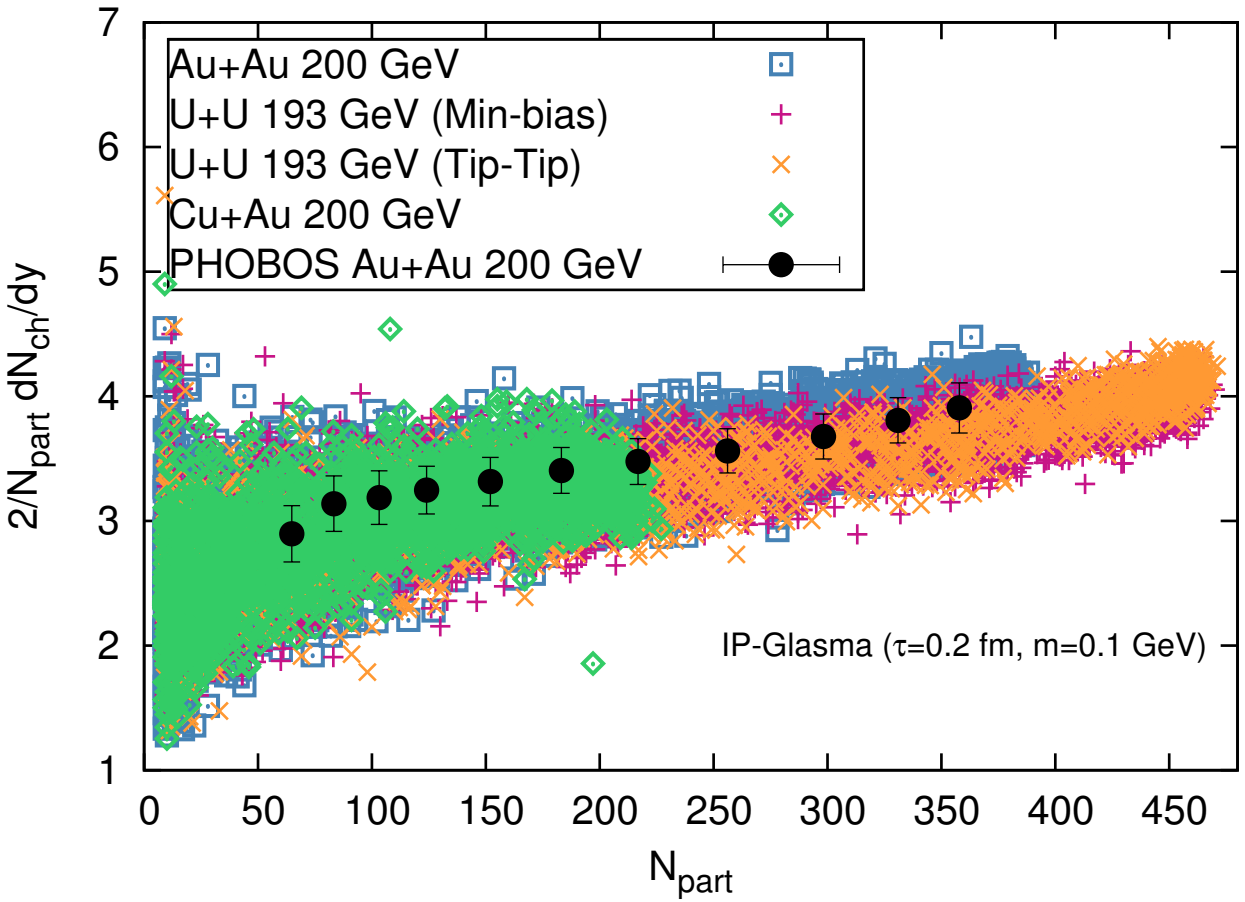
U+U (Random)

$\Phi_{1,2} = 1.9\pi, 1.7\pi, \Theta_{1,2} = \pi/2, 0.2\pi$



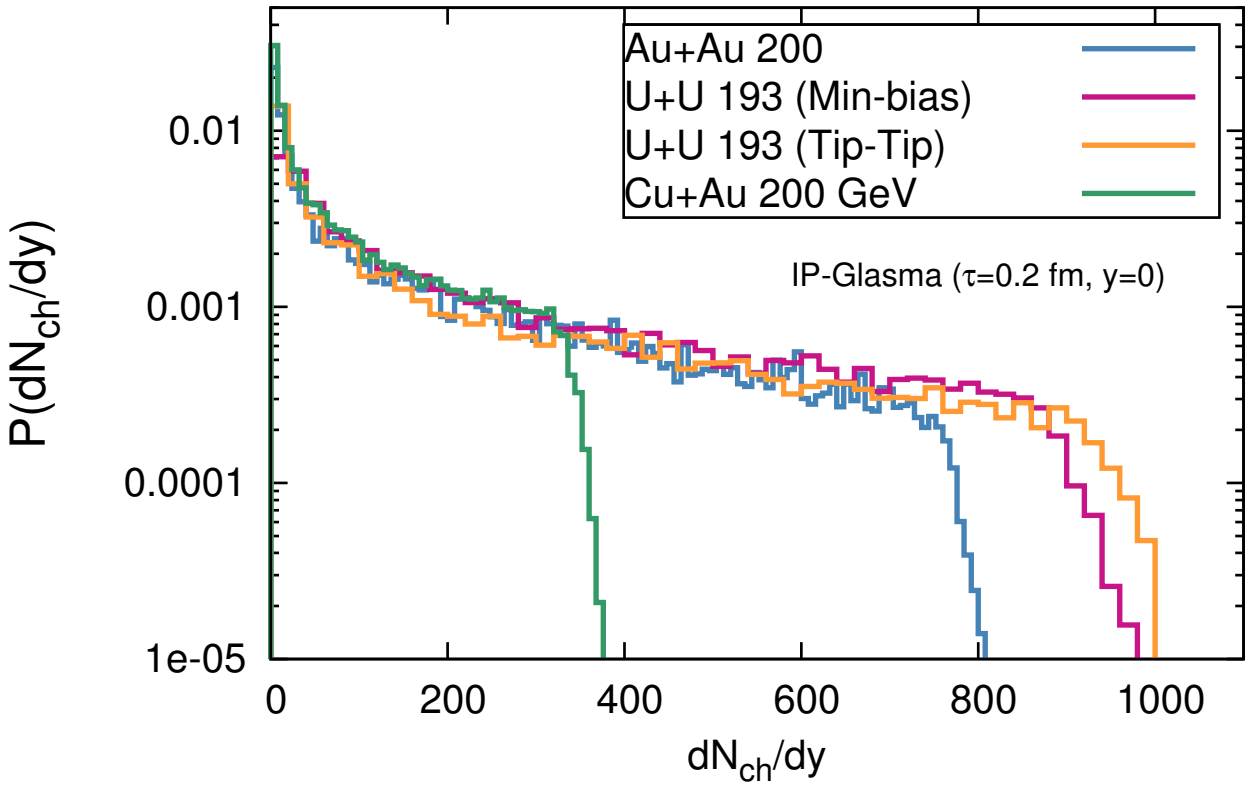
Multiplicity : centrality dependence

- dN_g/dy at time $\tau = 0.2$ fm in transverse Coulomb gauge $\partial_i A^i = 0$
- dN_g/dy scaled by $2/3$ to compare with data.
- Local Running coupling on each point on lattice $\alpha_s\left(Q_s^{max}(\mathbf{x}_\perp)\right)$



Multiplicity: Probability distributions

$P(dN_{ch}/dy)$ at $\tau = 0.2$ fm with linear distribution of $P(b)$



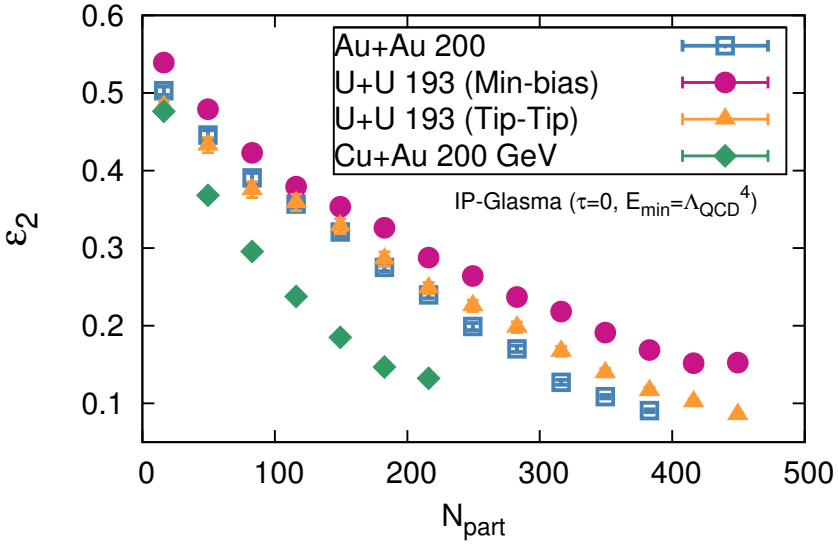
U+U min-bias & tip-tip are very close unlike 2-component model.

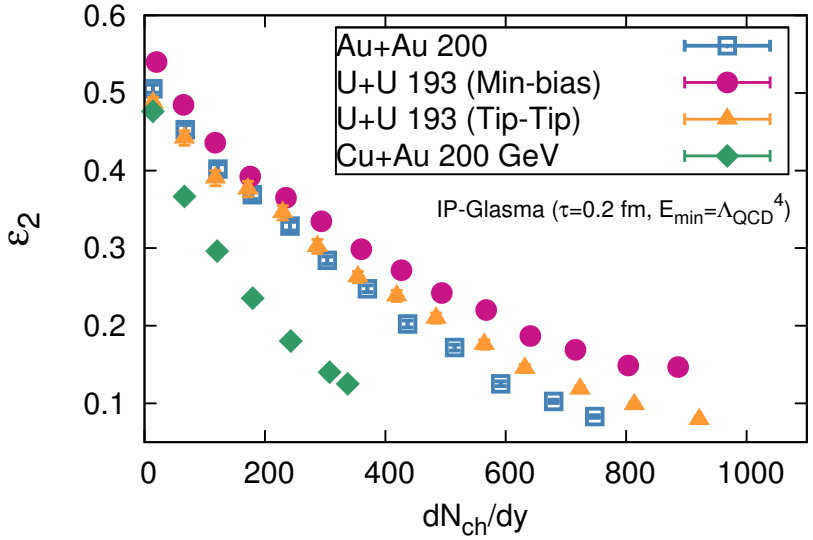
$\frac{dN_{ch}}{dy} \sim \frac{Q_s^2 S_\perp}{\alpha_S(Q_s^{max})}$, for Tip-Tip U+U, $Q_s^2 \uparrow$ but $S_\perp \downarrow$

Eccentricity for different systems

The spatial eccentricities that characterize the geometry

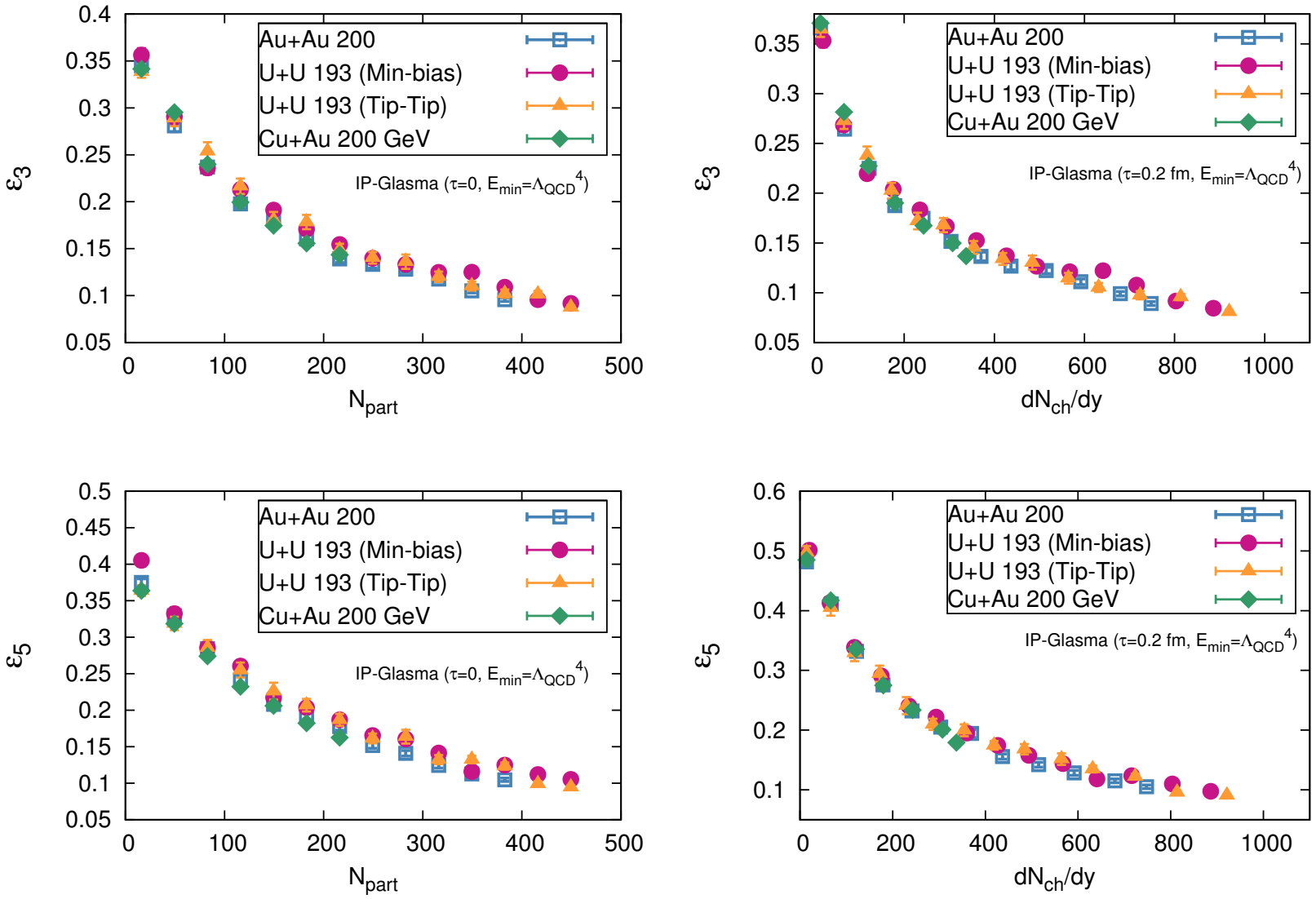
$$\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}, \ \langle \cdots \rangle \to \text{weight}\ \epsilon(\mathbf{x}_\perp, \tau)$$





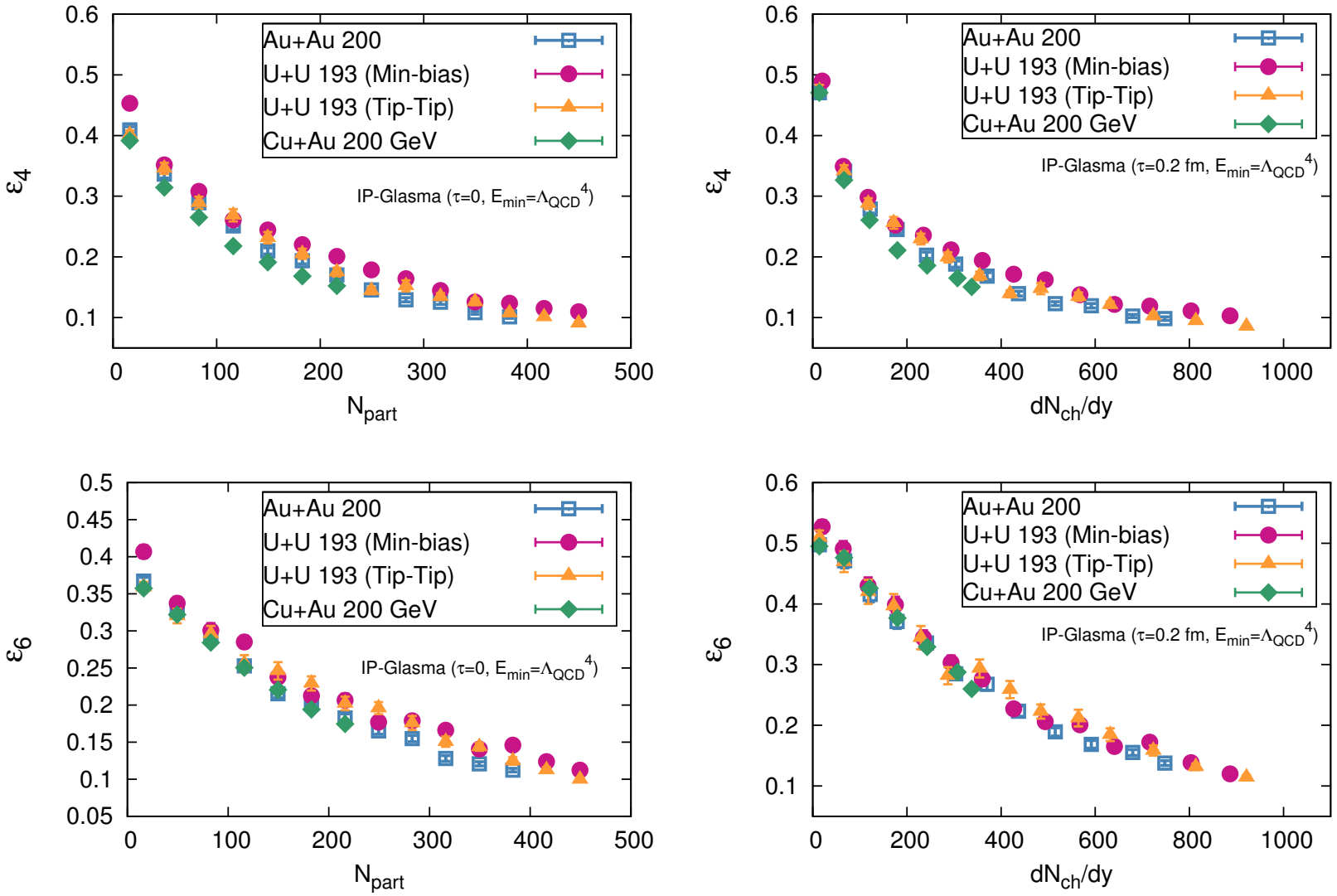
- ε_2 very sensitive to initial geometry of colliding system.
- larger transverse size $\to$ higher ε_2 for most central events.
- Tip-tip for U+U $\Leftrightarrow$ Au+Au

Higher order odd moments ϵ_3, ϵ_5

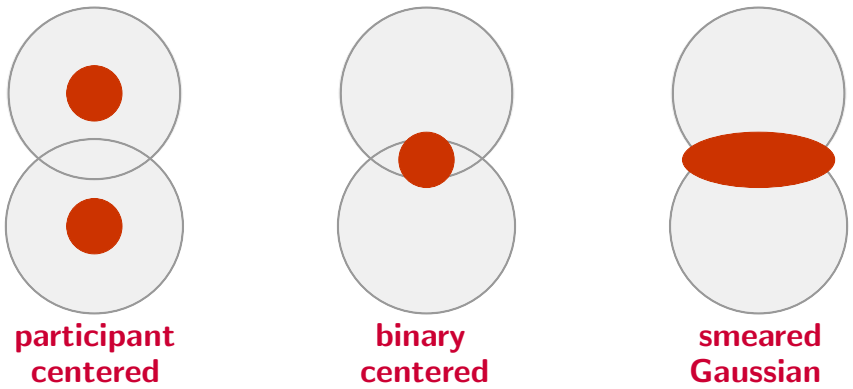


Fluctuation driven moments are very similar for different systems.

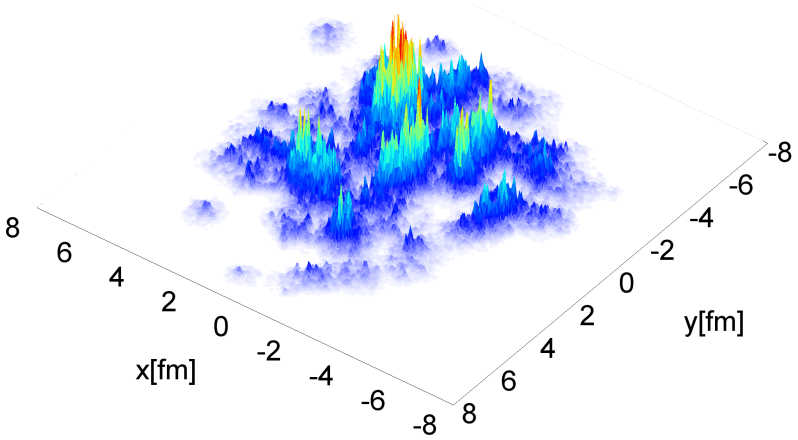
Higher order even moments ϵ_4, ϵ_6



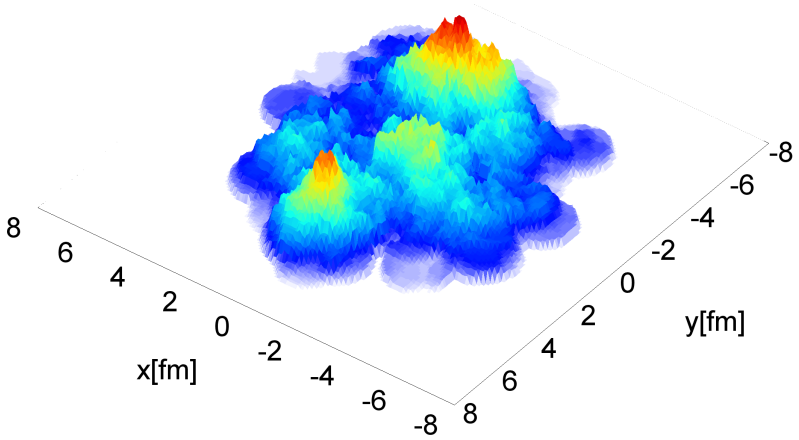
Comparison of U+U results with other models



MC-Glauber used implementation → participant centered
IP-Glasma → ϵ=0 outside overlap due to pure gauge fields.

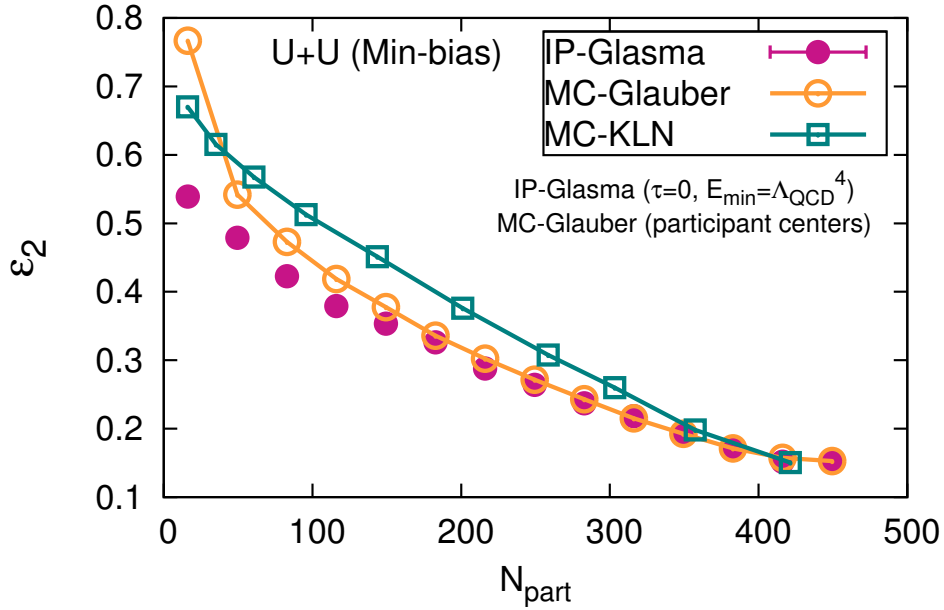


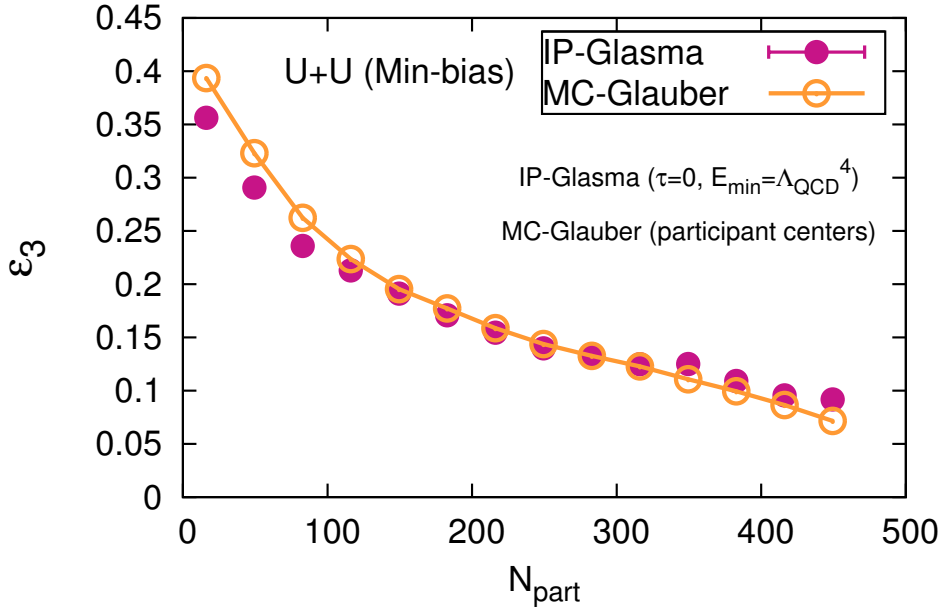
IP-Glasma
(Classical Yang-Mills)



MC-KLN
($k_\perp$-factorization)

Comparison of U+U results with other models





- Similar ε_2 for all models at large N_{part}.
- MC-KLN gives largest ε_2 for lower N_{part}.
- Fluctuation driven ε_3 is very similar for Glauber and IP-Glasma

MC-KLN : Hirano *et al* PRC 83, 021902(R) (2011)

Summary and outlook

- Initial state geometry and fluctuations studied in asymmetric systems using IP-Glasma model.
- Eccentricities are sensitive to system size and collision geometry.
- Fluctuation driven moments of ε_n are similar for different systems.
- Full hydro simulation with IP-Glasma initial conditions will be studied in future.