Gaining Insight with Bayesian Inference

EVAN SANGALINE (MICHIGAN STATE UNIVERSITY)
MADAI COLLABORATION (HTTP://MADAI.US)
JUNE 09, 2015

Cyber Enabled Discovery and Innovation
Part I
or “Determining the EOS and Viscosity”
I won’t talk about...

- The basic premise of Bayesian inference
- Dimensional reduction of experimental measurements
  - Principal Component Decomposition
- The details of our model
- Model emulation
  - Gaussian Process Interpolation
  - Validation
Because you’ve probably already heard it

**AGENDA**

<table>
<thead>
<tr>
<th>Session I:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>02:00-02:30</td>
<td>Accessing QCD Through Modeling of Heavy Ion Collisions</td>
<td>Scott Pratt, Michigan State University</td>
</tr>
<tr>
<td>02:30-03:00</td>
<td>The Initial Conditions of Heavy Ion Collisions</td>
<td>Prithwish Tribedy, BNL</td>
</tr>
<tr>
<td>03:00-03:30</td>
<td>Baysian Methods for Constraining Initial Conditions and Viscosity</td>
<td>Jonah Bernhard, Duke University</td>
</tr>
<tr>
<td>03:30-04:00</td>
<td><strong>Coffee Break</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>End of Session I</td>
<td></td>
</tr>
</tbody>
</table>

**Session II:**

| 04:00-04:30        | Experimental Overview of Correlations, Measurements, and Flow  | Jiangyong Jia, Stony Brook University                          |
| 04:30-05:00        | Baysian Methods for Determining the EOS and Viscosity          | Evan Sangaline, Michigan State University                      |
|                    | Adjourn                                                        |                                                                 |
Constraining the Shear Viscosity

**Prior**

- 100 Prior Samples
- extrapolated HRG (arXiv:1409.0010)
- $1/4\pi$

**Posterior**

- 100 Posterior Samples
- Posterior Mean $\pm 1\sigma$
- extrapolated HRG (arXiv:1409.0010)
- $1/4\pi$
Constraining the Equation of State

Prior

\[ c_s^2 (\text{speed of sound squared}) \]

Lattice: Hot QCD / BW (arXiv:1407.6387)

Hadron Gas

50 Prior Samples

Posterior

\[ c_s^2 (\text{speed of sound squared}) \]

Lattice: Hot QCD / BW (arXiv:1407.6387)

Hadron Gas

50 Posterior Samples
Constraining the Equation of State

Excellent agreement with lattice

Posterior
Bayesian approach is great for...

What does the data tell us about ____________?
What about...
Part II
or “Gaining Insight”
Very intuitive

A Common Approach

1. Pick a physics parameter of interest

2. Turn the knob in your model until you match experimental data

P. Romatschke and U. Romatschke PRL 99,172301

$\eta/s$
How do we restore that intuition?

- Visualization in more than 3 dimensions
  - e.g. projections, scatter-plot matrices, factorization, parallel coordinates, dimensional reduction/manifold learning

- Modifying the observable values and rerunning the MCMC
  - lots of knobs
  - expensive

- Sensitivity analysis

- Others
  - e.g. canonical correlation analysis
Stiff
Initially rising

 Initially falling
Soft

$c^2$ (speed of sound squared)

Lattice: Hot QCD / BW
(arXiv:1407.6387)

Green: Hadron Gas
Red: $X' = 1.0, X_0$ ratio = 1.5

$c^2$ (speed of sound squared)

Lattice: Hot QCD / BW
(arXiv:1407.6387)

Green: Hadron Gas
Red: $X' = 4.5, X_0$ ratio = 1.5

$c^2$ (speed of sound squared)

Lattice: Hot QCD / BW
(arXiv:1407.6387)

Green: Hadron Gas
Red: $X' = 4.5, X_0$ ratio = -0.5

$c^2$ (speed of sound squared)

Lattice: Hot QCD / BW
(arXiv:1407.6387)

Green: Hadron Gas
Red: $X' = 1.0, X_0$ ratio = -0.5
Scatter-Plot Matrices

STAR $v_2$

ALICE $v_2$

not useful

useful

$\frac{\eta}{s}$ $\frac{\partial \eta/s}{\partial \ln T}$
RHIC Data Only

Well constrained viscosity at $T_C$

Little constraint on the temperature dependence
RHIC Data Only

LHC Data Only

Poorly constrained viscosity at $T_C$

Tighter constraint on the temperature dependence
Both are well constrained

Preferred viscosity at $T_c$ is $\frac{2.26}{4\pi} \pm 0.07$
Can we estimate how the results would change for different experimental data?
Log-Likelihood Derivatives

- Normally store LL, parameters, and observables for each sample
- Can store additional information
  - How the likelihood depends on the experimental measurements/uncertainties

\[
\frac{\partial LL}{\partial z}
\]
Log-Likelihood Derivatives

With respect to experimental measurements

\[ \frac{\partial LL}{\partial y_{obs,i}} = (\Sigma^{-1}(y - y_{obs}))_i \]

\[ \Sigma = \text{combined model and measurement uncertainty covariance matrix} \]

With respect to experimental errors

\[ \frac{\partial LL}{\partial \sigma_{y_{obs,i}}} \approx \frac{1}{2} (y - y_{obs})^T \Sigma^{-1} \Delta \Sigma^{-1} (y - y_{obs}) \]

\[ \Delta_{j,k} = \begin{cases} 
2\sigma_{y_{obs,j}} & \text{if } i = j \land i = k \\
\sigma_{y_{obs,j}} \rho_{y_{obs,j},y_{obs,k}} & \text{if } i \neq j \land i = k \\
\sigma_{y_{obs,k}} \rho_{y_{obs,j},y_{obs,k}} & \text{if } i = j \land i \neq k \\
0 & \text{if } i \neq j \land i \neq k 
\end{cases} \]
Linearized Log-Likelihood Trace Reweighting

Consider a small change in either an experimental measurement or its uncertainty \( z \rightarrow z + \delta z \)

We can approximate the likelihood weighted expectation of any function \( f \) of observables and parameters as

\[
\langle f(x, y) \rangle \rightarrow \langle f(x, y) \rangle + \delta z \left[ \left( f(x, y) \frac{\partial LL}{\partial z} \right) - \langle f(x, y) \rangle \left( \frac{\partial LL}{\partial z} \right) \right]
\]

using the existing MCMC trace from the unperturbed case
Relationship Between Shear Viscosity and $v_2$

Extracted shear viscosity at $T_c$ can be approximated by $\langle (\eta/s)_0 \rangle \approx 0.183 - \delta v_2 5.95$

<table>
<thead>
<tr>
<th>STAR 20-30% fluctuation corrected $\langle v_2^2 \rangle_{pT}$</th>
<th>$\langle (\eta/s)_0 \rangle$ given $v_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>0.25</td>
</tr>
<tr>
<td>0.0814 (actual value)</td>
<td>0.18</td>
</tr>
<tr>
<td>0.9</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Implicitly, all parameter distributions are changing as we vary $v_2$

This is very different from varying $\eta/s$
An Important Question...

How should we allocate experimental resources to address physics goals?
Resolving Power: $\frac{\sigma_{obs}}{\sigma_{par}} \frac{\partial \sigma_{par}}{\partial \sigma_{obs}}$

useful

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$\left(\frac{\eta}{s}\right)_0$</th>
<th>$\left(\frac{\partial \eta/s}{\partial \ln T}\right)_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rhic200_cent0to5_STAR_RLONG_PION</td>
<td>0.0458210528304</td>
<td>0.22545980686</td>
</tr>
<tr>
<td>rhic200_cent20to30_STAR_V2_PION_PTWEIGHT</td>
<td>0.126717206018</td>
<td>-0.0177460504008</td>
</tr>
<tr>
<td>lhc2760_cent0to5_ALICE_RLONG_PION</td>
<td>0.046306698507</td>
<td>0.130951874936</td>
</tr>
<tr>
<td>lhc2760_cent20to30_ALICE_SPECTRA_PION_YIELD</td>
<td>0.0955331868317</td>
<td>0.358219778086</td>
</tr>
<tr>
<td>lhc2760_cent20to30_ALICE_V2_PION_PTWEIGHT</td>
<td>0.0330489825668</td>
<td>0.0867950762928</td>
</tr>
</tbody>
</table>
Resolving Power

Of 20-30% central ALIVE $v_2$

$$\frac{\sigma_{v_2}}{\sigma(\eta/s)_0} \frac{\partial \sigma(\eta/s)_0}{\partial \sigma_{v_2}} = 0.033$$

Of 20-30% central ALICE pion yield

$$\frac{\sigma_{N\pi}}{\sigma(\eta/s)_0} \frac{\partial \sigma(\eta/s)_0}{\partial \sigma_{N\pi}} = 0.095$$

Intuition tends to overestimate statistical significance

Much less than 1, the assumption if only varying $\eta/s$

$\sim 3x$ more significant (surprising?)

Of 20-30% central ALICE pion yield
Identifying Model Weaknesses

Contradiction?

\[
\frac{\partial L}{\partial \bar{O}_\text{obs}}
\]

\[
\sigma_{\text{obs}}
\]

\[
\text{rhic200_cent0to5_PHENIX_SPECTRA_PION_YIELD, lhc2760_cent0to5_ALICE_SPECTRA_PION_YIELD, lhc2760_cent2to30_ALICE_SPECTRA_PION_YIELD}
\]

\[
-3.78351109719 \quad -10.3162287547 \quad 3.08697230363
\]
Final Thoughts

- Bayesian methodology has proven fruitful

Three ways forward:
- New experimental data or analyses
- More accurate models and emulators
- Analysis of models and resulting posterior distributions
Backup slides
Parameterized Collision Model

- Smooth Glauber Initial Conditions
  - 10 parameters – (5 for 200 GeV, 5 for 2.76 TeV)
    - energy normalization
    - balance of wounded nucleon vs saturation picture
    - saturation scale
    - initial flow
    - stress energy tensor asymmetry

- Boost Invariant Israel-Stewart Hydro
  - 2 Equation of State Parameters
  - 2 Shear Viscosity Parameters
  - more on these later...

- Hadronic Cascade
  - begins at $T_C = 165$ MeV

- Analysis
  - Using the same cuts/methods as the experiments
Collection of Observations

- **16 Spectra Observables**
  - \( \langle p_T \rangle \) for \((\pi, k, \bar{p})\) X (0-5% centrality, 20-30% centrality) X (200 GeV PHENIX, 2.76 TeV ALICE)
  - \( \pi \) yields for (0-5% centrality, 20-30% centrality) X (200 GeV PHENIX, 2.76 TeV ALICE)

- **12 HBT Observables**
  - \( \pi \) (Rlong, Rout, Rside) X (0-5% centrality, 20-30% centrality) X (200 GeV PHENIX, 2.76 TeV ALICE)

- **2 Flow Observables**
  - 20-30% centrality \( v_2 \{2\} \) for (200 GeV STAR, 2.76 TeV ALICE)
  - Corrected for fluctuating initial conditions

20-30% centrality minimized effect of fluctuating initial conditions

Non-smooth initial conditions in progress
Model Emulation

Model is too computationally expensive for direct Markov chain Monte Carlo.

We need something faster...

Parameter space is explored using Latin Hypercube Sampling. 
~1000 model evaluations

\[ y \rightarrow \tilde{y} \equiv \frac{y - \langle y \rangle}{\sigma_y} \]

Perform principal component analysis by projecting \(y\) onto the eigenvectors of 
\[ \langle \tilde{y}\tilde{y}^T \rangle \]

ignoring those with negligible eigenvalues.

Normalize the data.
Gaussian Process Interpolation

Assume a prior over Gaussian processes to enforce smoothness.

Find posterior over functions based on consistency with training points.
Cross Validation and Consistency Check

Additional model runs are used to validate the emulation.

Emulation errors are negligible compared to the 5% model uncertainties.

The resulting posterior distributions are all consistent with the experimental measurements.
Shear viscosity parameterization

Viscosity at freeze-out ($\in [0,0.5]$)

\[ \frac{\eta}{s} = \left( \frac{\eta}{s} \right)_0 + \left( \frac{\partial \eta}{\partial \ln T} \right) \ln \frac{T}{T_C} \]

Temperature dependency of viscosity ($\in [0,3.0]$)

Encompasses many possibilities...
Speed of Sound Parameterization

\[ c_s^2 = c_{s, had}^2 + \left( \frac{1}{3} - c_{s, had}^2 \right) \frac{x^2 + X_0 x}{x^2 + X_0 x + X'} \quad \text{with} \quad x(\varepsilon) = \ln \frac{\varepsilon}{\varepsilon_{had}} \]

- Constrained to matched hadronic speed of sound at T=165 MeV
- Goes to 1/3 at large energy densities
- Positive definite:

\[ X_0 \text{ ratio} = \frac{X_0}{2X' \sqrt{3} c_{s, had}} > -1 \]
RHIC Data Only

LHC Data Only

Combined Data

\begin{itemize}
  \item RHIC Data Only
  \item LHC Data Only
  \item Combined Data
\end{itemize}