# Gluon Saturation

Yuri Kovchegov
The Ohio State University

# Outline

- Introduction: recapping the main principles of saturation physics.

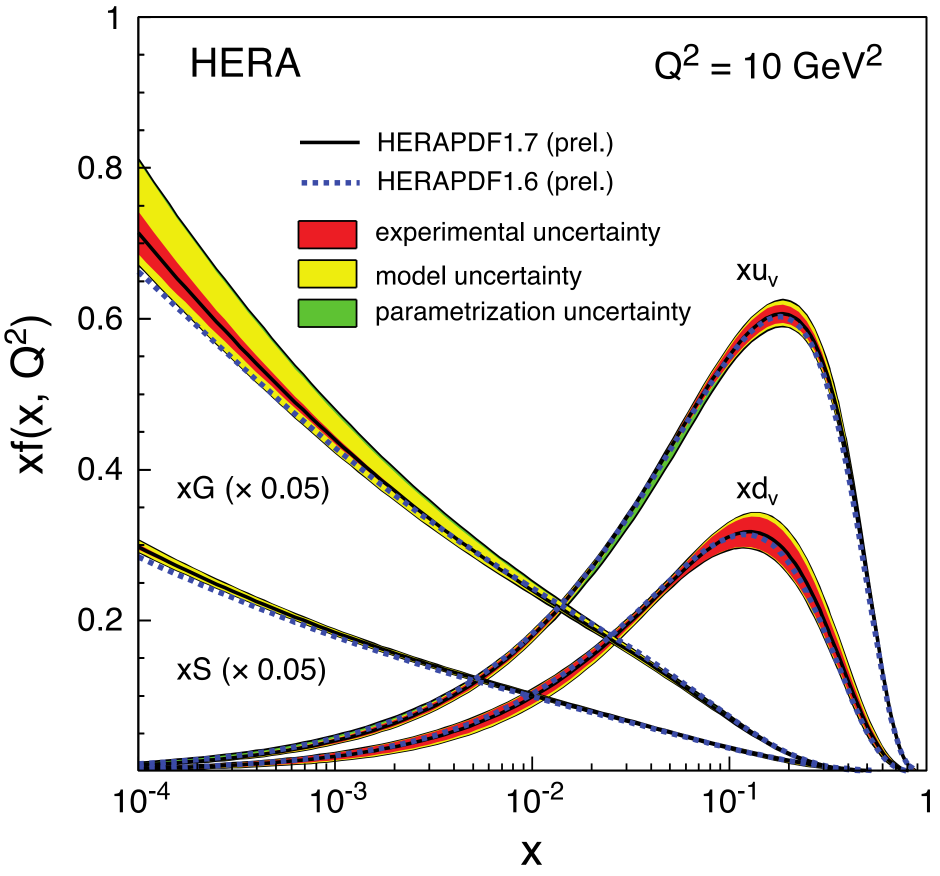
- Status report: a brief review of recent progress.

- EIC measurements needed to discover gluon saturation.

# Introduction

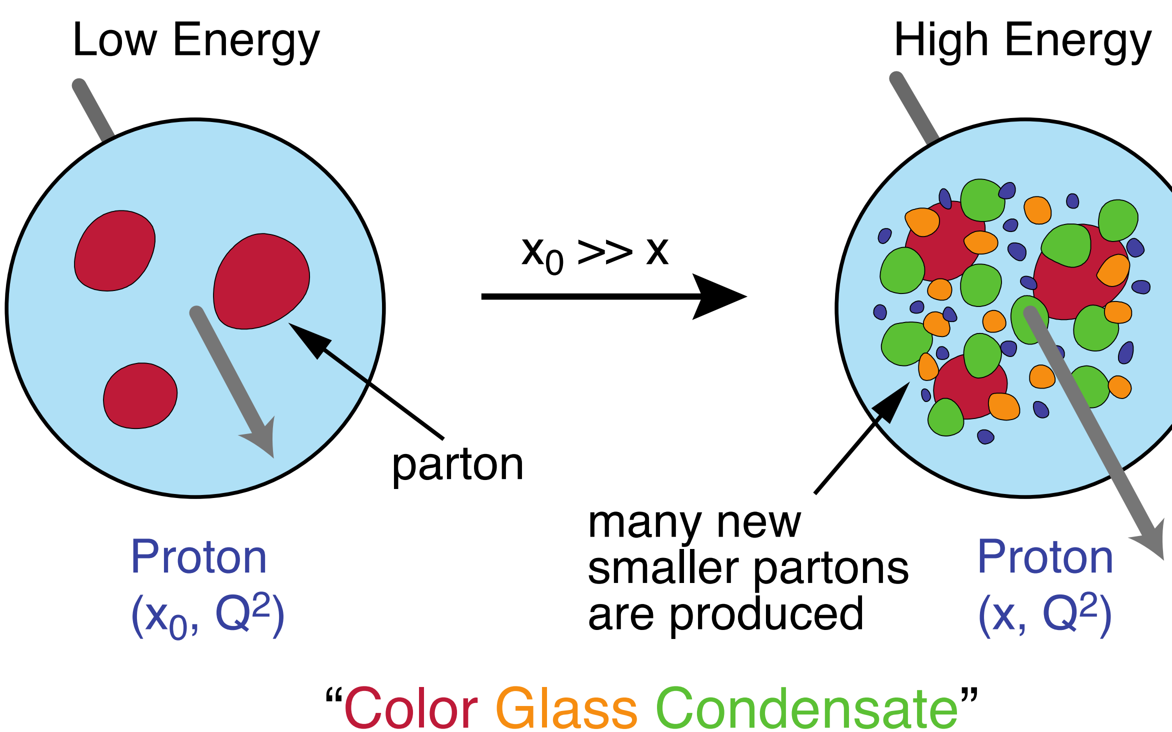
# Gluons at Small-x

- There is a large number of small-x gluons (and quarks) in a proton:



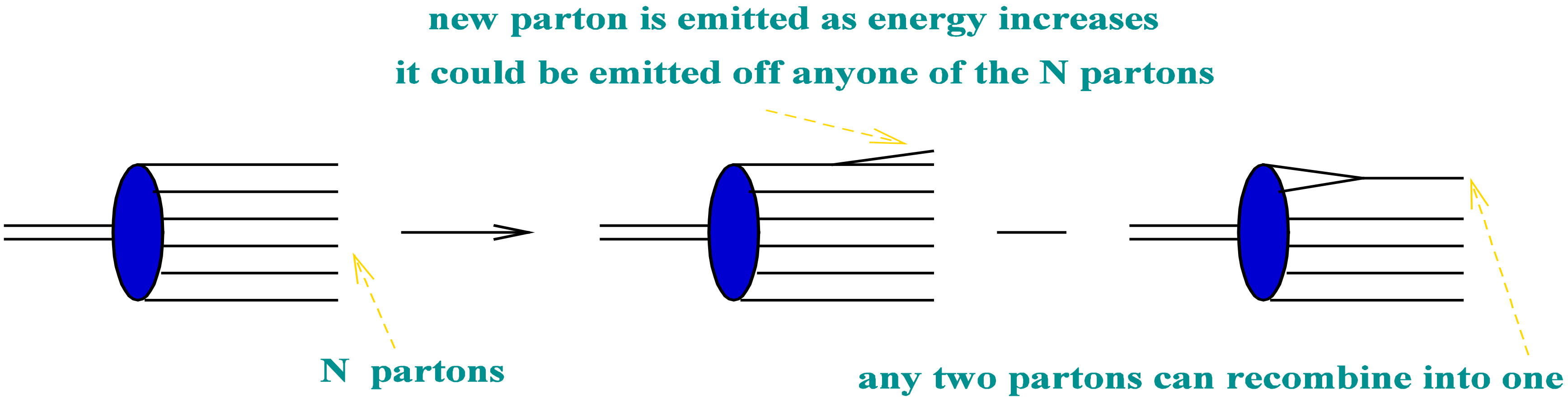
# High Density of Gluons

- High number of gluons populates the transverse extend of the proton or nucleus, leading to a very dense saturated wave function known as the Color Glass Condensate (CGC):



# Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.




$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s K_{BFKL} \otimes N(x, k_T^2) - \alpha_s [N(x, k_T^2)]^2$$

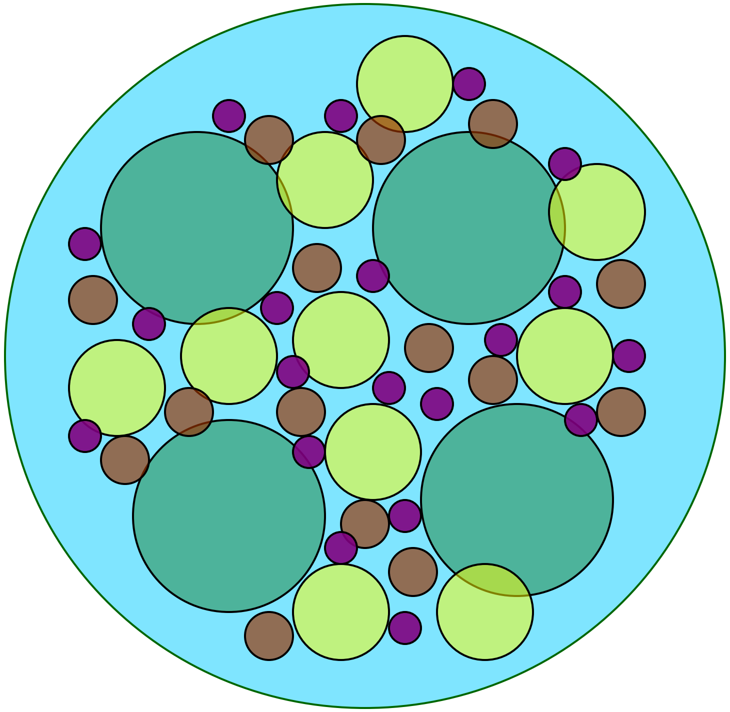
Number of parton pairs ~ $N^2$

I. Balitsky '96 (effective Lagrangian)
Yu. K. '99 (large $N_c$ QCD)
JIMWLK '98-'01 (beyond large-$N_c$)

# Nonlinear Evolution at Work

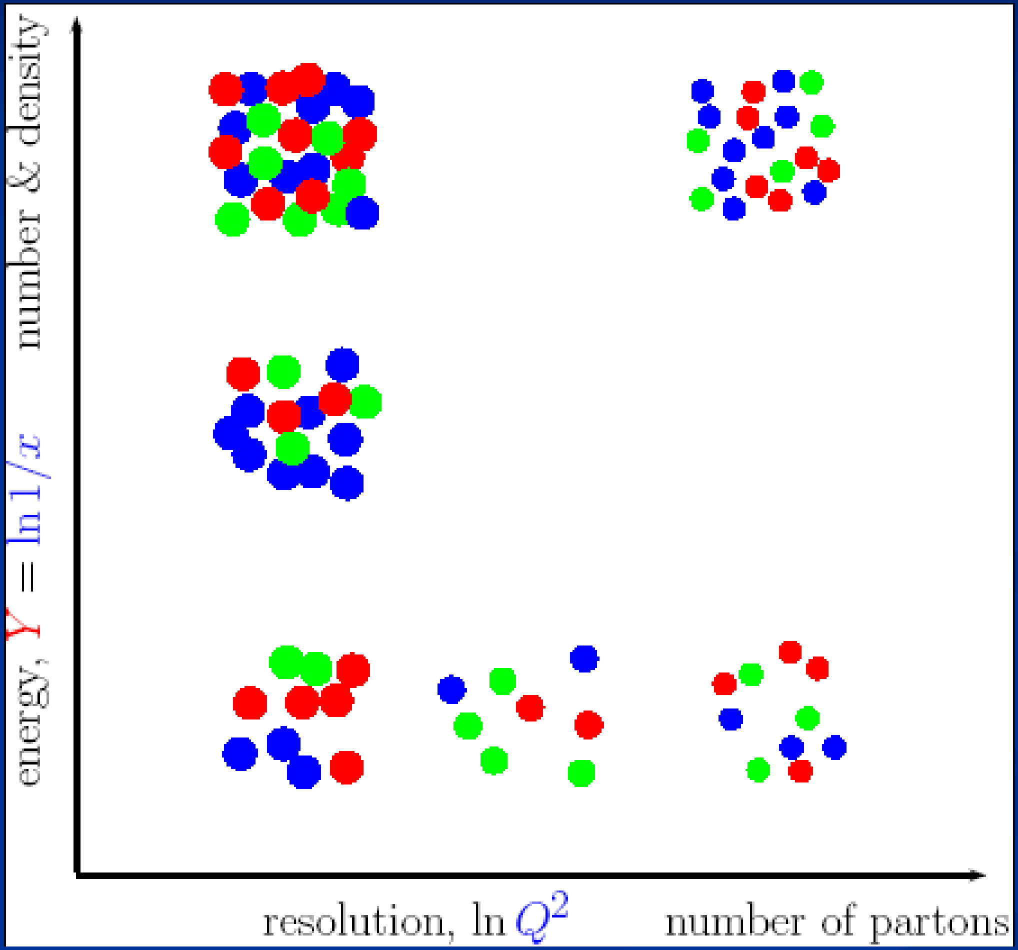
✓ First partons are produced overlapping each other, all of them about the same size.

✓ When some critical density is reached no more partons of given size can fit in the wave function. The proton starts producing smaller partons to fit them in.




Color Glass Condensate

# Map of High Energy QCD

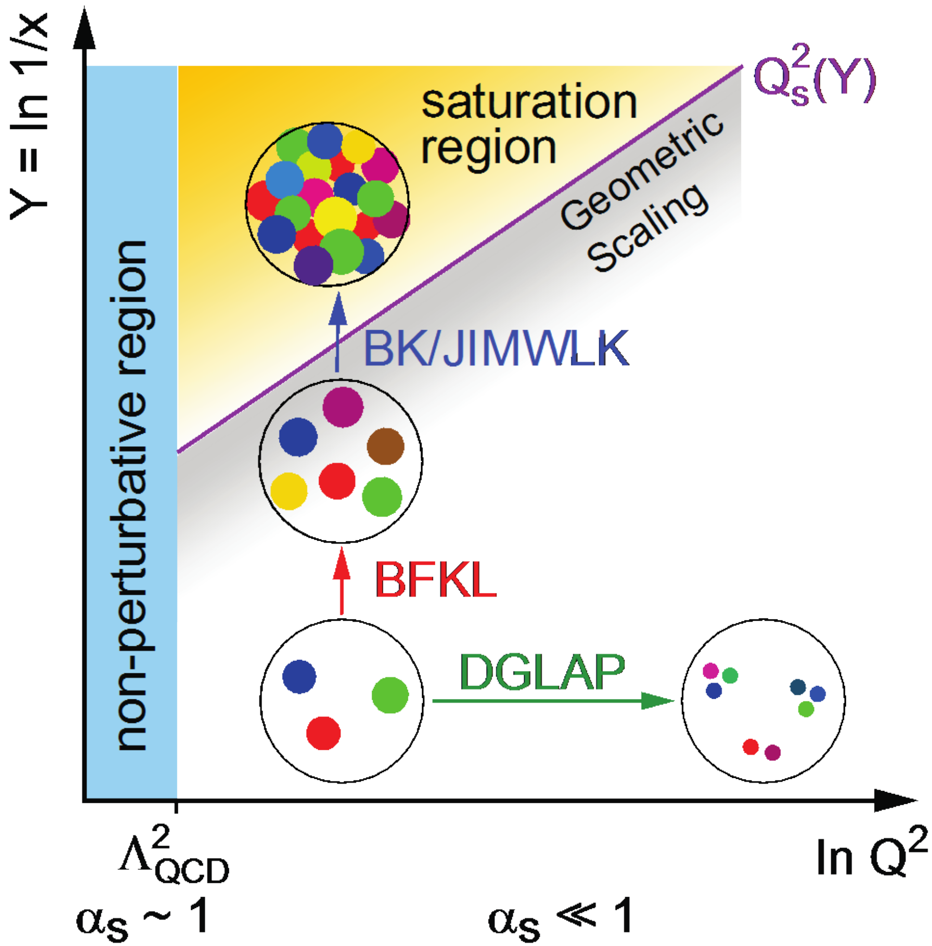



energy

size of gluons

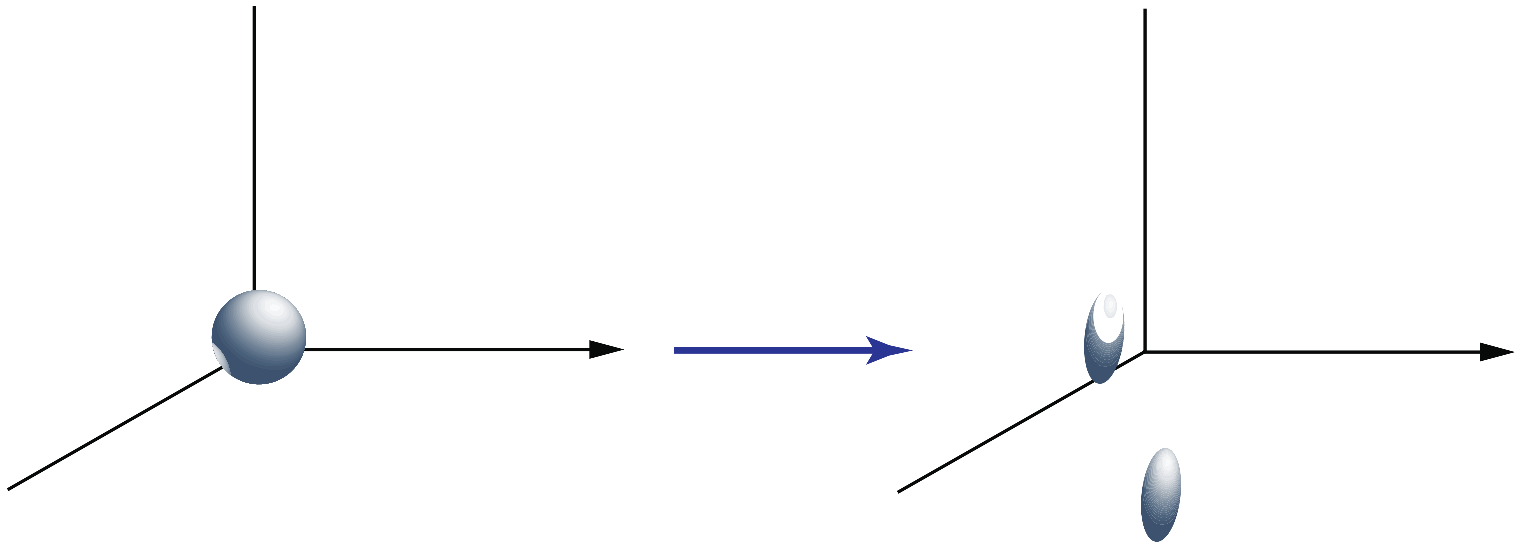
# Map of High Energy QCD

energy




size of gluons

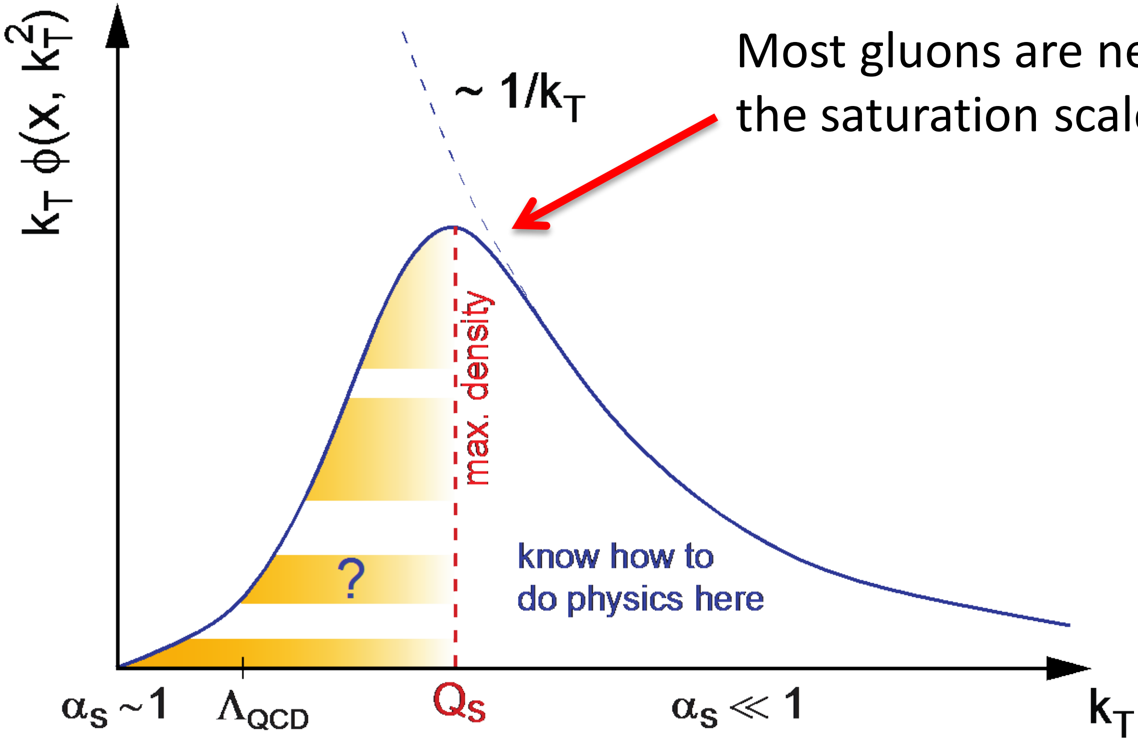
# McLerran-Venugopalan Model



- Large parton density gives a large momentum scale $Q_s$ (the saturation scale): $Q_s^2$ ~ # partons per unit transverse area.
- For $Q_s >> \Lambda_{QCD}$, get a theory at weak coupling $\alpha_s(Q_s^2) \ll 1$
- The leading gluon field is <u>classical</u>.

# Typical gluon "size"

Number of gluons (gluon TMD) times the phase space




Most gluons are near the saturation scale $Q_s$.

Gluon "size" = 1/transverse momentum

= $1/Q_s$

momentum transverse to the beam

# High Energy QCD: saturation physics

- The nonlinear BK/JIMWLK equations and the MV model lead to a large internal momentum scale $Q_S$ which grows with both energy s and nuclear atomic number A

$$Q_S^2 \sim A^{1/3} \; s^{\lambda}$$

such that

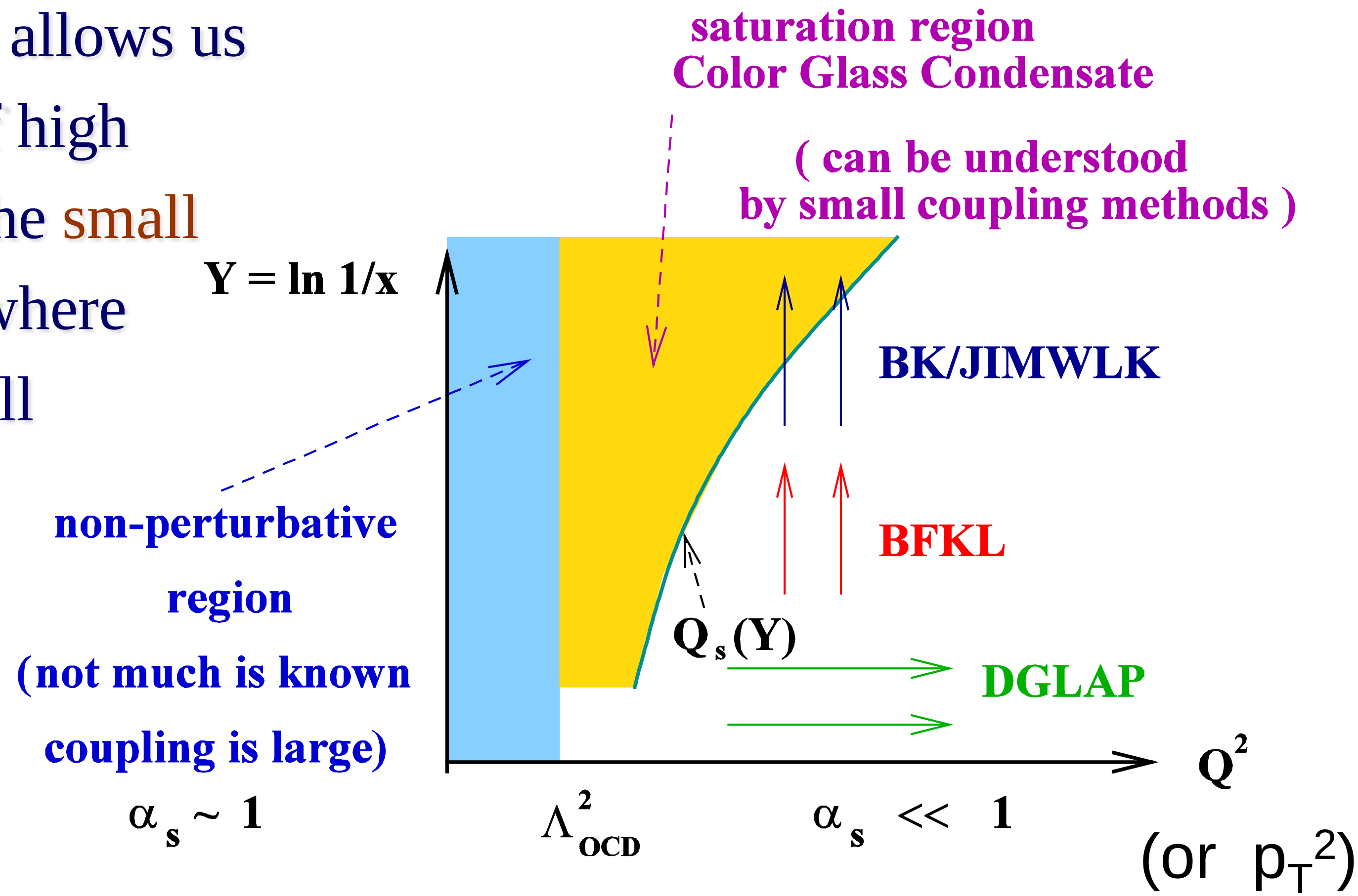
$$\alpha_S = \alpha_S(Q_S) << 1$$

and we can calculate total cross sections, particle multiplicities, etc, from <u>first principles</u>.

- Bottom line: everything is weakly-coupled, Feynman diagrams work! But: the system is dense and physics is nonlinear!

# Map of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!




Transition to saturation region is characterized by the saturation scale

$$Q_S^2 \sim A^{1/3}\left(\frac{1}{x}\right)^{\lambda}$$

# Recent Developments in the Saturation Physics

# Summary of Recent Progress

- Running coupling corrections to BK and JIMWLK evolutions have been calculated (YK, Weigert '06; Balitsky '06).
  - allowed for impressive predictions and good fits for AA and pA collisions (Albacete, Dumitru '10; Dusling, Venugopalan '12 (IP-Glamsa)) and DIS (Albacete et al '11)
- NLO BK and JIMWLK have been constructed (Balitsky, Chirilli '07; Grabovsky '13; Balitsky, Chirilli '13; Kovner, Lublinsky, Mulian '13, '14; Balitsky, Grabovsky '14).
- NLO BFKL solution has been found analytically (Chirilli, YK, '13).
- NLO corrections to particle production are being worked on (Chirilli, Xiao, Yuan '11, '12; Kang et al '14).
- Two-particle correlations have received a lot of attention. (Dumitru, Gelis, McLerran, Venugopalan '08; Dusling, Venugopalan; Kovner, Lublinsky '12; Levin and Rezaiean '12; YK, Wertepny '12, 13)

# What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) N(x_2, x_1, Y)]$$

$\alpha_S(???)$

In order to perform consistent calculations it is important to know the scale of the running coupling constant in the evolution equation.

There are three possible scales – the sizes of the "parent" dipole and "daughter" dipoles $x_{01}, x_{21}, x_{20}$ . Which one is it?

# Running Coupling BK

Here's the BK equation with the running coupling corrections (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{N_C}{2\pi^2} \int d^2 x_2$$

$$\times \left[ \frac{\alpha_S(1/x_{02}^2)}{x_{02}^2} + \frac{\alpha_S(1/x_{12}^2)}{x_{12}^2} - 2\, \frac{\alpha_S(1/x_{02}^2)\, \alpha_S(1/x_{12}^2)}{\alpha_S(1/R^2)} \frac{\mathbf{x}_{20} \cdot \mathbf{x}_{21}}{x_{02}^2\, x_{12}^2} \right]$$

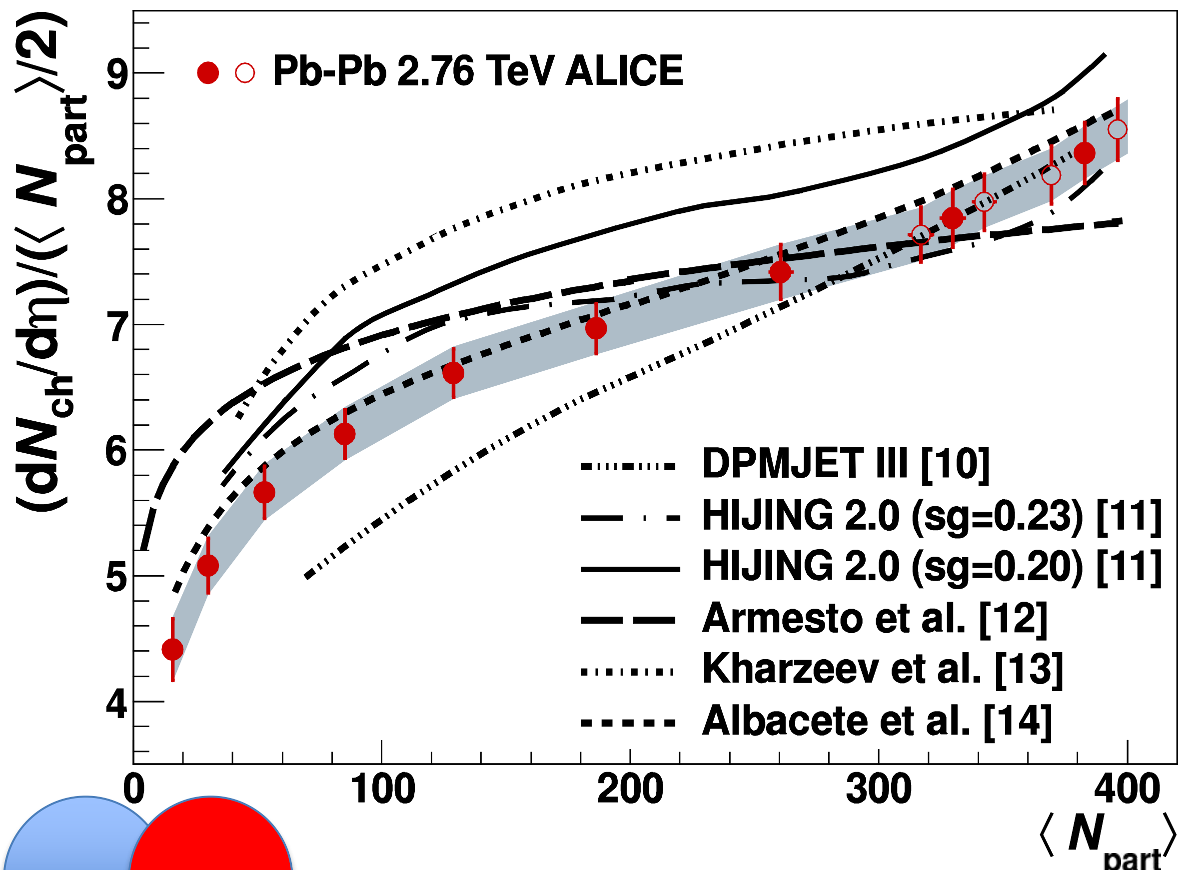
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\, N(x_2, x_1, Y)]$$

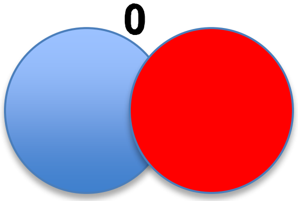
where

$$\ln R^2 \mu^2 = \frac{x_{20}^2 \ln(x_{21}^2 \mu^2) - x_{21}^2 \ln(x_{20}^2 \mu^2)}{x_{20}^2 - x_{21}^2} + \frac{x_{20}^2\, x_{21}^2}{\mathbf{x}_{20} \cdot \mathbf{x}_{21}} \frac{\ln(x_{20}^2 / x_{21}^2)}{x_{20}^2 - x_{21}^2}$$

# CGC Multiplicity Prediction

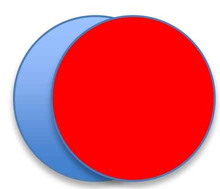
Number of hadrons
per nucleon-nucleon collision
at mid-rapidity.



CGC prediction by
Albacete and Dumitru '10
for LHC multiplicity
and its centrality
dependence was
quite successful.

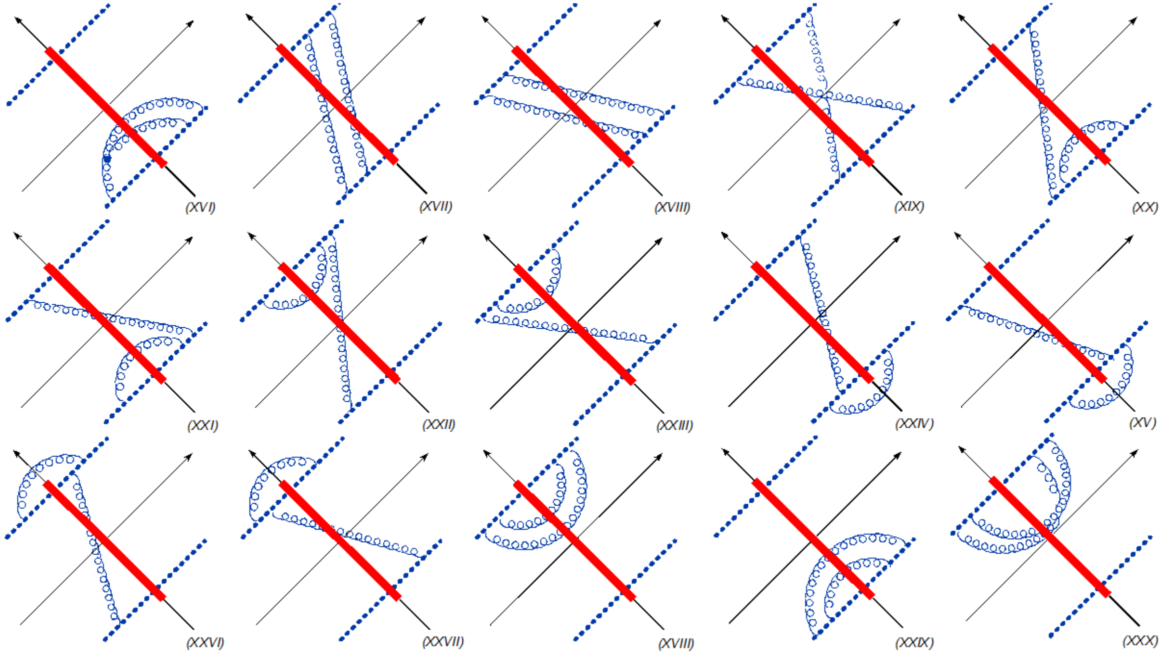
centrality



# NLO BK

- NLO BK evolution was calculated by Balitsky and Chirilli in 2007.
- It resums powers of $\alpha_s^2 Y$ (NLO) in addition to powers of $\alpha_s Y$ (LO).
- Here's a sampler of relevant diagrams (need kernel to order-$\alpha^2$):

Diagrams with 2 gluons interaction



# NLO BK

- The large-$N_C$ limit of evolution is

$$\frac{d}{d\eta}N(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int d^2z \frac{(x-y)^2}{X^2Y^2}\left\{1 + \frac{\alpha_s N_c}{4\pi}\left[\frac{11}{3}\ln(x-y)^2\mu^2 - \frac{11}{3}\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - 2\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\right]\right\}$$
$$\times [N(x,z) + N(z,y) - N(x,y) - N(x,z)N(z,y)]$$
$$+ \frac{\alpha_s^2 N_c^2}{8\pi^4}\int d^2z d^2z' \left\{-\frac{2}{(z-z')^4} + \left[\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)} + \frac{(x-y)^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)}\right.\right.$$
$$\left.\left. + \frac{(x-y)^2}{X^2Y'^2(z-z')^2}\right]\ln\frac{X^2Y'^2}{X'^2Y^2}\right\}[N(z,z') - N(x,z)N(z,z') - N(z,z')N(z',y) - N(x,z)N(z',y) + N(x,z)N(z,y)$$
$$+ N(x,z)N(z,z')N(z',y)]. \qquad (136)$$

(yet to be solved numerically)

# NLO JIMWLK

- Very recently NLO evolution has been calculated for other Wilson line operators (not just dipoles described by BK), most notably the 3-Wilson line operator (Grabovsky ‘13, Balitsky & Chirilli ’13, Kovner, Lublinsky, Mulian ’13, Balitsky and Grabovsky ‘14).

- The NLO JIMWLK Hamiltonian was constructed as well (Kovner, Lublinsky, Mulian ’13, ’14).

- However, the equations do not close, that is, the operators on the right hand side can not be expressed in terms of the operator on the left. Hence can’t solve.

- To find the expectation values of the corresponding operators, one has to perform a lattice calculation with the NLO JIMWLK Hamiltonian, generating field configurations to be used for averaging the operators.

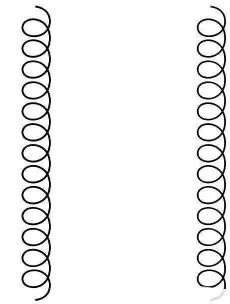
# NLO Dipole Evolution at any $N_C$

- NLO BK equation is the large-$N_C$ limit of (Balitsky and Chrilli '07)

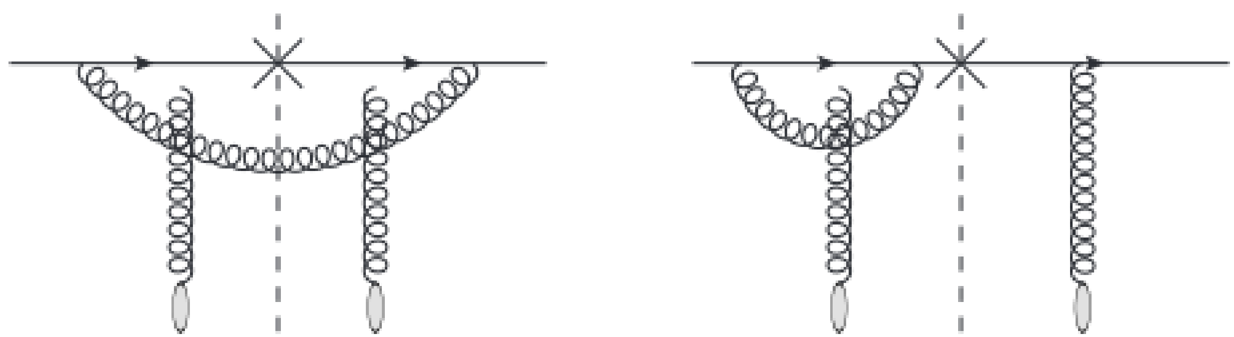
$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\} \qquad (5)\\
&= \frac{\alpha_s}{2\pi^2}\int d^2z\,\frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+\big(\frac{67}{9}-\frac{\pi^2}{3}\big)N_c-\frac{10}{9}n_f\\
&\qquad -2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\,[\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_y^\dagger\}-N_c\mathrm{Tr}\{\hat{U}_x\hat{U}_y^\dagger\}]\\
&+\frac{\alpha_s^2}{16\pi^4}\int d^2z\,d^2z'\Big[\Big(-\frac{4}{(z-z')^4}+\Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]}\\
&+\frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big)\\
&\qquad\times[\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_{z'}^\dagger\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}_y^\dagger\}-\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\hat{U}_{z'}U_y^\dagger\hat{U}_z\hat{U}_{z'}^\dagger\}-(z'\to z)]\\
&+\Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]-\frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\,\mathrm{Tr}\{\hat{U}_x\hat{U}_z^\dagger\}\mathrm{Tr}\{\hat{U}_z\hat{U}_{z'}^\dagger\}\mathrm{Tr}\{\hat{U}_{z'}\hat{U}_y^\dagger\}\\
&+4n_f\Big\{\frac{4}{(z-z')^4}-2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2-)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\mathrm{Tr}\{t^a\hat{U}_xt^b\hat{U}_y^\dagger\}[\mathrm{Tr}\{t^a\hat{U}_zt^b\hat{U}_{z'}^\dagger\}-(z'\to z)]\Big]
\end{aligned}
$$

# Quark Production

- Simplest case: quark produced in the fragmentation region of the projectile.
- The hybrid model (Dumitru, Jalilian ‘02):



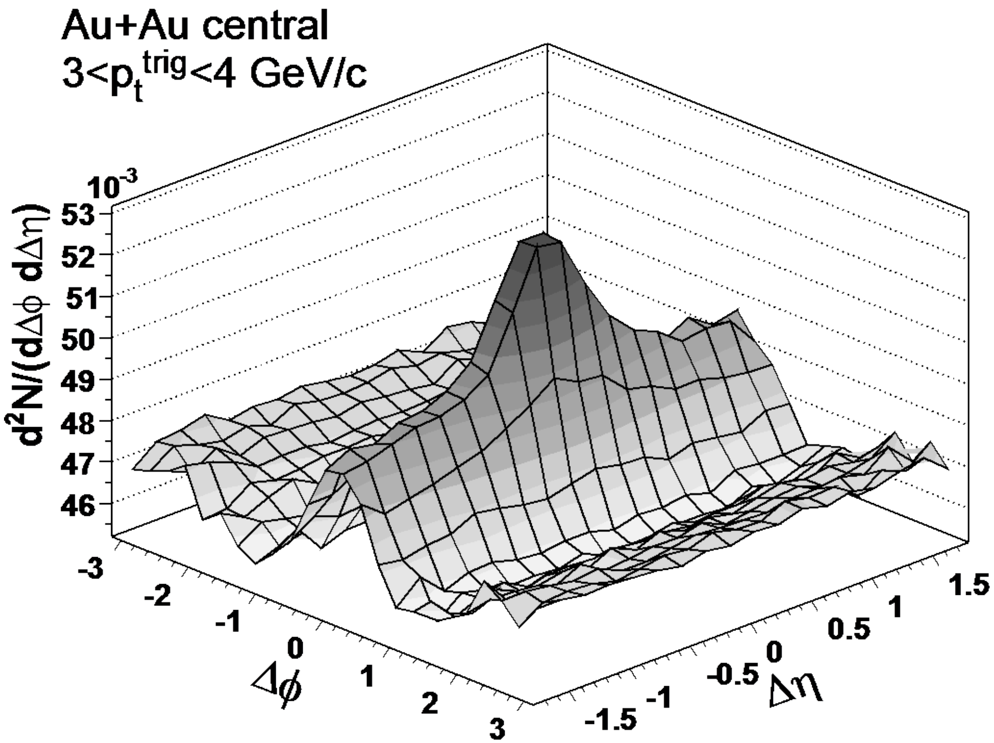
- NLO corrections were found by Chirilli, Xiao, Yuan ‘11, ’12:



- Hybrid model is not applicable at very large $p_T >> Q_S$.

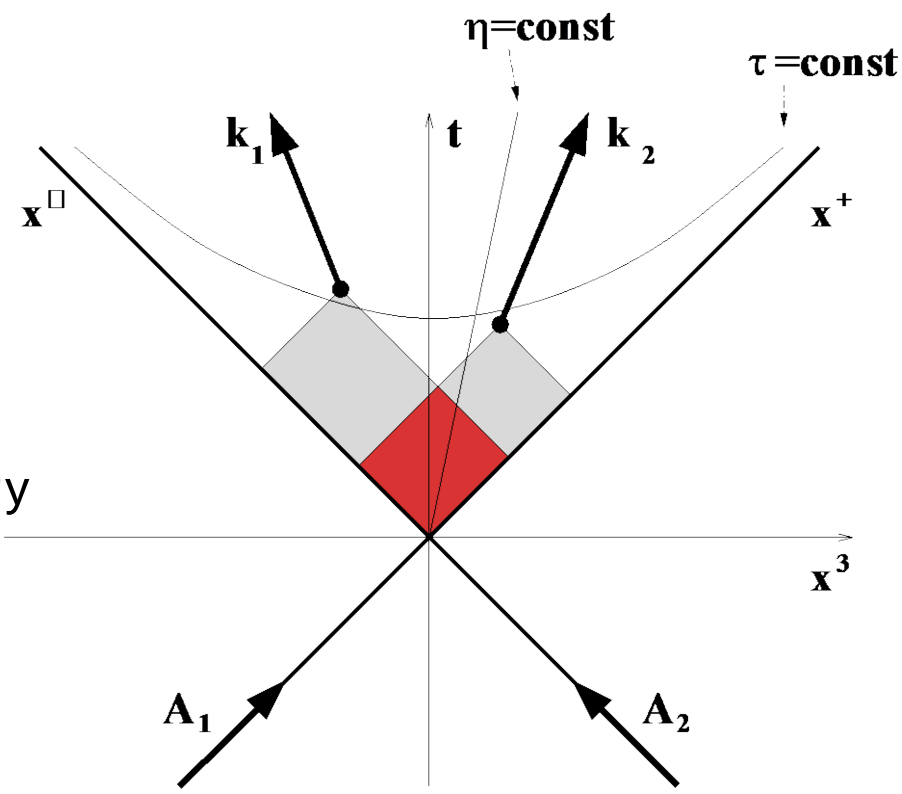
# Ridge in heavy ion collisions

- Heavy ion collisions, along with high-multiplicity p+p and p+A collisions, are known to have a long-range rapidity correlation known as 'the ridge':



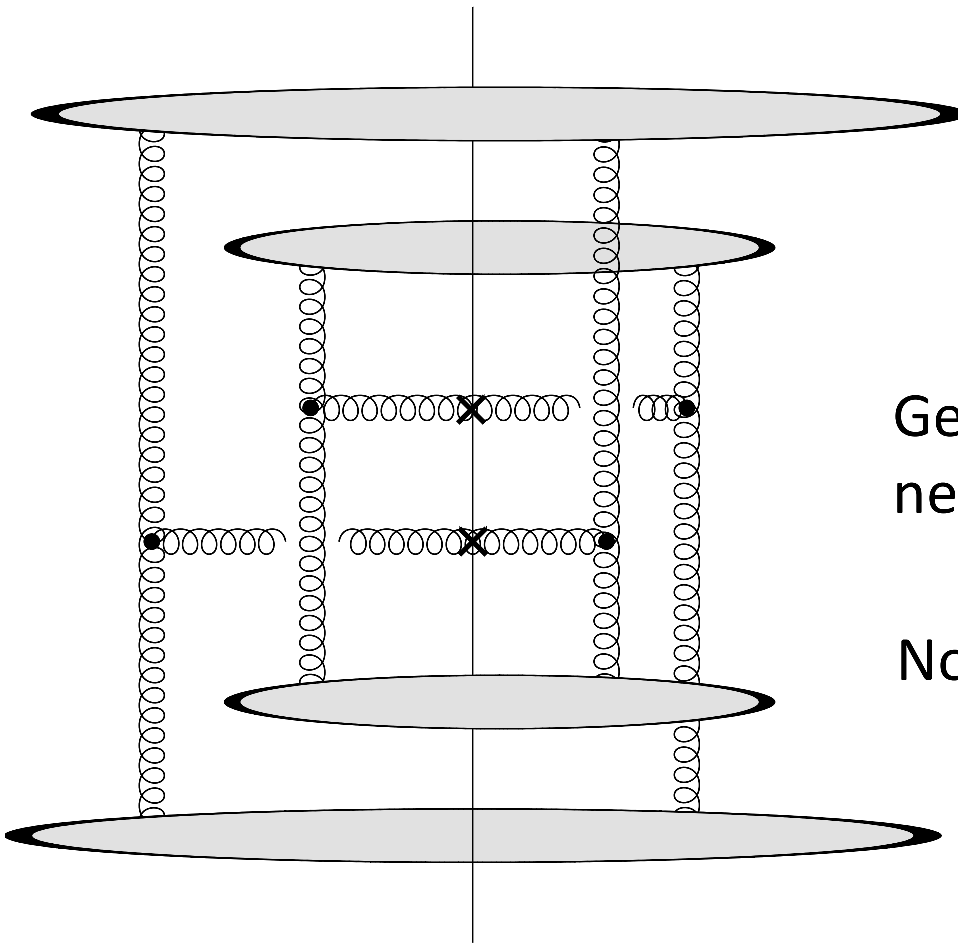
# Origin of rapidity correlations

Causality demands that long-range rapidity correlations originate at very early times (cf. explanation of the CMB homogeneity in the Universe)




Gavin, McLerran, Moschelli ‘08;
Dumitru, Gelis, McLerran, Venugopalan ‘08.

# “Glasma” graphs



Generate back-to-back and near-side azimuthal correlations.

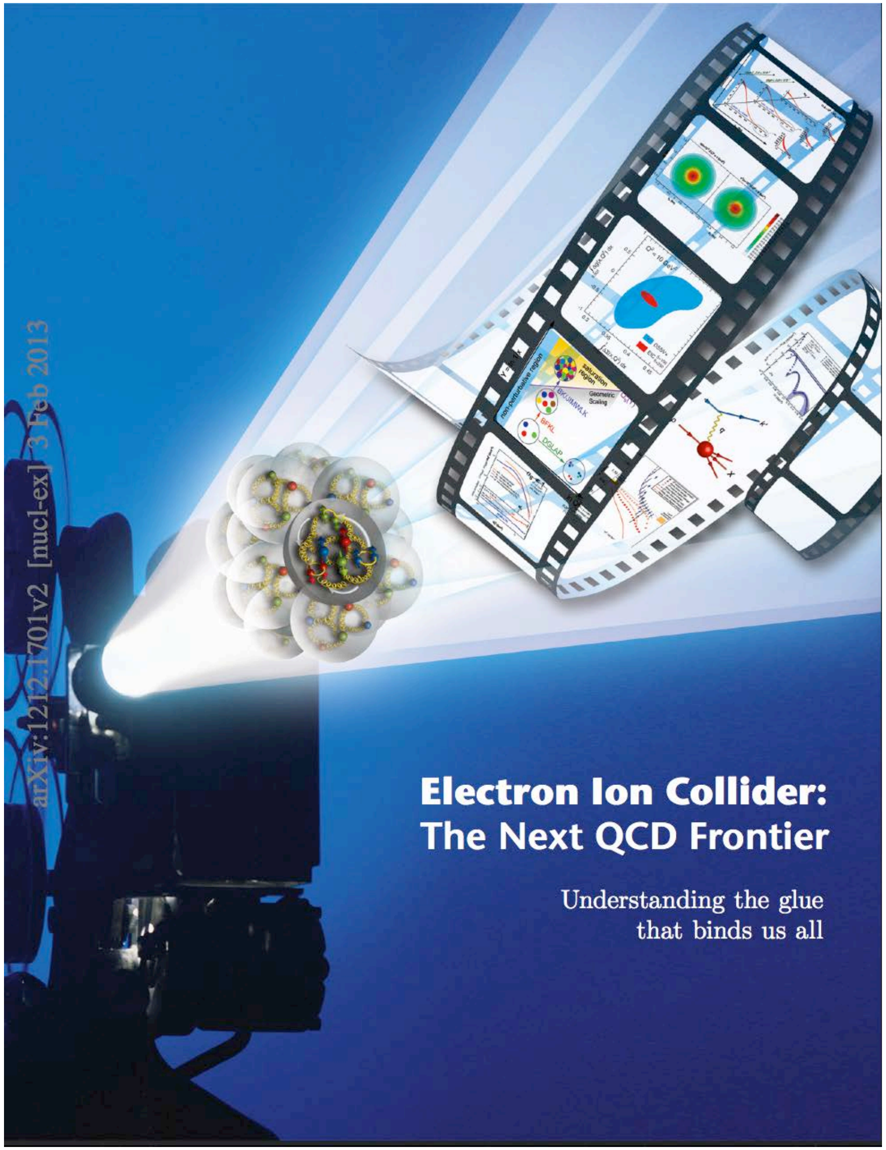
No odd harmonics though...

Dumitru, Gelis, McLerran, Venugopalan ‘08.

# Electron-Ion Collider Physics

# Electron-Ion Collider (EIC) White Paper

- EIC WP was finished in late 2012
- A several-year effort by a 19-member committee + 58 co-authors
- arXiv:1212.1701 [nucl-ex]
- EIC can be realized as eRHIC (BNL) or as ELIC (JLab)



# EIC Physics Topics

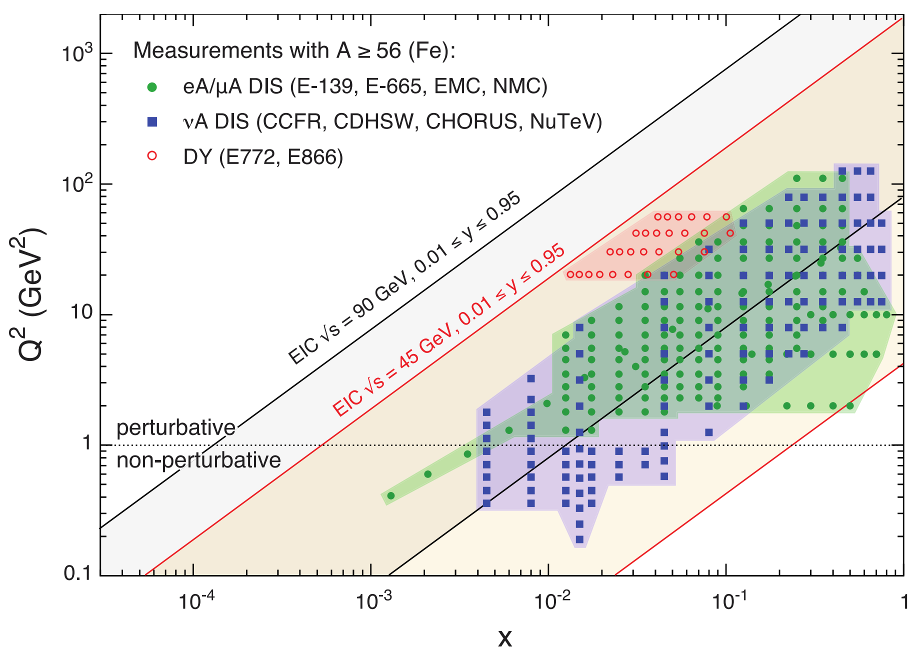
- Spin and Nucleon Structure
  - Spin of a nucleon
  - Transverse momentum distributions (TMDs)
  - Spatial imaging of quarks and gluons
- QCD Physics in a Nucleus
  - High gluon densities and saturation
  - Quarks and Gluons in the Nucleus
  - Connections to p+A, A+A, and cosmic ray physics

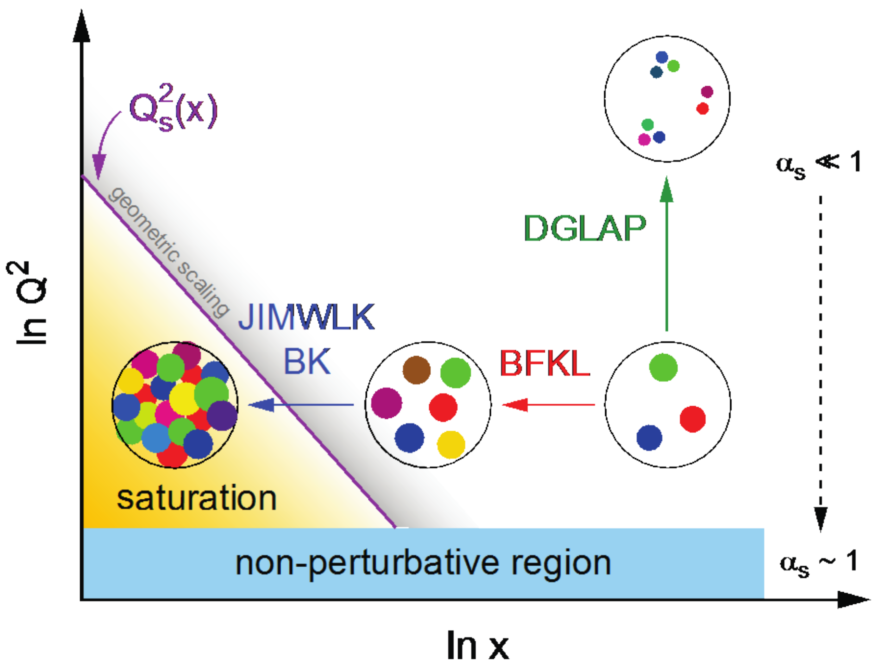
# Big Questions

- How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?
- Where does the saturation of gluon densities set it? What is the dynamics? Is it universal?
- How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?

# Can Saturation be Discovered at EIC?

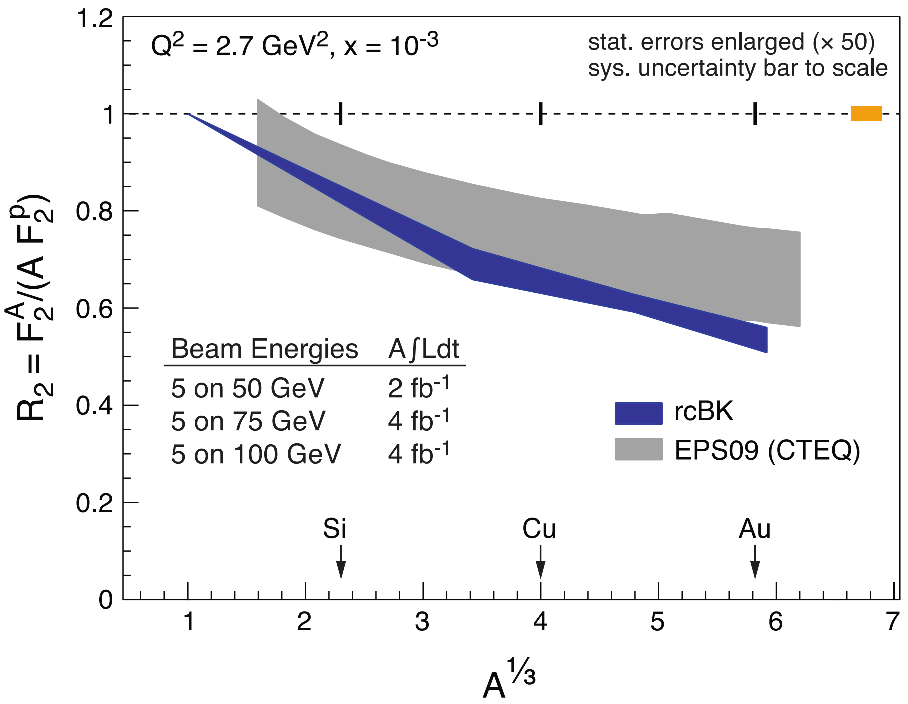
EIC would have an unprecedented small-x reach for DIS on large nuclear targets, allowing to seal the discovery of saturation physics and study of its properties:

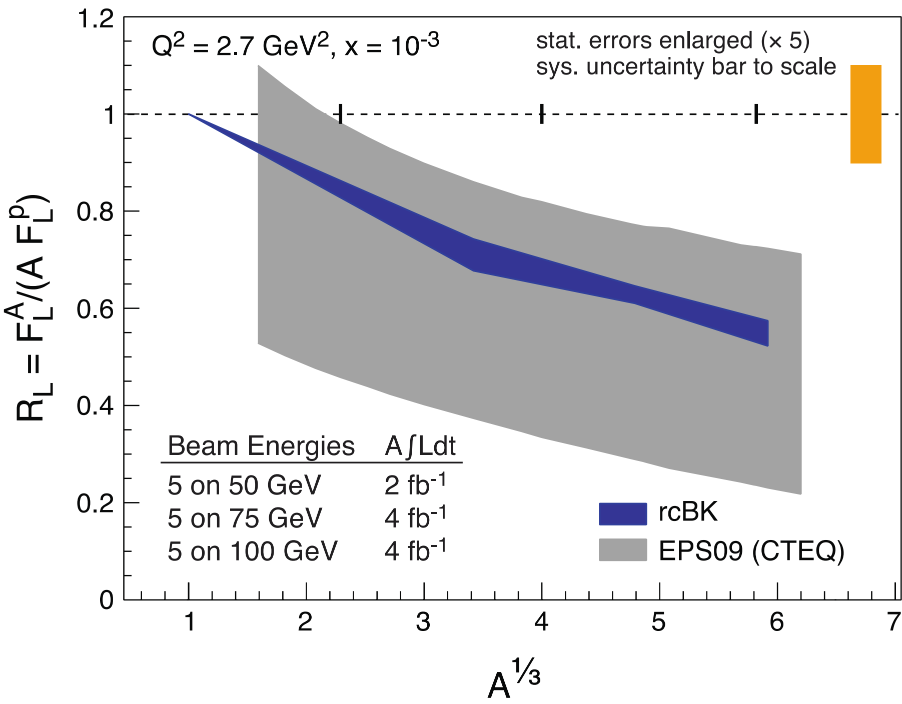


# Saturation Measurements at EIC

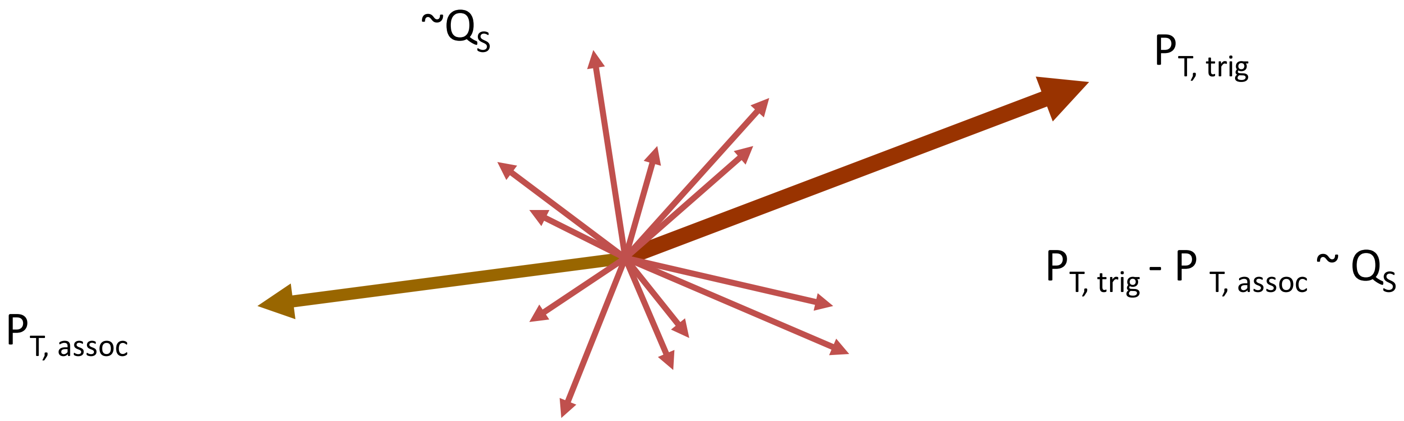
- Unlike DGLAP evolution, saturation physics predicts the x-dependence of structure functions with BK/JIMWLK equations and their A-dependence through the MV/GM initial conditions, though the difference with models for DGLAP initial conditions is modest.



# De-correlation

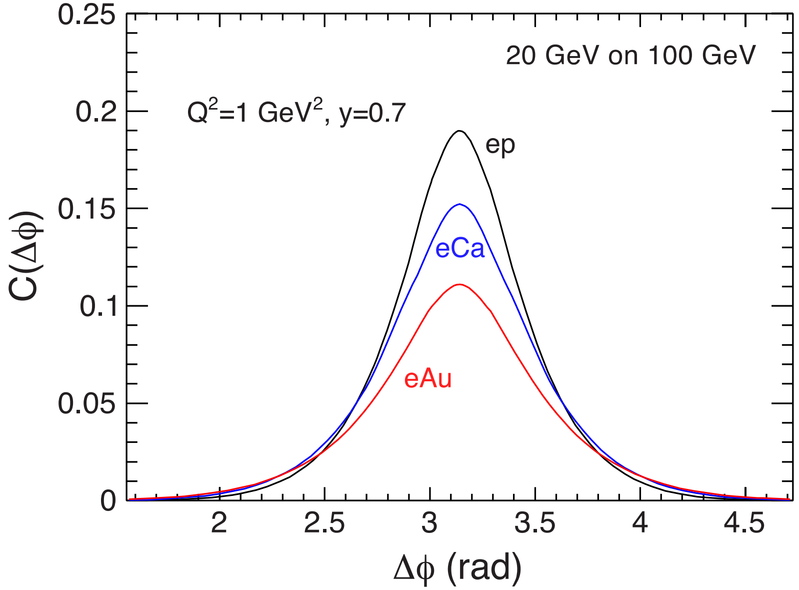
- Small-x evolution ↔ multiple emissions
- Multiple emissions → de-correlation.

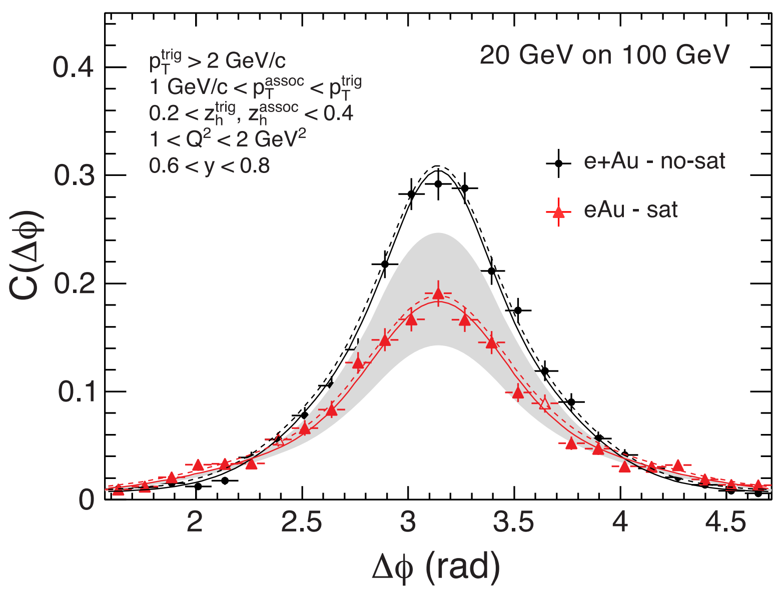



- B2B jets may get de-correlated in $p_T$ with the spread of the order of $Q_S$

# Di-hadron Correlations

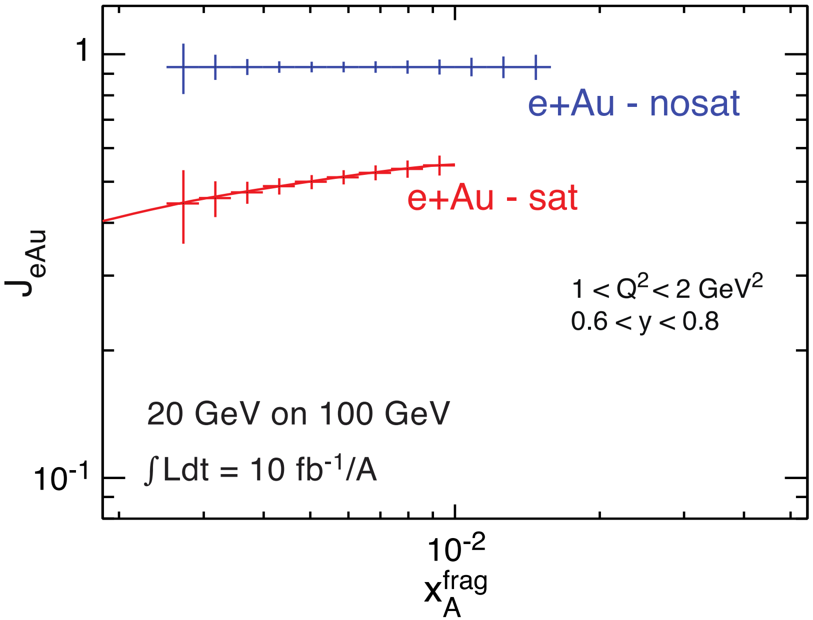
Depletion of di-hadron correlations is predicted for e+A as compared to e+p. (Domingue et al '11; Zheng et al '14)

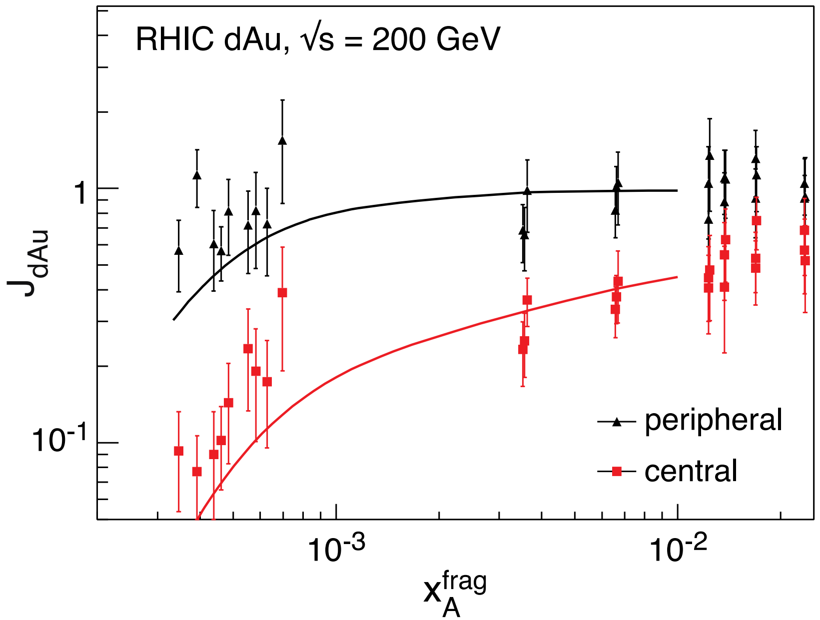


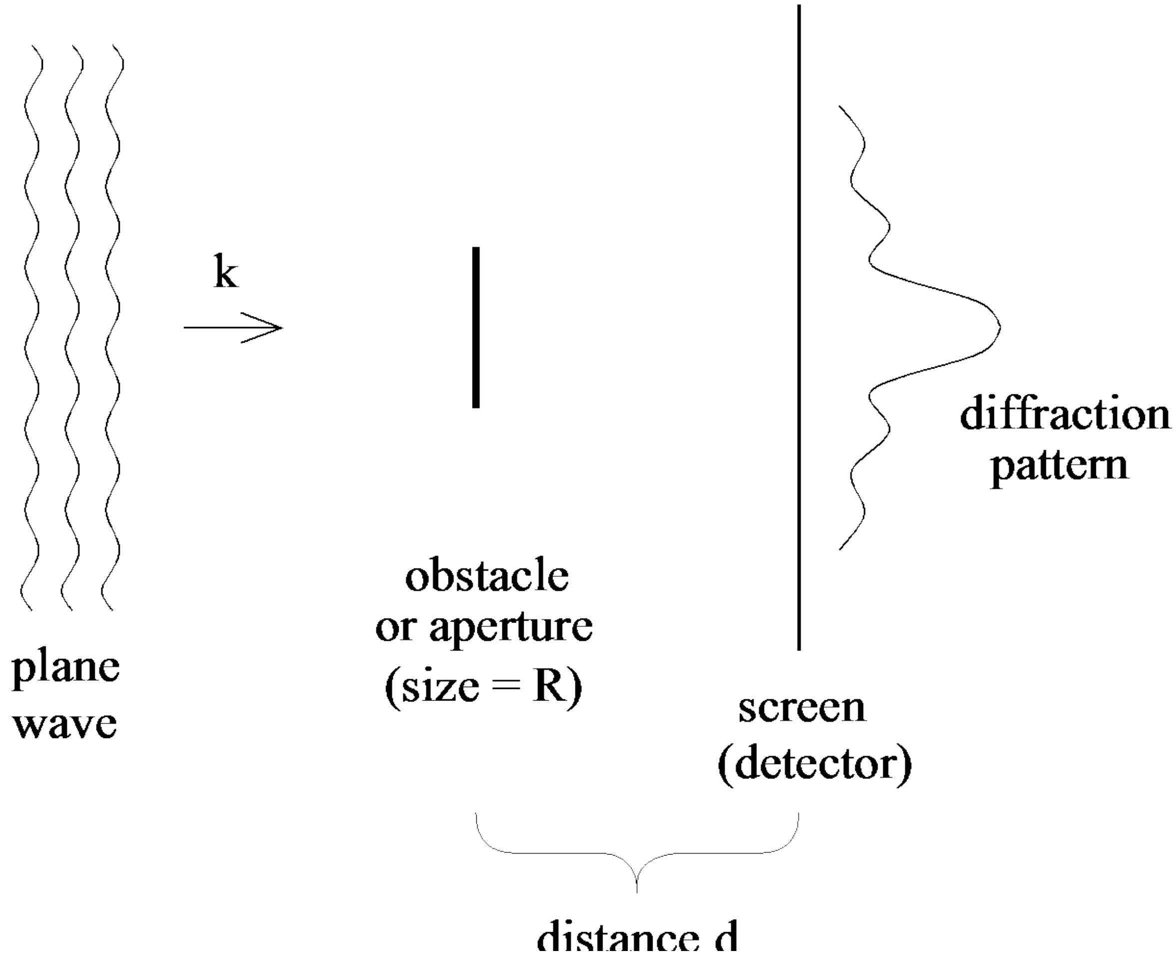
# Di-hadron Correlations

One would also expect a depletion of the away-side yield $J_{eA}$ in e+A as compared to e+p.



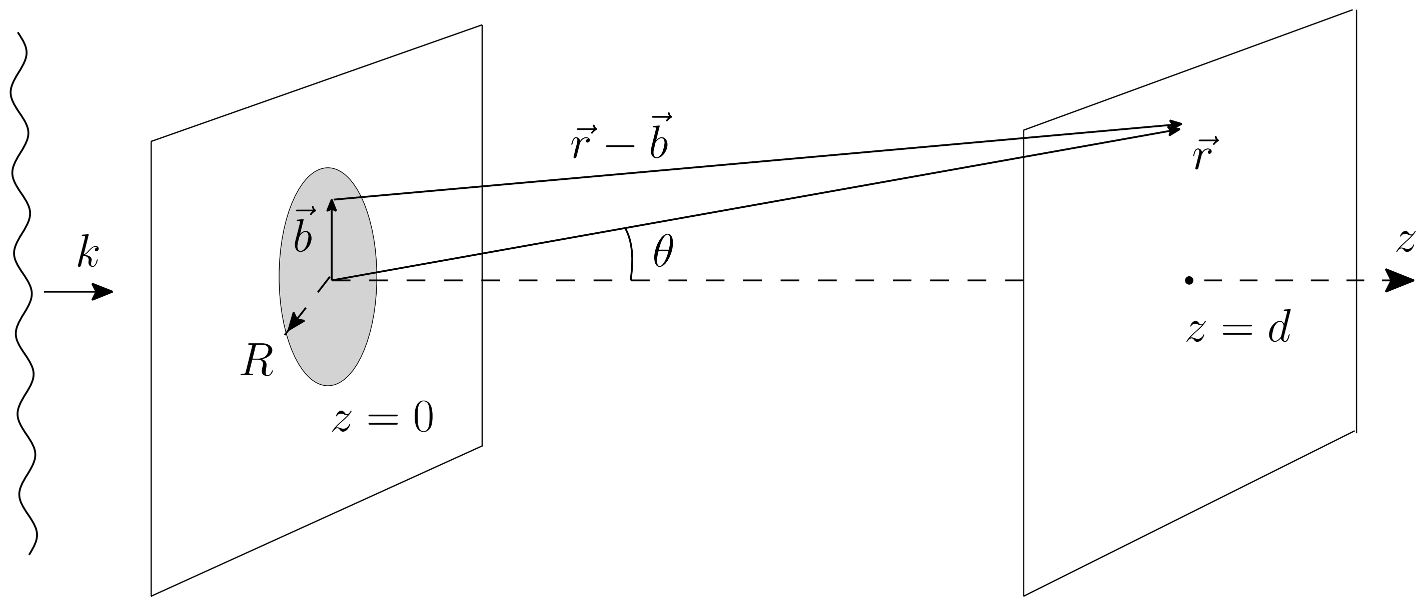



# Diffraction in optics




Diffraction pattern contains information about the size R of the obstacle and about the optical “blackness” of the obstacle.

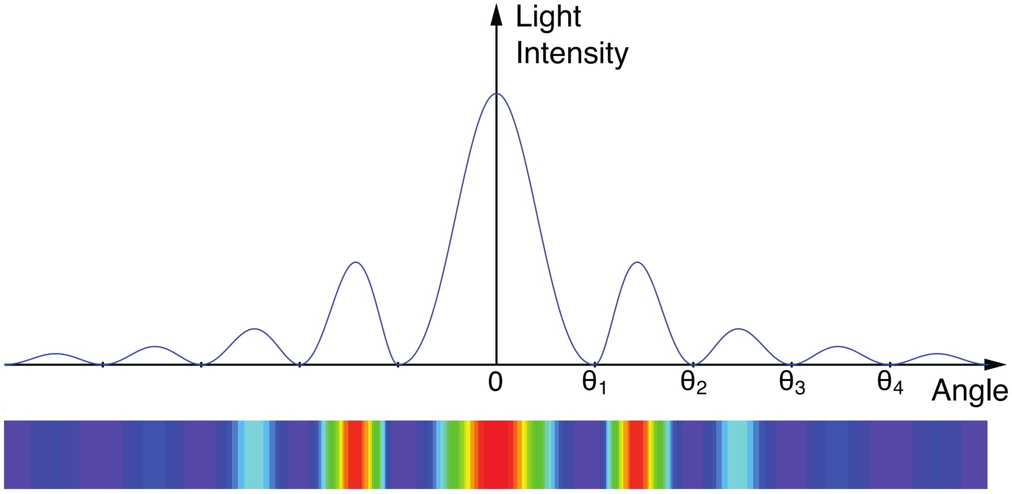
# Diffraction in optics and QCD

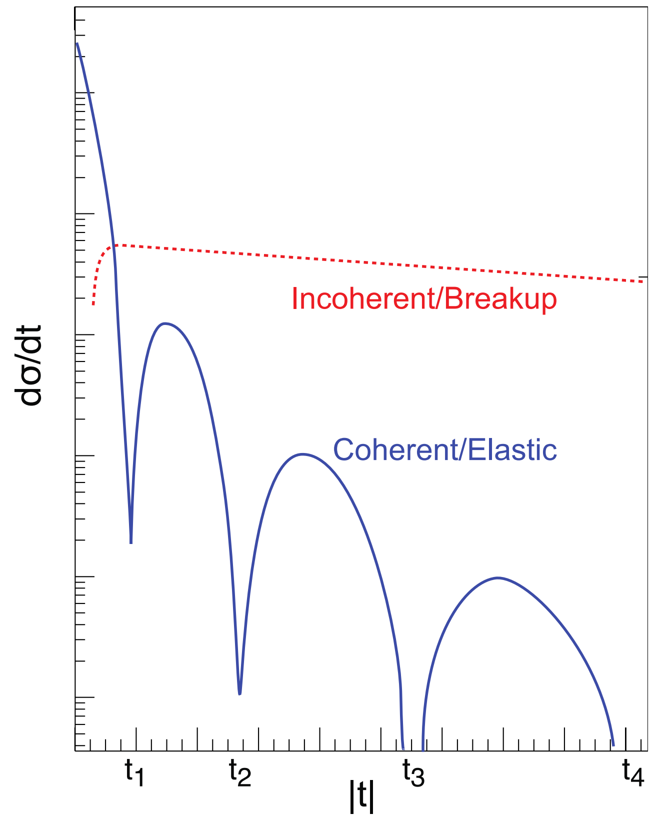



- In optics, diffraction pattern is studied as a function of the angle $\theta$.
- In high energy scattering the diffractive cross sections are plotted as a function of the Mandelstam variable $t = -(k \sin \theta)^2$.

# Optical Analogy

Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:

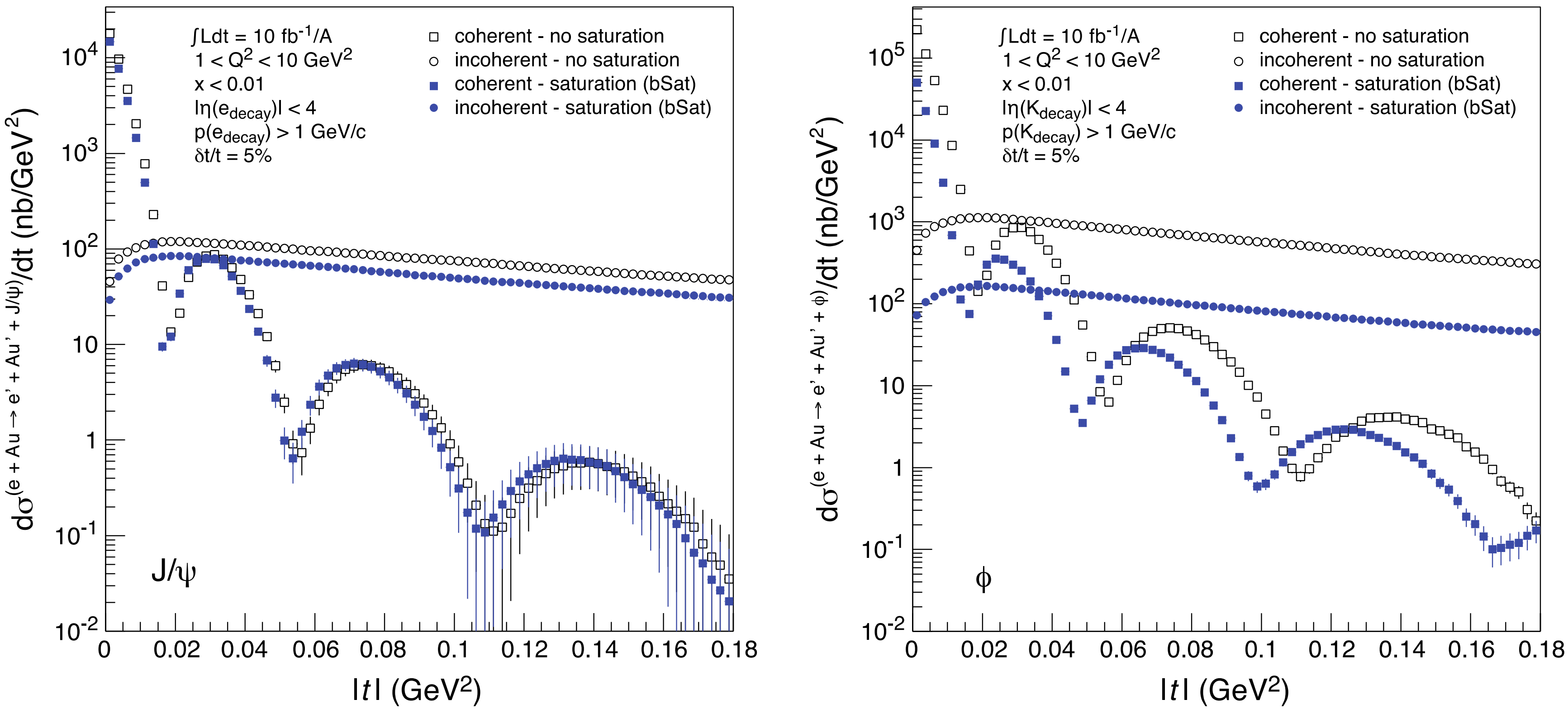



Coherent: target stays intact;

Incoherent: target nucleus breaks up, but nucleons are intact.

# Exclusive VM Production as a Probe of Saturation




Plots by T. Toll and T. Ullrich using the Sartre even generator (b-Sat (=GBW+b-dep+DGLAP) + WS + MC).

- J/psi is smaller, less sensitive to saturation effects
- Phi meson is larger, more sensitive to saturation effects
- High-energy EIC measurement (most likely)

# Diffraction on a black disk

- For low $Q^2$ (large dipole sizes) the black disk limit is reached with N=1

$$\sigma_{tot} < 2\pi R^2$$

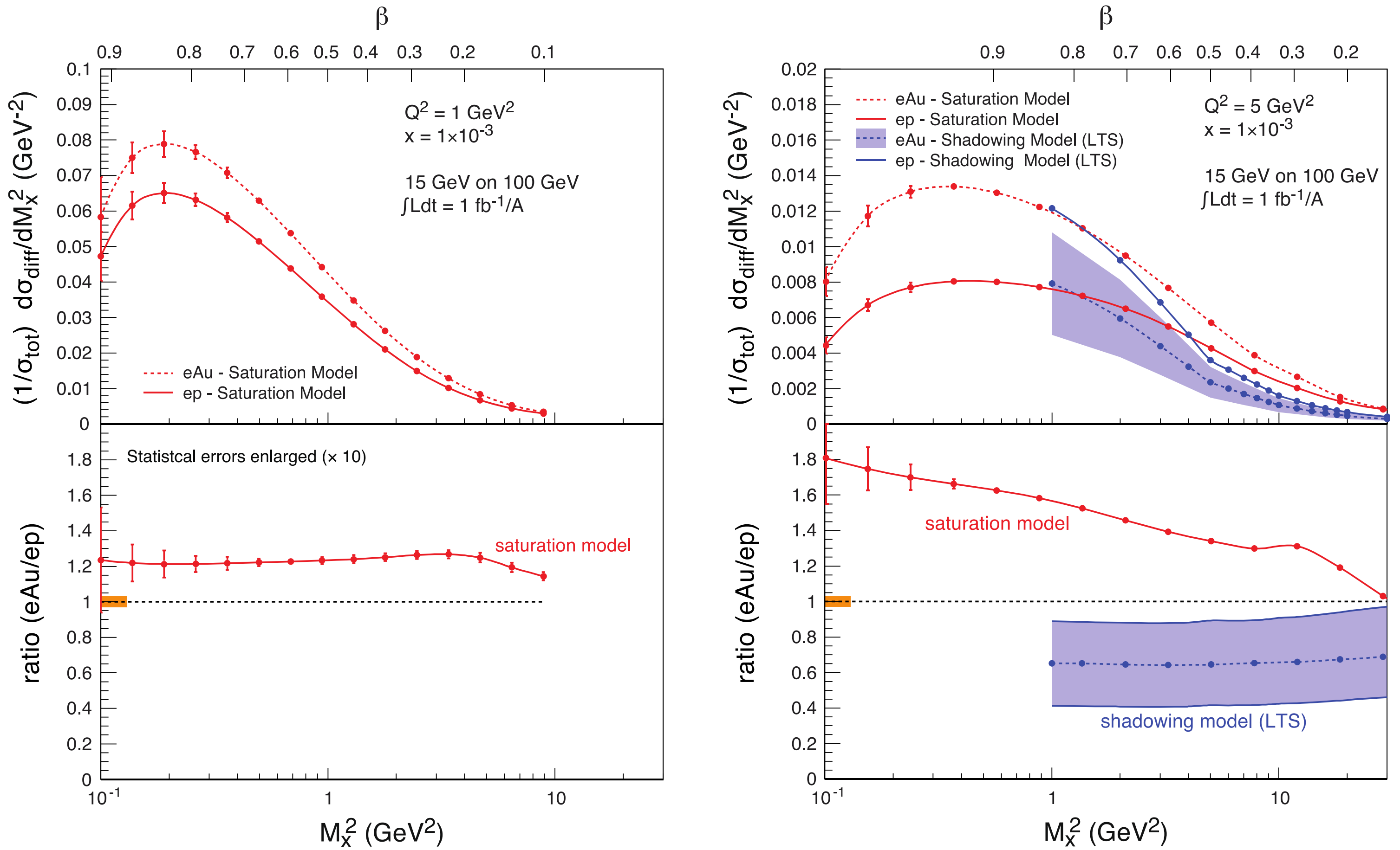
- Diffraction (elastic scattering) becomes a half of the total cross section

$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\int d^2b\, N} \longrightarrow \frac{1}{2}$$

- Large fraction of diffractive events in DIS is a signature of reaching the black disk limit! (at least for central collisions)

# Diffractive over total cross sections

- Here's an EIC measurement which may distinguish saturation from non-saturation approaches:

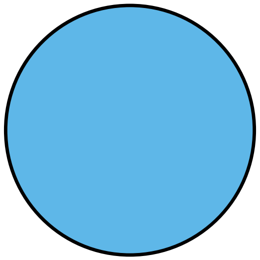



Saturation = Kowalski et al '08, plots generated by Marquet

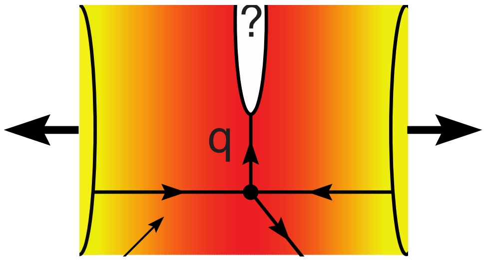
Shadowing = Leading Twist Shadowing (LTS), Frankfurt, Guzey, Strikman '04, plots by Guzey

# Energy Loss in Cold Nuclear Matter

- EIC would be able to measure the energy loss of quarks in a cold nuclear matter, complementing the RHIC and LHC measurements of energy loss in hot QCD plasma:



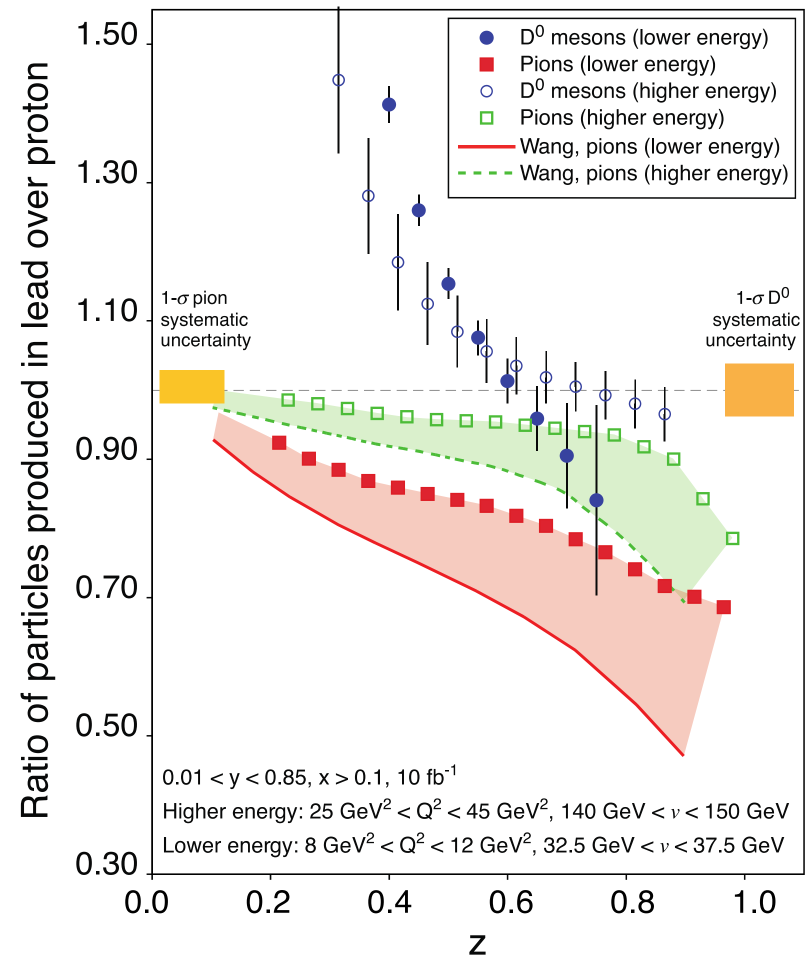
atter




hot nuclear
matter

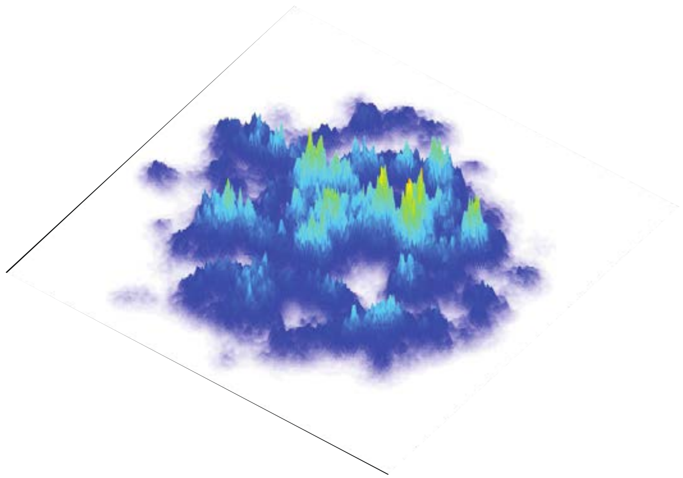
# Energy Loss in Cold Nuclear Matter

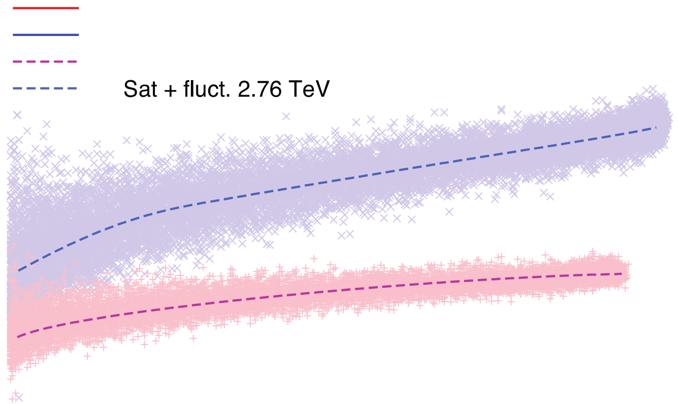
- By studying quark propagation in cold nuclear matter we can learn important information about hadronization and may even measure qhat in the cold nuclear medium:



# Connections to Heavy Ion Physics

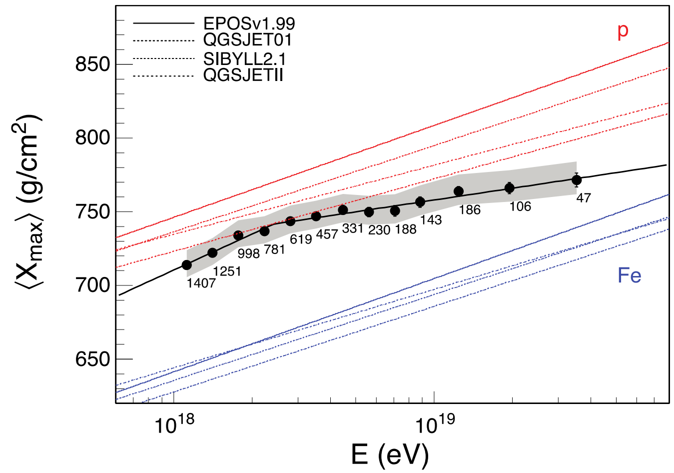
- CGC Physics plays an important role in the early-time dynamics of heavy ion collisions
- By exploring it at EIC we would get a better handle on formation of QGP and on fluctuations





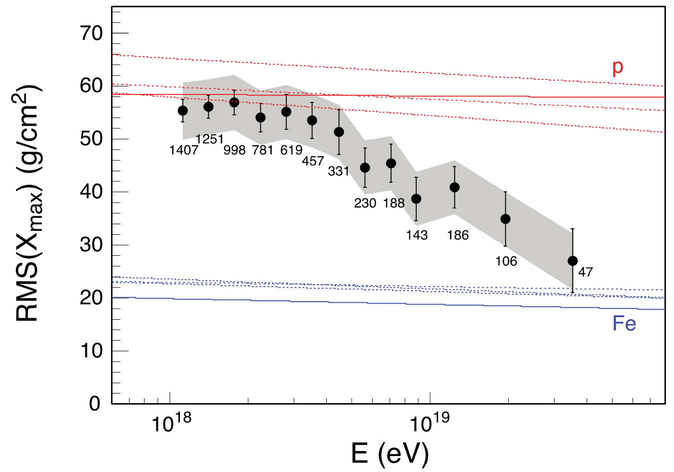
# Connections to Cosmic Rays

- There is a known problem in Auger data indicating that cosmic rays behave like protons at lower energies and like nuclei at higher energies, according to the existing QCD Monte-Carlos.




- It could be that the problem is with our understanding of QCD at these super-high energies.

- Perhaps saturation physics, with input from EIC, could help improve our understanding of the Auger data.

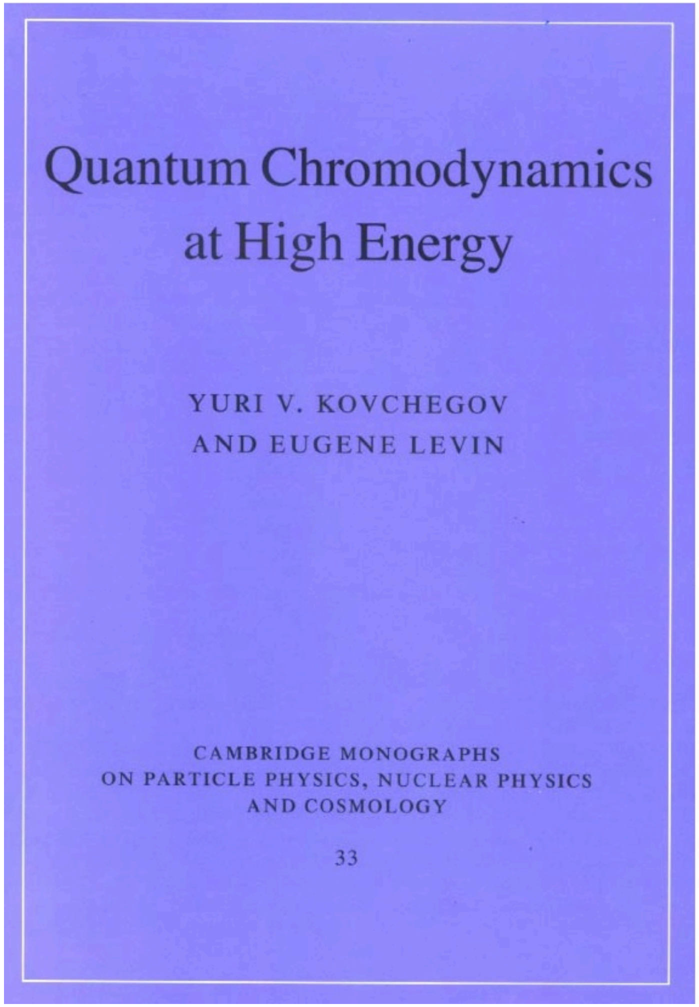


# Conclusions

- In recent years there has been a lot of theoretical progress on calculating saturation physics observables with NLO precision.
- While running coupling corrections generated successful phenomenology, NLO corrections are not yet there.
- There is a number of LO+rc saturation predictions for e+A collisions at EIC: structure functions, di-hadrons, diffraction.
- EIC may complete the discovery of saturation/CGC physics and study its properties, significantly improving our understanding of QCD in nucleons and nuclei.

# Backup Slides

# A reference




Published in September 2012
by Cambridge U Press