Dynamical Models for Quarkonium in Thermal Bath

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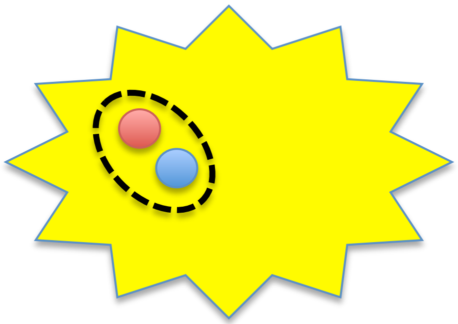
References:
- Y.A., A.Rothkopf, PRD 85 (2012) 105011
- Y.A., PRD 87 (2013) 045016
- Y.A., PRD 91 (2015) 056002

Heavy-ion collisions

- Classic questions
  - Did we observe deconfinement?
  - Did we observe restoration of chiral SSB?
  - Is thermal equilibrium really achieved?
- And many more modern topics



At present, we can only do consistency checks.

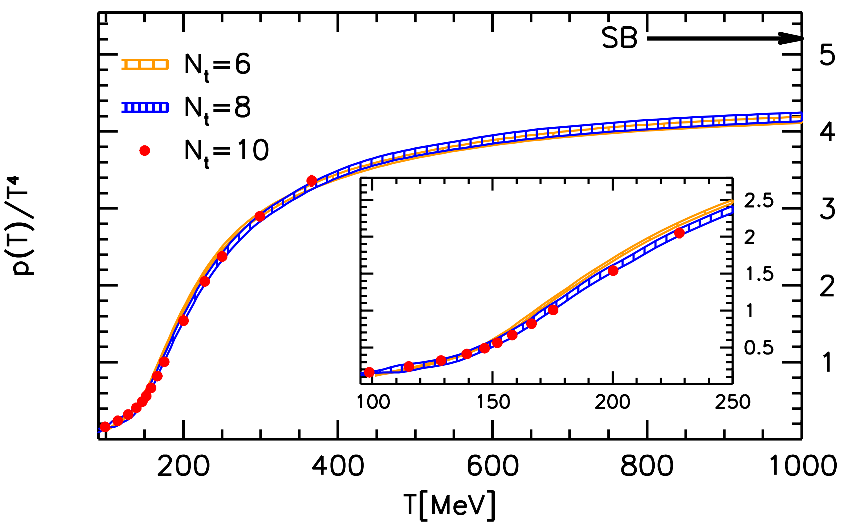
- Hydro simulation with EoS that has deconfinement works or not
- **Quarkonium simulation** with screened potential on hydro backgrounds works or not

Outline

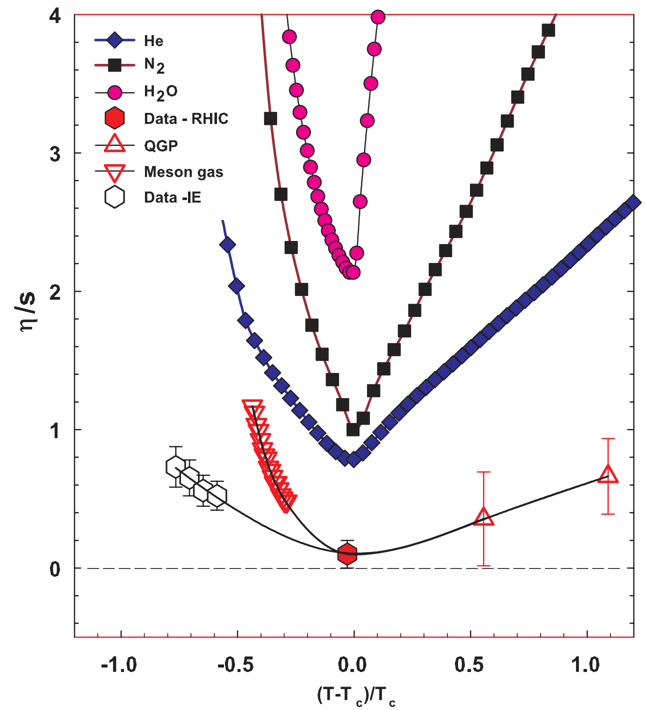
1. Introduction and a little more
2. Annual Users' Guide of Stochastic Potentials
   – Once it gets popular, I will do it every year!
3. Summary

1. Introduction +

- What is quark-gluon plasma?
  - Phase transition? Crossover?
  - Perfect liquid? So?



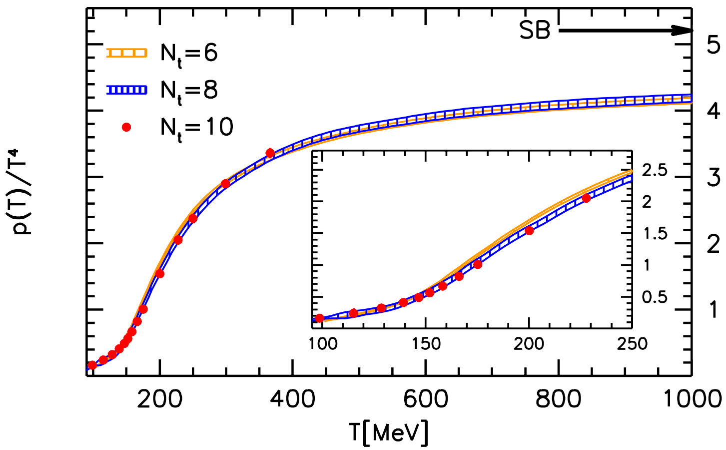
Budapest-Wuppertal (10)



Lacey (06)

No clear definition

- Then think physically
  - What is characteristic of QGP?
    - Liberation of color degrees of freedom



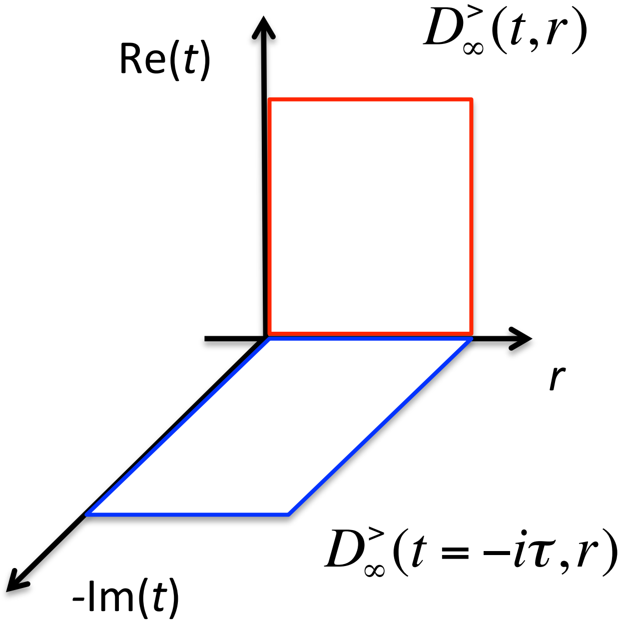
Color accounts for a lot!

- $\frac{7}{8} \times 4N_f \times N_c \approx 10.5N_c$
- $2\left(N_c^2 - 1\right) = 16$

- Restoration of chiral symmetry breaking

Color degrees of freedom

- In vacuum, colors are not liberated
  - In the static limit, one can refer to "potential"



Wilson loop for singlet potential energy

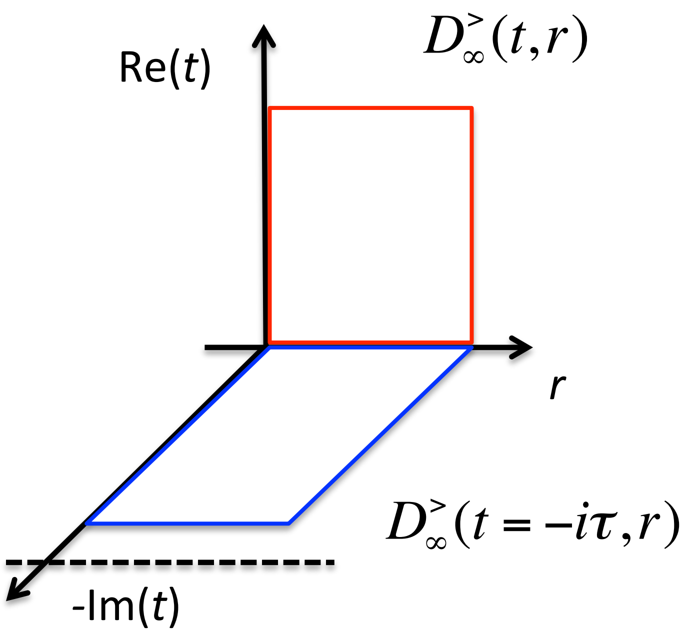
- $D^{>}_{\infty}(t,r) \sim \exp\left[-iV(r)t\right]$
- $D^{>}_{\infty}(t=-i\tau,r) \sim \exp\left[-V(r)\tau\right]$

→ Linear potential

*Assuming energy gap

Potential at finite-T

- Above Tc, colors are screened



Wilson loop for singlet potential energy

- $D^{>}_{\infty}(t,r) \sim \exp\left[-iV(r)t\right]$
- $D^{>}_{\infty}(t=-i\tau,r) \sim \exp\left[-V(r)\tau\right]$

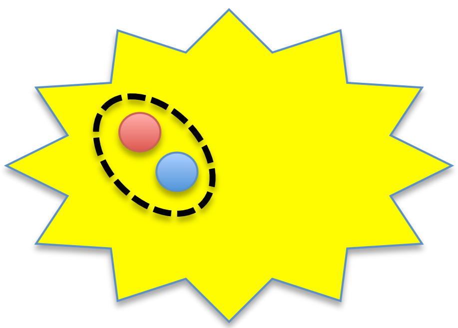
One cannot assume energy gap due to collisions

→ Continuum spectrum

→ Complex potential (Alex's talk)

2. Users' Guide of Stochastic Potential

- Modeling quarkonium in thermal bath
  - Extension of potential model in the vacuum



$$i\frac{\partial}{\partial t}D^{>}_{\infty}(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]D^{>}_{\infty}(t,r)$$

$$\rightarrow i\frac{\partial}{\partial t}\psi(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]\psi(t,r)\ ?$$

Violates HQ number conservation

Stochastic potential in a nutshell

- A stochastic infinitesimal time step

$$\psi(t+\Delta t,r)=\exp\left[-i\Delta t\left\{V_{\mathrm{Re}}(r)+\Theta(t,r)\right\}\right]\psi(t,r),$$

$$\left\langle\Theta(t,r)\right\rangle=0,\quad \left\langle\Theta(t,r)\Theta(t',r')\right\rangle=\underline{\Gamma(r,r')}\,\delta_{tt'}/\Delta t,\quad \Theta\sim\left(\Delta t\right)^{-1/2}$$

Can have off-diagonal components

- Stochastic Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(t,r)=\left\{V_{\mathrm{Re}}(r)-\frac{i}{2}\underline{\Gamma(r,r)}+\Xi(t,r)\right\}\psi(t,r),$$

Diagonal part

$$\Xi(t,r)\equiv\Theta(t,r)\underbrace{-\frac{i\Delta t}{2}\left\{\Theta(t,r)^2-\left\langle\Theta(t,r)^2\right\rangle\right\}}_{\text{irrelevant}}\cong\Theta(t,r),\quad \left\langle\Xi(t,r)\right\rangle=0$$

Complex potential

- What is $D^{>}_{\infty}(t,r)$?
  - Quarkonium wave function averaged over gauge configurations
    - $D^{>}_{\infty}(t,r)$ evolution needs not be unitary

$$i\frac{\partial}{\partial t}\psi(t,r)=\left\{V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r,r)+\Xi(t,r)\right\}\psi(t,r)$$

$$\Rightarrow i\frac{\partial}{\partial t}\left\langle\psi(t,r)\right\rangle=\left\{V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r,r)\right\}\left\langle\psi(t,r)\right\rangle,$$

$$\Leftrightarrow i\frac{\partial}{\partial t}D^{>}_{\infty}(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]D^{>}_{\infty}(t,r)$$

Recipes for stochastic potential

$$\boxed{\begin{aligned}&V_{\mathrm{Re}}(r)+\Theta(t,r),\\&\left\langle\Theta(t,r)\Theta(t',r')\right\rangle=\Gamma(r,r')\delta_{tt'}/\Delta t,\end{aligned}}$$

- Potential : Real part of the complex potential
- Noise correlations
  - Diagonal: Imaginary part of the complex potential

$$i\frac{\partial}{\partial t}\left\langle\psi(t,r)\right\rangle=\left\{V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r,r)\right\}\left\langle\psi(t,r)\right\rangle,$$

$$\Leftrightarrow i\frac{\partial}{\partial t}D_{\infty}^{>}(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]D_{\infty}^{>}(t,r)$$

No recipe for off-diagonal correlation from complex potential

Off-diagonal correlation

- Correlation length ~ medium resolution scale
  - e.g. One-body stochastic Schrödinger equation in a uniform medium

$$\psi(t+\Delta t,r)=\exp\left[-i\Delta t\left\{-\frac{\nabla^2}{2M}+\theta(t,r)\right\}\right]\psi(t,r),$$

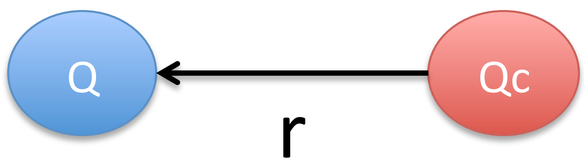
$$\left\langle\theta(t,r)\right\rangle=0,\quad \left\langle\theta(t,r)\theta(t',r')\right\rangle=\gamma(r-r')\delta_{tt'}/\Delta t,$$

- $\gamma(r-r')=\gamma\,?$ Uniform random phase → no physical effect
- $\gamma(r-r')=\gamma\exp\left[-\Delta r^2/l_{\mathrm{corr}}^2\right]$ Decoherence over distance l_{corr}

Essentially, this leads to quarkonium melting!

Perturbation theory

- Noise correlations
  - do not know where Q and Qc are



$\theta(t,r)$ $\qquad$ $\theta(t,0)$

$$\Theta(t,r) = \theta(t,r) - \theta(t,0)$$

$$\langle \theta(t,r) \rangle = 0, \quad \langle \theta(t,r)\theta(t',r') \rangle = \gamma(r-r')\delta_{tt'}/\Delta t,$$

$$\rightarrow \langle \Theta(t,r)\Theta(t',r') \rangle = \cdots, \quad \langle \Theta(t,r)^2 \rangle = 2\left[\gamma(0) - \gamma(r)\right]/\Delta t > 0$$

r-dependence of the imaginary part of the potential arises due to interference of noises at Q and Qc

Numerical simulation

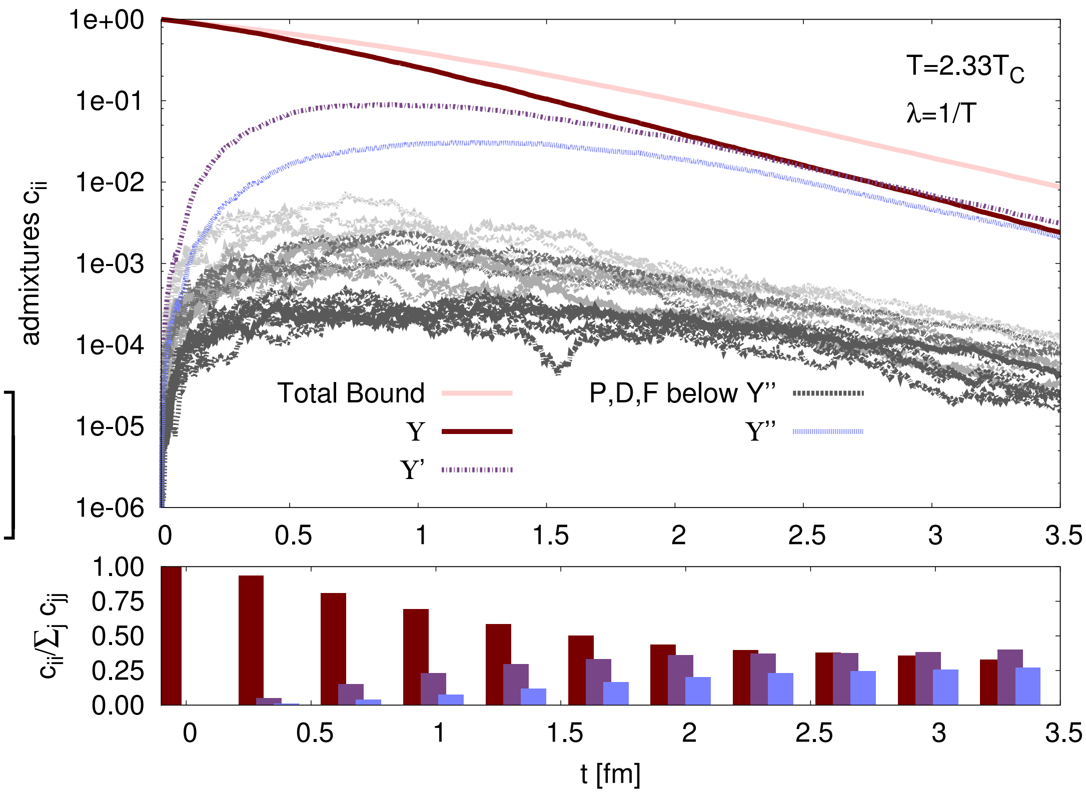
- Static & uniform matter

Rothkopf (14)

$$V_{\mathrm{Re}}(r) = -\frac{g^2 C_F}{4\pi}\left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r}\right]$$

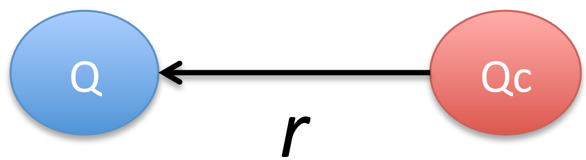
$$\Gamma(r,r) = \frac{g^2 T C_F}{4\pi}\phi(m_{\mathrm{D}} r),$$

$$\Gamma(r,r') = \sqrt{\Gamma(r,r)\Gamma(r',r')}\exp\left[-\frac{|r-r'|^2}{2\lambda^2}\right]$$



Quarkonium color?

- According to perturbation theory



$\theta^a(t,r)t^a$ $\qquad$ $\theta^a(t,0)(-t^{a*})$

$$\Theta(t,r) = \theta^a(t,r)t^a \otimes 1 + \theta^a(t,0)1 \otimes (-t^{a*})$$

$$\left\langle \theta^a(t,r) \right\rangle = 0, \quad \left\langle \theta^a(t,r)\theta^b(t',r') \right\rangle = \gamma(r-r')\delta^{ab}\delta_{tt'}/\Delta t,$$

– Singlet → Octet → Singlet → …

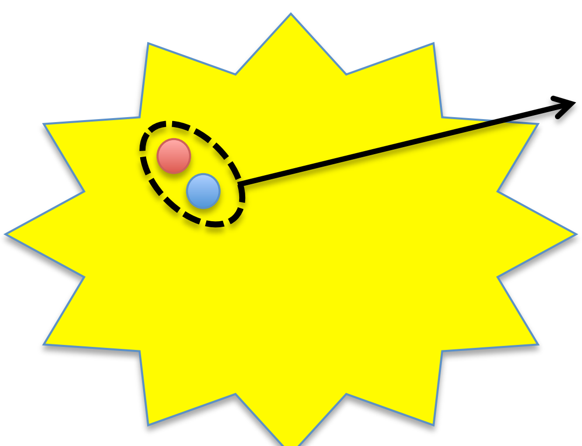
Limitations: no dissipation

- Hamiltonian formalism
  - Stochastic potential ~ random time-dependent potential
  - Irreversible process cannot be described

- Applicable to
  - Study of quarkonium melting until HQ sector approaches close to equilibration (KE_Q~T)

Hydro + QQc

- Propagation and decoherence in hydro background
  - Quarkonium feels *T*(*t*) along its trajectory
    - Can explain upsilon yields?
    - Can explain centrality dependence?



Y.A. Nonaka, Rothkopf, in progress

3. Summary

- Users' guide for the stochastic potential
  - Imaginary part of the potential comes from thermal noise
  - Its off-diagonal correlation is important for decoherence, i.e. melting of quarkonium.
  - Application to phenomenology: Hydro + QQc