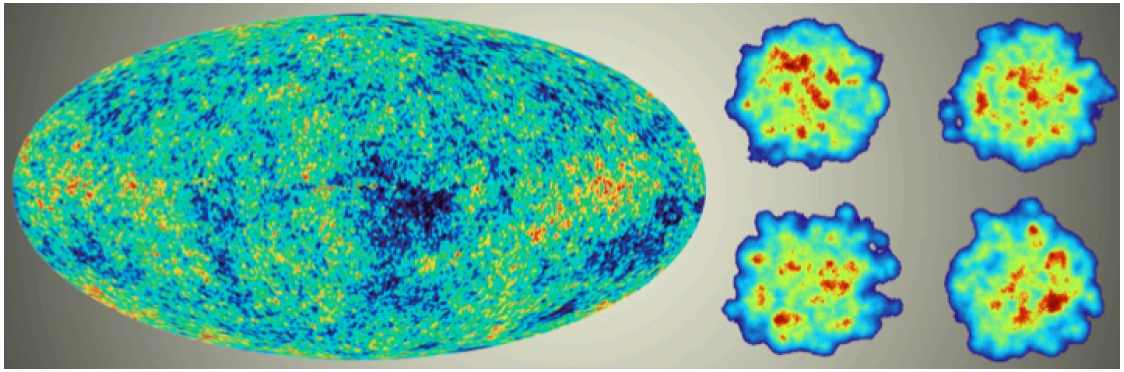
What have we learned from event-by-event flow measurement?

Jiangyong Jia

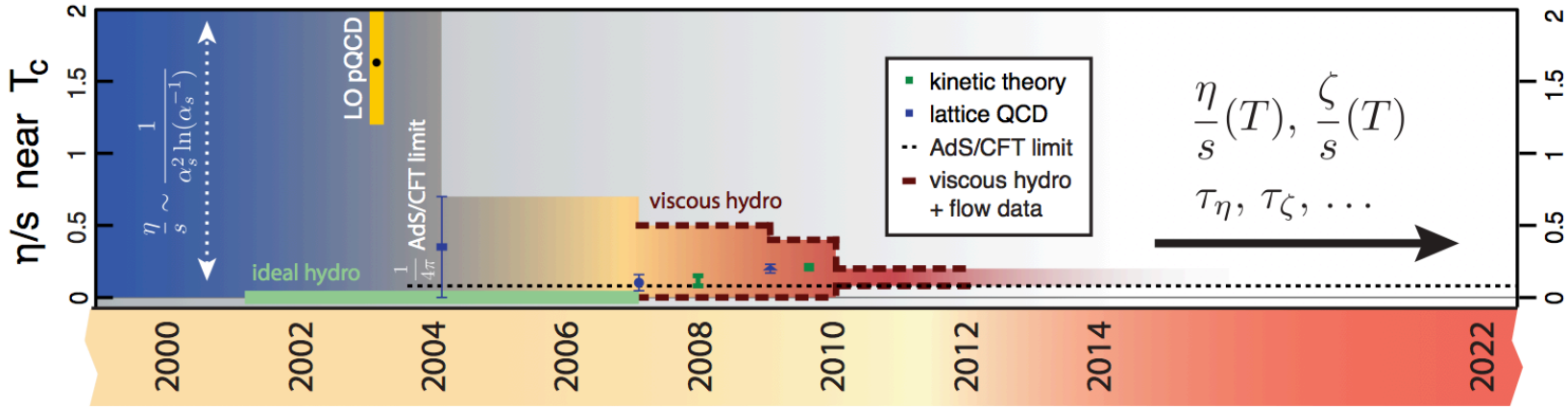
RHIC/AGS User Workshop

Recent accomplishments

Improved the space-time picture of bulk matter evolution



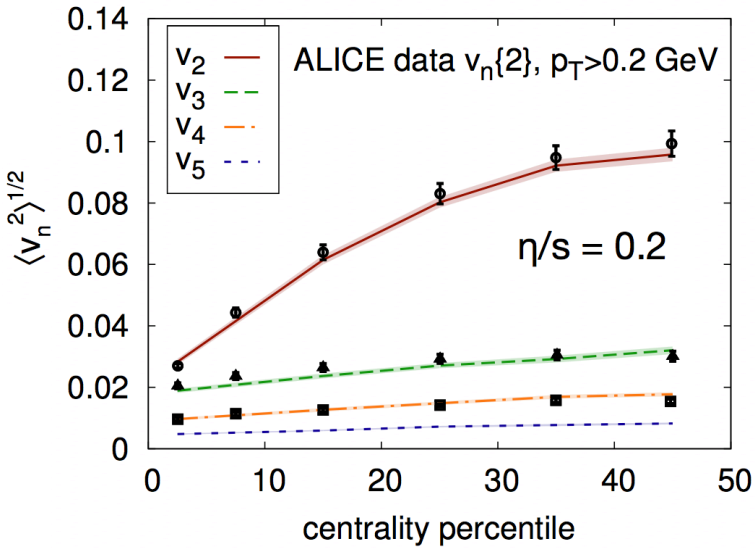
Improved extraction of the medium properties

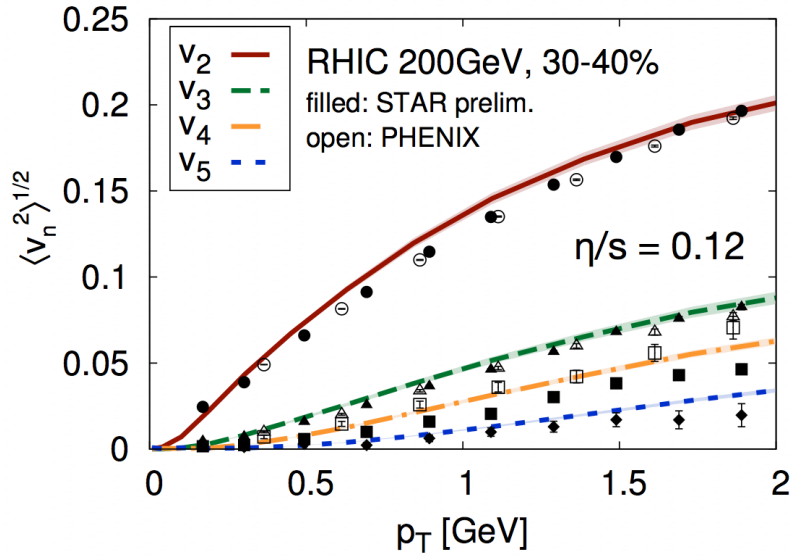


Challenge: simultaneous determination of two unknowns

Future experimental measurements

- Experimental error bar no-longer the limiting factors on the η/s

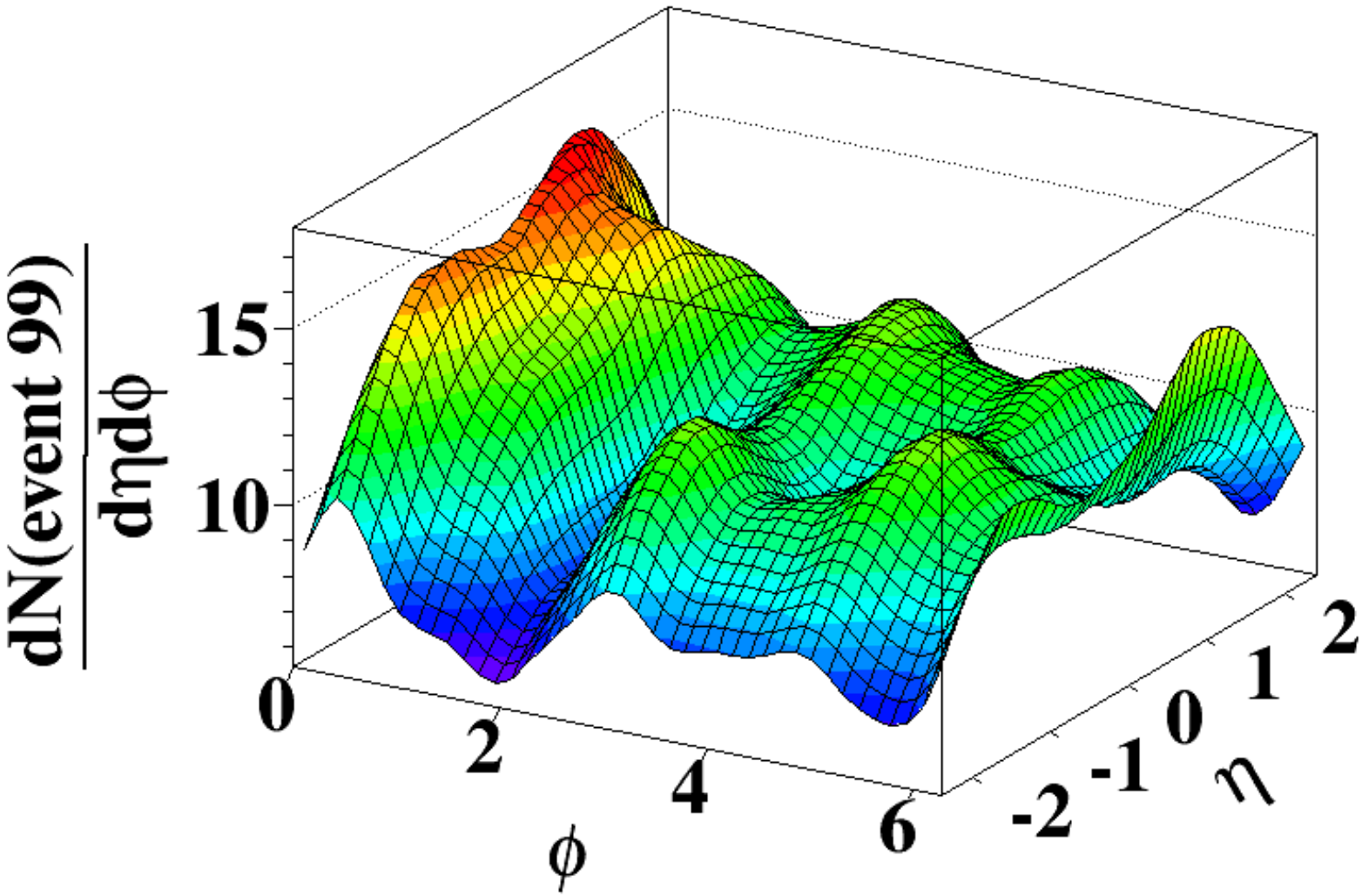




- Measurement aim to provide a picture that is as complete as possible.
  - Event-by-event flow observables→ initial fluctuation & flow correlation
  - Event shape engineering & engen-mode ana.→ geometry & hydro. response
  - Longitudinal fluctuations→ particle production & early time dynamics
  - Medium response→ relaxation mechanism for local perturbation
  - PID v_n → hadronization and hadronic transport
- Detailed baseline for system size scan & beam energy scan studies

EbyE 3+1D hydro with AMPT initial condition

Significant transverse and longitudinal fluctuations

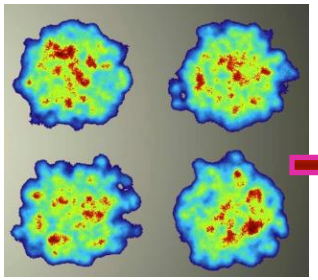


Curtsey of L.Pang and X.N Wang, EbyE 3D hydro+AMPT condition

Event-by-event observables

$$\frac{dN}{d\phi} \propto 1 + \sum_n 2\mathrm{v}_\mathrm{n} \cos n\left(\phi - \Phi_n\right)$$

Many little bangs



1104.4740, 1209.2323,1203.5095 ,1312.3572

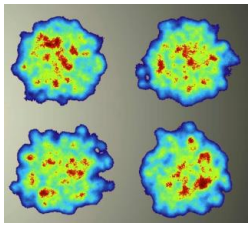
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

Also access via multi-particle correlations

$$\boxed{\begin{aligned}&\langle \cos(n_1\phi_1 + n_2\phi_2 ... + n_m\phi_m)\rangle = \\ &\qquad \langle v_{n_1} v_{n_2} ... v_{n_m} \cos(n_1\Phi_{n_1} + n_2\Phi_{n_2} ... + n_m\Phi_{n_m})\rangle\end{aligned}} \quad \Sigma n_i = 0$$

Event-by-event observables

Many little bangs



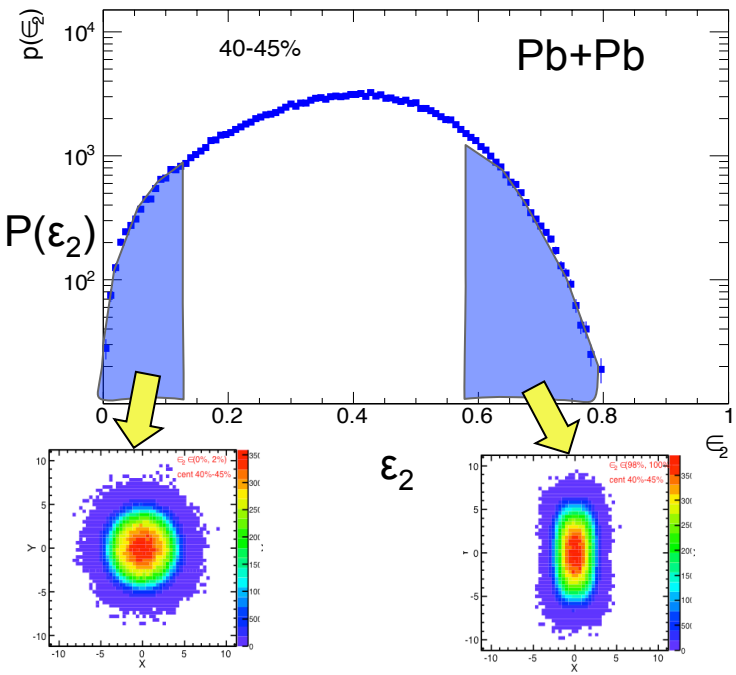
1104.4740, 1209.2323,1203.5095 ,1312.3572

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\rm evts}} \frac{dN_{\rm evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

	pdf's	Moments / cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1,2,...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle - \langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2 \rangle \langle v_n^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle - \langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$	yes

Event shape selection

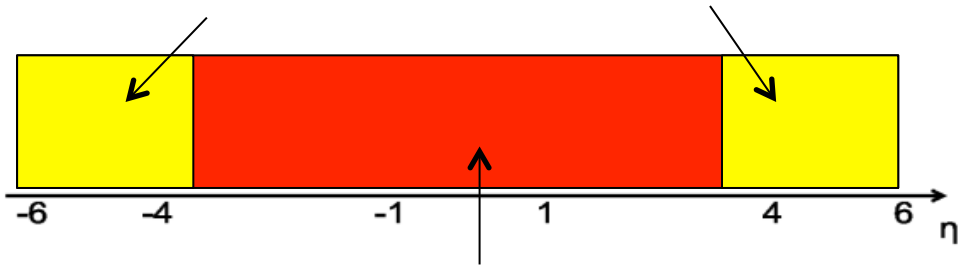
arXiv:1208.4563 arxiv:1311.7091



Fourier expansion of forward E_T distri.:

$$2\pi \frac{dE_T}{d\phi} = \Sigma E_T \left[1 + 2 \sum_n v_n^{\text{obs}} \cos n\left(\phi - \Phi_n^{\text{obs}}\right) \right]$$

Select events with certain v_2^{obs} or v_3^{obs}



measure $v_n(p_T)$ and $p(v_n, v_m, \ldots, \Phi_n, \Phi_m, \ldots)$

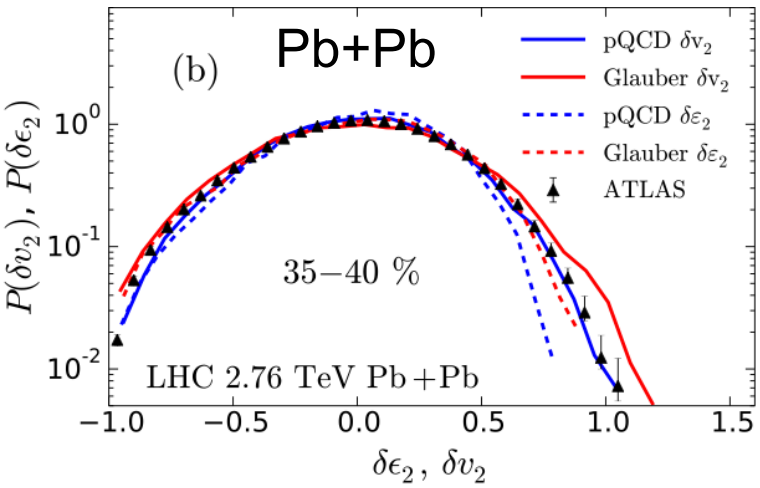
- 0th order event-shape selection: Centrality by ΣE_T (system size)
- 2nd order event-shape selection: ellipticity by v_2^{obs} (system shape)
- 3rd order event-shape selection: triangularity by v_3^{obs} (system shape)

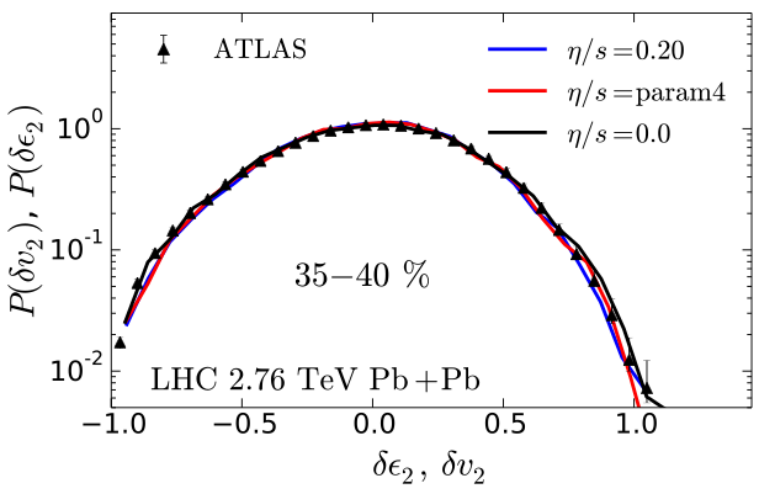
Applications

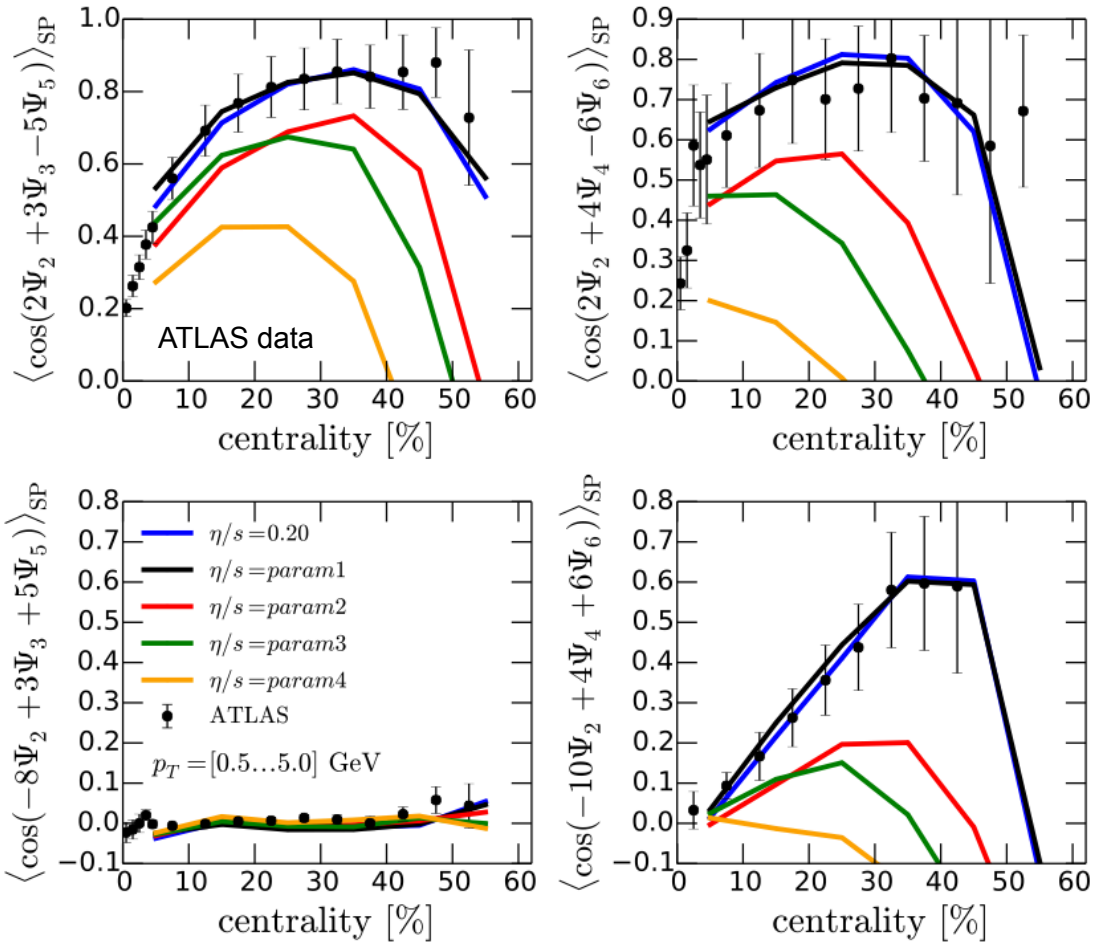
Current measurements of E-by-E distributions

$p(v_2)$, $p(v_3)$, $p(v_4)$

$p(\Phi_n,\Phi_m)$ and $p(\Phi_n,\Phi_m,\Phi_L)$



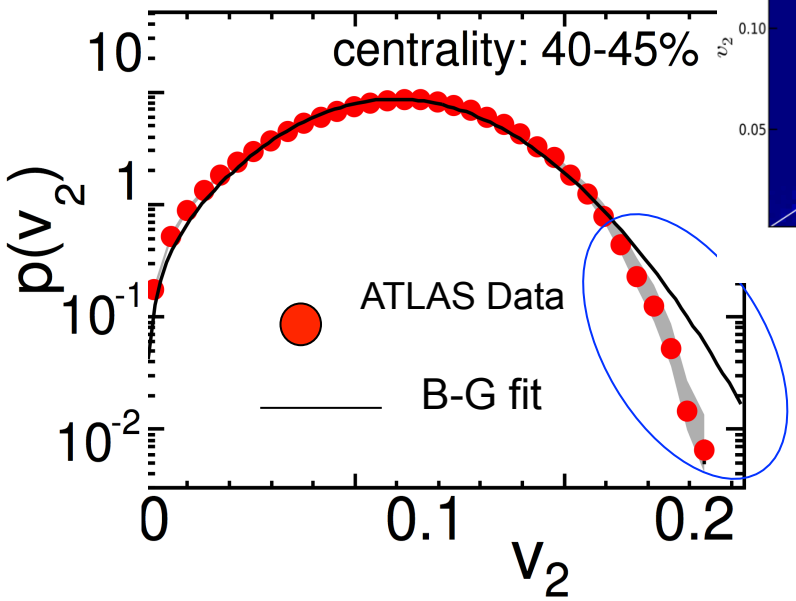


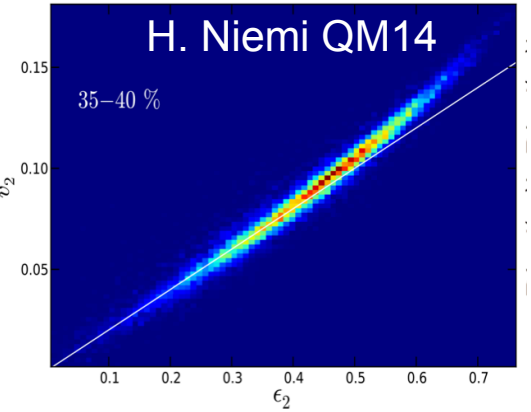


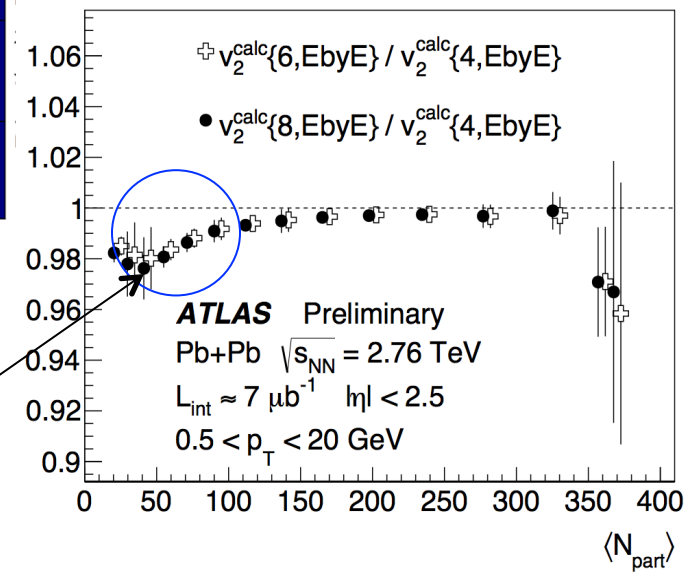
1505.02677

Constraint initial condition & medium properties!!

Non-Gaussianity in the $p(v_2)$ distribution







Non-Gaussian behavior

$\langle v_2 \rangle = k \langle \varepsilon_2 \rangle$

- Reflected by a 1-2% change beyond 4th order cumulants

Non-BG in $p(\varepsilon_2)$ or non-linearity of response for large ε_2

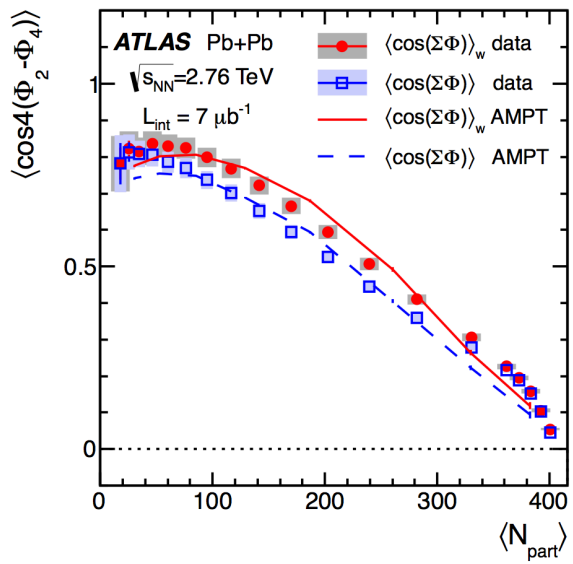
How are (ε_n,Φ_n^*) transferred to (v_n, Φ_n)?

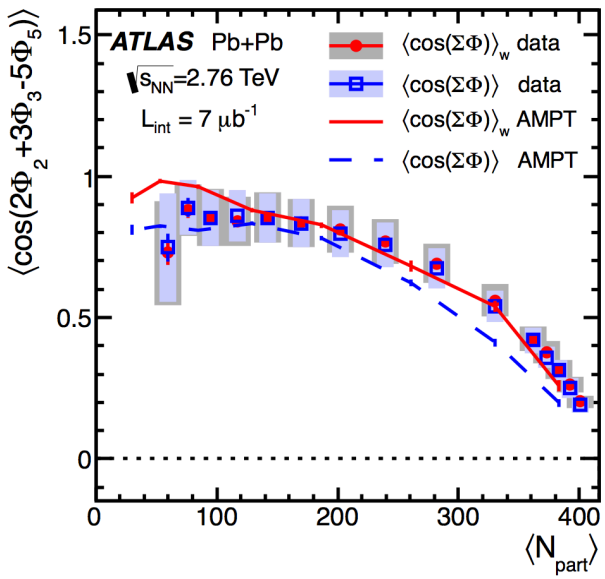
- Flow response is linear for v_2 and v_3: arXiv:1403.0489 arXiv:1111.6538

$$v_2 e^{i2\Phi_2} \propto \epsilon_2 e^{i2\Phi_2^*}, \quad v_3 e^{i3\Phi_3} \propto \epsilon_3 e^{i3\Phi_3^*}$$

- Higher-order flow arises from EP correlations., e.g. <cos4(Φ_2-Φ_4)>:

$$v_4 e^{i4\Phi_4} \approx c_0\, \epsilon_4 e^{i4\Phi_4^*} + c_1 \left(v_2 e^{i2\Phi_2}\right)^2$$



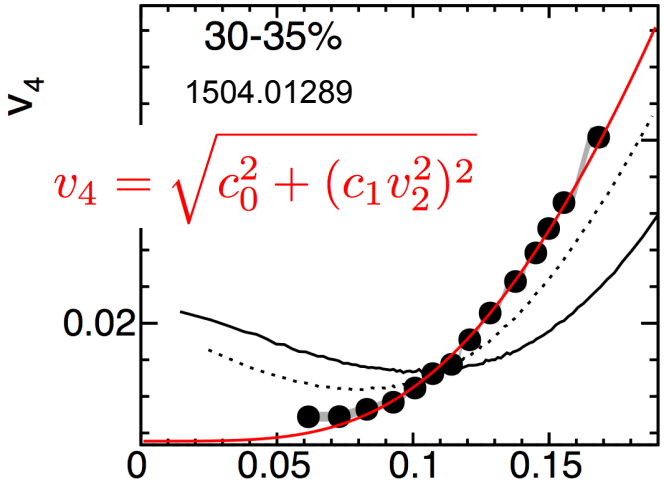


$$v_5 e^{i5\Phi_5} \approx c_0\, \epsilon_5 e^{i5\Phi_5^*} + c_1\, v_2 v_3 e^{i(2\Phi_2+3\Phi_3)}$$

$$v_4 \approx \sqrt{c_0^2 + (c_1 v_2^2)^2}$$

$$v_5 \approx \sqrt{c_0^2 + (c_1 v_2 v_3)^2}$$

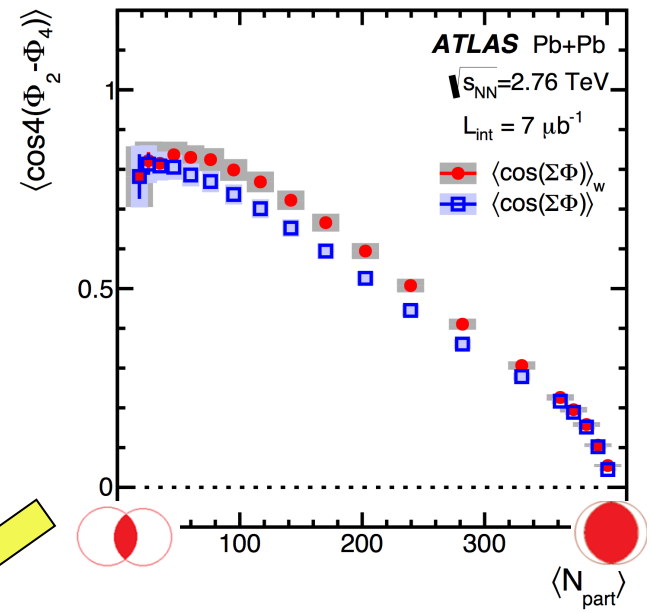
v_4-v_2 correlation from event-shape engineering



$$v_4 = \sqrt{c_0^2 + (c_1 v_2^2)^2}$$

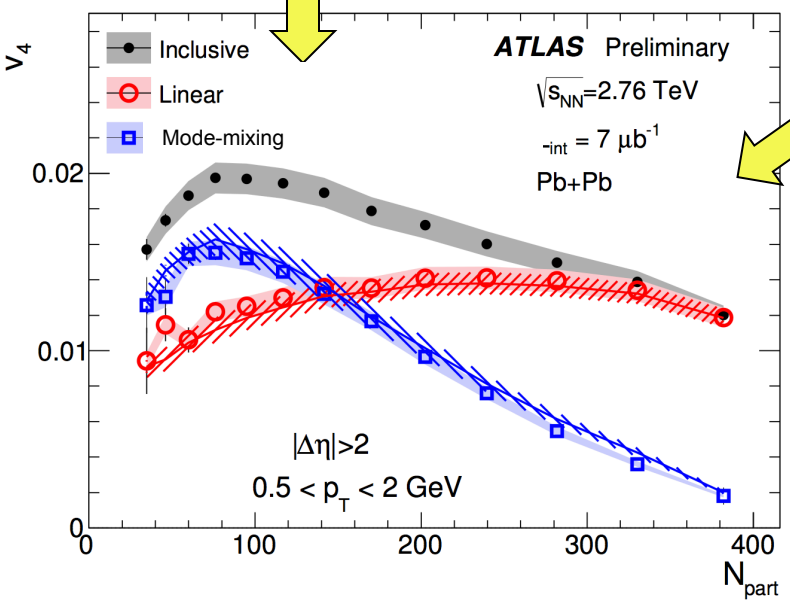
— Glauber

- - MC-KLN



$$v_4 e^{i4\Phi_4} = c_0 e^{i4\Phi_4^*} + c_1 \left(v_2 e^{i2\Phi_2}\right)^2 + \ldots$$

v_2



$$v_4^{NL} = v_4 \langle\cos 4(\Phi_2 - \Phi_4)\rangle$$

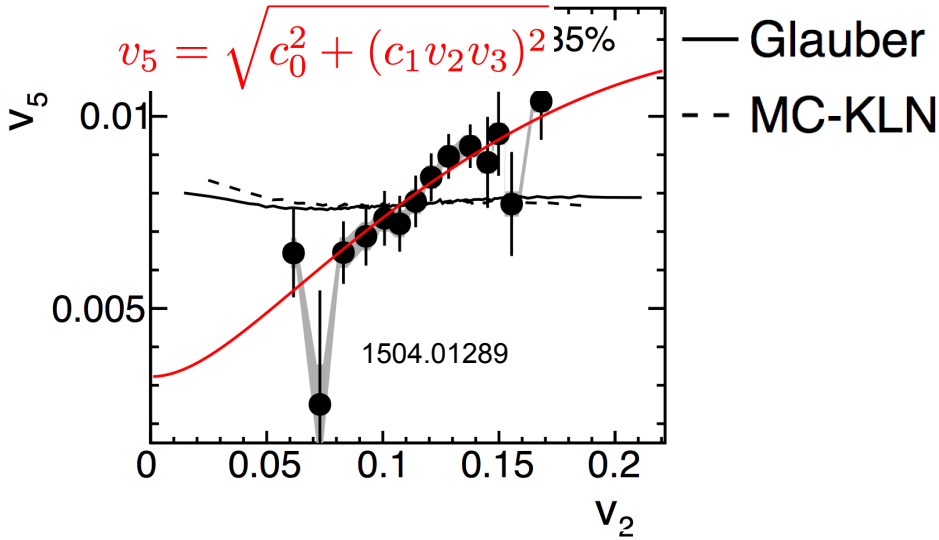
$$v_4^{L} = \sqrt{v_4^2 - (v_4^{NL})^2}$$

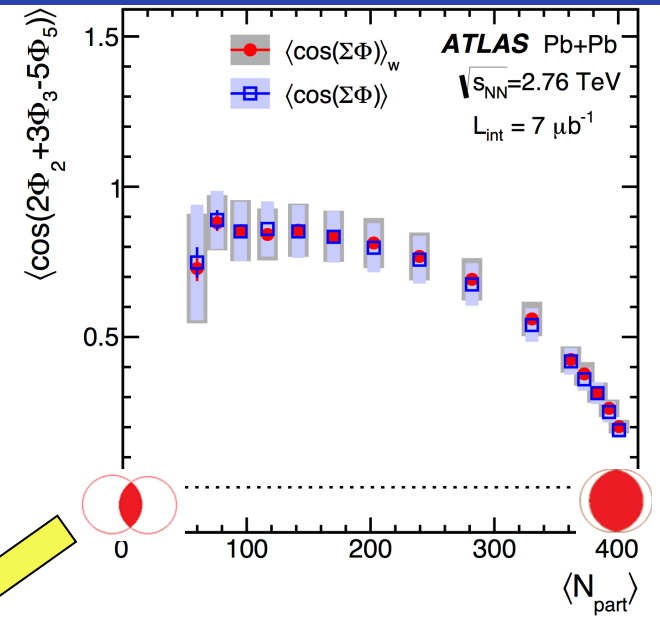
Compare well with expectation from EP correlations.

higher-order mode-mixing negligible

Separate geometry and mode-mixing components!!

v_5-v_2 correlation from event-shape engineering

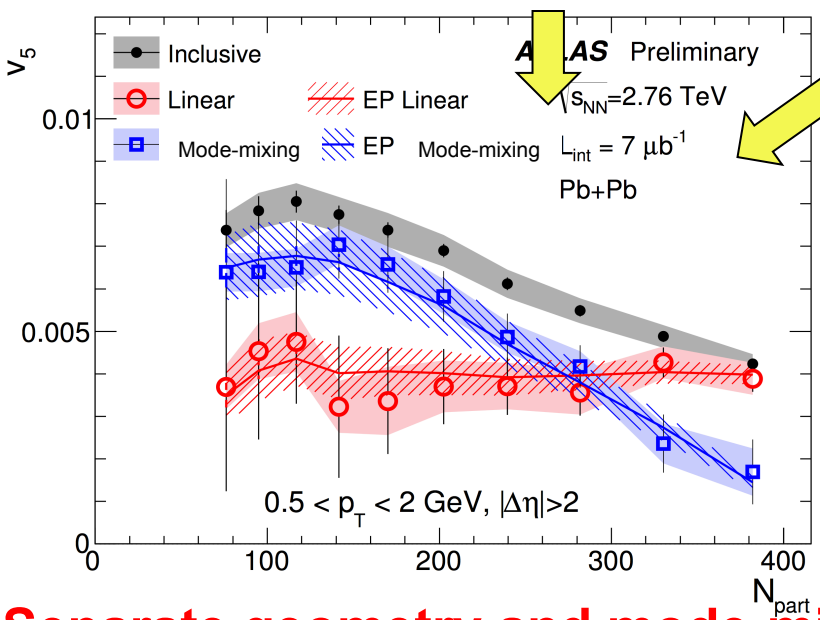




$$v_5 e^{i5\Phi_5} \propto c_0 e^{i5\Phi_5^*} + c_1 v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$

$$v_5^{NL} = v_5 \langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle$$

$$v_5^{L} = \sqrt{v_5^2 - (v_5^{NL})^2}$$

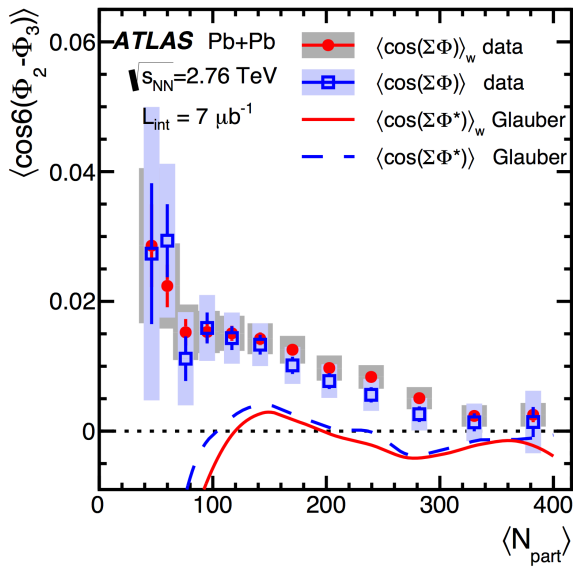


Compare well with expectation from EP correlations.
higher-order mode-mixing negligible

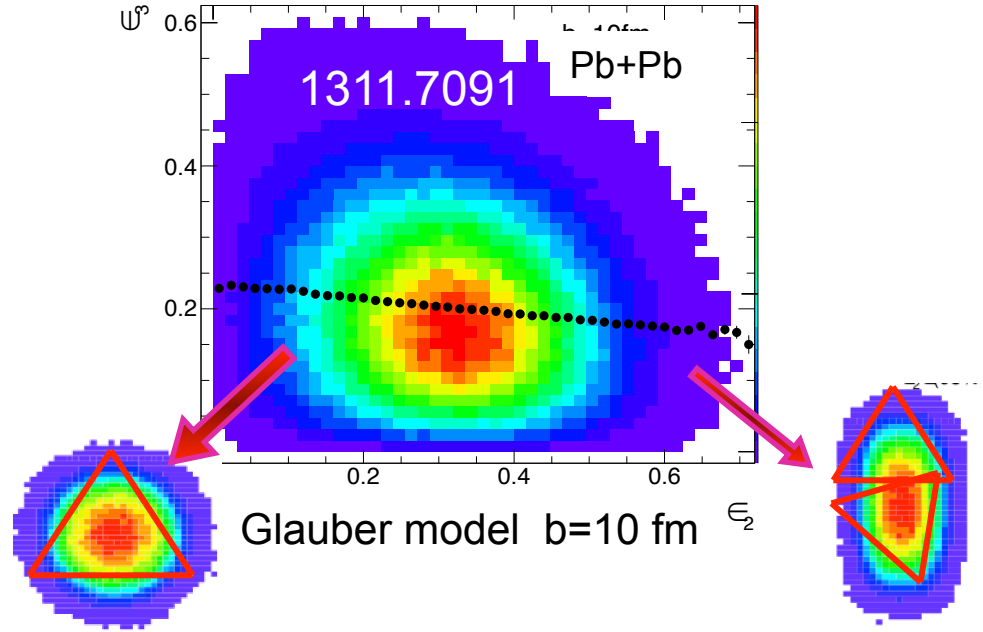
Separate geometry and mode-mixing components!!

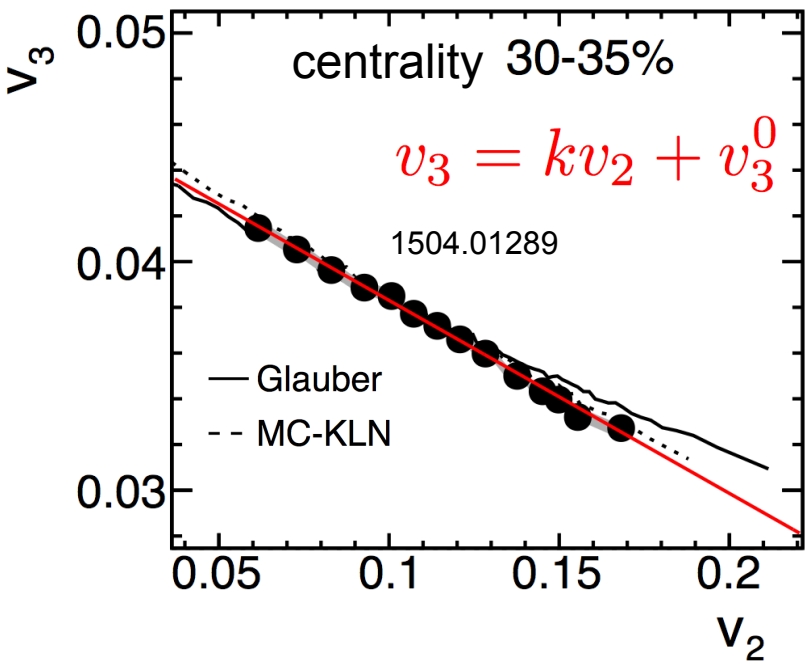
v_3-v_2 correlation from event-shape engineering

Very weak EP correlations

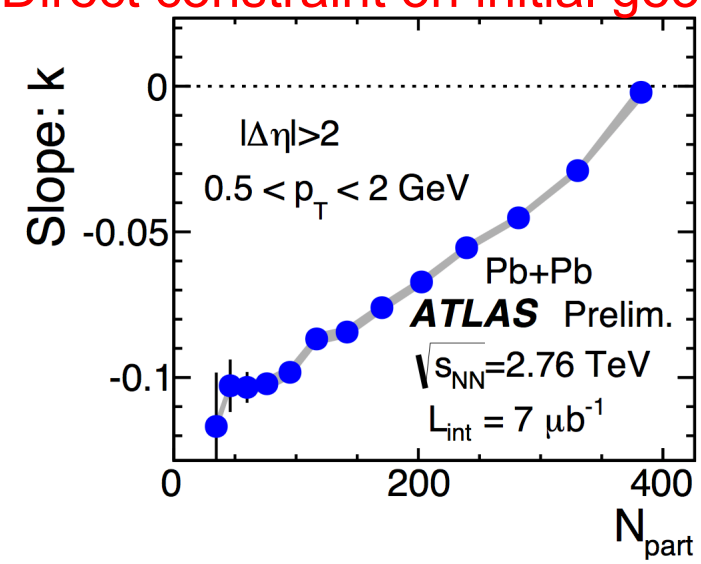


Significant magnitude anti-correlations

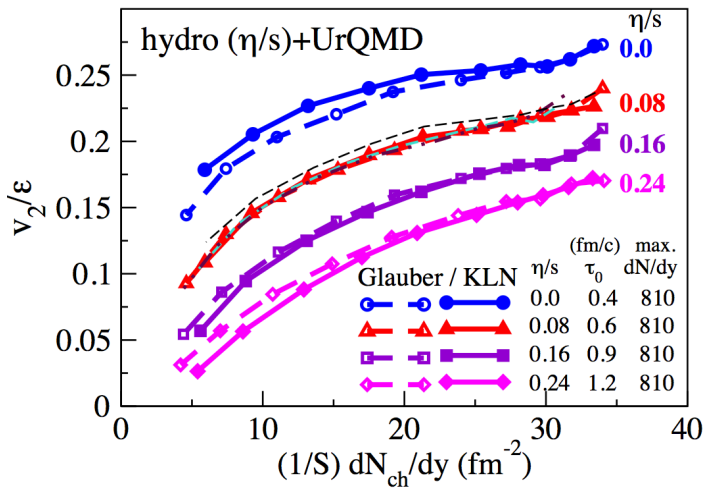




Direct constraint on initial geometry



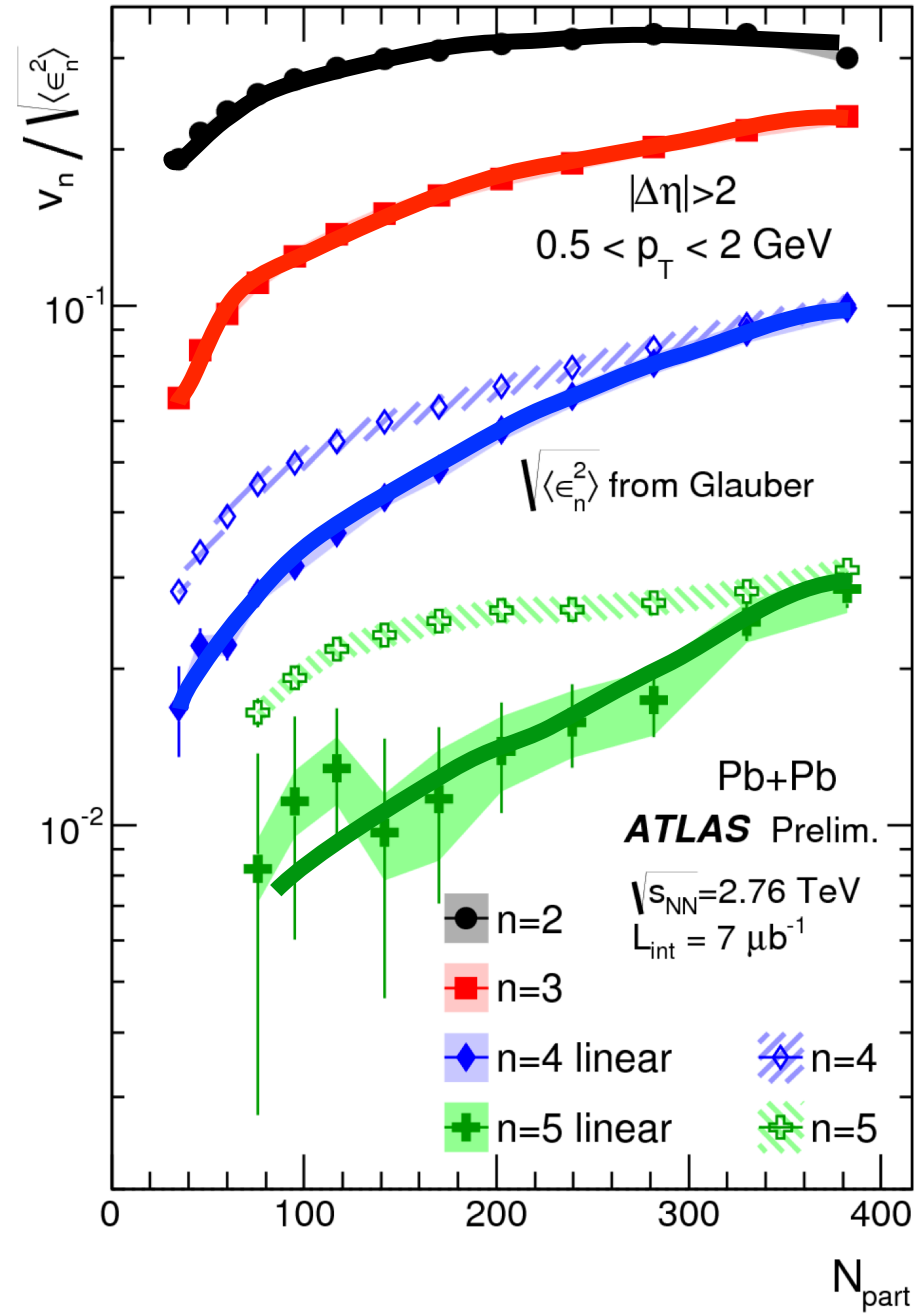
Eccentricity scaling: v_n/ε_n



Teaney and Yan

Viscous correlation: $$-\frac{\Delta w_n}{w_n^{\mathrm{id}}} \sim n^2 \frac{\eta}{s}$$

Viscous damping expected to be larger for larger n



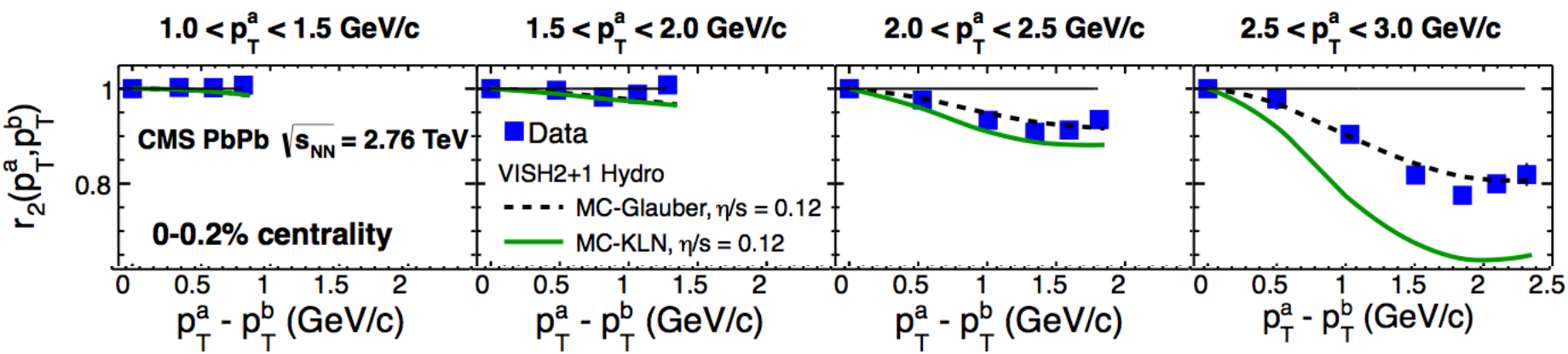
Even more detailed observables

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

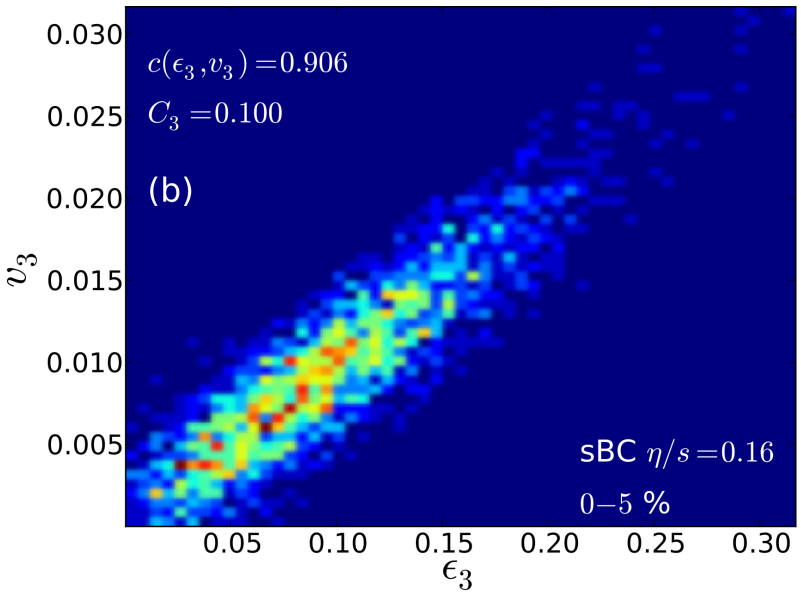
- Fluctuations can also be differential in p_T, η, PID etc.
  - $v_n(p_T,\eta,\ldots)$ & $\Phi_n(p_T, \eta, \ldots)$
  - Averaged out in most analyses!
- Example: p_T dependent flow fluctuation

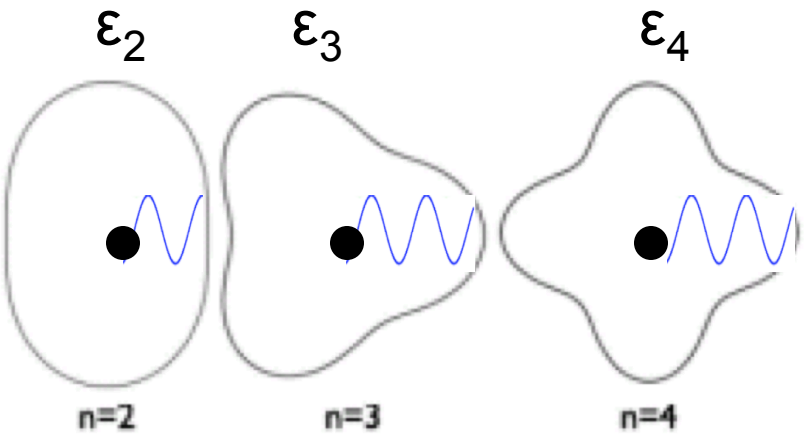
$$r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}} = \frac{\langle v_n(p_T^a) v_n(p_T^b) \cos\left[n\left(\Psi_n(p_T^a) - \Psi_n(p_T^b)\right)\right]\rangle}{\sqrt{\langle v_n^2(p_T^a)\rangle \langle v_n^2(p_T^b)\rangle}}$$

CMS 1503.01692



Transverse flow fluctuations





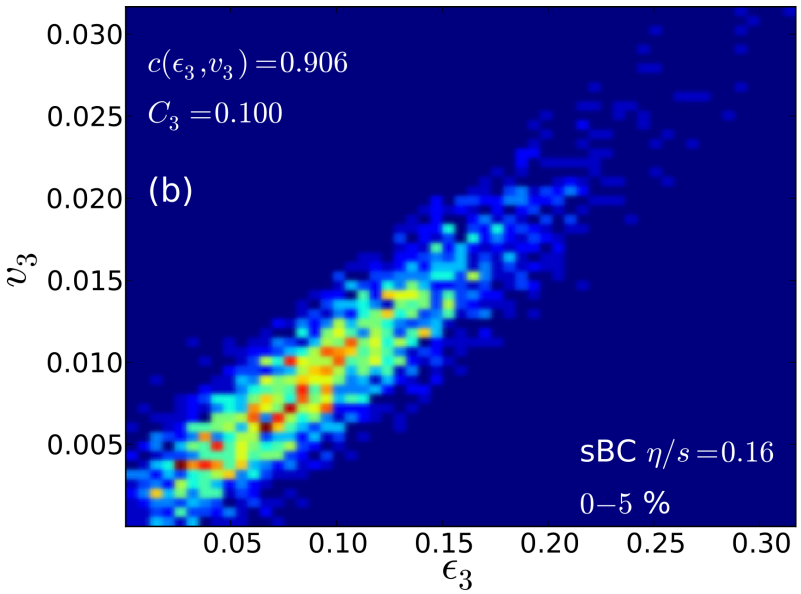
- No-unique mapping between ε_n and v_n, though $\langle v_n \rangle \propto \langle \varepsilon_n \rangle$
  - ε_n defined by different r^m weights → radial degrees of freedom.
- Related to flow angle and amplitude fluctuates in p_T

$$v_n(p_T)e^{in\Phi(p_T)} \qquad V_{n\Delta}(p_T^a, p_T^b) \neq v_n(p_T^a)v_n(p_T^b)$$

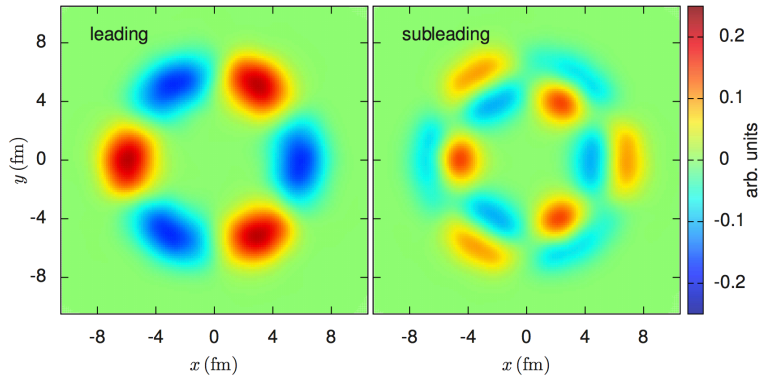
- Decompose the $v_n(p_T)$ into principle modes, each mode factorize

$$V_{n\Delta}(p_T^a, p_T^b) = \sum_\alpha \langle M(p_T^a)M(p_T^b) \rangle v_n^{(\alpha)}(p_T^a)v_n^{(\alpha)}(p_T^b)$$

Transverse flow fluctuations



arXiv:1410.7739,1501.03138



JeanYves, Rajeev, Derek, Aleksas

$$v_3 \propto c_1 e^{i3\Phi_3^1} + c_2 e^{i3\Phi_3^2}$$

Magnitude and phase of leading and subleading modes fluctuates independently.

- No-unique mapping between ε_n and v_n.
  - ε_n defined by different r^m weights → radial degrees of freedom.
- Related to flow angle and amplitude fluctuates in p_T

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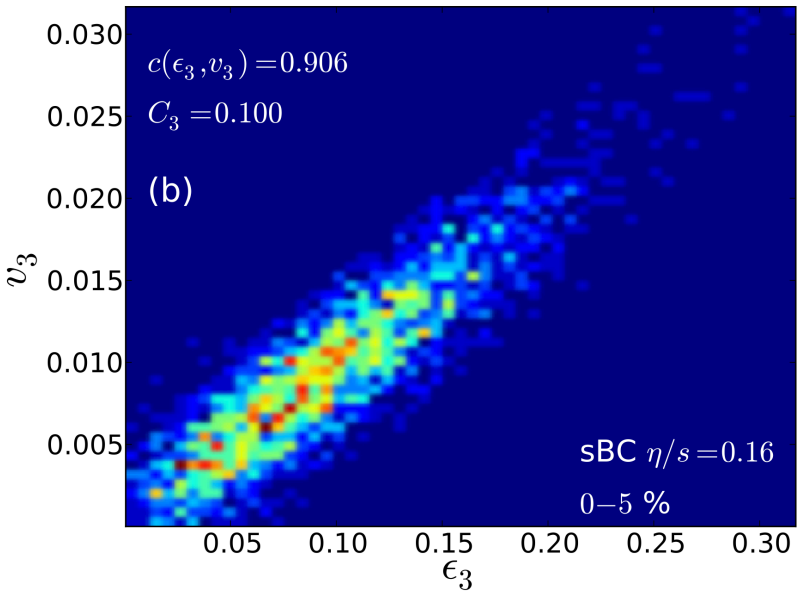
Transverse flow fluctuations



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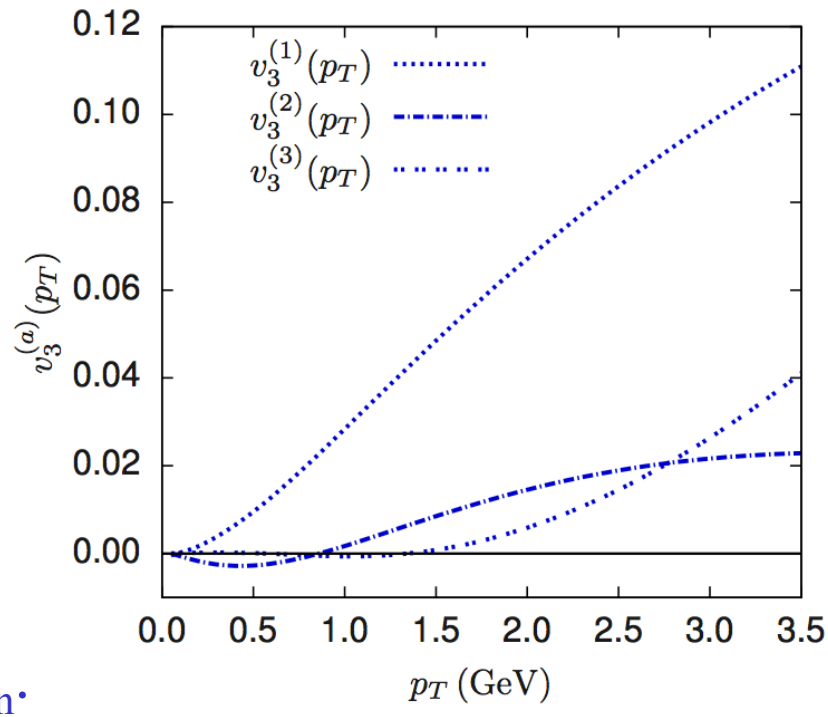

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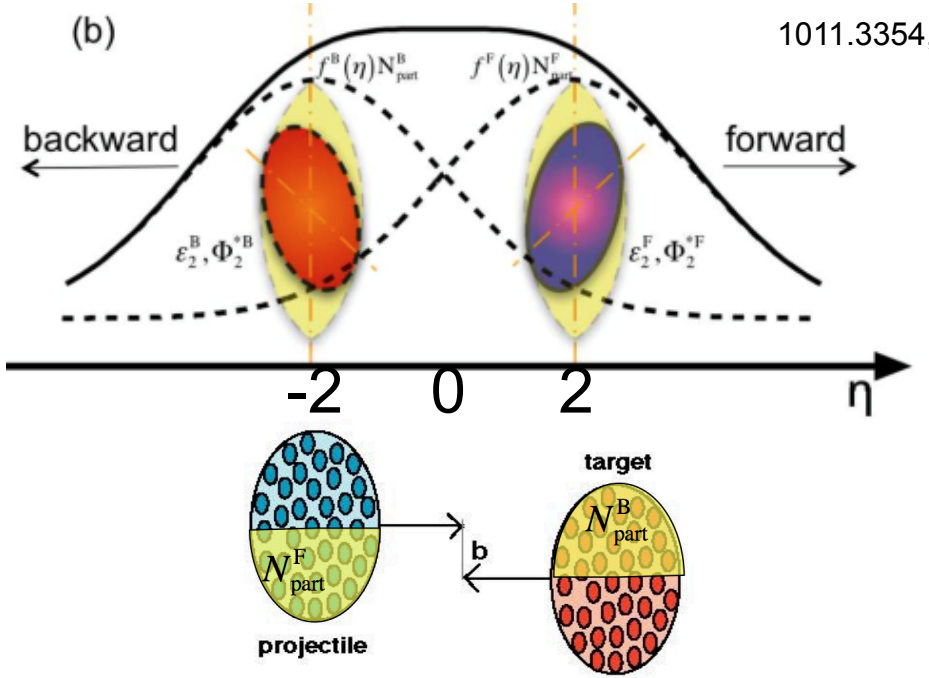
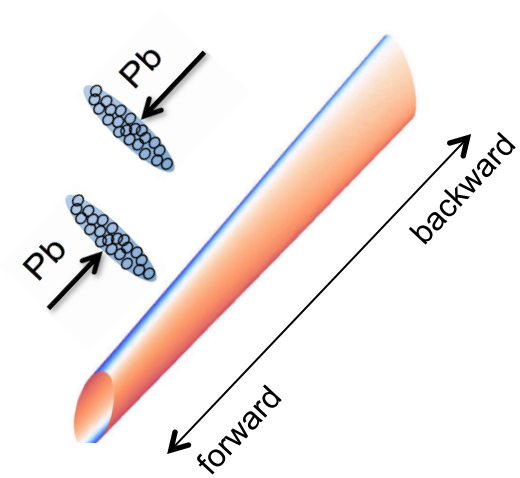
- Decompose the $v_n(p_T)$ into principle modes, each mode factorize

$$V_{n\Delta}(p_T^a, p_T^b) = \sum_\alpha < M(p_T^a)M(p_T^b) > v_n^{(\alpha)}(p_T^a)v_n^{(\alpha)}(p_T^b)$$

Longitudinal flow fluctuations



1011.3354, 1403.6077



- Shape of participants in two nuclei not the same due to fluctuation

$$\varepsilon_n, \Phi_n^* N_{\text{part}} \quad \varepsilon_n^{\text{F}}, \Phi_n^{*\text{F}} N_{\text{part}}^{\text{F}} \quad \varepsilon_n^{\text{B}}, \Phi_n^{*\text{B}} N_{\text{part}}^{\text{B}}$$

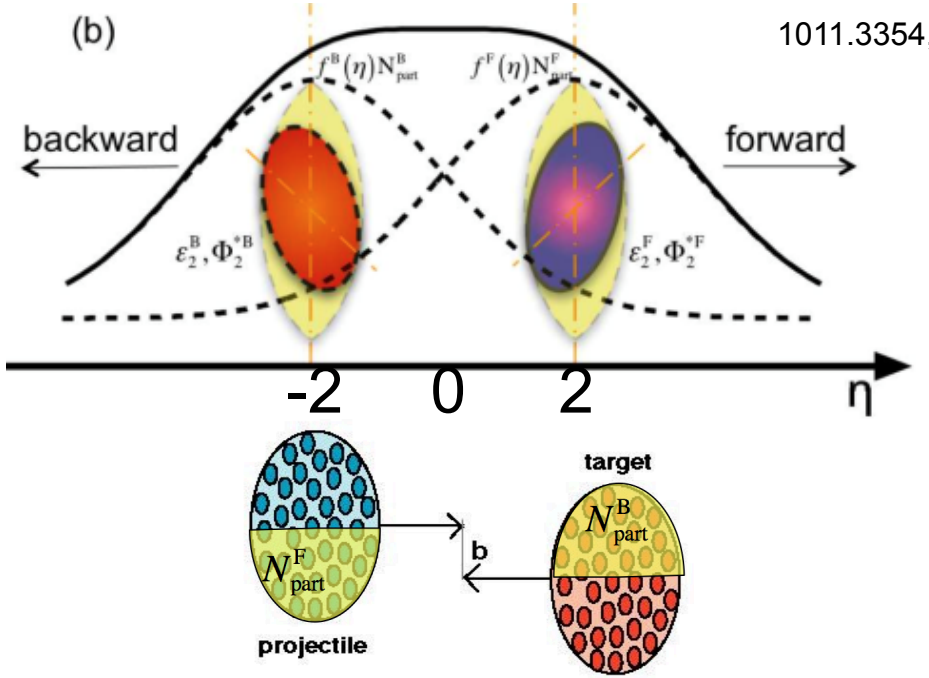
$$\boxed{\varepsilon_n^{\text{F}}, \Phi_n^{*\text{F}} \neq \varepsilon_n^{\text{B}}, \Phi_n^{*\text{B}}}$$

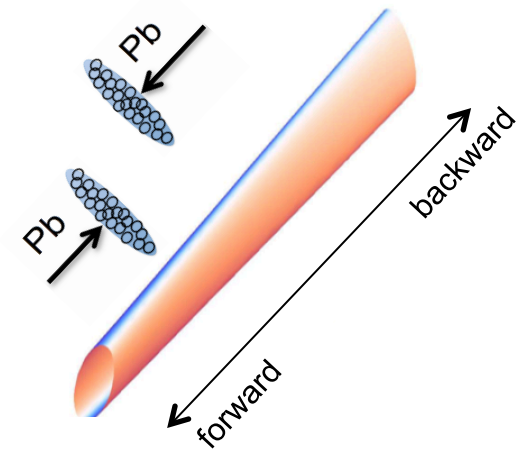
- Wounded nucleon model: particles are produced by independent fragmentation of wounded nucleons, emission function f(η) not symmetric in η

Bialas et.al

Longitudinal flow fluctuations

1011.3354, 1403.6077





- Eccentricity vector interpolates between $\vec{\epsilon}_n^{\mathrm{F}}$ and $\vec{\epsilon}_n^{\mathrm{B}}$

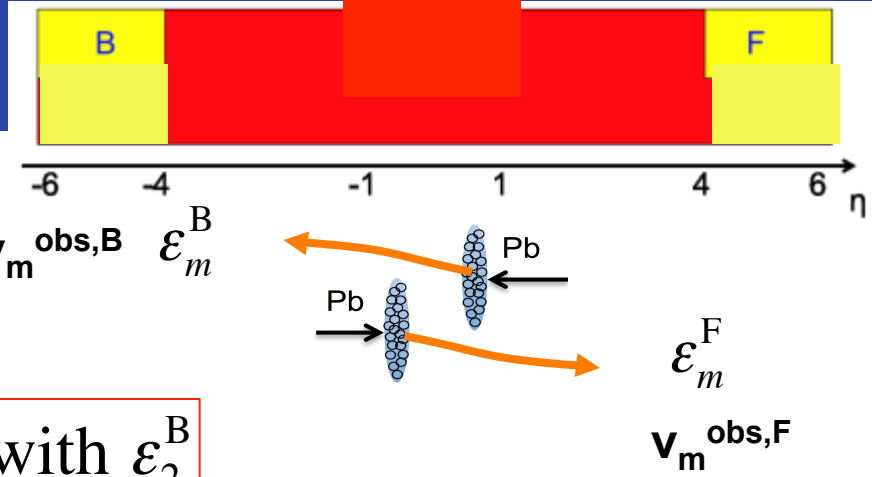
$$\vec{\epsilon}_n^{\mathrm{tot}}(\eta) \approx \alpha(\eta)\vec{\epsilon}_n^{\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\mathrm{B}} \equiv \epsilon_n^{\mathrm{tot}}(\eta) e^{in\Phi_n^{*\mathrm{tot}}(\eta)}$$

Asymmetry:	$\varepsilon_n^{\mathrm{F}} \neq \varepsilon_n^{\mathrm{B}}$
Twist:	$\Phi_n^{*\mathrm{F}} \neq \Phi_n^{*\mathrm{B}}$

- Hence $\boxed{\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\mathrm{B}}\right]}$ for n=2,3

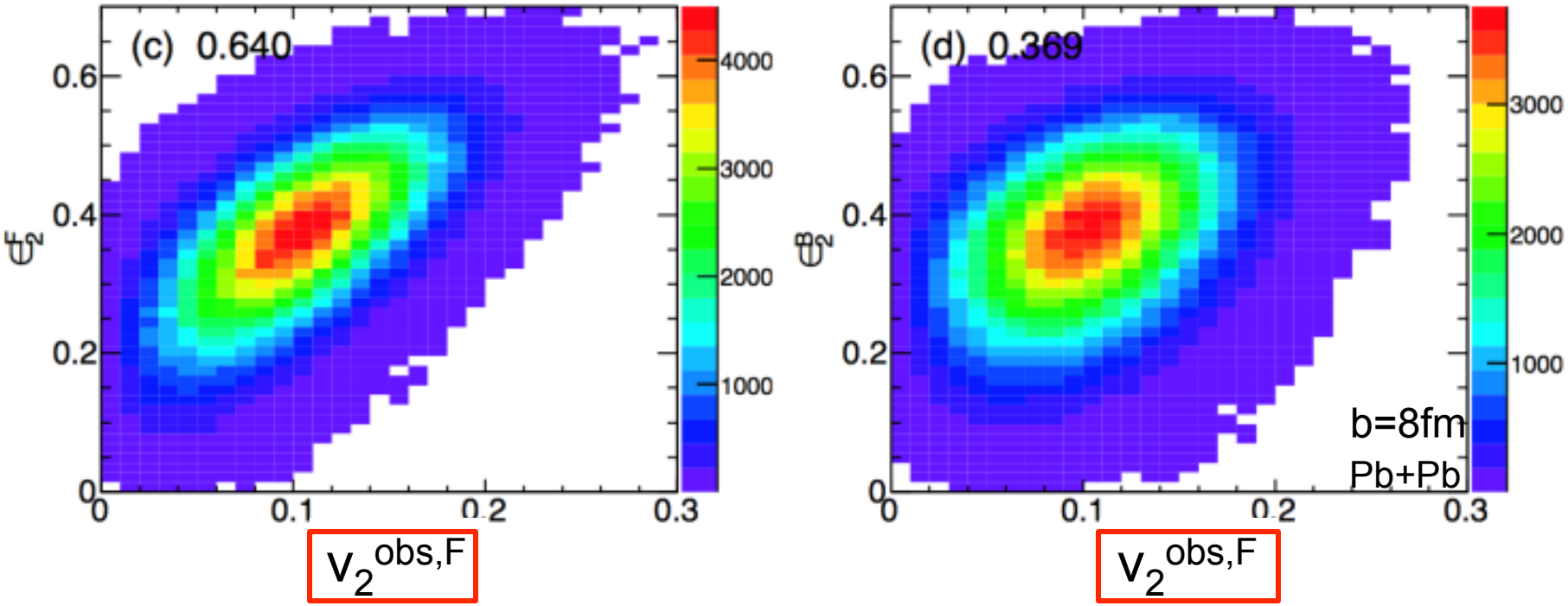
  - Picture verified in AMPT simulations, magnitude estimated 1403.6077

What AMPT tell us?



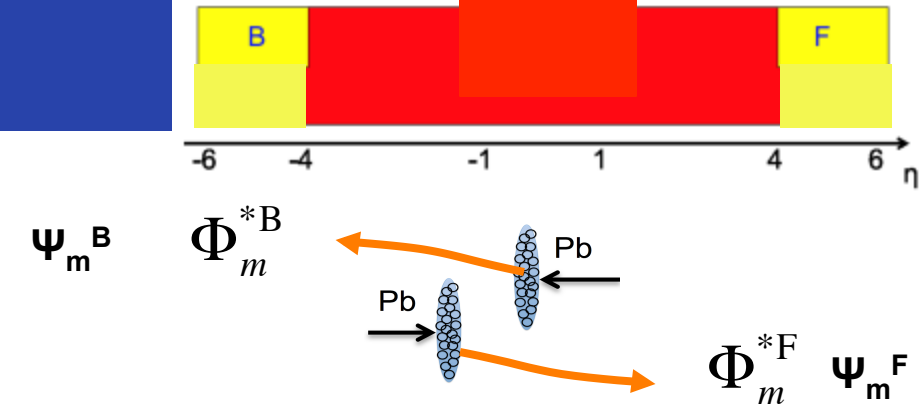
- F/B asymmetry in initial geometry appears as asymmetry of flow

$v_2^{obs,F}$ more correlated with ε_2^F than with ε_2^B



Initial FB asymmetry survives to final state!!

What AMPT tell us?

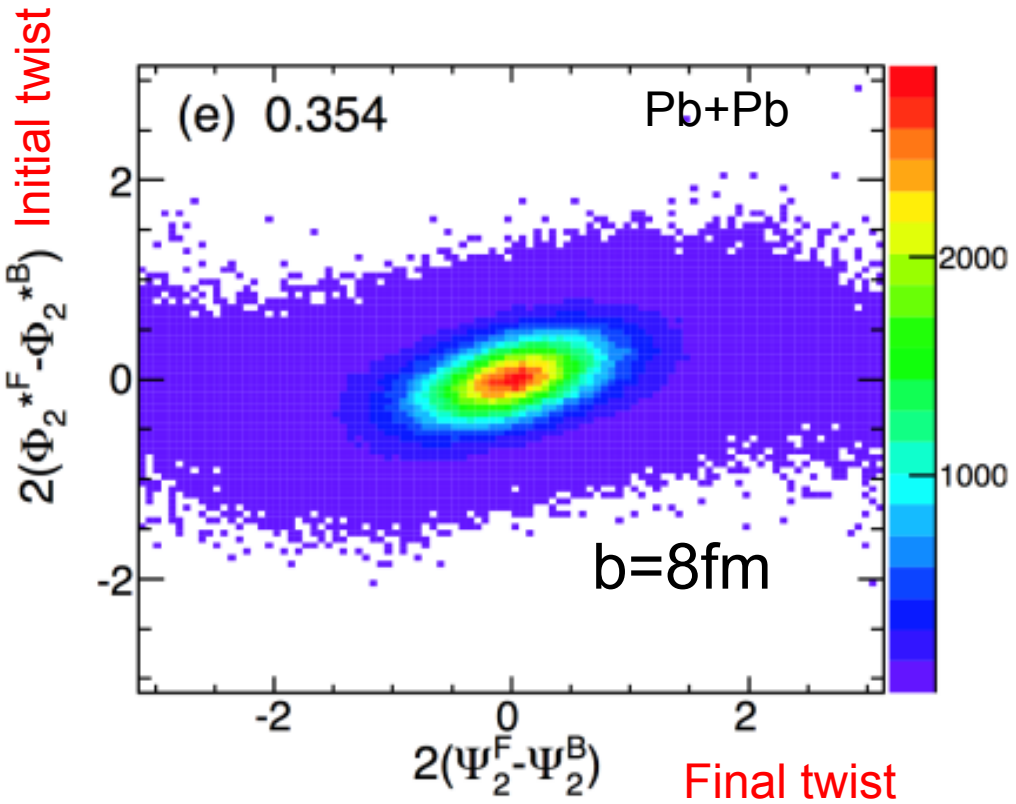


- Twist in initial geometry appears as twist in flow
  - Participant plane angles:

$$\Phi_n^{*F} \quad \Phi_n^{*B}$$

  - Final state event-plane angles

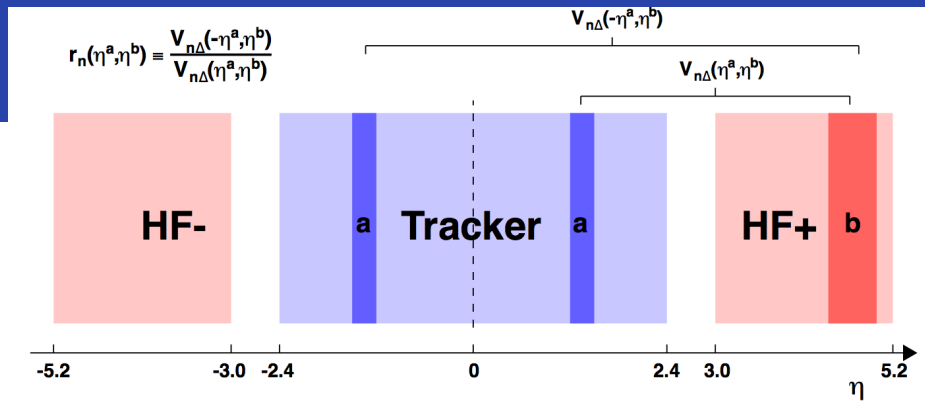
$$\Psi_n^F \quad \Psi_n^B$$



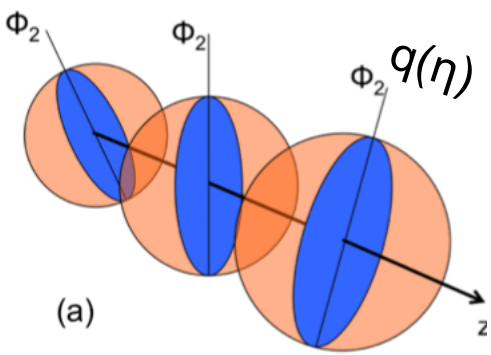
Initial twist survives to final state!!

CMS measurement

$$R_2 = \frac{\langle q(-\eta) q^*(\eta_{\rm ref}) \rangle}{\langle q(\eta) q^*(\eta_{\rm ref}) \rangle}$$



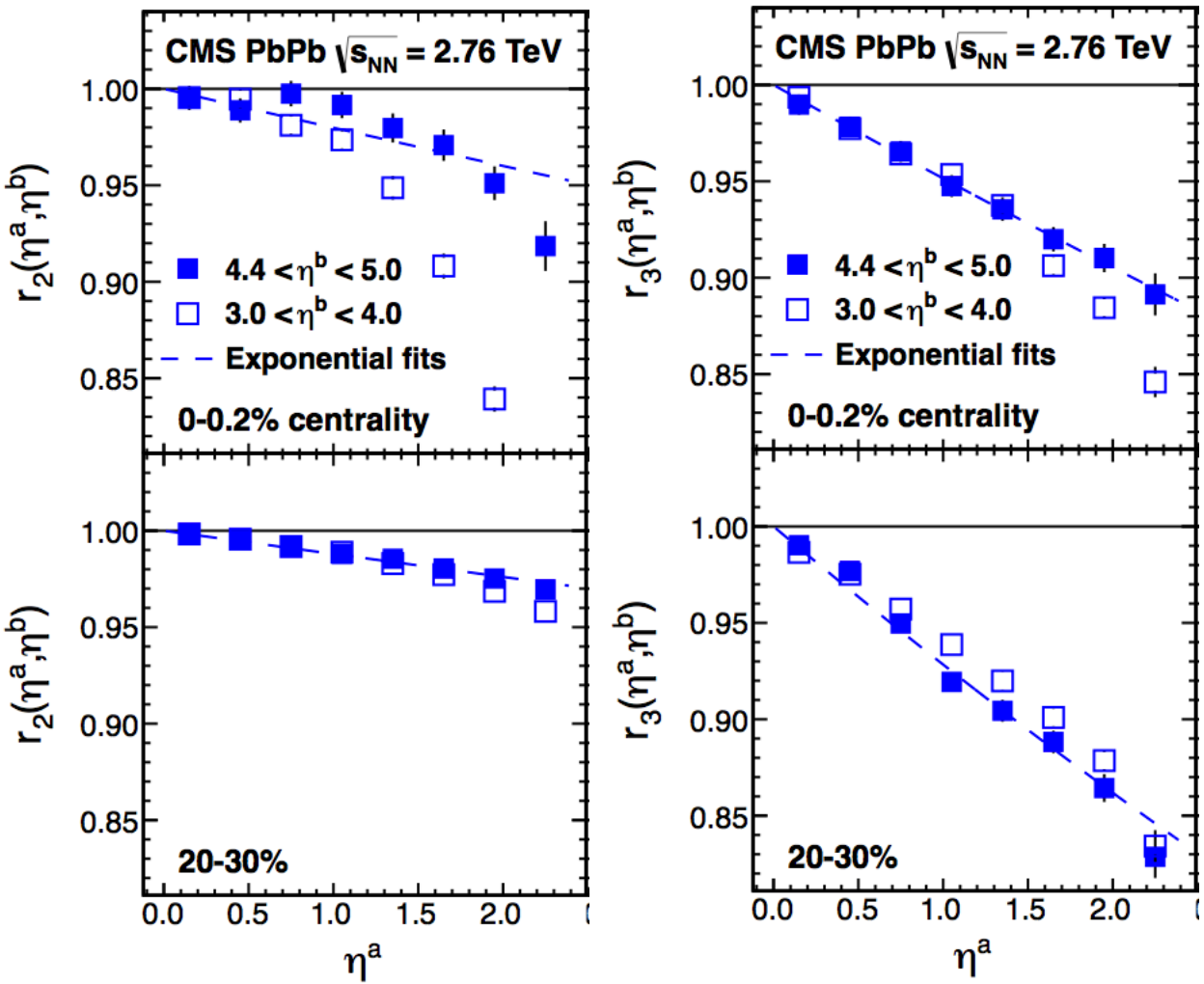
Flow vector: $q_n = v_n^{obs} e^{in\Psi_n}$



1503.01692

Separate contributions from twist and asymmetry necessary!!

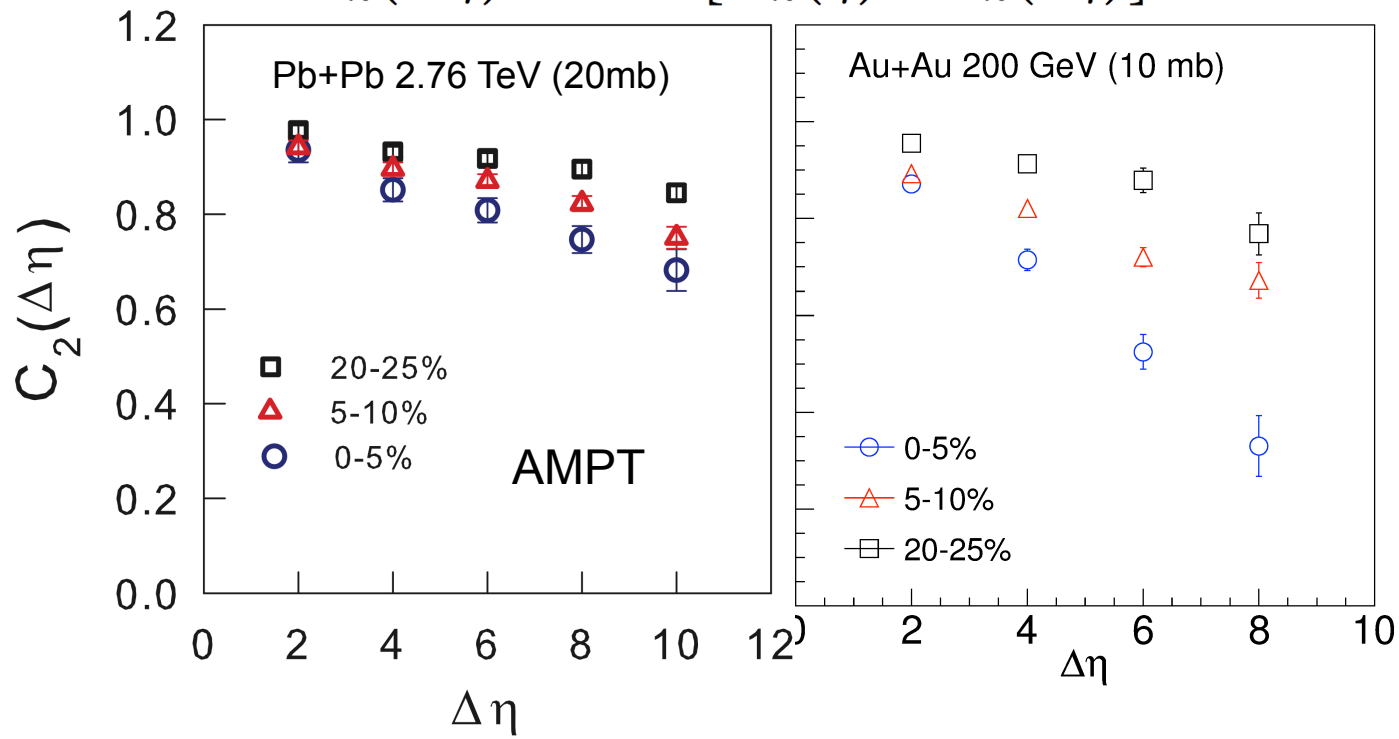
See also 1503.07425



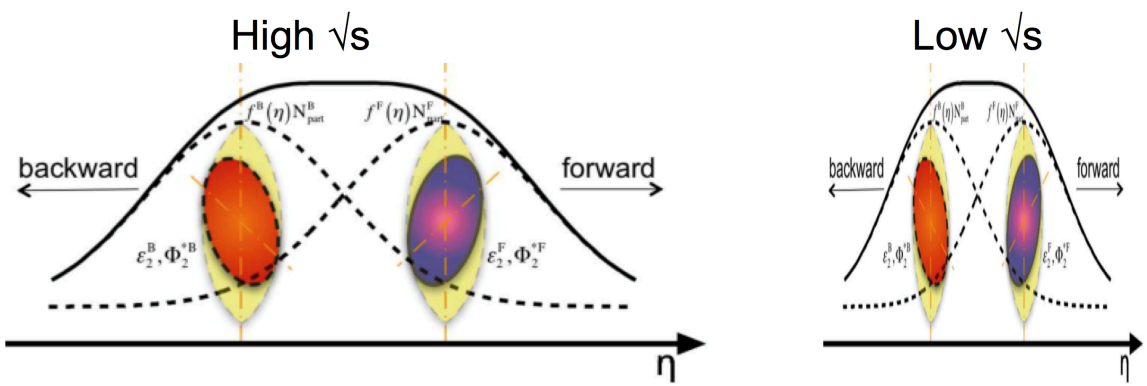
Twist

Longgang etc,
arxiv1410.8690

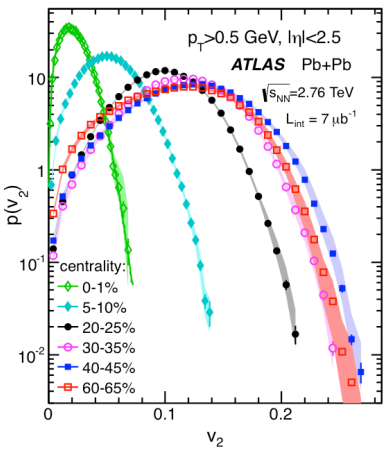
$$C_n(\Delta\eta) \approx \cos n\left[\Psi_n(\eta) - \Psi_n(-\eta)\right]$$

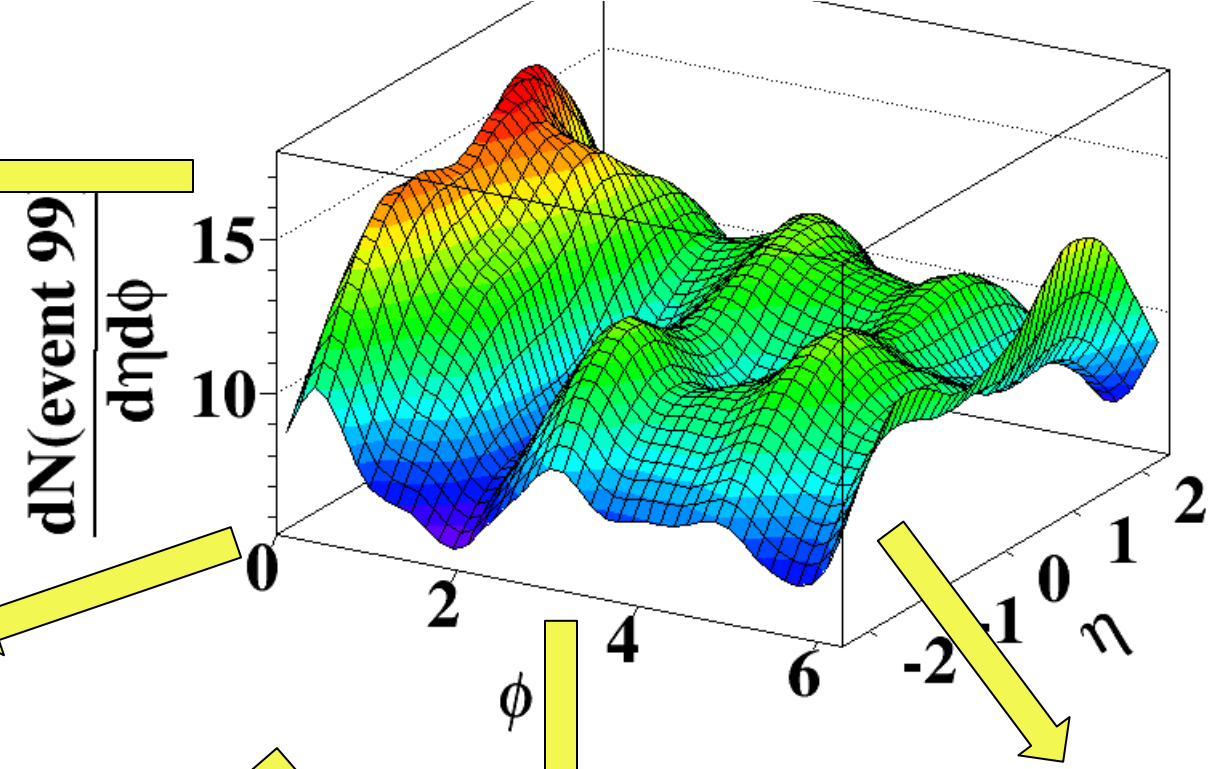


- Strong centrality and √s dependence, need both LHC & RHIC

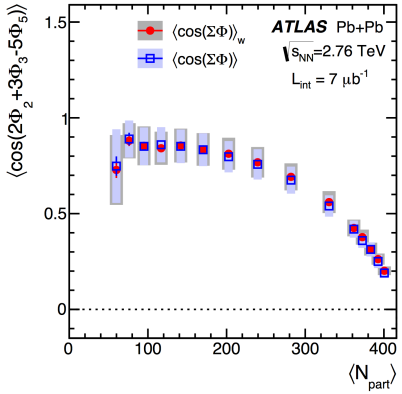


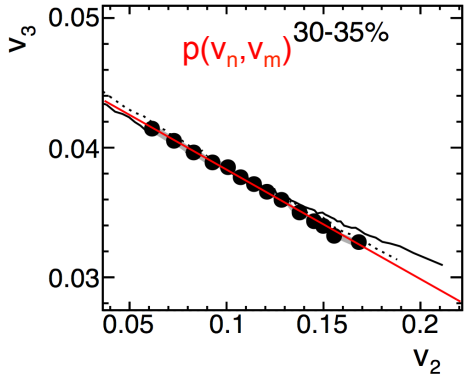
$p(v_n)$





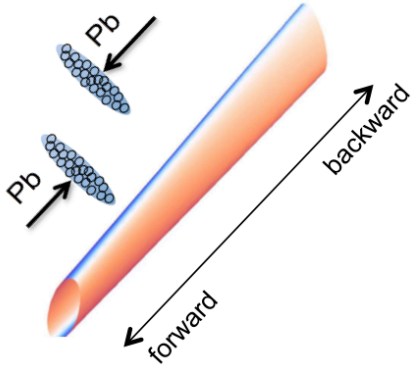
$p(\Phi_n,\Phi_m)$, $p(\Phi_n,\Phi_m,\Phi_L)$





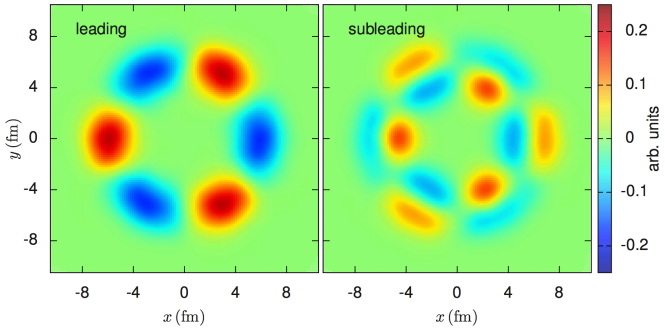
Longitudinal fluctuations

$$\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\mathrm{B}}\right]$$



Transverse fluctuations

$$v_3(p_T) \propto c_1(p_T)e^{i3\Phi_3^1(p_T)} + c_2(p_T)e^{i3\Phi_3^2(p_T)}$$



Summary

- Future measurements should focus on obtaining a complete picture
- Event-shape fluctuation contains lots of information

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\rm evts}} \frac{dN_{\rm evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

  - Separate initial state and final state effects
  - Applicable to other collisions system Cu+Au and U+U

- Rich patterns forward/backward EbyE flow fluctuations:

$$\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\rm F} + (1-\alpha(\eta))\vec{\epsilon}_n^{\rm B}\right]$$

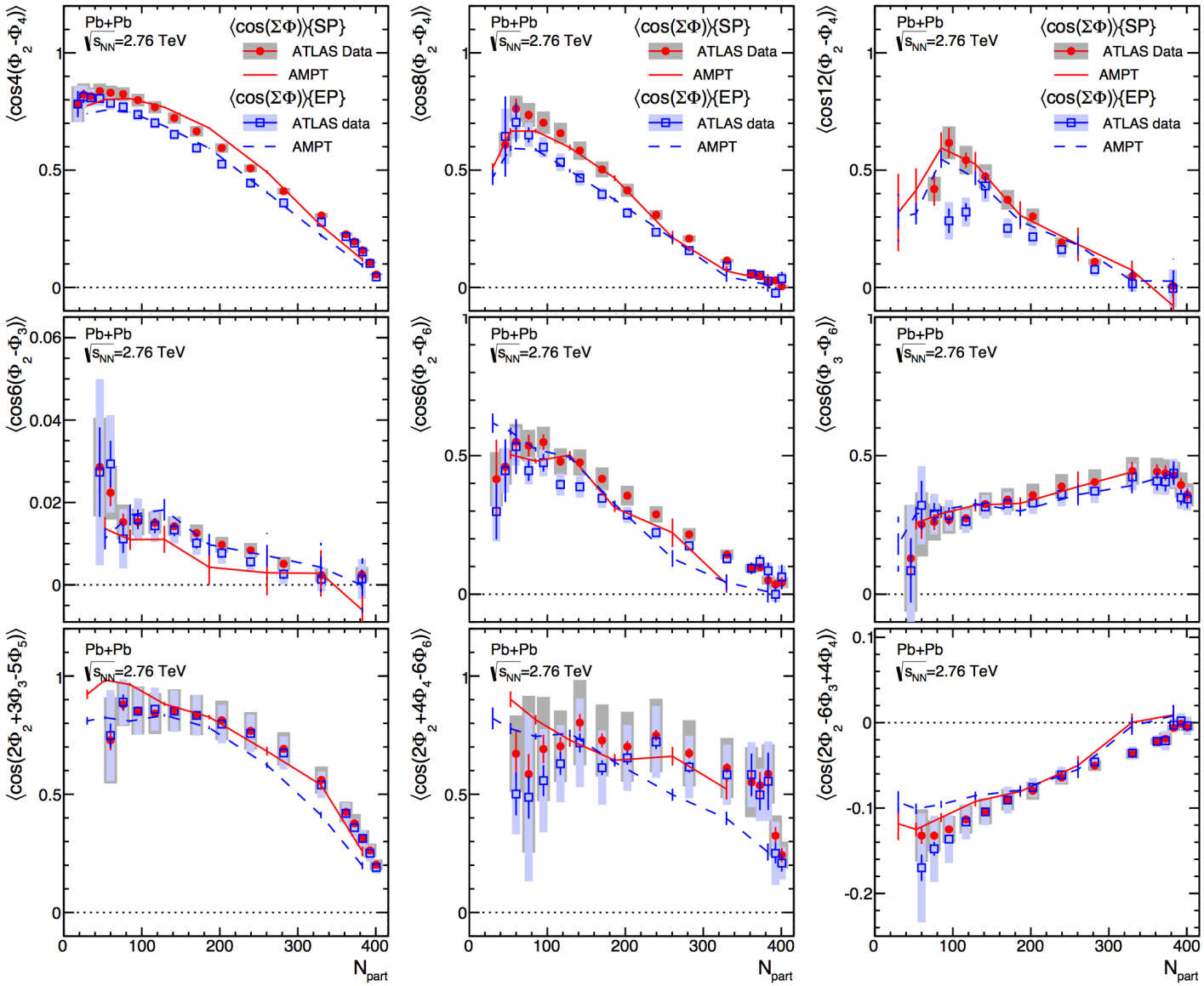
Event-shape selection and event twist techniques

  - New avenue to study initial state fluctuations, particle production and collective expansion dynamics

Backup

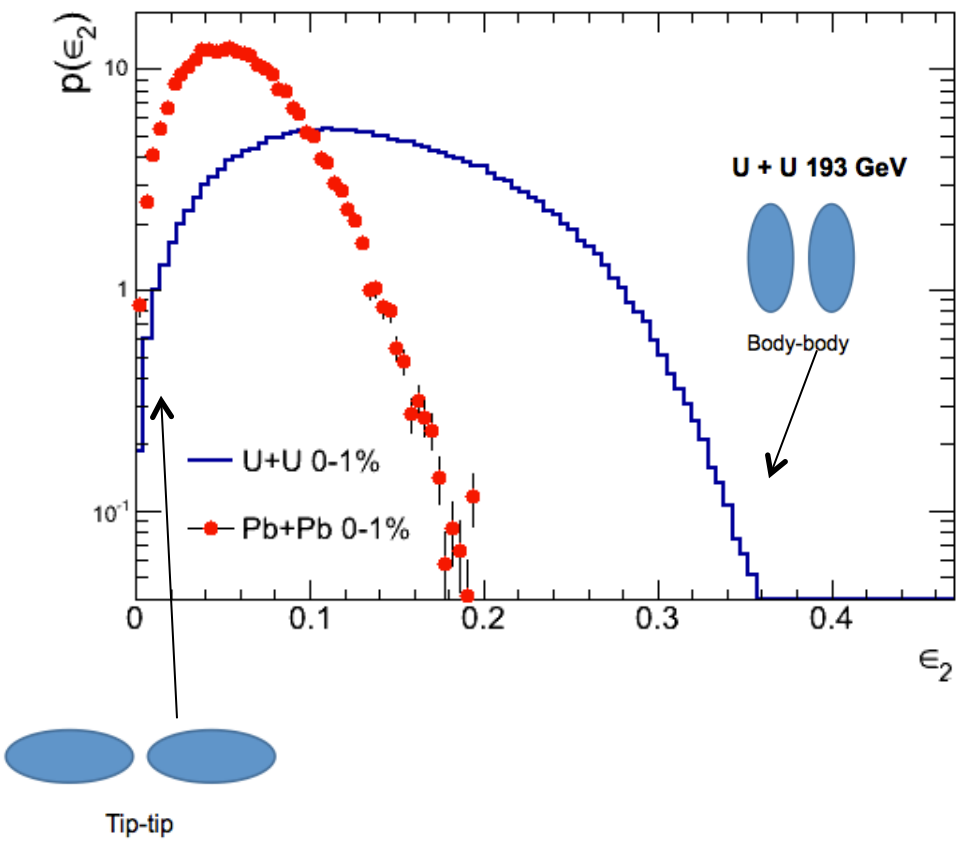
EP correlation: How are $(\varepsilon_n, \Phi_n^*)$ transferred to (v_n, Φ_n)?

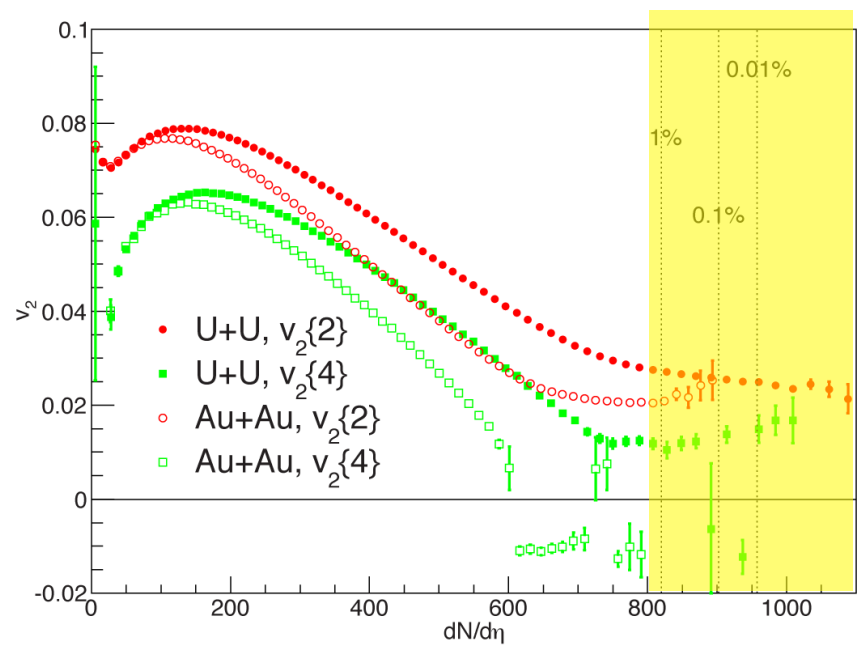
- EP correlation probes into the mode-mixing (14 correlators from ATLAS)



Exploring the collision geometry & flow response

- Expect broader $p(v_2)$ in U+U than Au+Au in ultra-central collisions
  - e.g→ Distinguish tip-tip vs body-body contribution





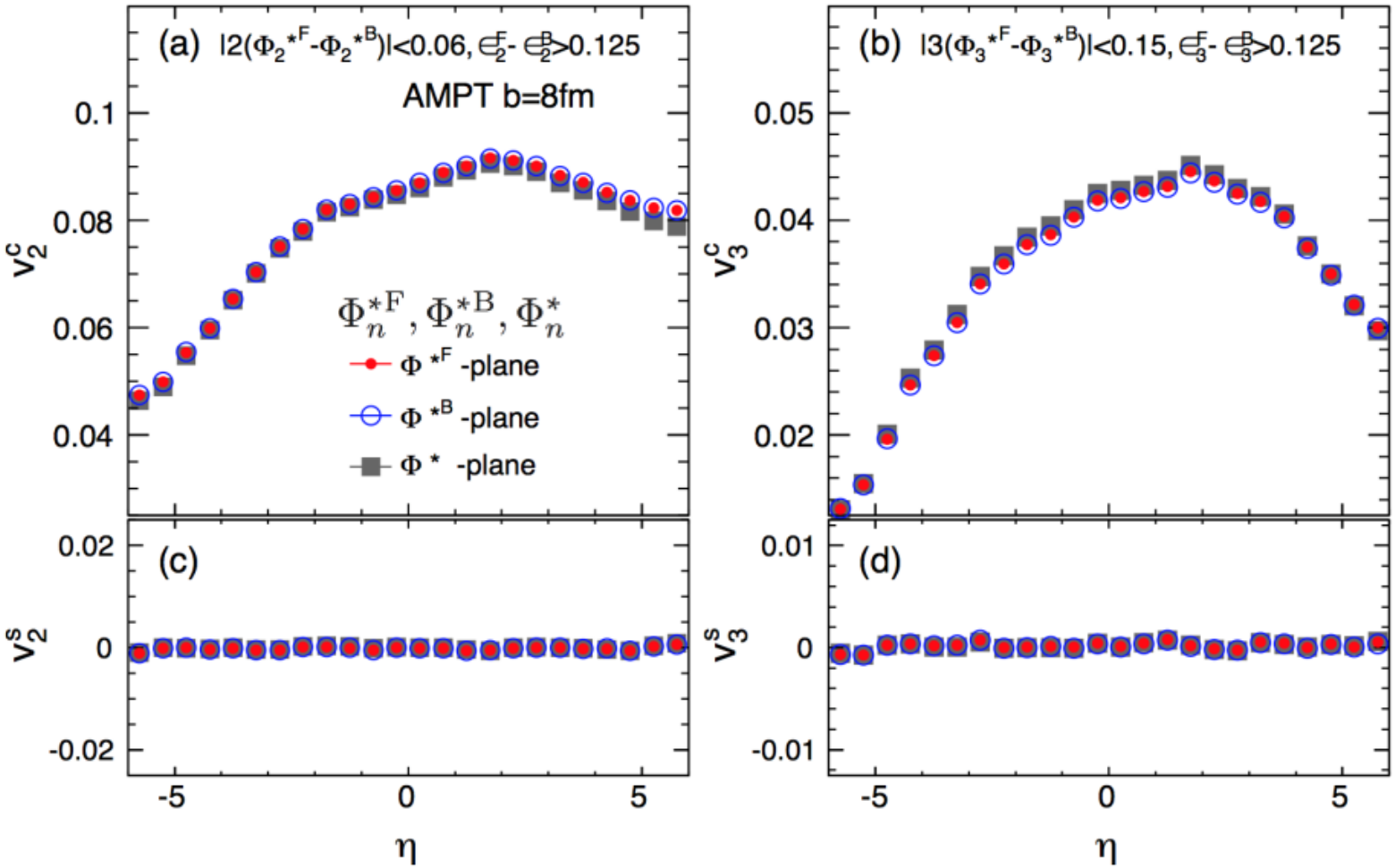
measure $p(v_2)$

Asymmetry

Select events with same PP but different eccentricity:

$$\Phi_n^{*F} \approx \Phi_n^{*B} \qquad \varepsilon_n^F > \varepsilon_n^B$$

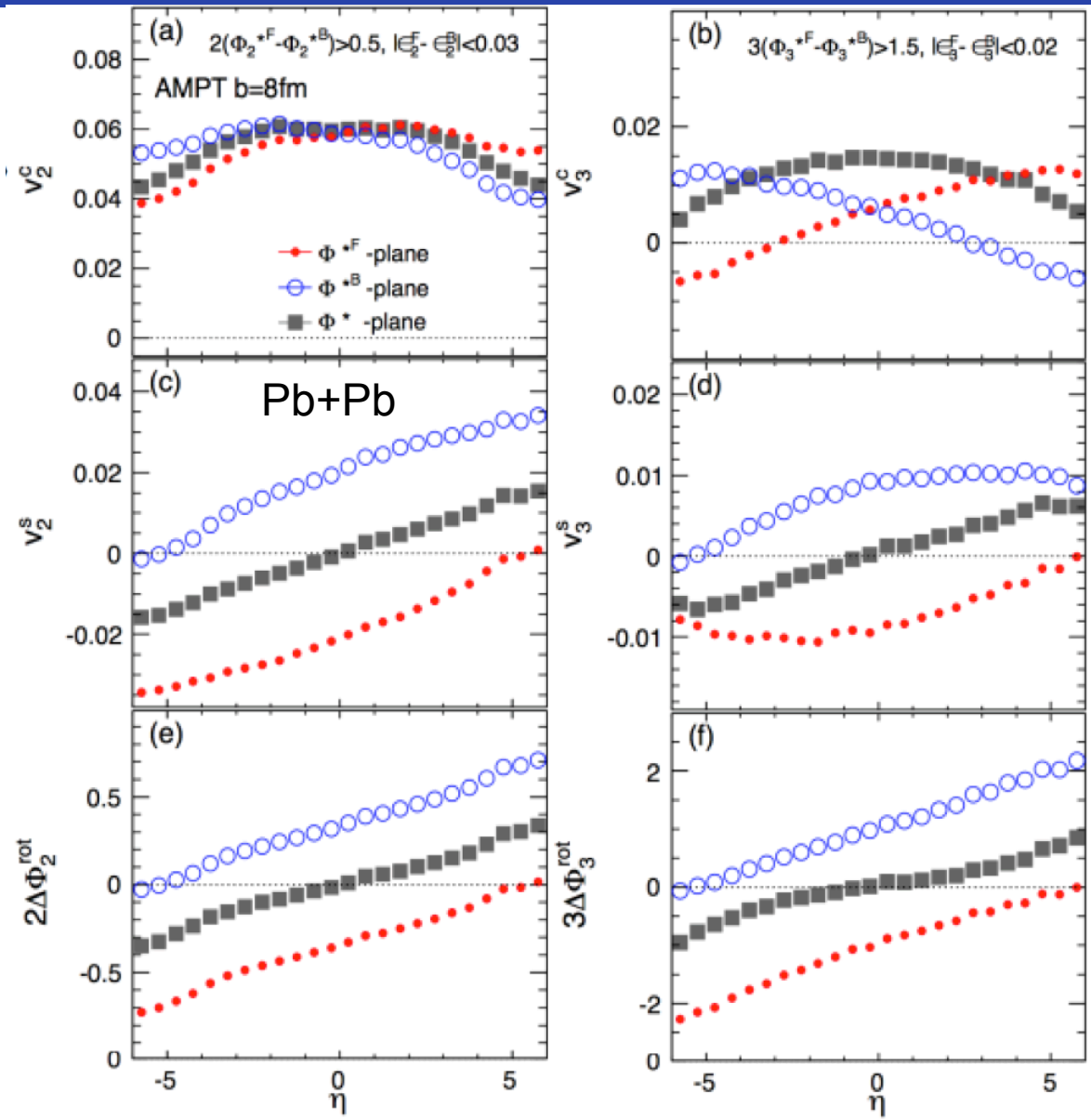
Observe clear F/B flow asymmetry:



Twist

$$v_n^c(\eta) = \langle \cos n\,(\phi(\eta) - \Theta_n) \rangle$$
$$v_n^s(\eta) = \langle \sin n\,(\phi(\eta) - \Theta_n) \rangle$$

Select events with same eccentricity but different PP:

$$\Phi_n^{*F} > \Phi_n^{*B} \qquad \varepsilon_n^F \approx \varepsilon_n^B$$

Observe clear F/B twist of EP angle: