A scaling relation in p+A and A+A collisions

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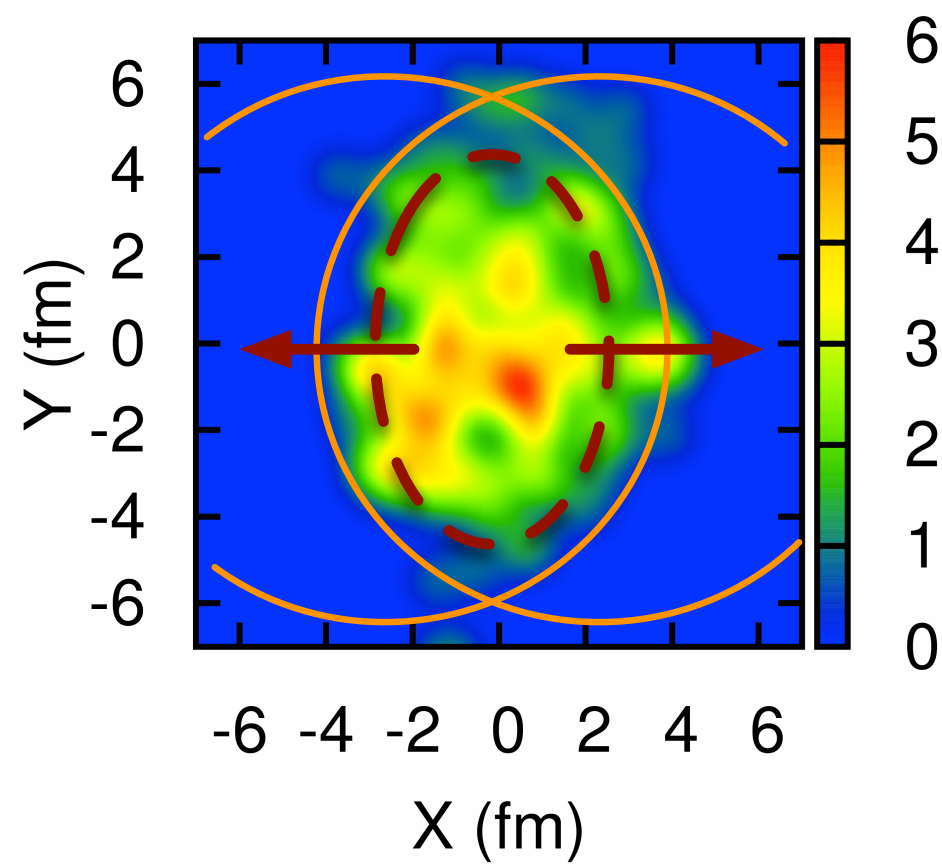
- Gokce Basar and DT: arXiv:1312.6770. Thanks Thomas Schaefer.

Outline

1. Review hydro in A+A to set notation
2. Review what is seen in p+A and amazing similarity to A+A results
   - Striking similarity shocked the heavy ion world.
3. Give a zeroth order theory/explanation for the similarity.

Ultra-quick review: flow harmonics and fluctuations in AA



Quantify the init. e-dense w. eccentricity:

$$\epsilon_n = -\frac{\langle r^n \cos(2(\phi_s - \Phi_n))\rangle}{\langle r^n \rangle}$$

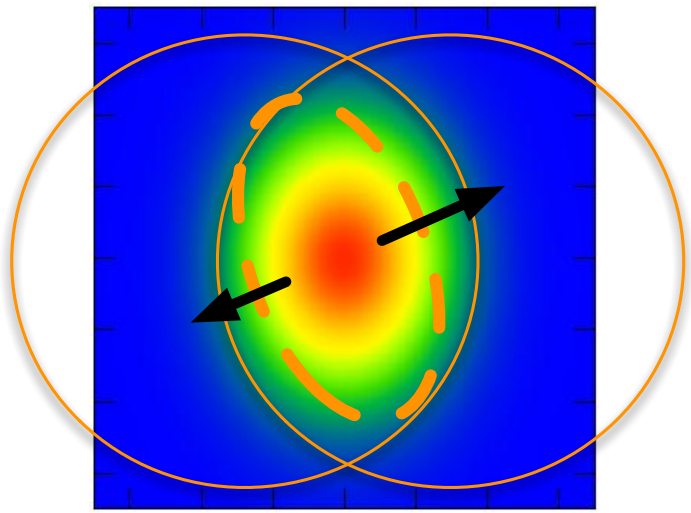
and orientation angle Φ_n:

$\Phi_n =$ orientation angle of n-th distortion:

Expand the observed momentum spectra in a fourier series:

$$\frac{dN}{d\phi_{\mathbf{p}}} = N\,(1 + \; 2\,v_2 \cos(2(\phi_{\mathbf{p}} - \Psi_2)) \; + \; \ldots + v_1, v_3, v_4, v_5, \mathrm{etc})$$

Linear response theory for v_2 and v_3



In a linear response approximation the v_2 comes from geometric correl in the initial state:

$$v_2 e^{i2\Psi_2} = \underbrace{k_2\ \epsilon_2 e^{i2\Phi_2}}_{\text{Linear response to deformation}}$$

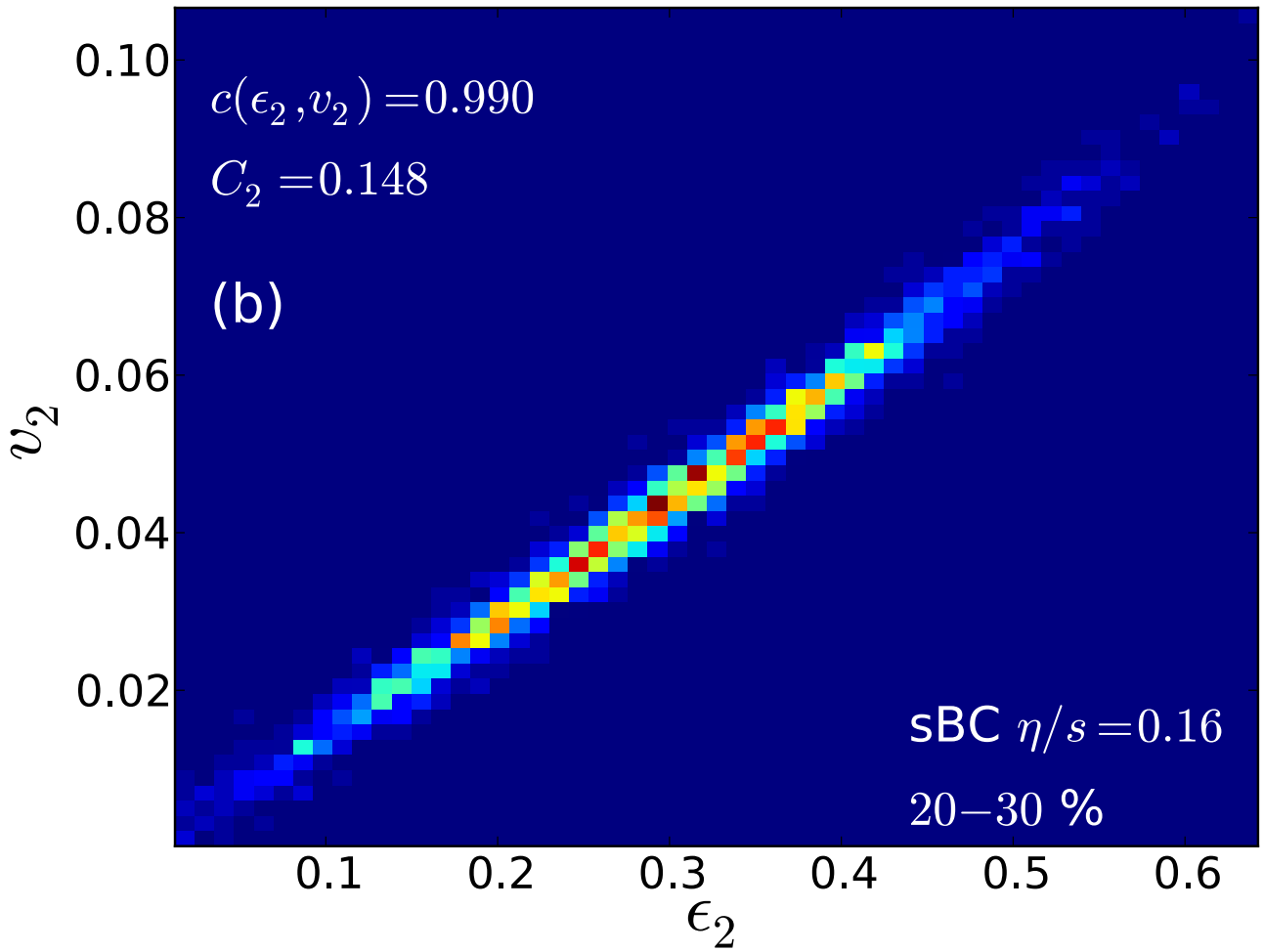
So the rms v_2 is a combination of the linear response coefficients and geometry

$$\sqrt{\langle v_2^2 \rangle} = \underbrace{v_2\{2\}}_{\text{rms } v_2} = \underbrace{k_2}_{\text{response}} \times \underbrace{\sqrt{\langle \epsilon_2^2 \rangle}}_{\text{geometric flucts}}$$

Response coefficients k_2, k_3 computed with hydro or kinetics and depend on $\ell_{\rm mfp}/L$

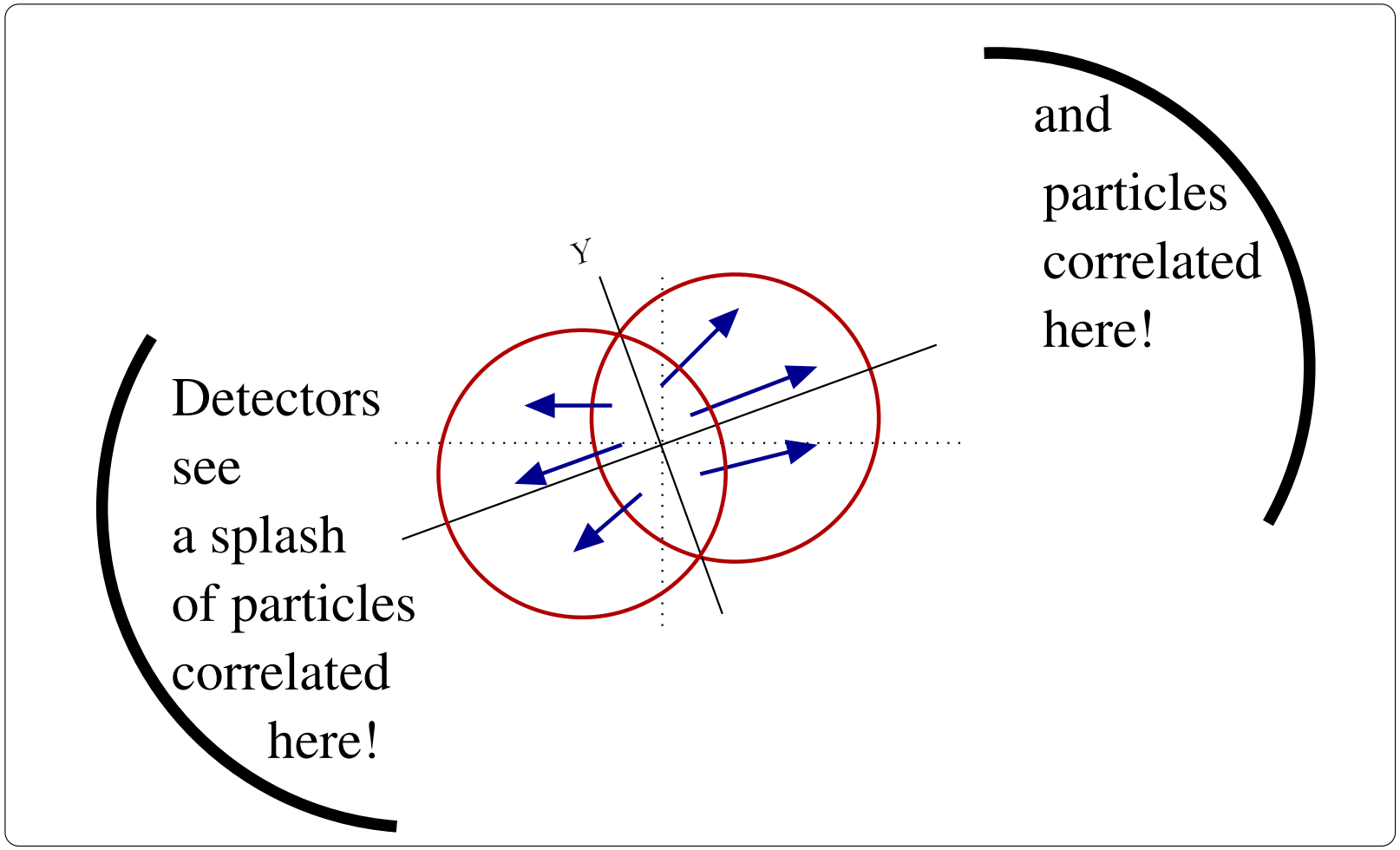
Comparing linear response and event by event hydro

(H. Niemi et al, arXiv:1212.1008)



ϵ_2 is a good predictor of the hydrodynamic response

Measuring flow in AA

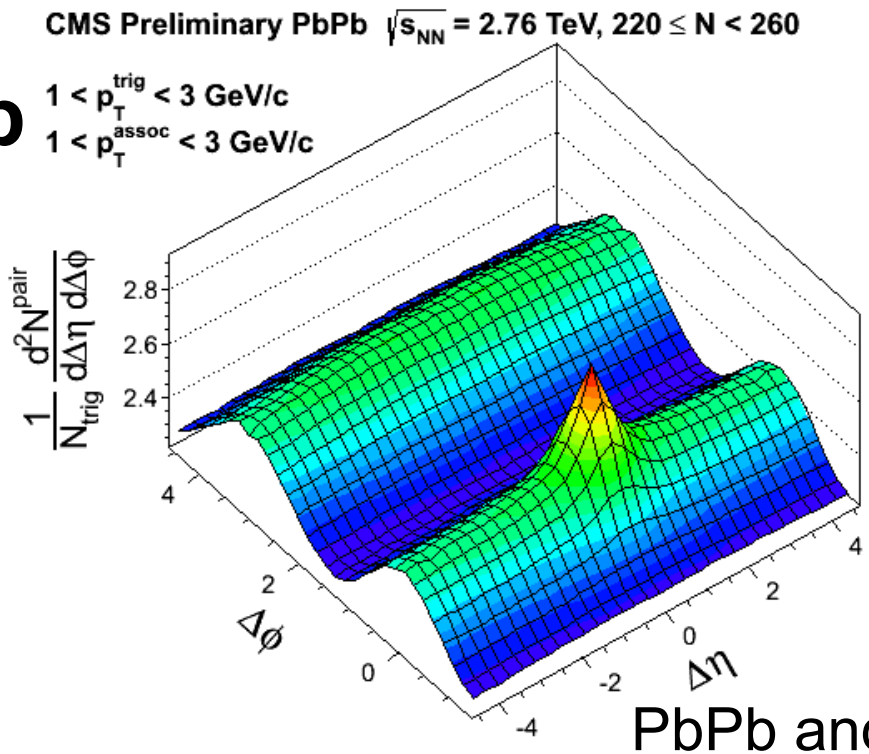


Angular Correlations in AA generally attributed to flow:

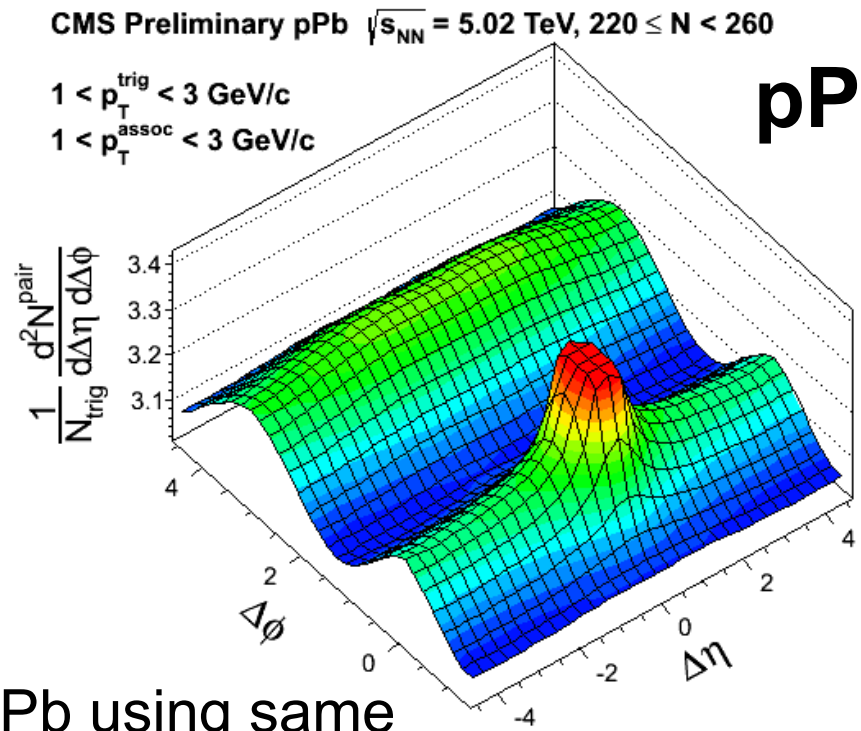
$$C(\phi_1 - \phi_2) = \frac{dN}{d\phi_1 d\phi_2} \Big/ \left(\frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \right)$$

Comparison of Peripheral PbPb to pPb

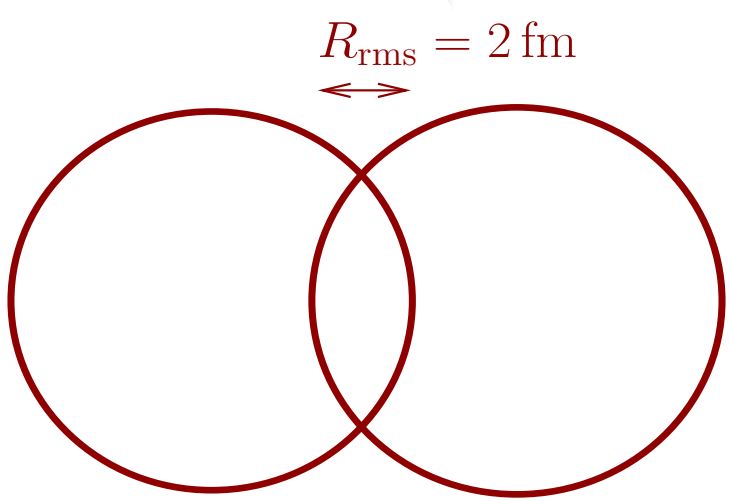
PbPb

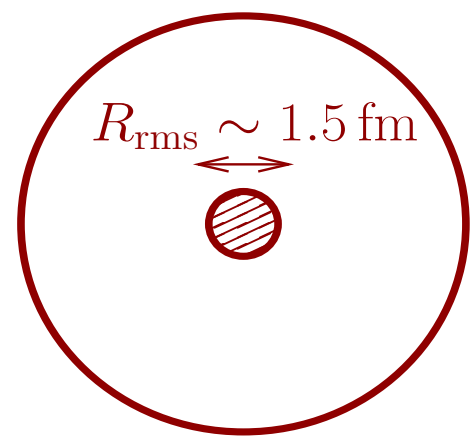


pPb

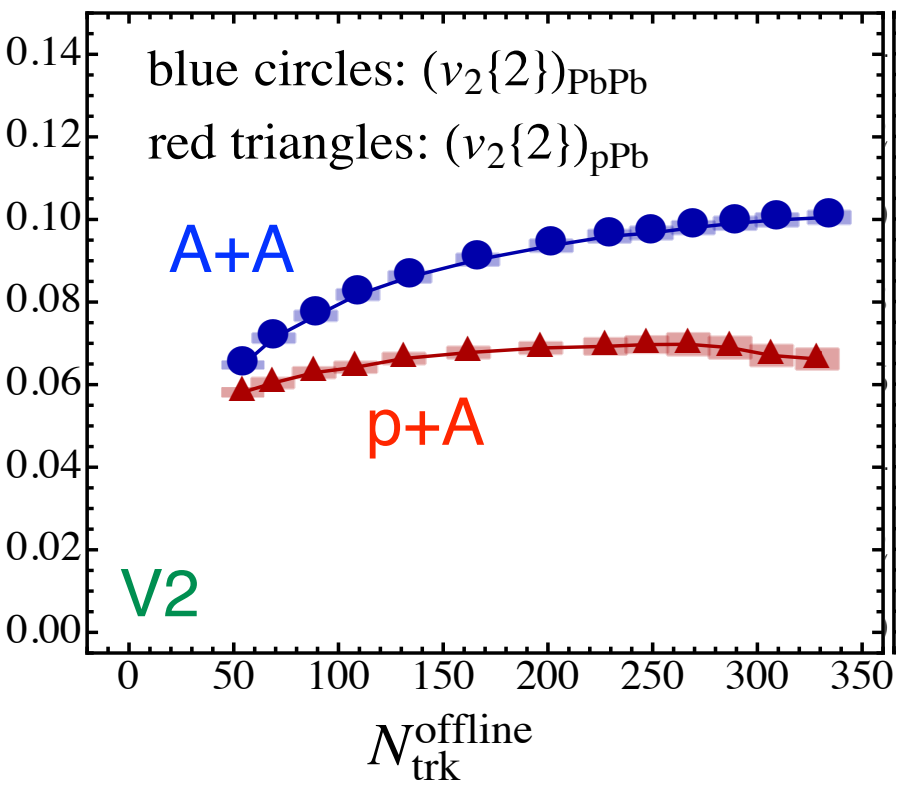


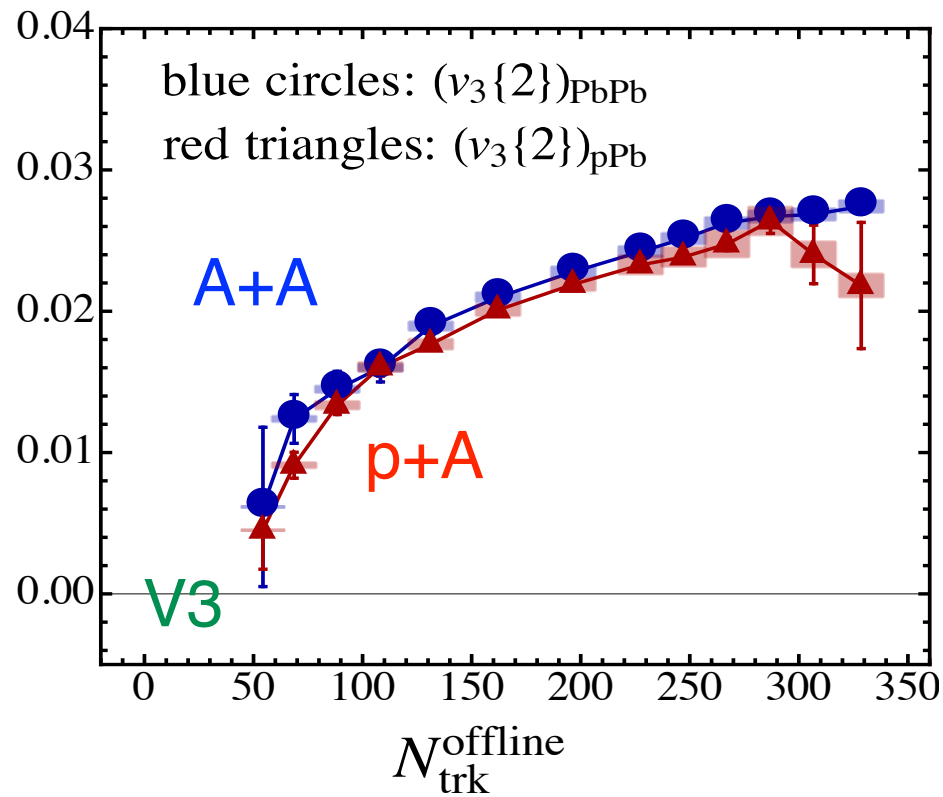
PbPb and pPb using same multiplicity selection, 220 < N < 260





Can extract a v_2 and a v_3 from these data . . . (CMS Data)





Questions:

1. Is the mechanism for the correlation the same in pPb and PbPb?
2. Is hydrodynamics the origin of pPb correlations?
3. How important are non-equilibrium effects for pPb? PbPb?

Tried to answer these questions with Gokce Basar

Flow in $p + A$ and $A + A$:

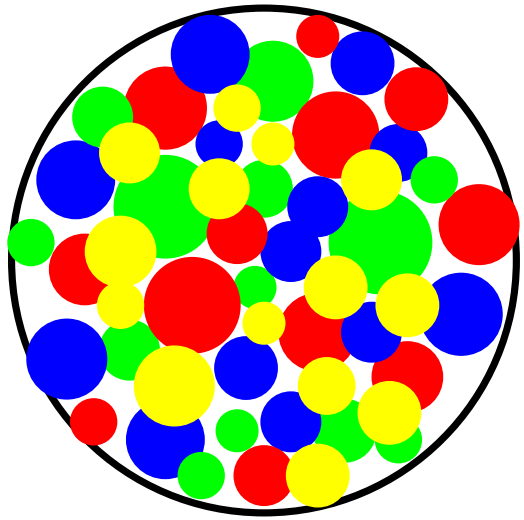
$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same?

2. Is the geometry or ϵ_2 the same? ϵ_3?

Part I: Flow Response Coefficients

What is the $\ell_{\rm mfp}/L$ in high multiplicity p+A and A+A?



1. Throw $N_{\rm clust}$ clusters in in the transverse plane:

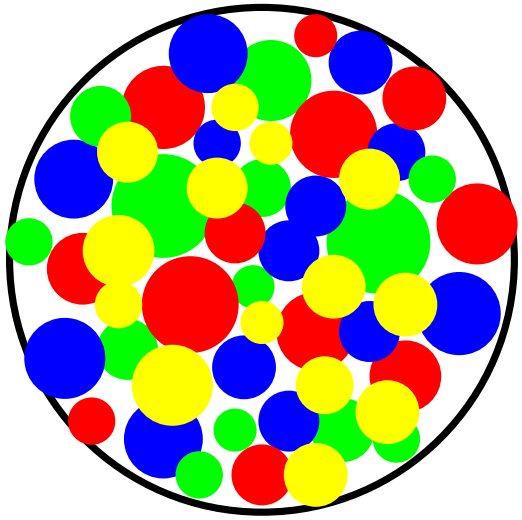
   - This defines a momentum scale (the saturation momentum)

$$Q_s^2 \propto \frac{N_{\rm clust}}{\pi L_T^2} \qquad (L_T = \text{transverse size})$$

2. Assume that this is the only momentum scl for 1D expansion, $1/Q_s \ll \tau \ll L_T$

$$\ell_{\rm mfp} \propto \frac{1}{Q_s} \propto \frac{1}{T_{\rm o}}$$

What is the $\ell_{\rm mfp}/L$ in high multiplicity p+A and A+A?



3. Assume that the multiplicity is proportional to the number of clusters

$$\frac{dN}{dy} \propto N_{\rm clust}$$

4. Find that $\ell_{\rm mfp}/L_T$ (at early time) is fixed by dN/dy

$$\frac{\ell_{\rm mfp}}{L_T} \propto \frac{1}{T_o L_T} \propto \frac{1}{Q_s L_T} \propto \frac{1}{\sqrt{dN/dy}}$$

$\ell_{\rm mfp}/L_T$ is the same in p+A and A+A at fixed multiplicity!

What is the mean $\ell_{\rm mfp}/L_T$ when the flow develops, at $\tau \sim L_T$?

1. Need to estimate $\ell_{\rm mfp}/L_T$ at time $\tau \propto L_T$

2. Using, Bjorken estimate:

$$\ell_{\rm mfp} \propto \frac{1}{T(\tau)} \propto \frac{1}{T_o}\left(\frac{\tau}{\tau_o}\right)^{1/3}$$

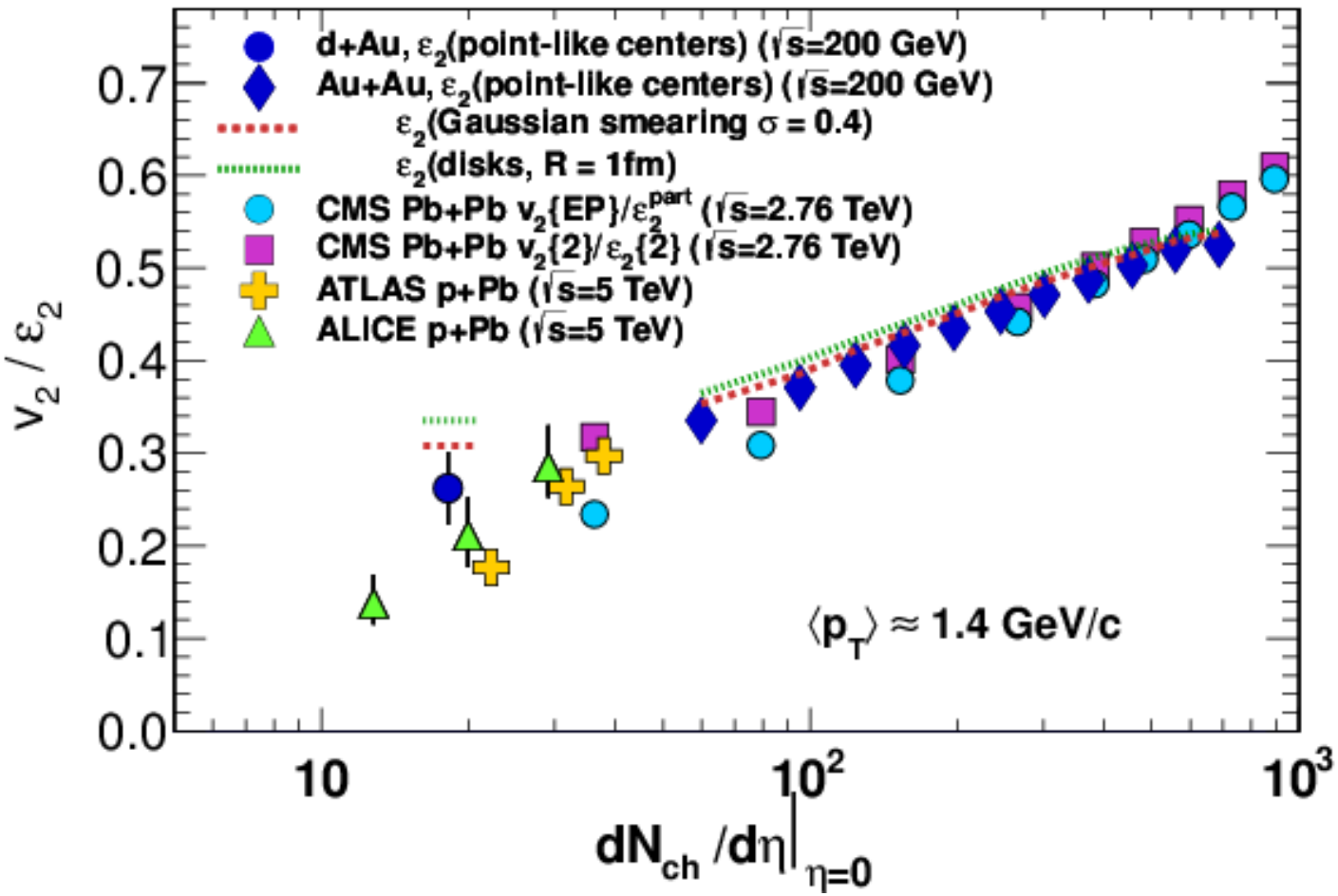
Find after algebra

$$\left.\frac{\ell_{\rm mfp}}{L}\right|_{\tau\sim L_T} \propto \frac{1}{\left(\frac{dN}{dy}\right)^{1/3}}$$

Find (again) that $\ell_{\rm mfp}/L_T$ is the same in p+A and A+A at fixed multiplicity

v_2/ϵ_2 the same at fixed multiplicity – compare RHIC and LHC AA events

Ann Sickles – Quark Matter



v_2/ϵ_2 same at fixed multiplicity

Flow in $p + A$ and $A + A$:

$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same?

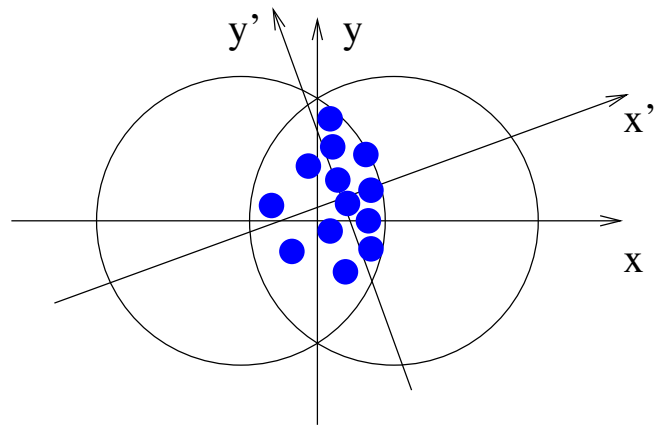
   Yes. The $\ell_{\rm mfp}/L_T$ is the same at fixed multiplicity

2. Is the geometry or ϵ_2 the same? ϵ_3?

Part II. Geometric fluctuations

Understanding the fluctuating geometry in A+A?

Bhalerao and Ollitrault '06



- Throw $N_{\rm clust}$ according to the average nuclear distribution with statistics:

$$\langle \delta n(\boldsymbol{x}) \delta n(\boldsymbol{y}) \rangle = \bar{n}(\boldsymbol{x}) \delta^{(2)}(\boldsymbol{x} - \boldsymbol{y}) \, .$$

- Calculate $\epsilon_2\{2\} = \sqrt{\langle \epsilon_2^2 \rangle}$

$$\langle \epsilon_2^2 \rangle_{AA} = \underbrace{\epsilon_s^2}_{\text{ave. geometry}} + \underbrace{\langle \delta \epsilon_2^2 \rangle}_{\text{fluctuations}} \qquad \langle \delta \epsilon_2^2 \rangle_{AA} \equiv \frac{\langle r^4 \rangle_{AA}}{N_{\rm clust} \langle r^2 \rangle_{AA}^2}$$

Reproduces the results of more complex Glauber models

Understanding the fluctuating geometry in p+A?

- In p+A the average geometric eccentricity is zero, $\epsilon_s = 0$

$$\underbrace{\left\langle \epsilon_2^2 \right\rangle_{pA}^2}_{\text{only flucts}} = \underbrace{\langle \delta\epsilon_2^2 \rangle_{pA}}_{\text{only flucts}} \equiv \frac{\left\langle r^4 \right\rangle_{pA}}{N_{\text{clust}} \left\langle r^2 \right\rangle_{pA}^2}$$

- So to compare to p+A, we scale out the average geometry out of the A+ A system

$$\underbrace{(v_2\{2\})_{\text{PbPb,rscl}}}_{\text{only flucts}} \equiv \sqrt{1 - \frac{\epsilon_s^2}{\left\langle \epsilon_2^2 \right\rangle_{AA}}}\,(v_2\{2\})_{\text{PbPb}}\,.$$

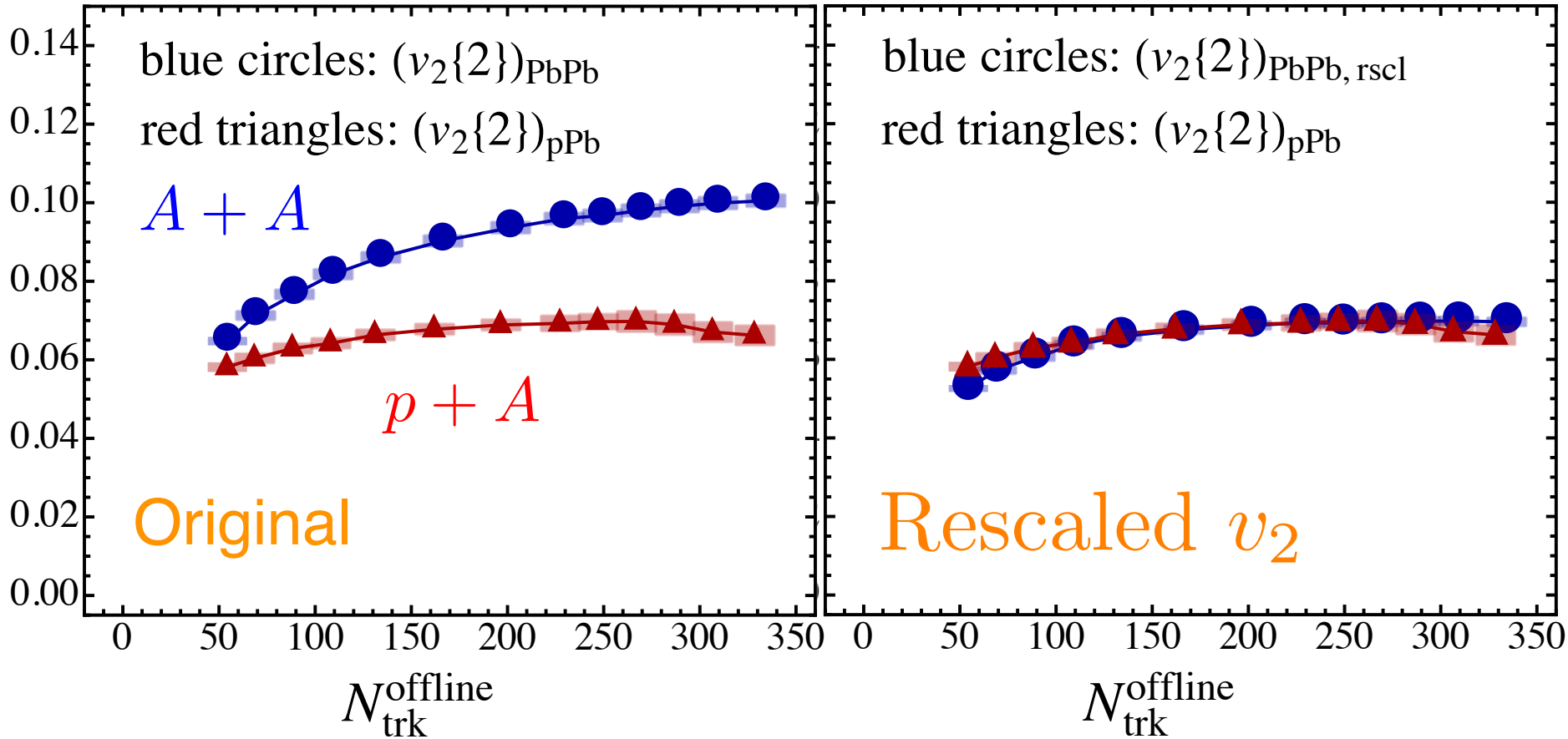
- With this rescaling we expect

$$(v_2\{2\})_{\text{PbPb,rscl}} \equiv k_2 \sqrt{\left\langle \delta\epsilon_2^2 \right\rangle_{AA}}$$

$$(v_2\{2\})_{pA} \equiv k_2 \sqrt{\left\langle \delta\epsilon_2^2 \right\rangle_{pA}}$$

Lets compare $(v_2\{2\})_{\text{PbPb,rscl}}$ to $(v_2\{2\})_{\text{pPb}}$ ...

Comparing fluctuation driven v_2 . . . (CMS Data)



Fluctuation driven v_2's are equal to 5% accuracy!

Ratio of fluctuation v_2's and eccentricities

- Fluctuation driven v_2

$$\frac{(v_2\{2\})_{\rm pPb}}{(v_2\{2\})_{\rm PbPb}} = \frac{k_2}{k_2}\sqrt{\frac{\langle\delta\epsilon_2^2\rangle_{pA}}{\langle\delta\epsilon_2^2\rangle_{AA}}} = \sqrt{\frac{(\langle r^4\rangle\,/\,\langle r^2\rangle^2)_{pA}}{(\langle r^4\rangle\,/\,\langle r^2\rangle^2)_{AA}}}$$

- Don't know the radial profile in pA. Does it matter ?

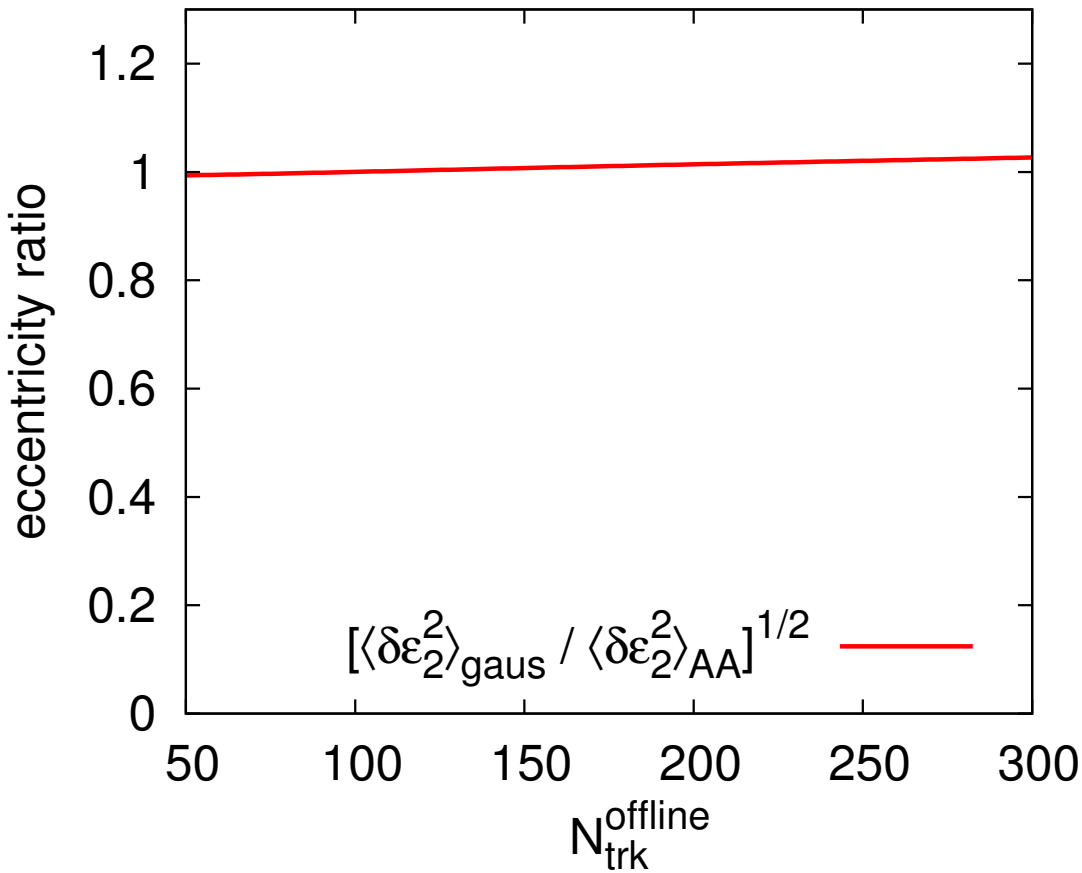
- Ratio of fluct-driven v_2 is determined by a square root of a double ratio!

  – Very different profiles give the almost same answer

$$\sqrt{\frac{\langle\delta\epsilon_2^2\rangle_{\rm hard-sphere}}{\langle\delta\epsilon_2^2\rangle_{\rm Gaussian}}} \approx 0.85$$

- A gaussian profile for pA seems most likely to me

Comparing a gaussian profile in pA to the nuclear (woods-saxon) geometry

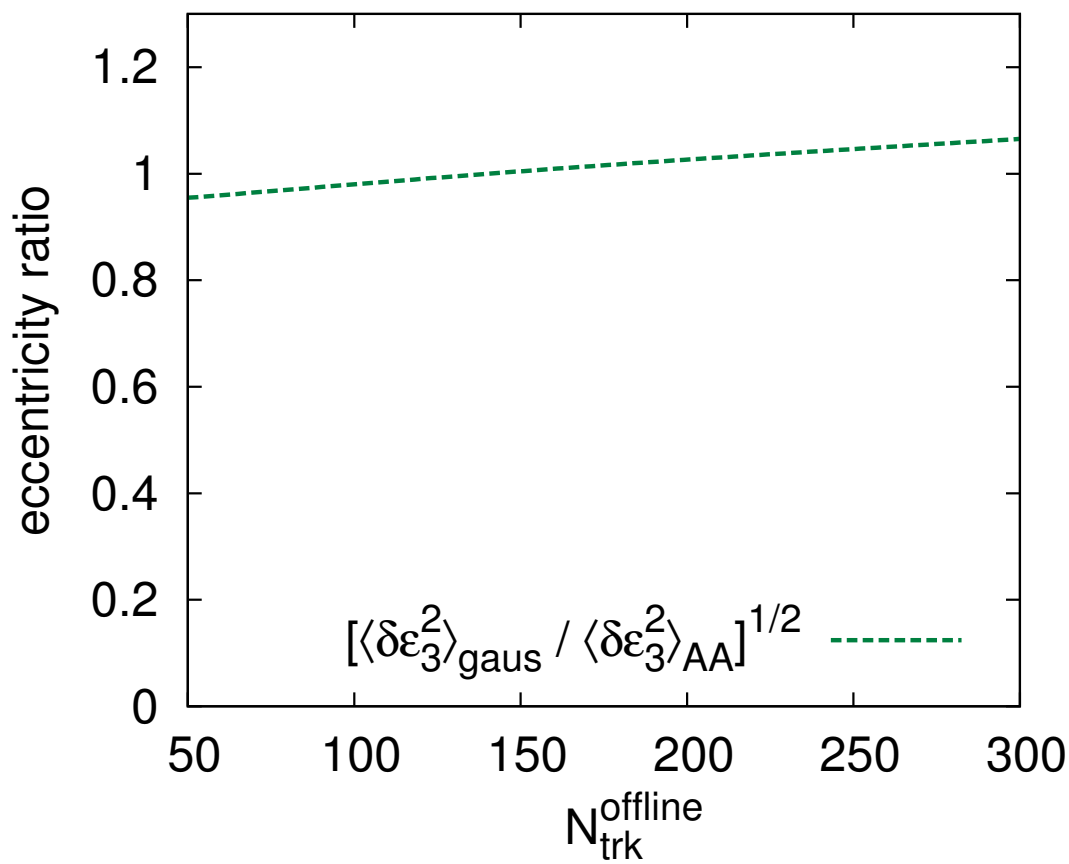


Expect $p + A$ and $A + A$ to have the same eccentricities for $\delta\epsilon_2$

and for a conformal response . . . the same v_2

v_3 works the same way . . .

$$\frac{(v_3\{2\})_{pA}}{(v_3\{2\})_{AA}} = \underbrace{\frac{k_3\sqrt{\left\langle \delta\epsilon_3^2 \right\rangle_{pA}}}{k_3\sqrt{\left\langle \delta\epsilon_3^2 \right\rangle_{AA}}}}_{k_3 \text{ same}} = \underbrace{\left[\frac{\left\langle r^6 \right\rangle_{pA} / \left\langle r^3 \right\rangle_{pA}^2}{\left\langle r^6 \right\rangle_{AA} / \left\langle r^3 \right\rangle_{AA}^2}\right]^{1/2}}_{\text{root of geometric double ratio}}$$



Flow in $p + A$ and $A + A$:

$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same?

   Yes. The $\ell_{\rm mfp}/L_T$ is the same at fixed multiplicity

2. Is the geometry or ϵ_2 the same? ϵ_3?

   Yes, for any independent cluster type model after taking out ave. geom.

Expect the v_n's the same

Transverse momentum dependence of elliptic and triangular flow: p+A and A+A

$$\frac{dN}{p_T dp_T d\phi_{\mathbf{p}}} = \frac{N_o}{2\pi}\left(1 + 2v_2(p_T)\cos(2(\phi_{\mathbf{p}} - \Psi_2)) + \ldots\right)$$

- Conformal scaling argument:

$$(v_2(p_T)) = \underbrace{\xi_2}_{\text{response coefficient}} \times \underbrace{(\epsilon_2)}_{\text{geometry}} \times \underbrace{f(p_T/\langle p_T\rangle)}_{\text{universal function at fixed dN/dy}}$$

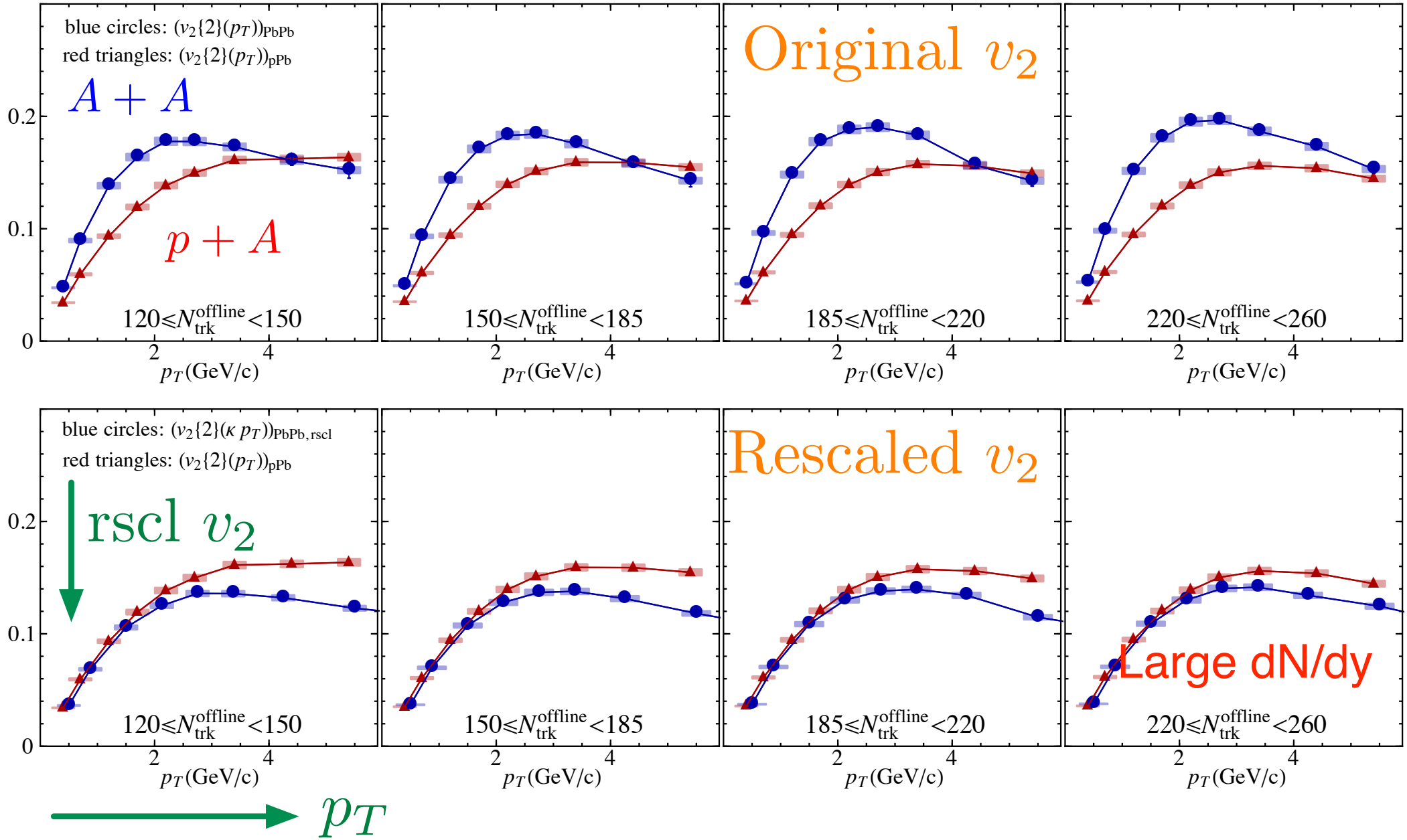
- So with the measured ratio: (ALICE)

$$\kappa = \frac{\langle p_T\rangle_{pA}}{\langle p_T\rangle_{AA}} = \frac{L_{AA}}{L_{pA}} \simeq 1.3$$

- Expect:

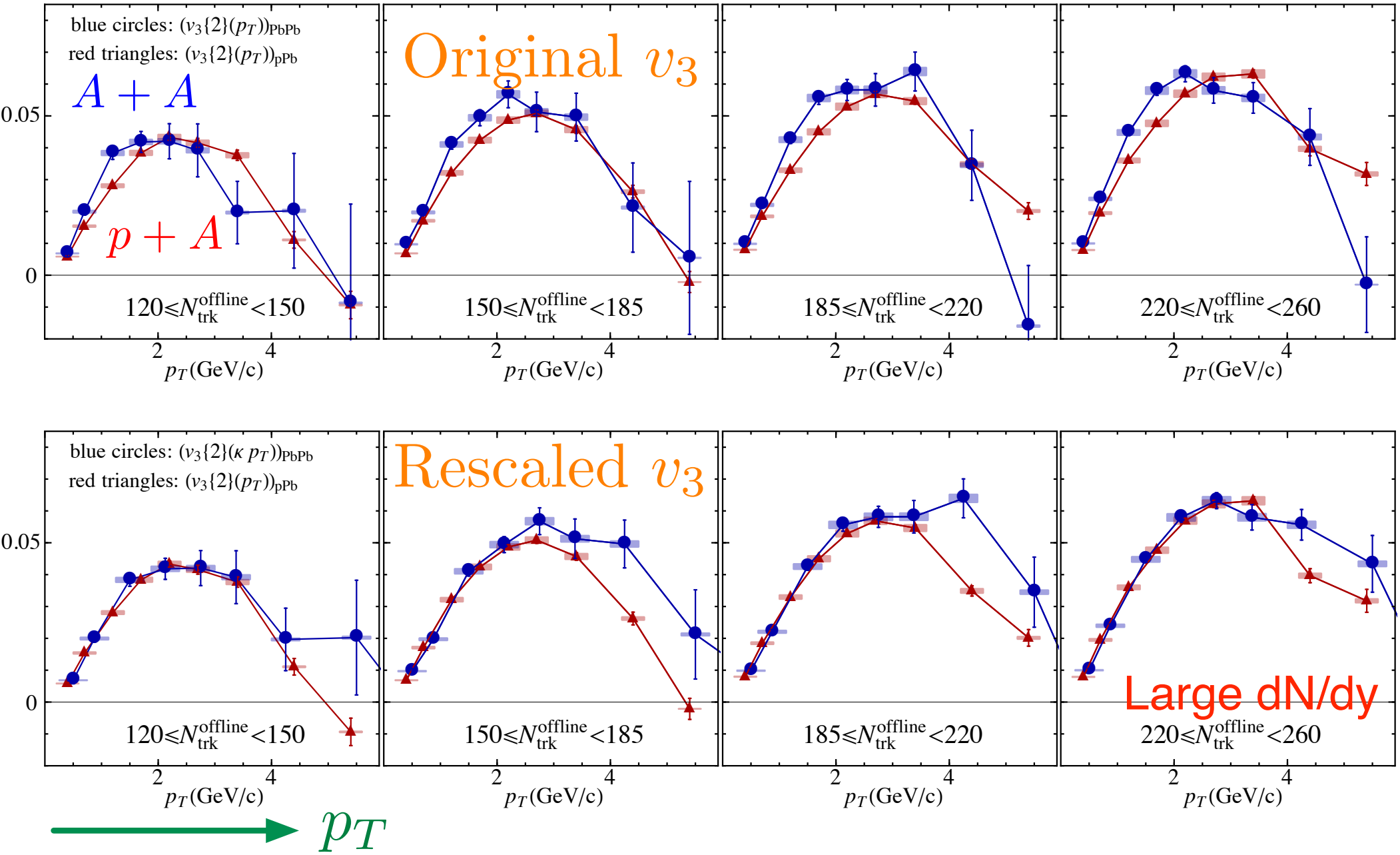
$$\left[v_2\{2\}(p_T)\right]_{pPb} = \left[v_2\{2\}\left(\frac{p_T}{\kappa}\right)\right]_{PbPb,\mathrm{rscl}}$$

V_2 in p+A and A+A



The rescaling induces a common slope for typical pt

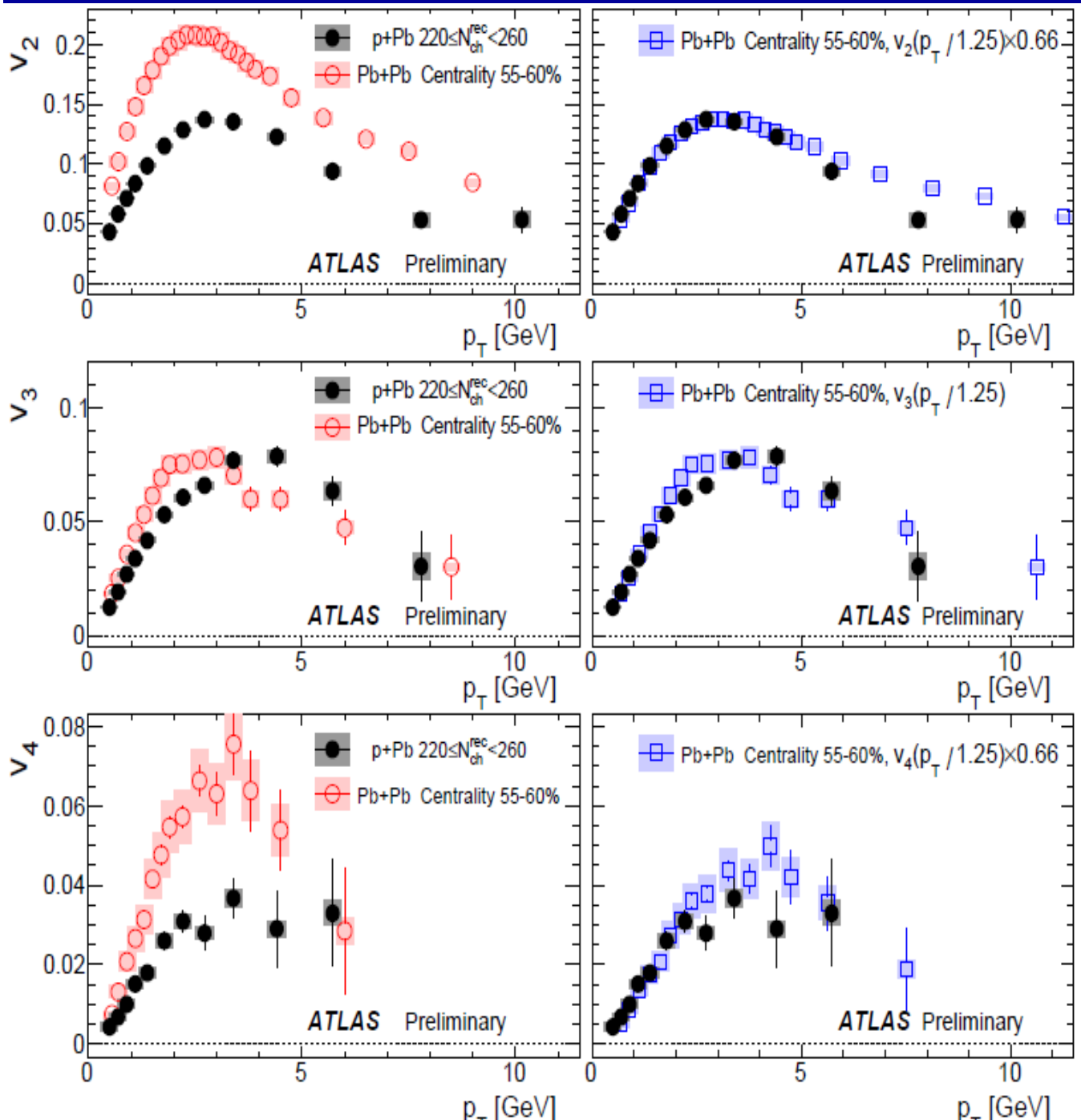
V_3 in p+A and A+A



p_T

The rescaling induces a common slope for typical pt

v_n scaling between the p+Pb and Pb+Pb systems.

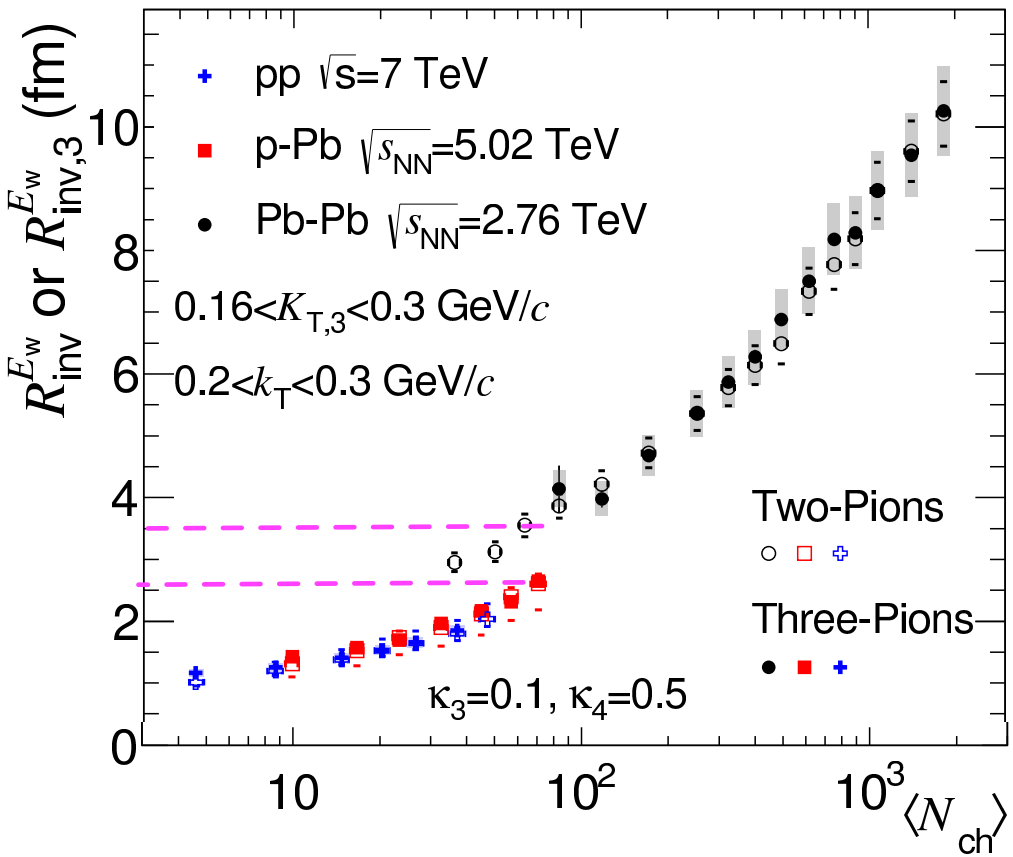


- v_2 values, after scaling the p_T axis, differ only by a scale factor between the two systems.
- Suggests a similar origin for v_2 in the two systems and similar medium response to initial geometry?

HBT Radii and Conformal Rescaling (ALICE Collaboration)

- From the confomal analysis

$$\frac{R_{AA}}{R_{pA}} = \frac{\langle p_T \rangle_{pA}}{\langle p_T \rangle_{AA}} = 1.3$$

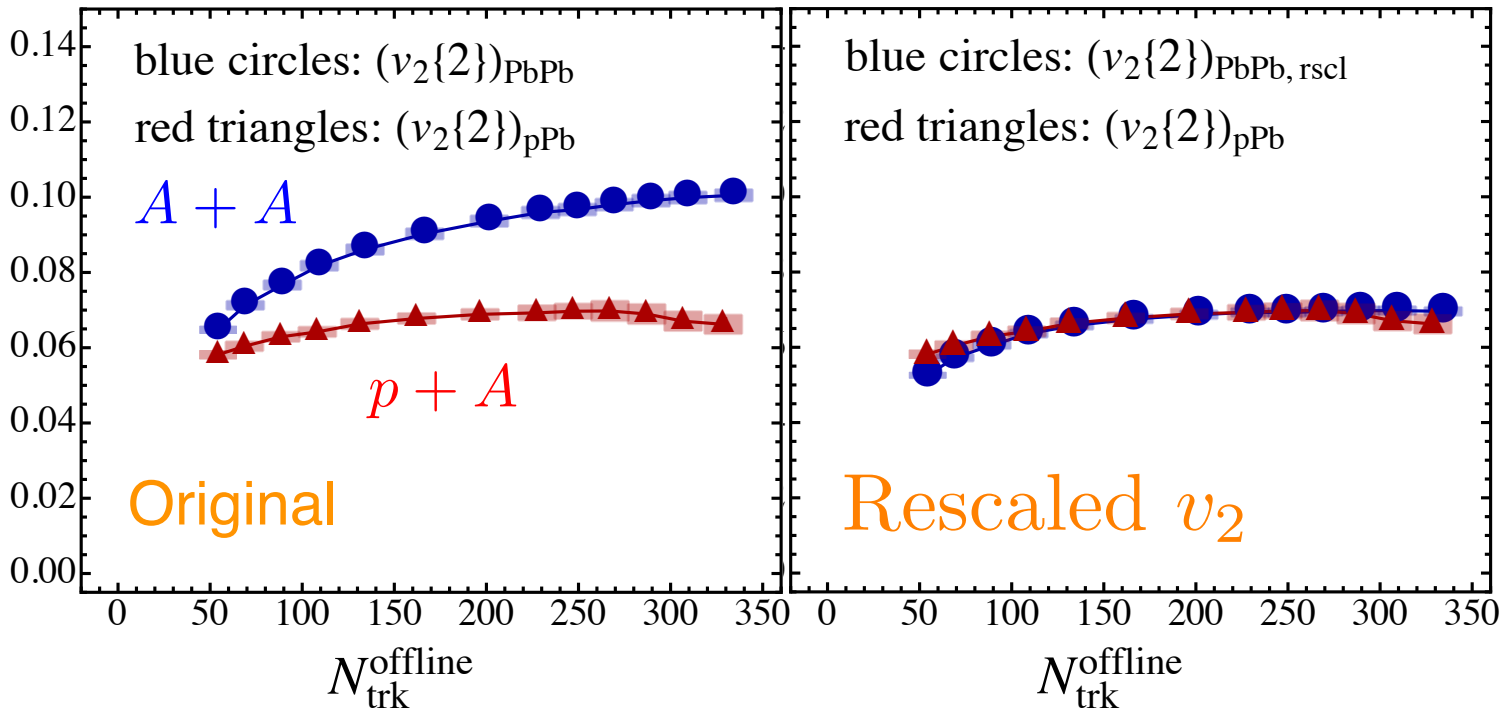


- Find:

$$\frac{R_{AA}}{R_{pA}} = \frac{3.5}{2.6} = 1.35$$

Conformal scaling seems to be consistent with HBT analysis

Recap:



- Motivated entirely by geometry
  - Either a complete accident or

The correlations in p+A are driven by a (conformal) response to geometry

Conclusions:

1. The correlations reflect a conformal response to the geometry
   - He3 can be a good test
2. In a conformal response, the response coeff are equal in p+A and A+A
   - $\ell_{\rm mfp}/L$ same in p+A and A+A at fixed dN/dy
   - Evidence for a conformal scaling from the p_T dependence
   - The conformal scaling is consistent with HBT increase of $R_{AA}/R_{pA} \simeq 1.35$
3. Not necessary to fine tune the initial conditions to get the same v_2 to 5% accuracy

$$\underbrace{\sqrt{\frac{\left(\langle r^4\rangle \,/\, \langle r^2\rangle^2\right)_{pA}}{\left(\langle r^4\rangle \,/\, \langle r^2\rangle^2\right)_{AA}}}}_{\text{root of a double ratio}}$$

4. The conformal dynamics is not obviously hydrodynamics (conf. kinetics is fine)
   - Running hydro in these peripheral bins is hard (AdS?)

All (conformal) non-equilibrium dynamics will tend to conformal hydrodynamics