Conserved Charge Fluctuations and Correlations from Lattice QCD and the Beam Energy Scan

Frithjof Karsch

Brookhaven National Laboratory & Bielefeld University

OUTLINE

– the QCD phase diagram

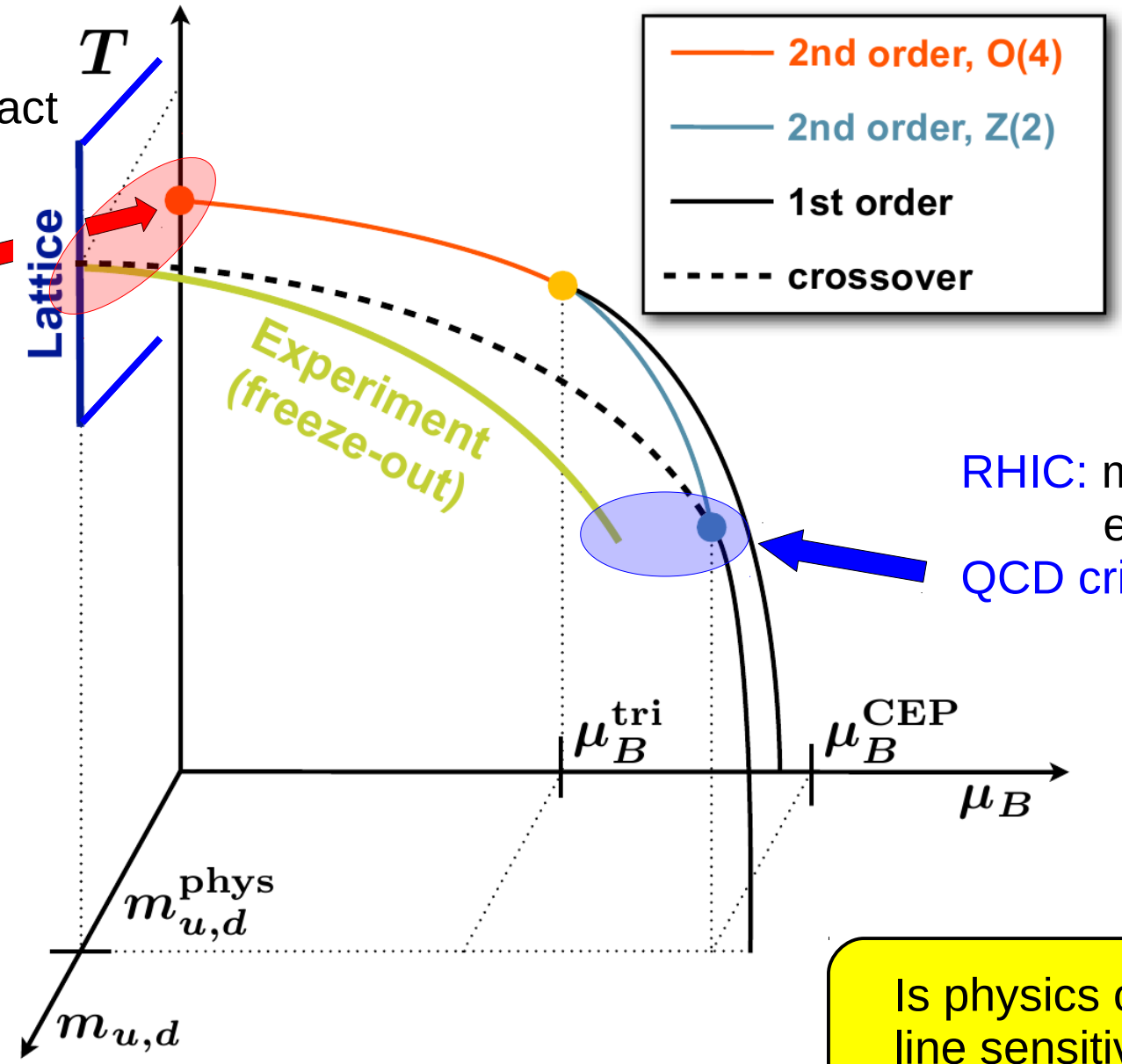
– EoS at non-zero baryon chemical potential

– cumulant ratios of conserved charge fluctuations

– the QCD critical point

Chiral critical point and QCD critical endpoint

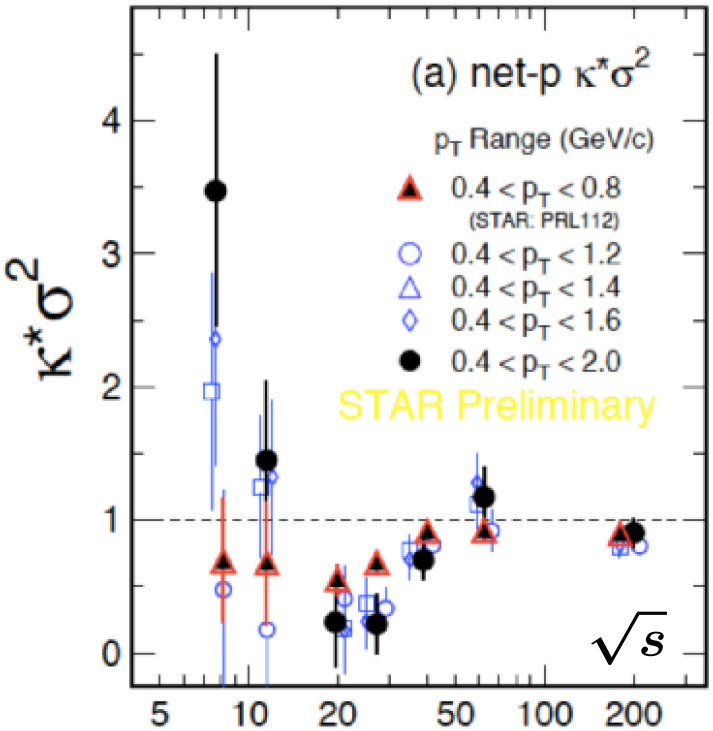
LHC: may establish contact with the QCD chiral PHASE transition



RHIC: may establish evidence for the QCD critical end point

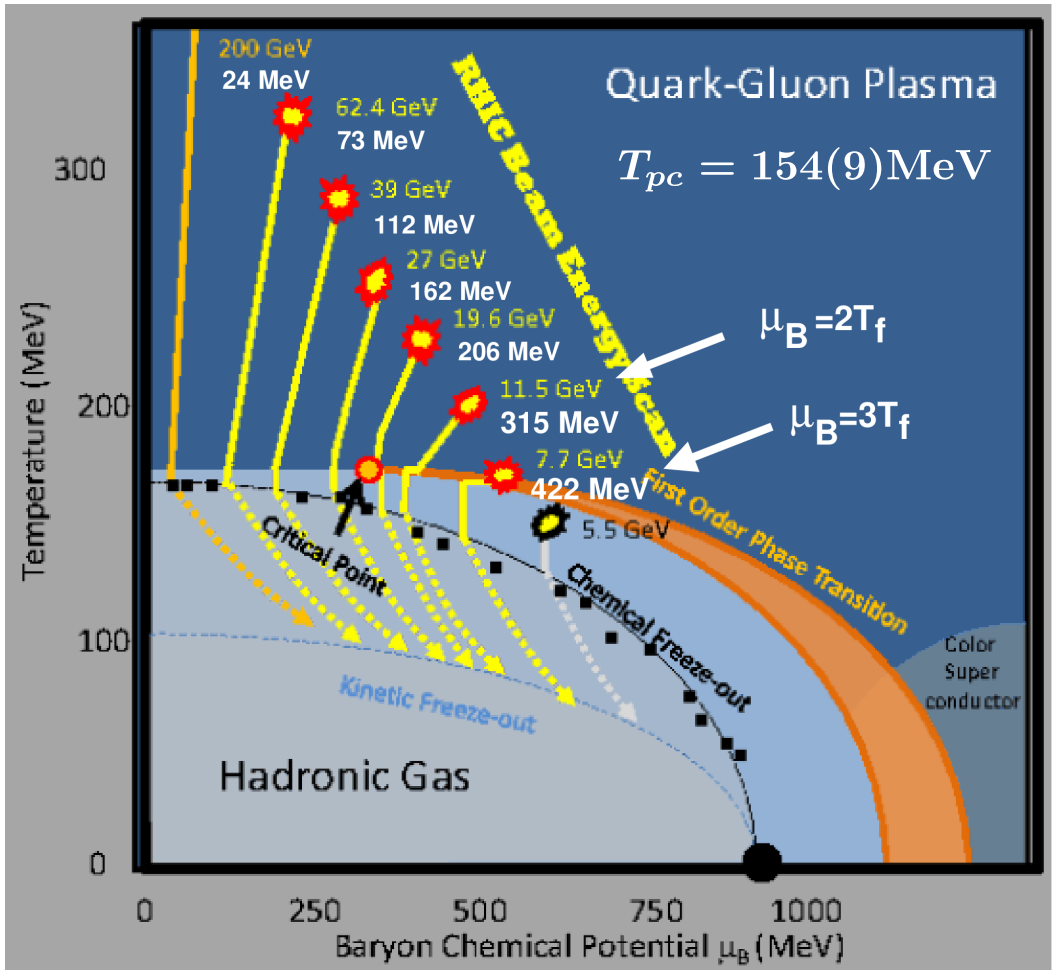
Is physics on the freeze-out line sensitive to critical behavior?

Exploring the QCD phase diagram



2.0 1.3 0.7 μ_B/T

X. Luo (STAR Collaboration), PoS CPOD2014 (2014) 019

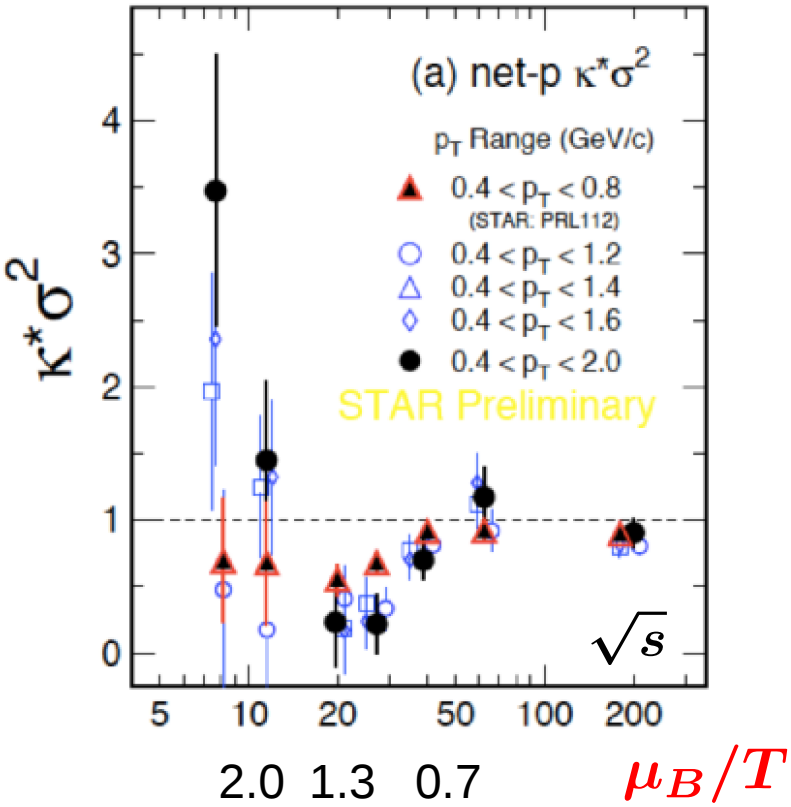


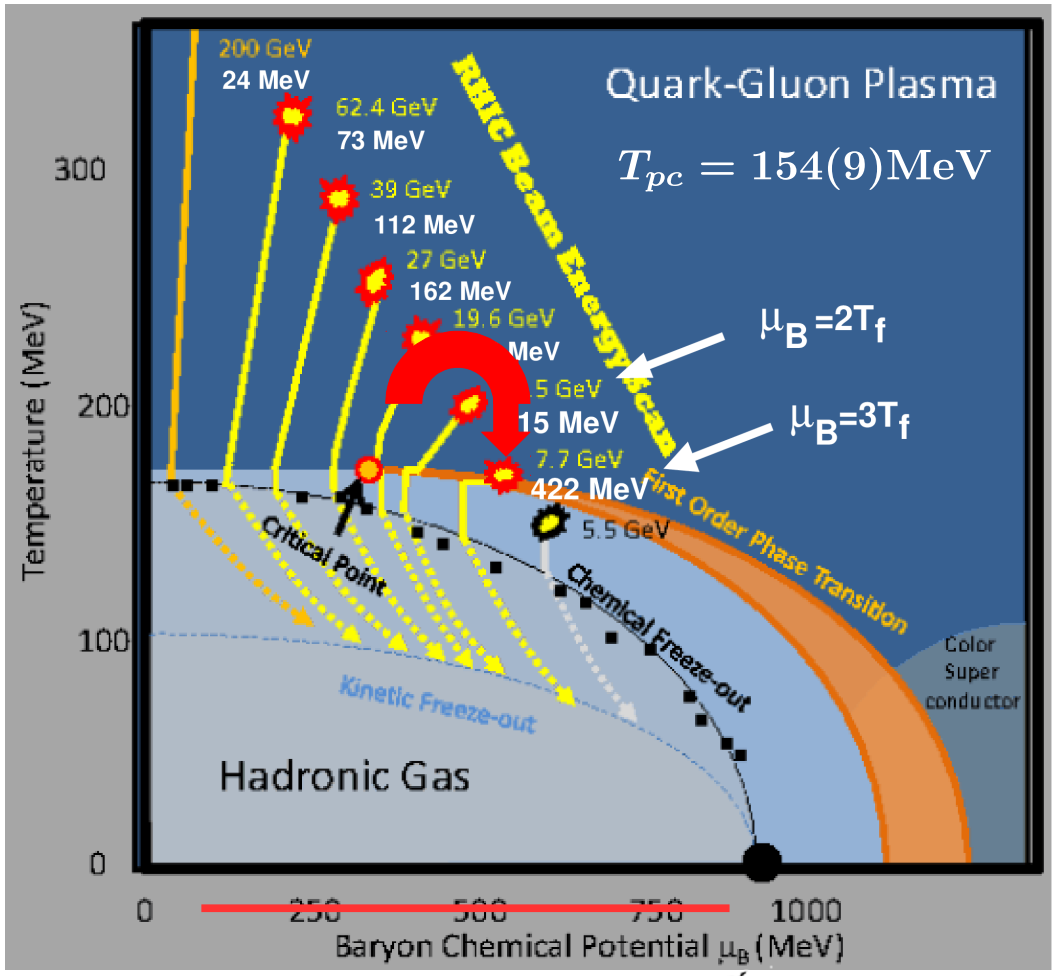
observable consequences:

conserved charge fluctuation

controlled by T_f, μ_B, μ_S (...and μ_Q)

Exploring the QCD phase diagram





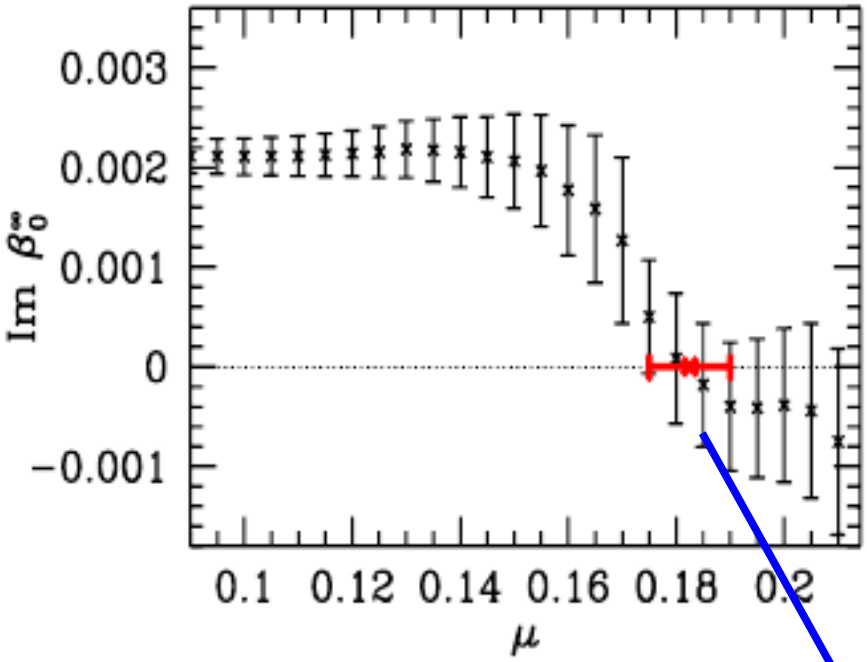
equilibrium thermodynamics:

conserved charge fluctuation

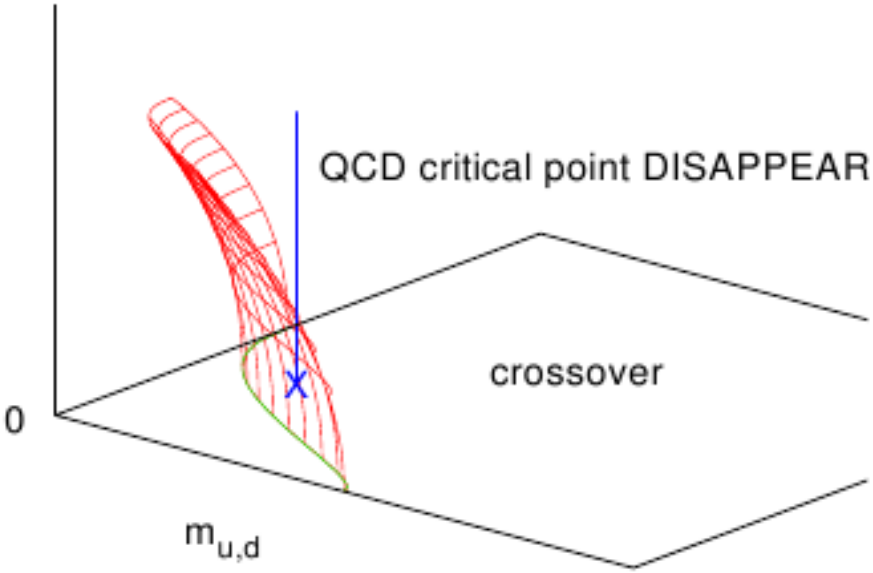
controlled by T_f, μ_B, μ_S

Where is the critical point?

LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

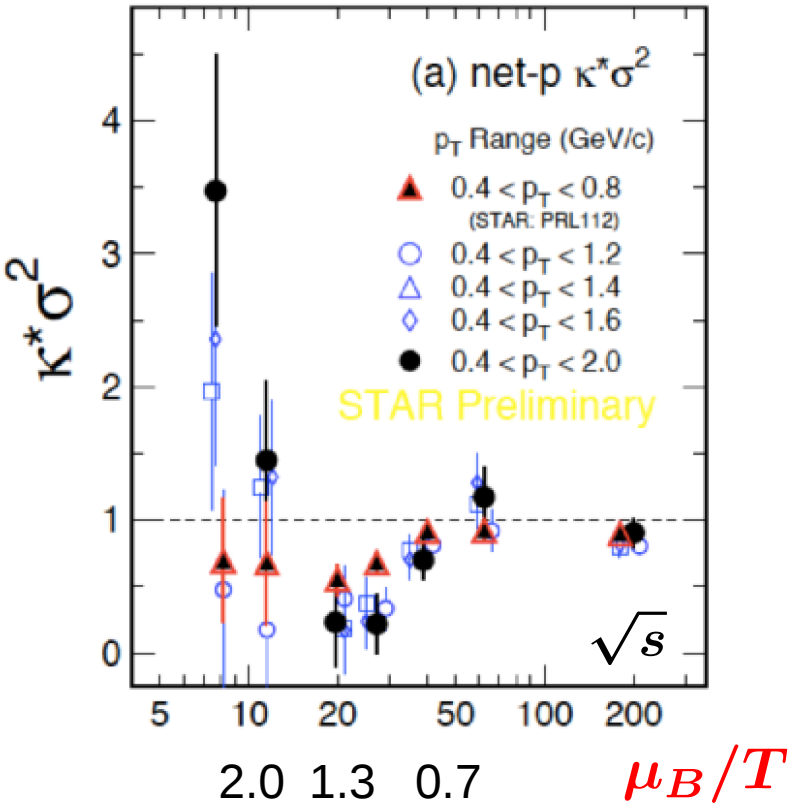
these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude

critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 10 years no progress along this line

Exploring the QCD phase diagram



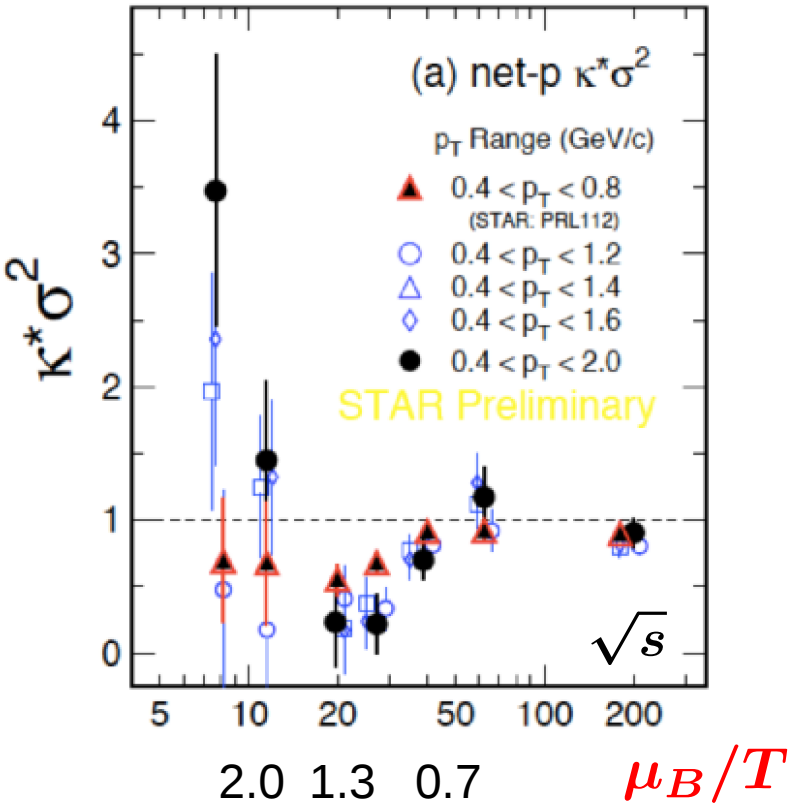
More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

equilibrium thermodynamics:
conserved charge fluctuation
controlled by T_f, μ_B, μ_S

Where is the critical point?

Exploring the QCD phase diagram



More moderate questions:

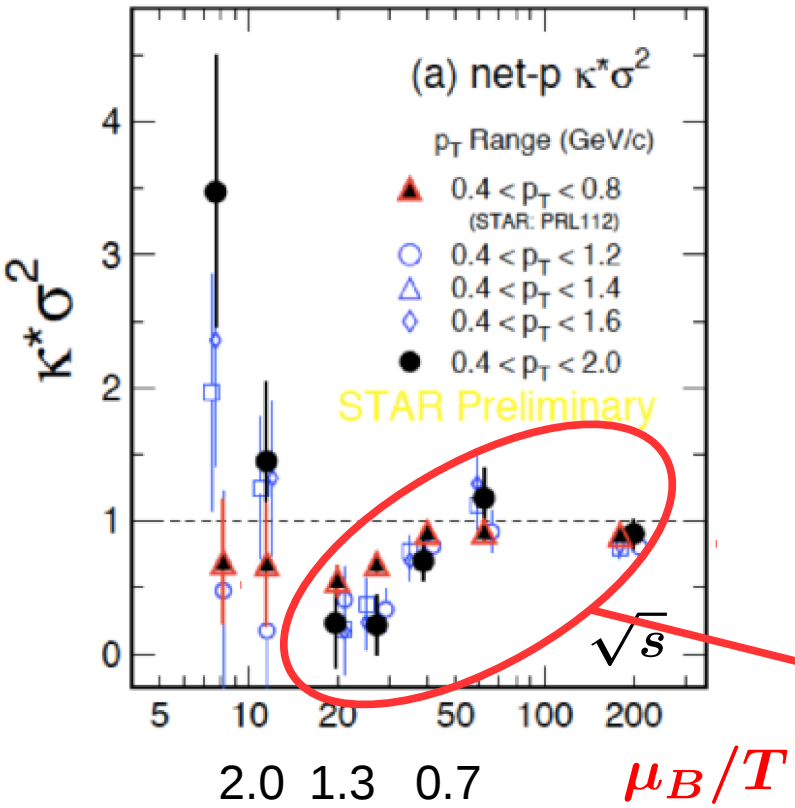
preliminary answer (modulo well known details like proton vs baryon fluctuations, acceptance cuts, ….)

- For $\sqrt{s} \geq 20\ \mathrm{GeV}$:
  Structure of net-electric charge and net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood

equilibrium thermodynamics:
conserved charge fluctuation
controlled by T_f, μ_B, μ_S

Where is the critical point?

Exploring the QCD phase diagram



?

More moderate questions:

preliminary answer (modulo well known details like proton vs baryon fluctuations, acceptance cuts, ….)

- For $\sqrt{s} \geq 20\ \mathrm{GeV}$:
  Structure of net-electric charge and net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a 6th-order Taylor expansion**

equilibrium thermodynamics:
conserved charge fluctuation
controlled by T_f, μ_B, μ_S

Where is the critical point?

Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$

$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu=0}$$

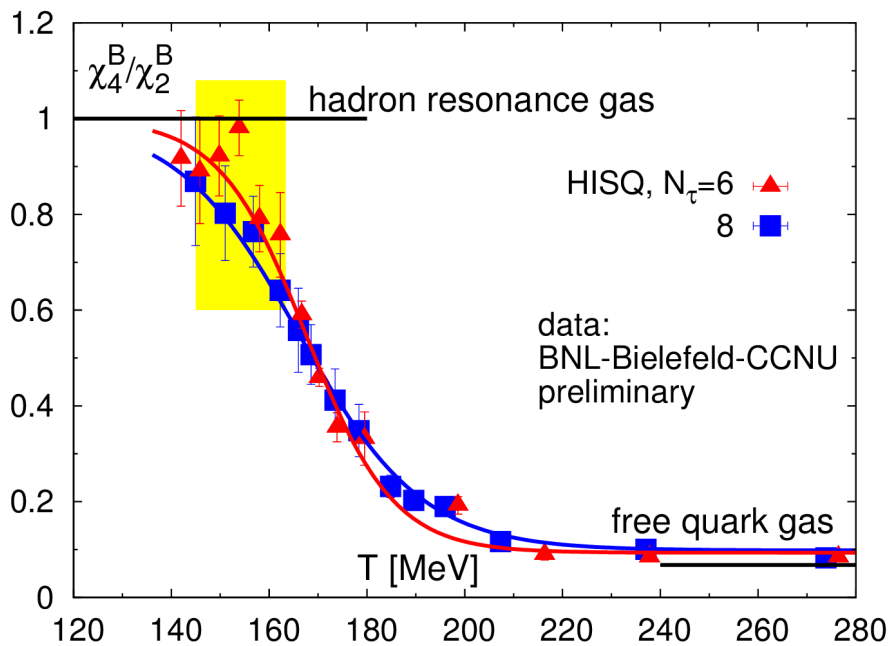
conserved charge fluctuations: $$\chi_n^X(T,\mu_B,...) = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$$

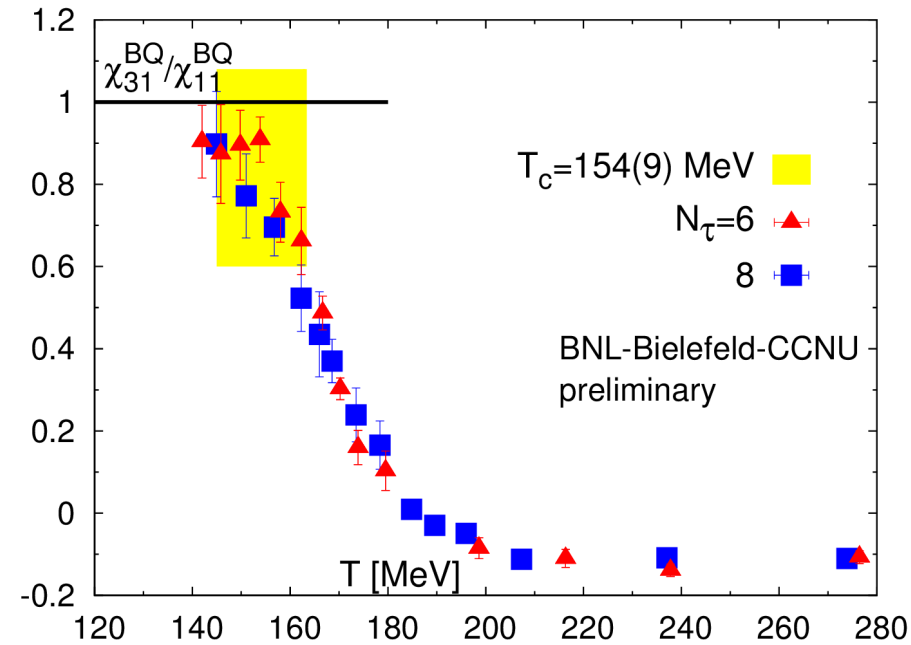
$$X = B,\ Q,\ S$$

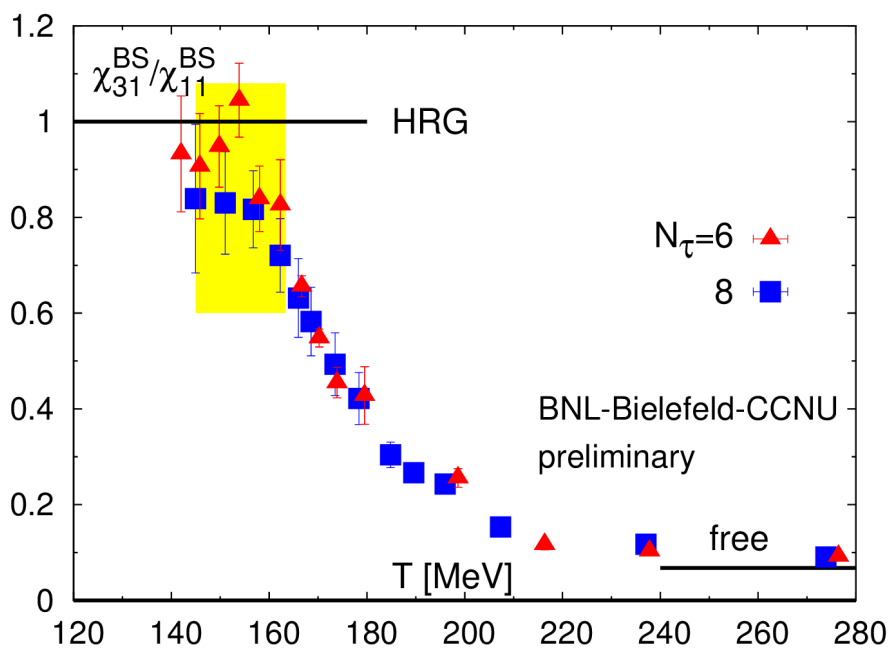
$$\frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \quad S_X\sigma_X = \frac{\chi_3^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \quad \kappa_X\sigma_X^2 = \frac{\chi_4^X(T,\mu)}{\chi_2^X(T,\mu)}$$

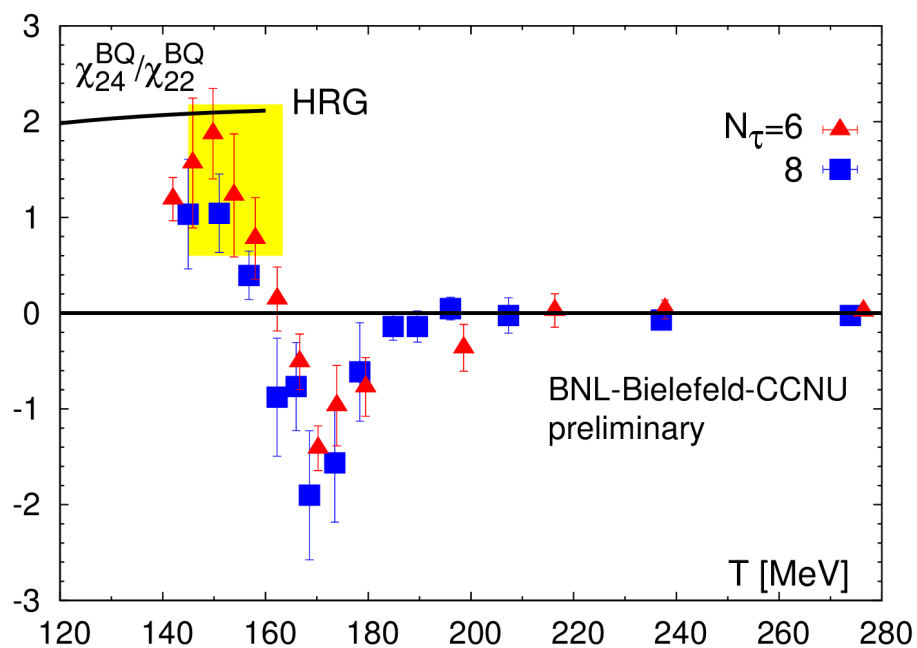
Strong interaction matter in the crossover region (close to the freeze-out line) is not a Hadron Resonance Gas – although HRG sometimes is a good approximation

Some 4th and 6th order cumulants









Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

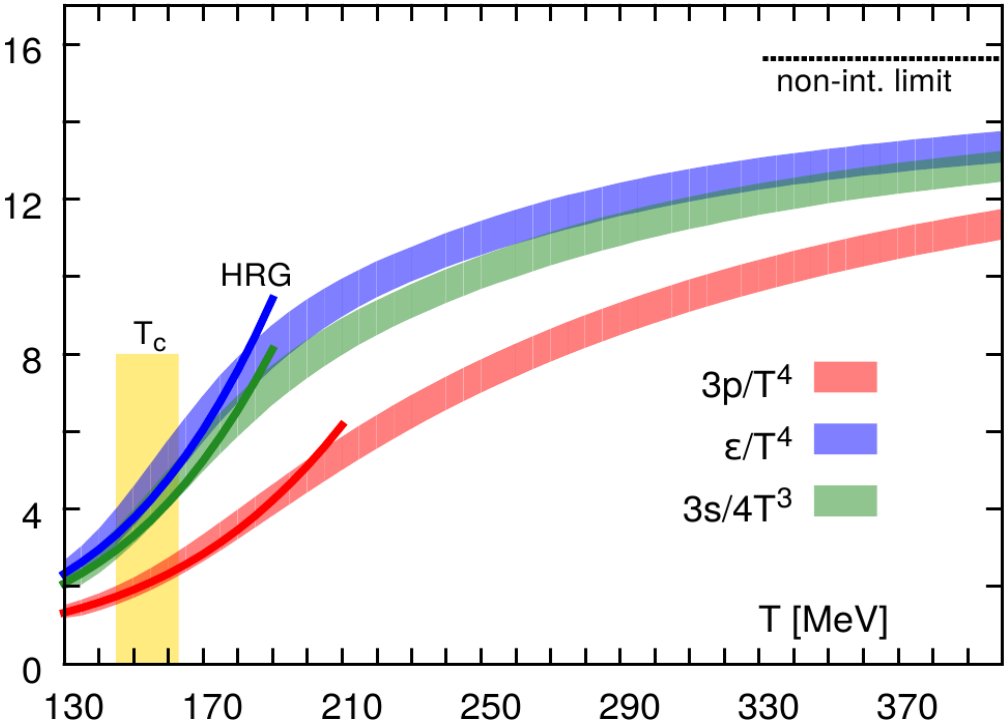
$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

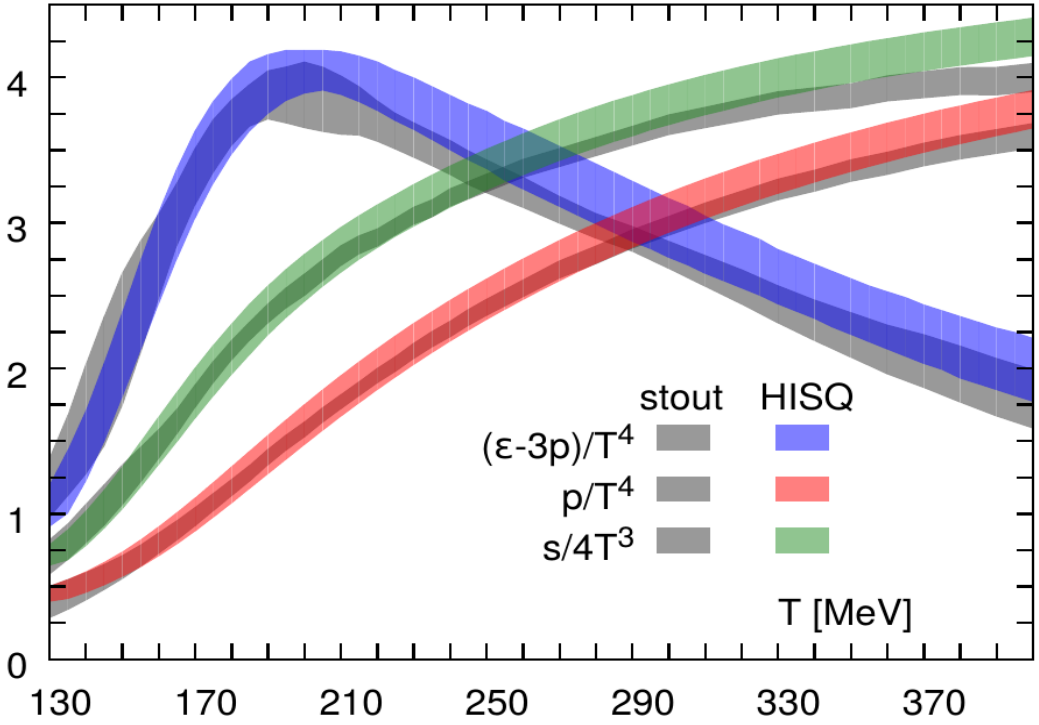
for simplicity: $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

Equation of state of (2+1)-flavor QCD

pressure, **entropy** & **energy** density





A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

– improves over earlier hotQCD calculations:
A. Bazavov et al., Phys. Rev. D80, 014504 (2009)

– consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

– up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

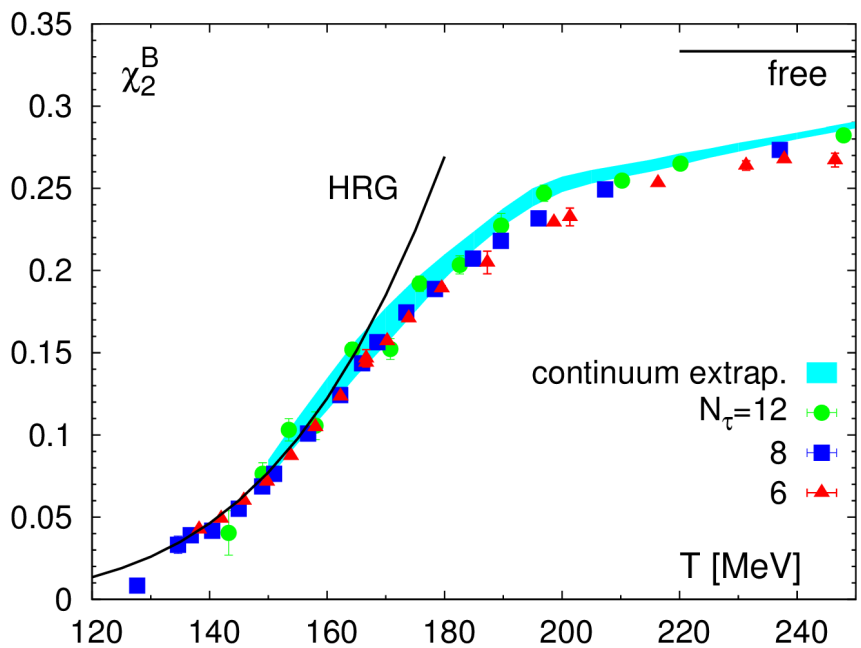
Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

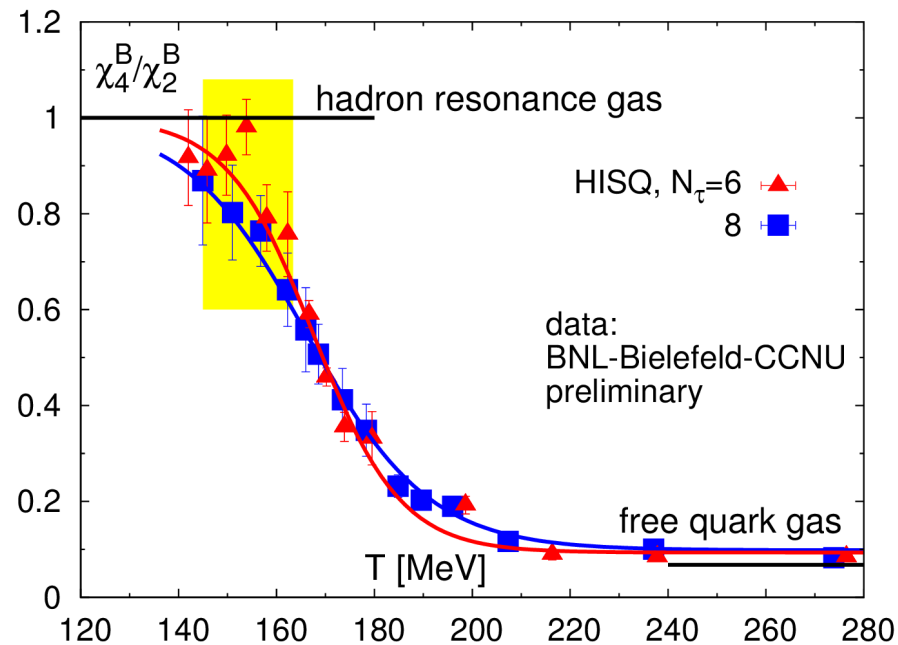
variance of net-baryon number distribution

kurtosis*variance

$\kappa_B \sigma_B^2$



leading order correction agrees well with HRG in crossover region

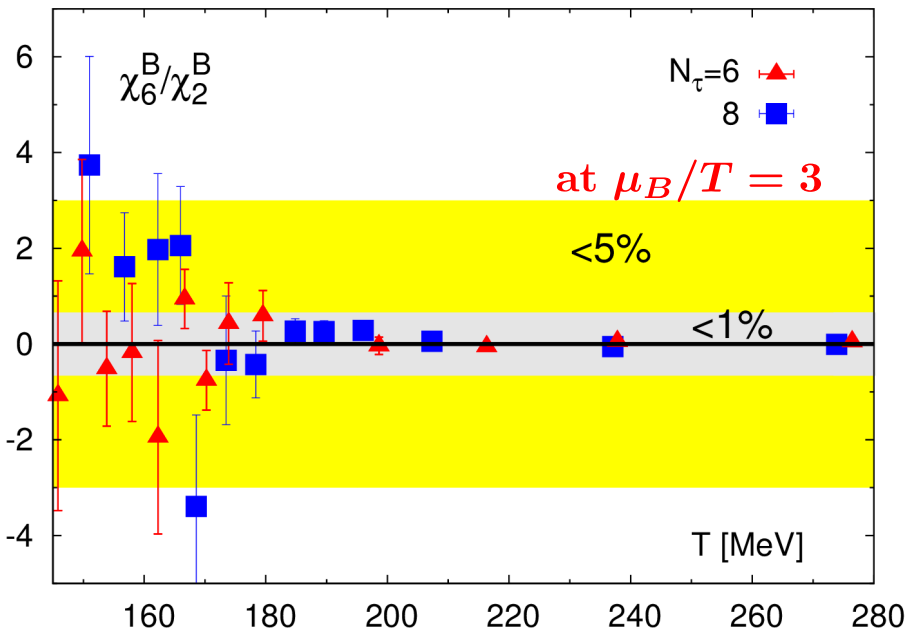


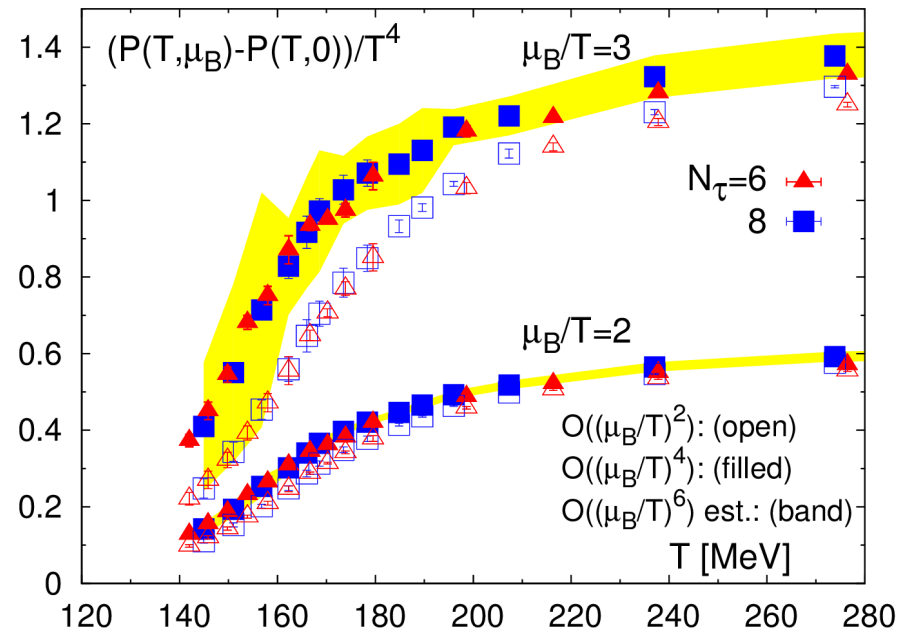
~20% deviations from HRG in crossover region

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right) + \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^4$$

estimating the $\mathcal{O}((\mu_B/T)^6)$ correction:

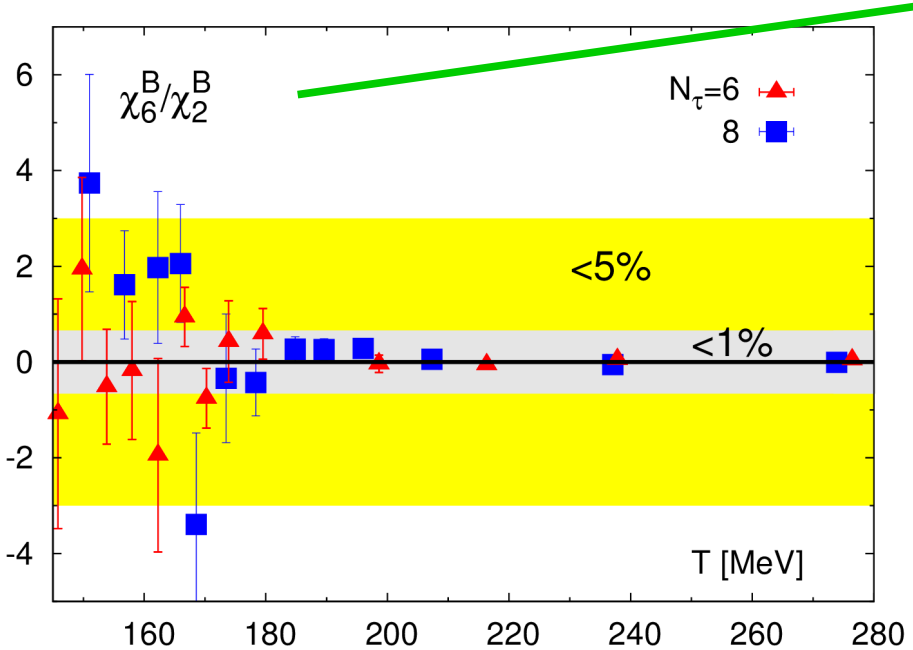


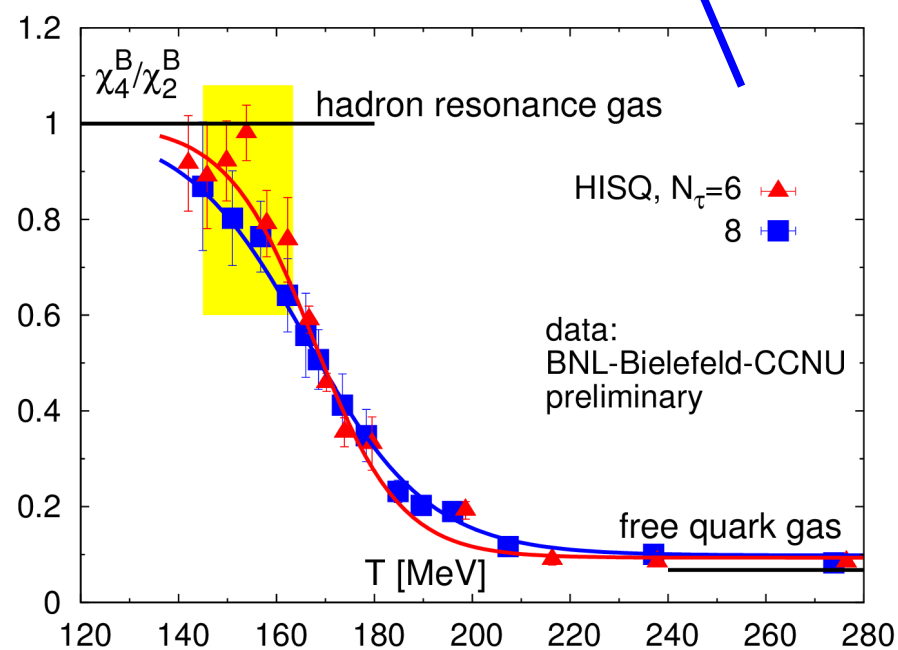


The EoS is well controlled for $\mu_B/T \leq 2$

Conserved charge fluctuations and freeze-out

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right) + \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^4$$





Taylor coefficients control also leading terms in several ratios of conserved charge fluctuations

Conserved charge fluctuations and freeze-out

for simplicity: $\mu_S = \mu_Q = 0$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

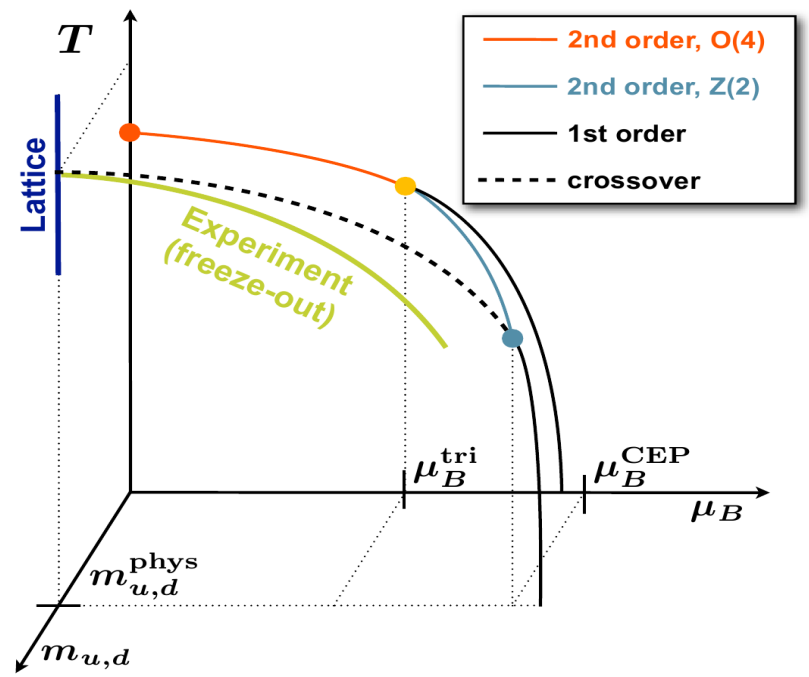
$$\kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\equiv \left(\frac{\chi_4^B}{\chi_2^B}\right)_{T_f(0)} - \kappa_f T_f(0) \left(\frac{\chi_4^B}{\chi_2^B}\right)' \left(\frac{\mu_B}{T}\right)^2$$

freeze-out line

$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_f \left(\frac{\mu_B}{T}\right)^2\right)$$

$$\kappa_f \simeq 0$$



all $\chi_n \equiv \chi_n(T)$ eventually need to be expanded in T

Conserved charge fluctuations and freeze-out

for simplicity: $\mu_S = \mu_Q = 0$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

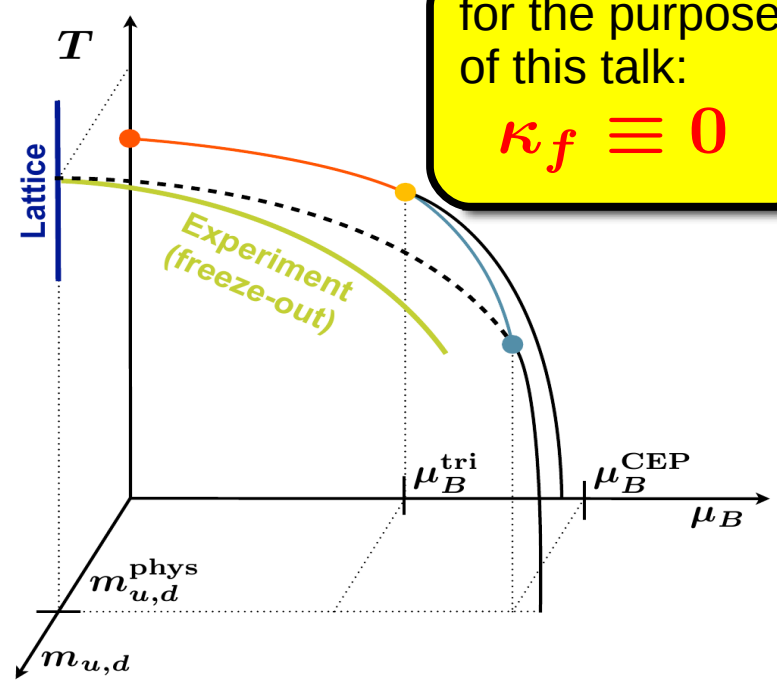
$$\equiv \left(\frac{\chi_4^B}{\chi_2^B}\right)_{T_f(0)} - \kappa_f T_f(0) \left(\frac{\chi_4^B}{\chi_2^B}\right)' \left(\frac{\mu_B}{T}\right)^2$$

freeze-out line

$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_f \left(\frac{\mu_B}{T}\right)^2\right)$$

$$\kappa_f \simeq 0$$

for the purpose of this talk: $\kappa_f \equiv 0$



all $\chi_n \equiv \chi_n(T)$ eventually need to be expanded in T

QCD vs. HRG and simple scaling arguments

a simple question: How do cumulant ratios change when we increase the baryon chemical potential, in particular

$$\frac{M_B}{\sigma_B^2} > S_B \sigma_B \quad \text{or} \quad \frac{M_B}{\sigma_B^2} < S_B \sigma_B \quad ?$$

QCD vs. HRG and simple scaling arguments

a simple question: How do cumulant ratios change when we increase the baryon chemical potential, in particular

$$\frac{M_B}{\sigma_B^2} > S_B\sigma_B \quad \text{or} \quad \frac{M_B}{\sigma_B^2} < S_B\sigma_B \quad ?$$

i) HRG: $\frac{M_B}{\sigma_B^2} \equiv S_B\sigma_B$ for all μ_B/T

ii) 3-d, Z(2) Ising universality,
e.g. sigma models with a critical point:

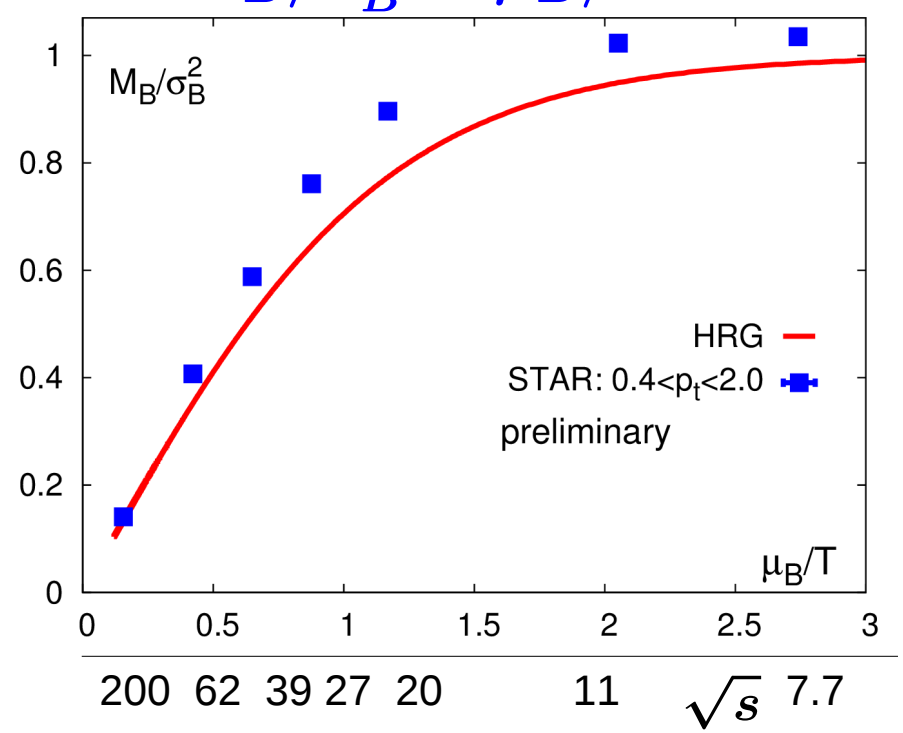
M. Stephanov, PRL 102 (2009) 032301;
S. Mukherjee, R. Venugopalan, Y. Yin, arXiv:1506.00645

correlation length diverges at the critical point, higher moments diverge with larger exponents

$$M_B \equiv \chi_1^B \sim \xi_{eq}^{-1/2} + const. \ , \ \sigma_B^2 = \chi_2^B \sim \xi_{eq}^2$$

$$S_B\sigma_B^3 = \chi_3^B \sim \xi_{eq}^{9/2} \ , \ \kappa_B\sigma_B^4 \sim \chi_4^B \sim \xi_{eq}^7$$

$M_B/\sigma_B^2 \sim \mu_B/T$!!



QCD

a simple QCD prediction: $\frac{M_B}{\sigma_B^2} > S_B \sigma_B$ for small μ_B/T

QCD:

LO Taylor expansion: $$\frac{M_B}{\sigma_B^2} - S_B \sigma_B = \left(1 - \frac{\chi_4^B}{\chi_2^B}\right) \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

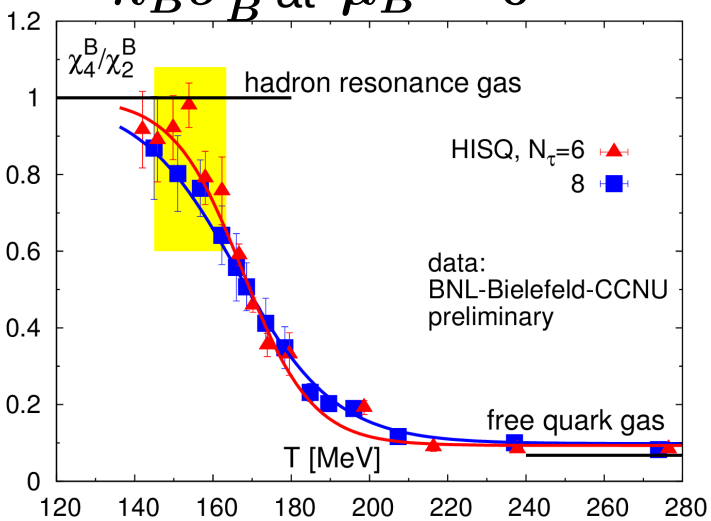
QCD

a simple QCD prediction: $\frac{M_B}{\sigma_B^2} > S_B\sigma_B$ for small μ_B/T

QCD:

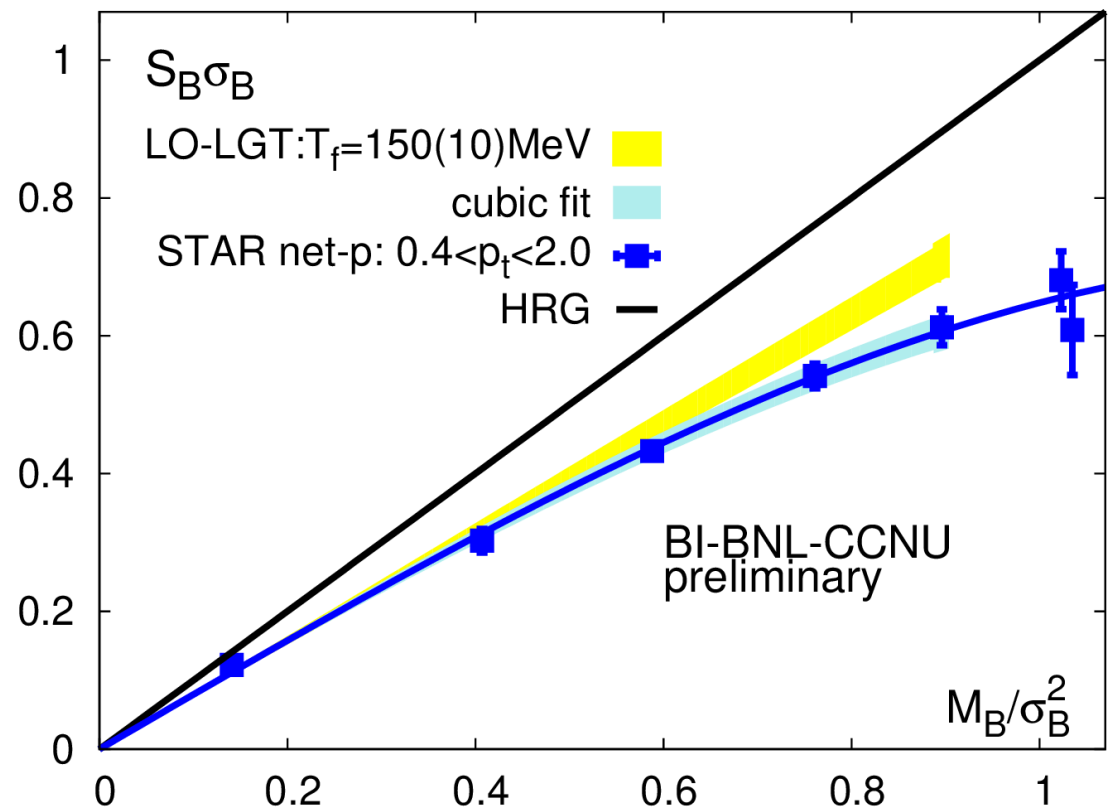
LO Taylor expansion: $$\frac{M_B}{\sigma_B^2} - S_B\sigma_B = \left(1 - \frac{\chi_4^B}{\chi_2^B}\right)\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$\kappa_B\sigma_B^2$ at $\mu_B = 0$



fit: $$\frac{\chi_4^P(T)}{\chi_2^P(T)} = 0.794(3)$$

caution: net-proton is only a proxy for net-baryon



QCD

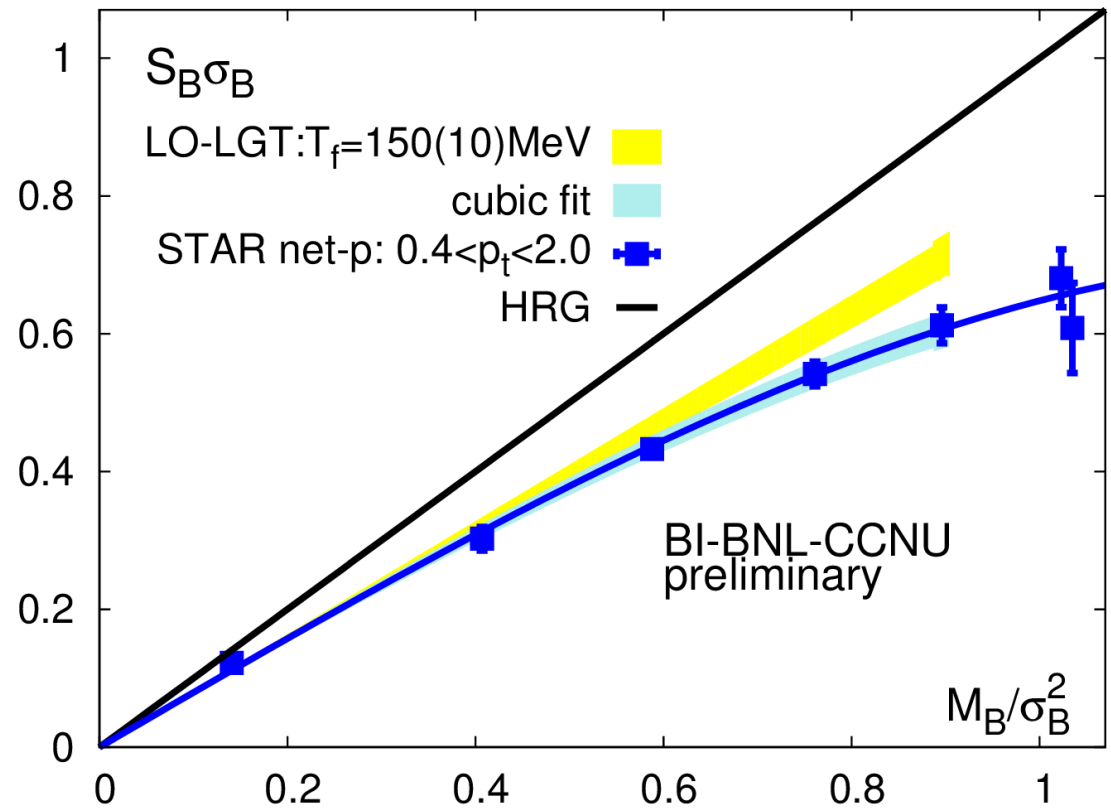
a simple QCD prediction: $\frac{M_B}{\sigma_B^2} > S_B\sigma_B$ for small μ_B/T

QCD:

LO Taylor expansion: $$\frac{M_B}{\sigma_B^2} - S_B\sigma_B = \left(1 - \frac{\chi_4^B}{\chi_2^B}\right)\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

– What does the curvature in the data tell us?

– Is it consistent with QCD expectations?



caution: net-proton is only a proxy for net-baryon

Skewness in Next-to-Leading Order

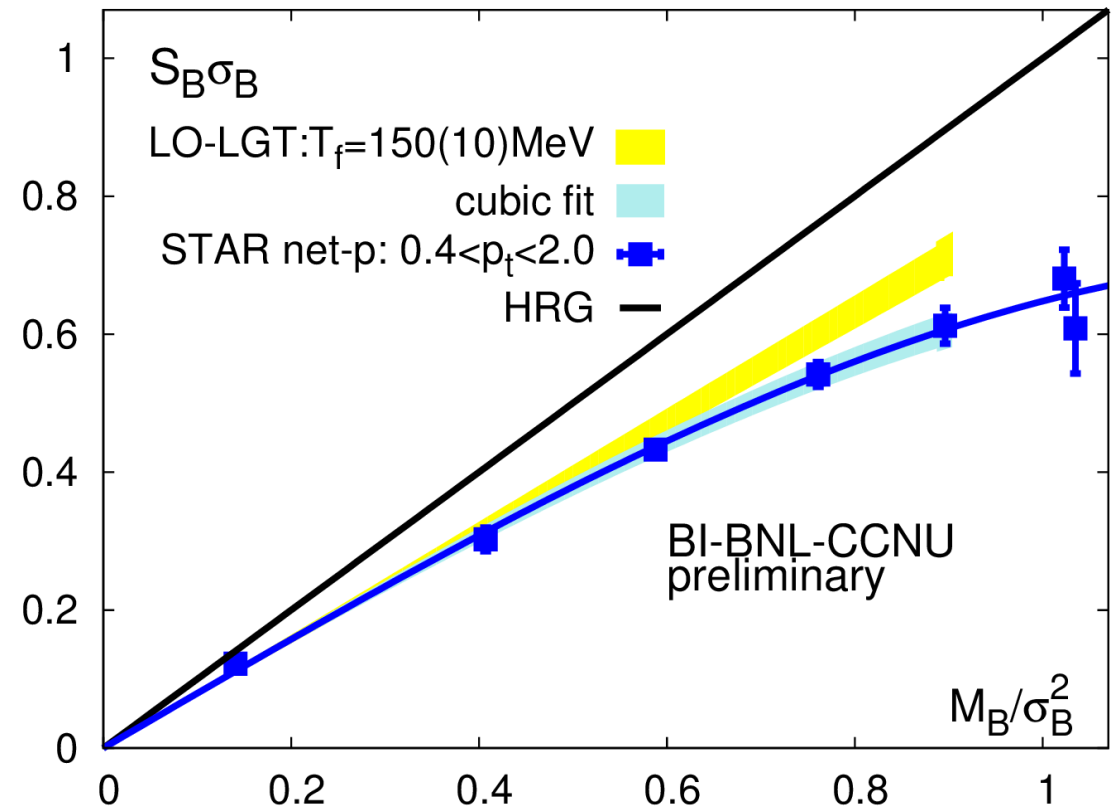
NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

curvature is negative

$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 < 1$$

$$\sim 0.64$$



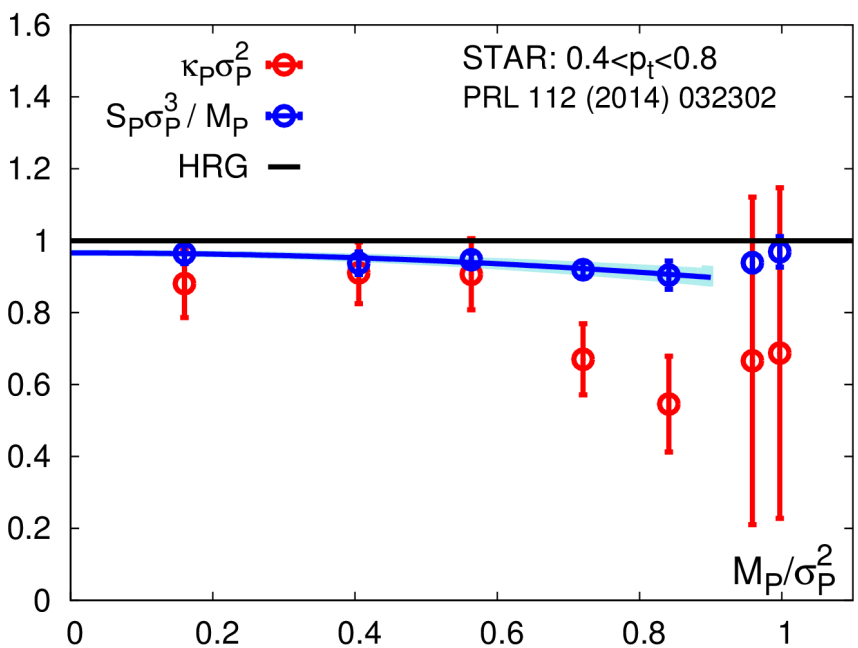
L.Chen (STAR Collaboration),
Nucl. Phys. A904-905 (2013) 471c,

$$\frac{\chi_6^P}{\chi_2^P} \simeq 0.50(15) \quad \text{at} \quad \sqrt{s} = 200\ \text{GeV}$$

Consequences for the kurtosis on the freeze-out line

$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 < 1 \quad \Rightarrow \quad \frac{S_B\sigma_B^3}{M_B} > \kappa_B\sigma_B^2$$ in NLO Taylor expansion

$$\kappa_B\sigma_B^2 - \frac{S_B\sigma_B^3}{M_B} = \frac{1}{3}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^2$$

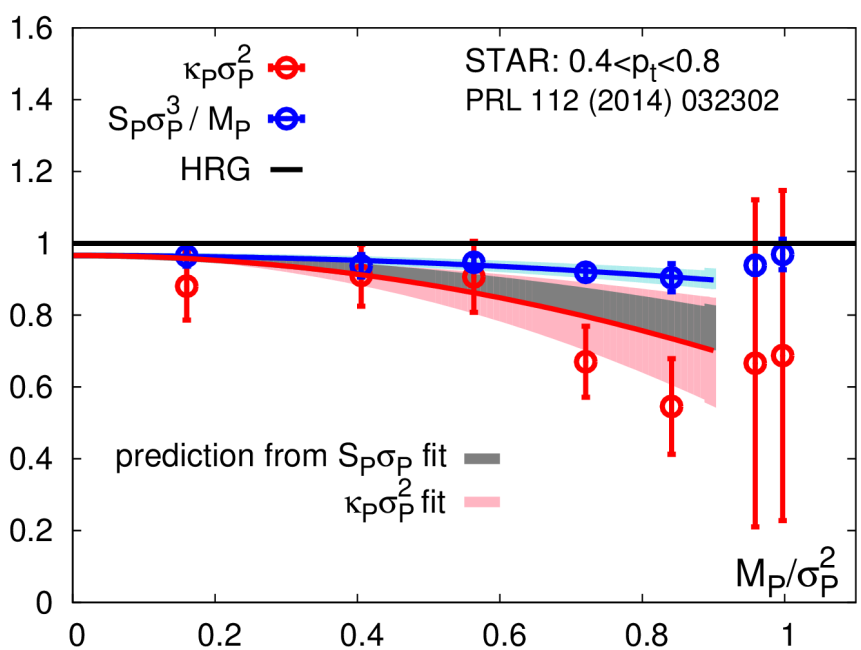


$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 \quad \Leftrightarrow \quad \frac{S_B\sigma_B^3}{M_B} > \kappa_B\sigma_B^2$$

Consequences for the kurtosis on the freeze-out line

$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 < 1 \quad \Rightarrow \quad \frac{S_B\sigma_B^3}{M_B} > \kappa_B\sigma_B^2$$ in NLO Taylor expansion

$$\kappa_B\sigma_B^2 - \frac{S_B\sigma_B^3}{M_B} = \underbrace{\frac{1}{3}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)}_{\text{twice the curvature term of } S_B\sigma_B} \left(\frac{M_B}{\sigma_B^2}\right)^2$$



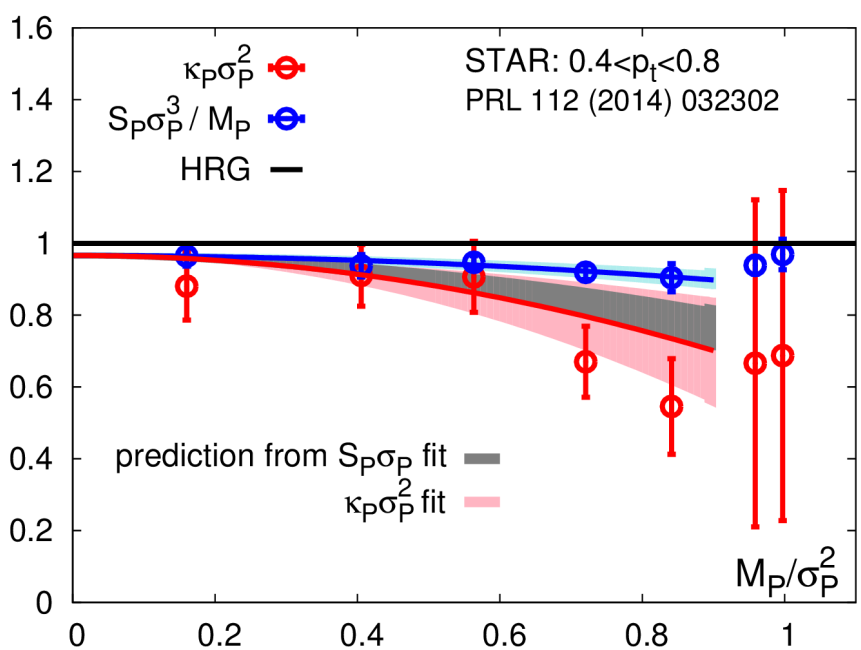
$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 \quad \Leftrightarrow \quad \frac{S_B\sigma_B^3}{M_B} > \kappa_B\sigma_B^2$$

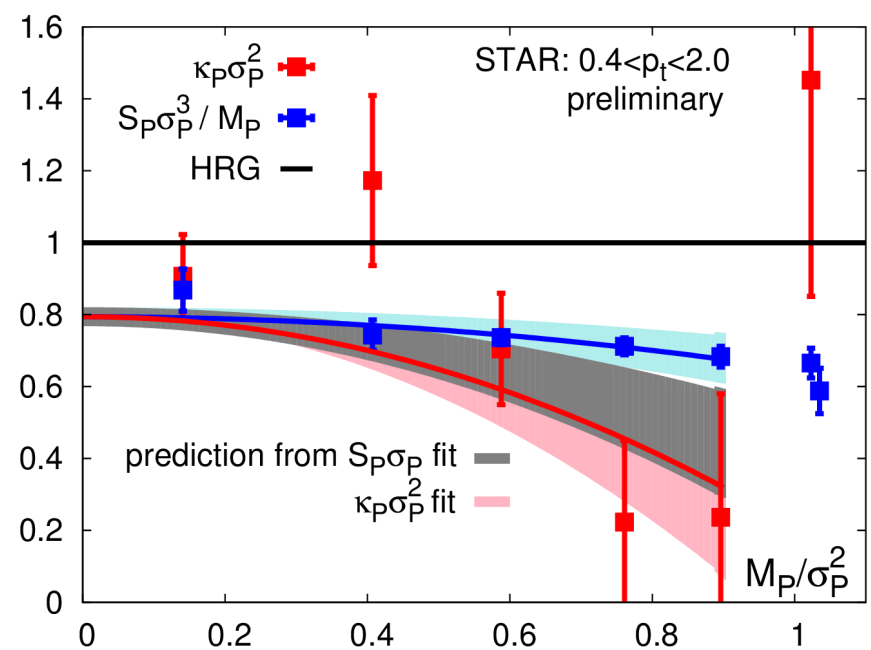
Consequences for the kurtosis on the freeze-out line

$$\frac{\chi_6^B}{\chi_2^B} < \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 < 1 \quad \Rightarrow \quad \frac{S_B\sigma_B^3}{M_B} > \kappa_B\sigma_B^2$$ in NLO Taylor expansion

$$\kappa_B\sigma_B^2 - \frac{S_B\sigma_B^3}{M_B} = \frac{1}{3}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^2$$

$\sqrt{s} = 7.7\mathrm{GeV}$

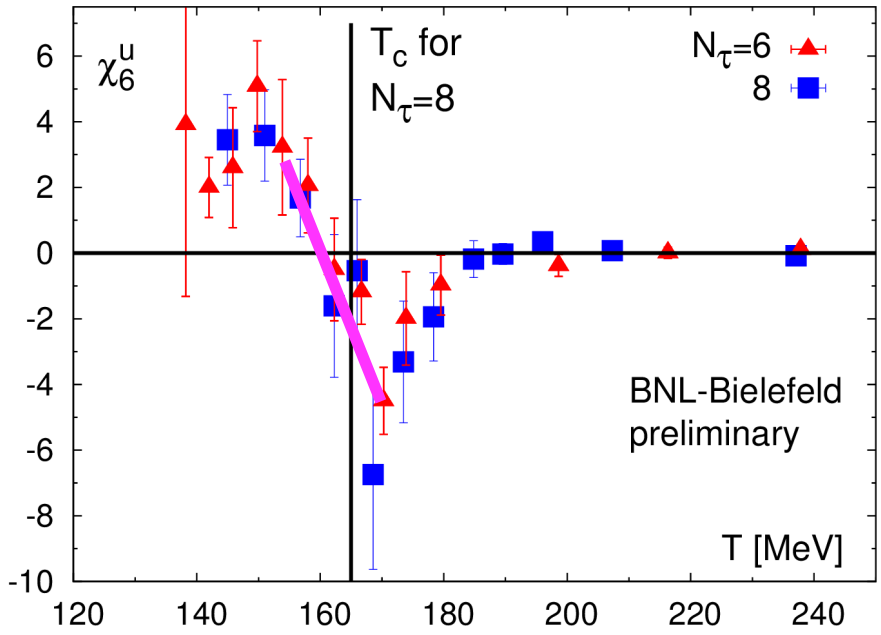




Generic structure of 6th order cumulants

6-th order net "up-ness" fluctuations – unlike baryon number fluctuations statistically under control;
– **qualitatively similar**

6th order cumulants change rapidly in the crossover region



6th order fluctuations change sign in the transition region at $\mu_B = 0$
– it is conceivable that they stay small on the freeze-out line at small values of the chemical potential
– they are expected to be large and positive close to the critical point

Consequences for the kurtosis on the freeze-out line

intercept at $M_B/\sigma_B^2 = 0$ determines freeze-out temperature

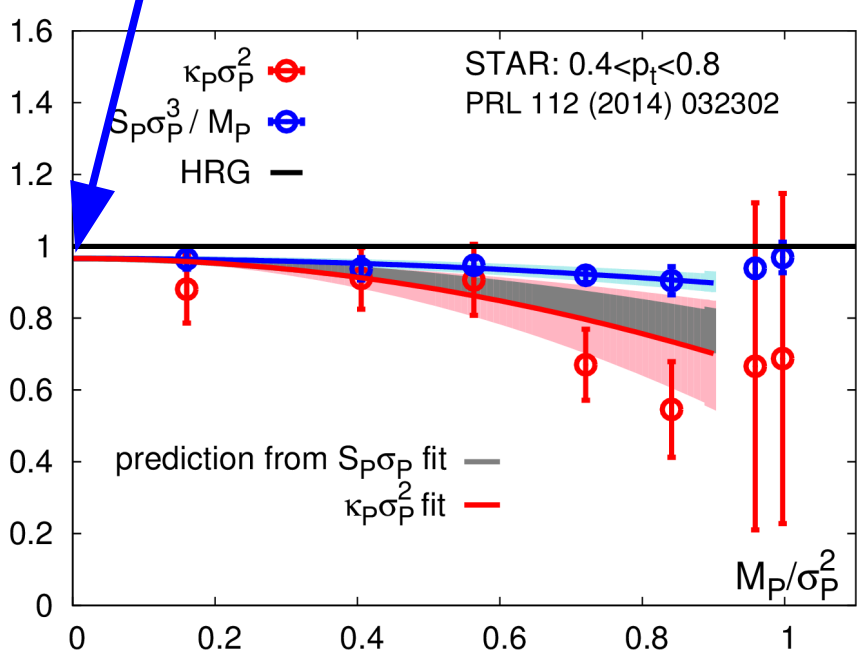
$$T_f(\mu_B = 0)$$

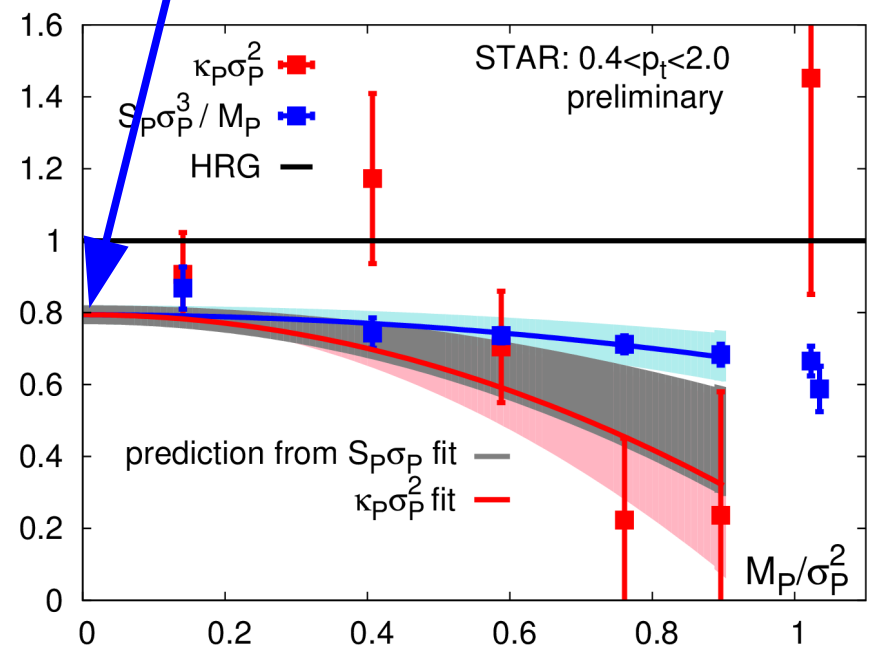
smaller ratio
→ larger T_f

quadratic fit:

$$\frac{\chi_4^P(T_f(0))}{\chi_2^P(T_f(0))} = 0.966(8)$$

$$\frac{\chi_4^P(T_f(0))}{\chi_2^P(T_f(0))} = 0.794(3)$$





Cumulant ratios of conserved net-charge fluctuations

$$R_{31}^B = \frac{\chi_4^B + s_1\chi_{31}^{BS} + q_1\chi_{31}^{BQ}}{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}} + \mathcal{O}(\mu_B^2)$$

where

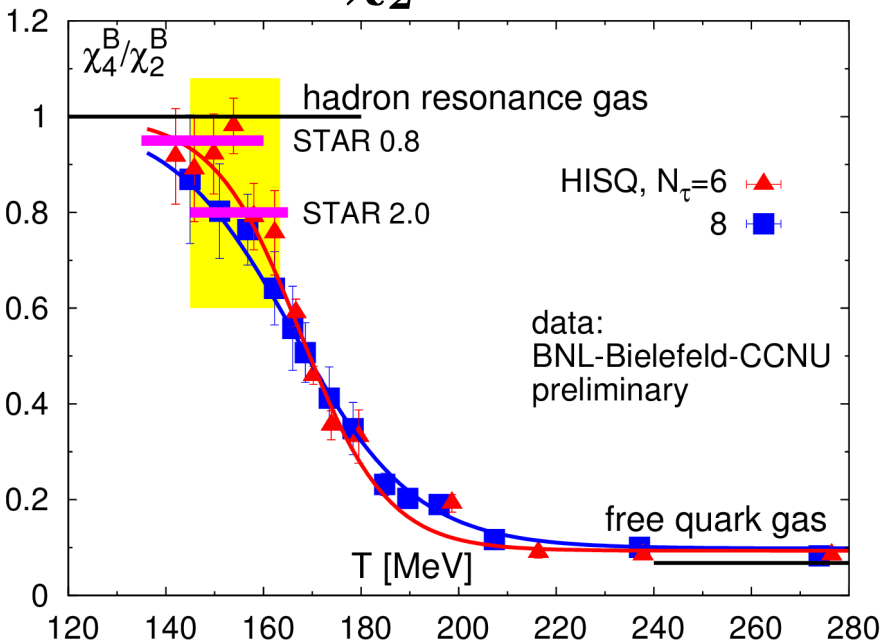
$$s_1 = \frac{\mu_S}{\mu_B}\ ,\ q_1 = \frac{\mu_Q}{\mu_B}$$

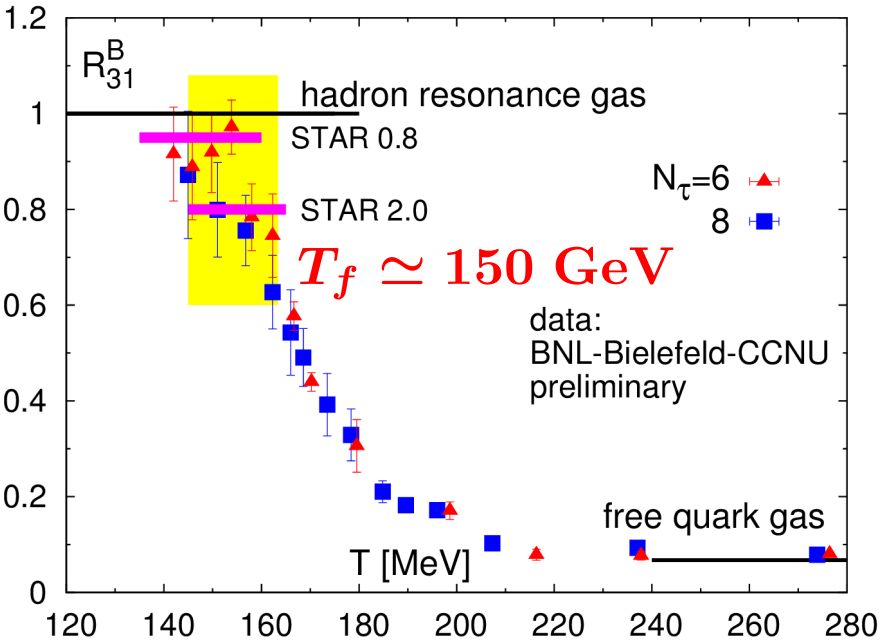
are fixed through strangeness neutrality and charge/baryon number constraints realized in a HIC

$$\langle n_S\rangle = 0\ ;\langle n_Q\rangle/\langle n_B\rangle \simeq 0.4$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^2)$$

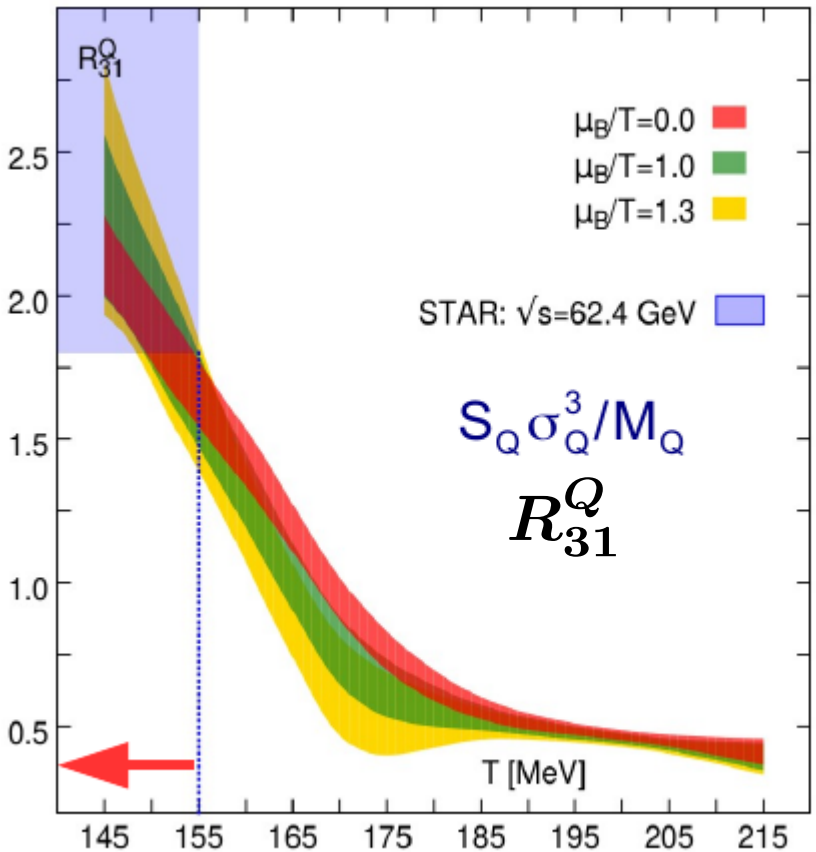
$$\frac{S_B\sigma_B^3}{M_B} = R_{31}^B + \mathcal{O}(\mu_B^2)$$

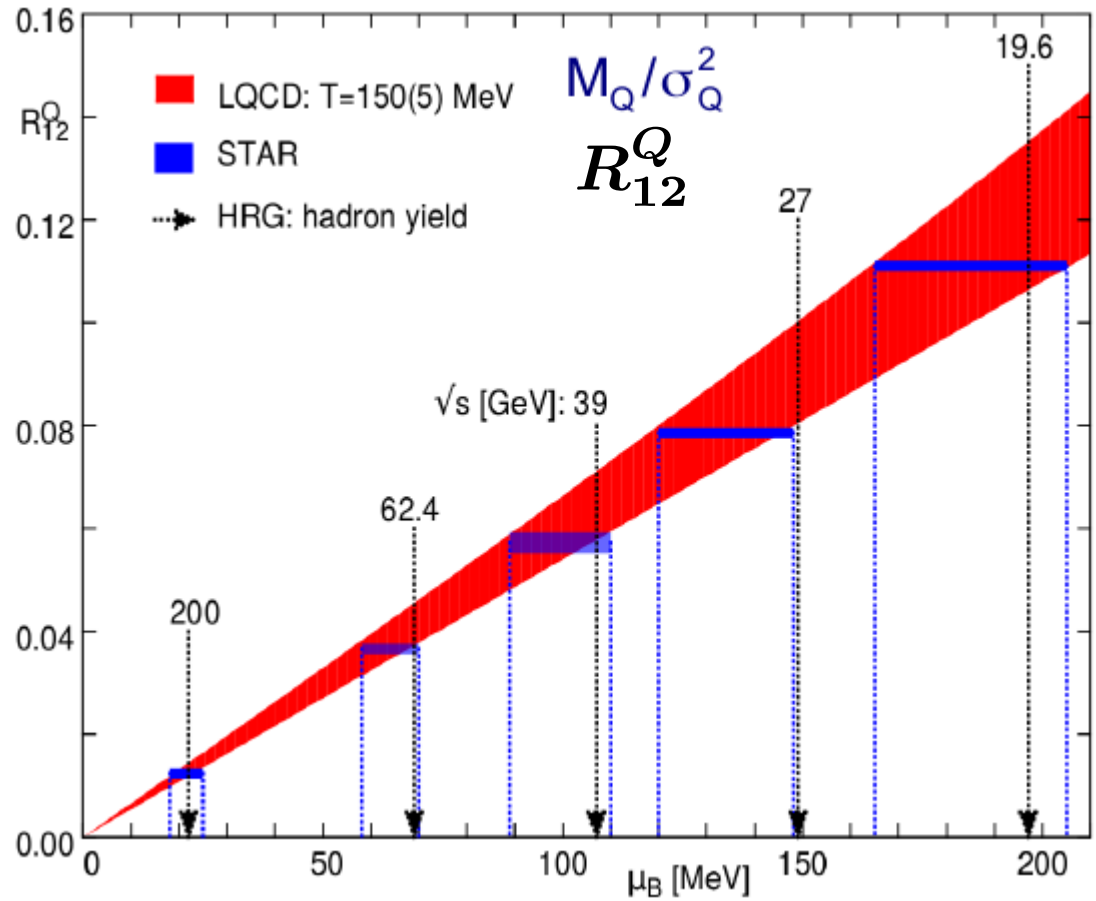




Freeze-out parameter from conserved charge fluctuations

cumulant ratios of electric charge fluctuations





constraints freeze-out temperature

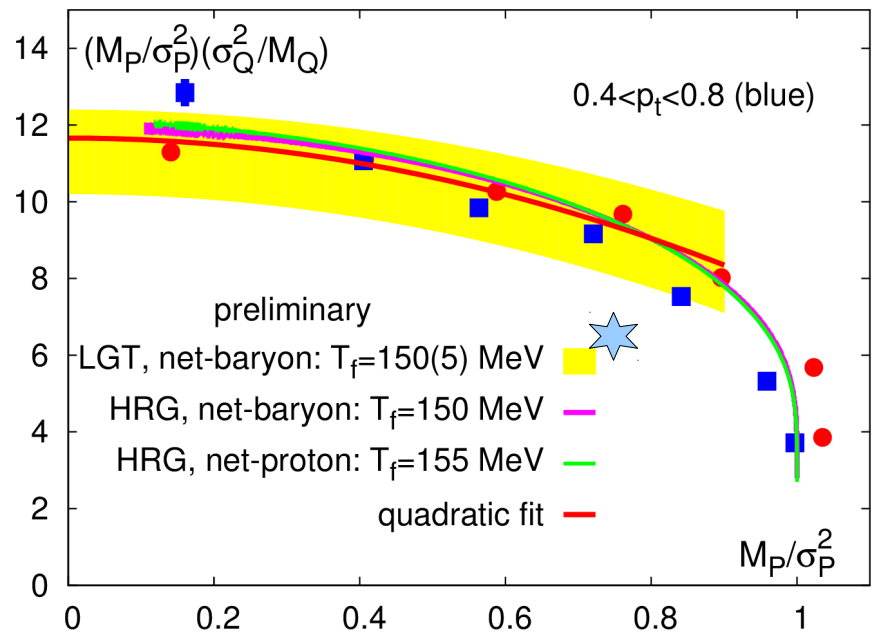
$$T_f \simeq (150 \pm 5)\ \mathrm{MeV}$$

determines freeze-out chemical potential

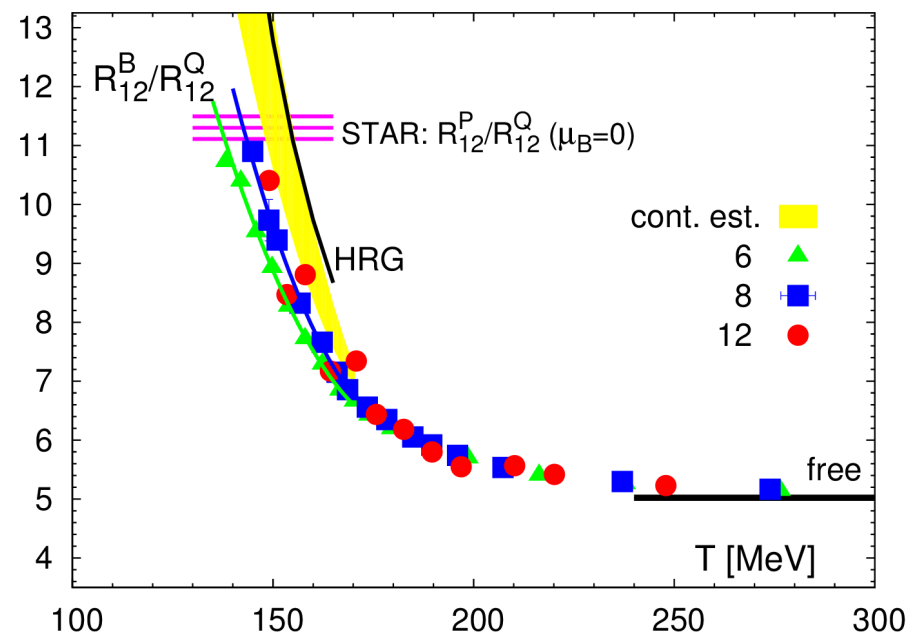
BI-BNL, PRL 109, 192302 (2012)
S. Mukherjee, M. Wagner, PoS CPOD2013 (2013) 039

Mean net-charge over variance ratios

$$\frac{M_B/\sigma_B^2}{M_Q/\sigma_Q^2} = \left(\frac{R_{12}^B}{R_{12}^Q}\right)_0 + \mathcal{O}\left(\left(M_B/\sigma_B^2\right)^2\right)$$



lattice QCD (preliminary)



NLO LQCD correction assumes negligible contribution from curvature of the freeze-out line

net-proton fluctuations as a proxy for net-baryon fluctuations:
systematic effects in some observables;
may shift freeze-out temperature ~5 MeV

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T, \mu_B)$ to be monotonically growing with μ_B/T

$\Updownarrow$

at T_{CP} : $\kappa_B \sigma_B^2 = \frac{\chi_4^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

– Taylor expansion can not be used close to the critical point

Estimates of the radius of convergence

a challenging prediction from susceptibility series for standard staggered fermions:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

suggests large deviations from HRG in the hadronic phase

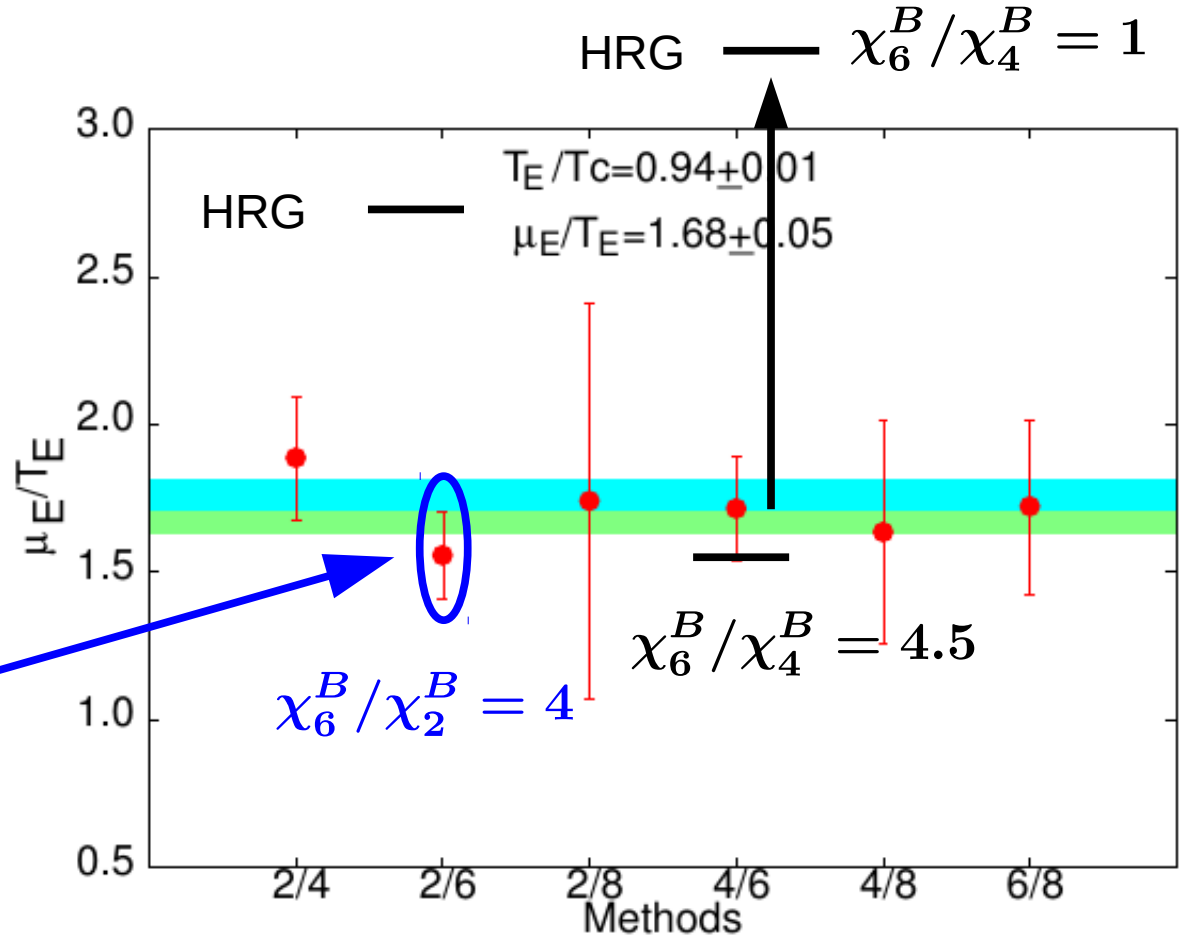
huge deviations from HRG in 6th order cumulants!

S. Datta et al., PoS Lattice2013 (2014) 202

suggests a critical point for $\mu_B/T < 2$

at present, we cannot rule it out!

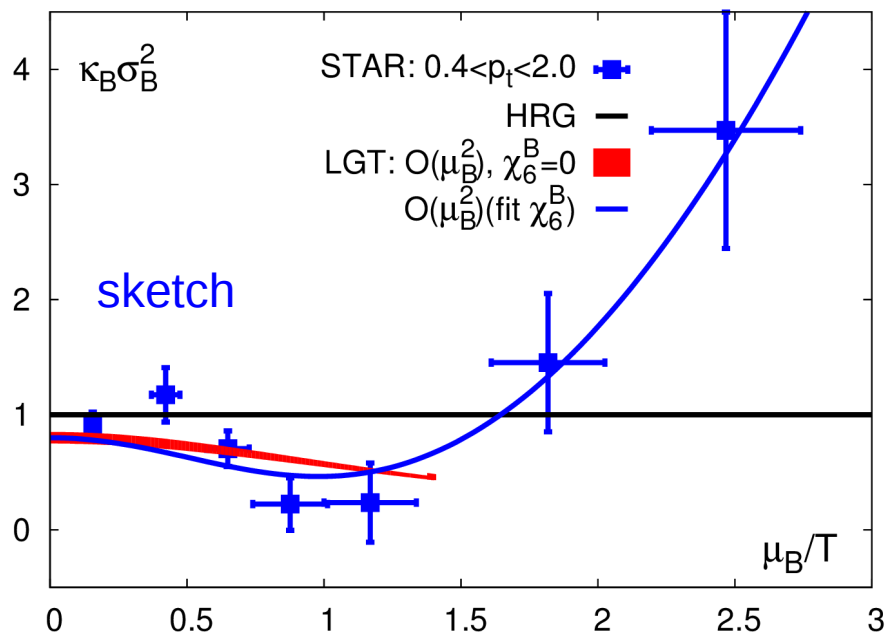
BNL-Bielefeld-CCNU



Kurtosis*variance on the freeze-out line

$$\kappa_B \sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2} \simeq \frac{\chi_4^B}{\chi_2^B} + \frac{1}{2}\left(\left(\frac{\chi_4^B}{\chi_2^B}\right)^2 - \frac{\chi_6^B}{\chi_2^B}\right)\left(\frac{\mu_B}{T}\right)^2$$

$\frac{\chi_6^B}{\chi_2^B}$ changes sign in crossover region, can get large at lower T (larger μ_B/T)



consistent treatment requires knowledge of T-dependence of

$\frac{\chi_4^B}{\chi_2^B}$, $\frac{\chi_6^B}{\chi_2^B}$ on the freeze-out line

and control over NNLO correction

ansatz: $\frac{\chi_6^B}{\chi_4^B} = a_6 + b_6 \left(\frac{\mu_B}{T}\right)^2$

Conclusions

- in the range $\mathbf{20GeV} \leq \sqrt{s} \leq \mathbf{200GeV}$

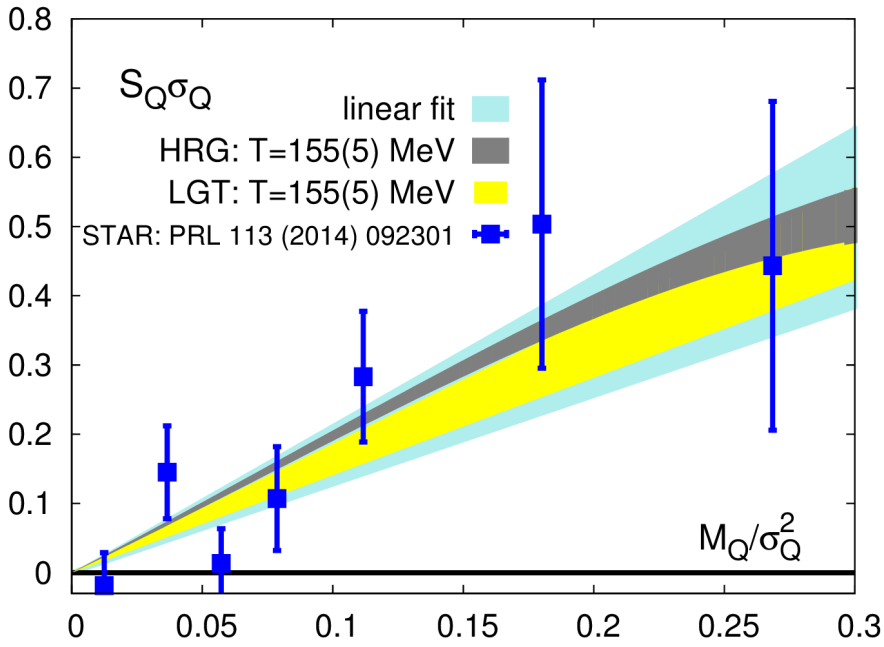
  the pattern seen in the beam energy dependence of up to 4th order cumulants of electric charge and proton (baryon) number fluctuations can be understood in terms of **QCD equilibrium thermodynamics**

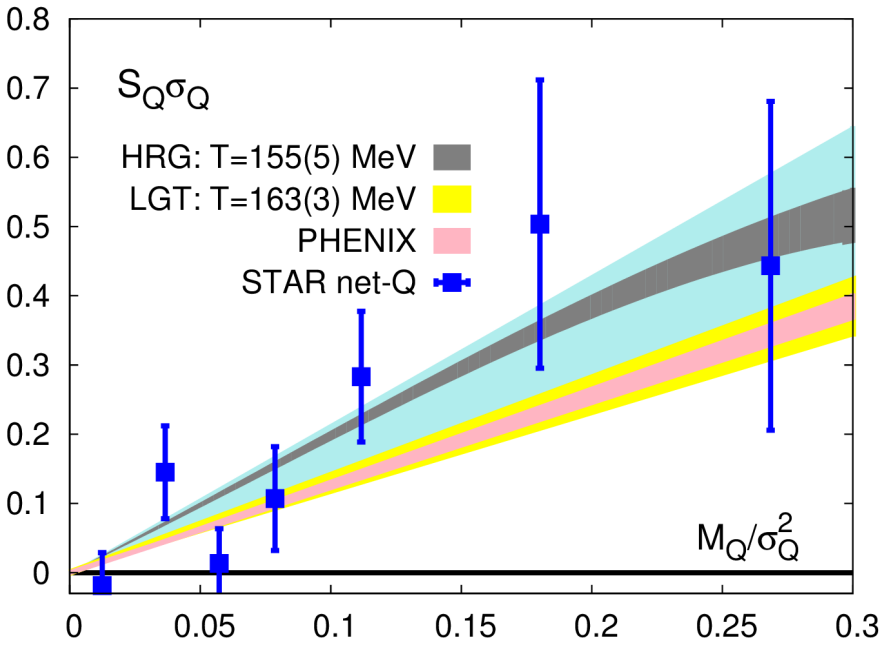
- experimental data at $\sqrt{s} = \mathbf{11.5GeV}$ may still be consistent with this pattern

- **QCD equilibrium thermodynamics** sets a baseline for the discussion of systematic effects which, of course, need to be taken into account for a more quantitative comparison

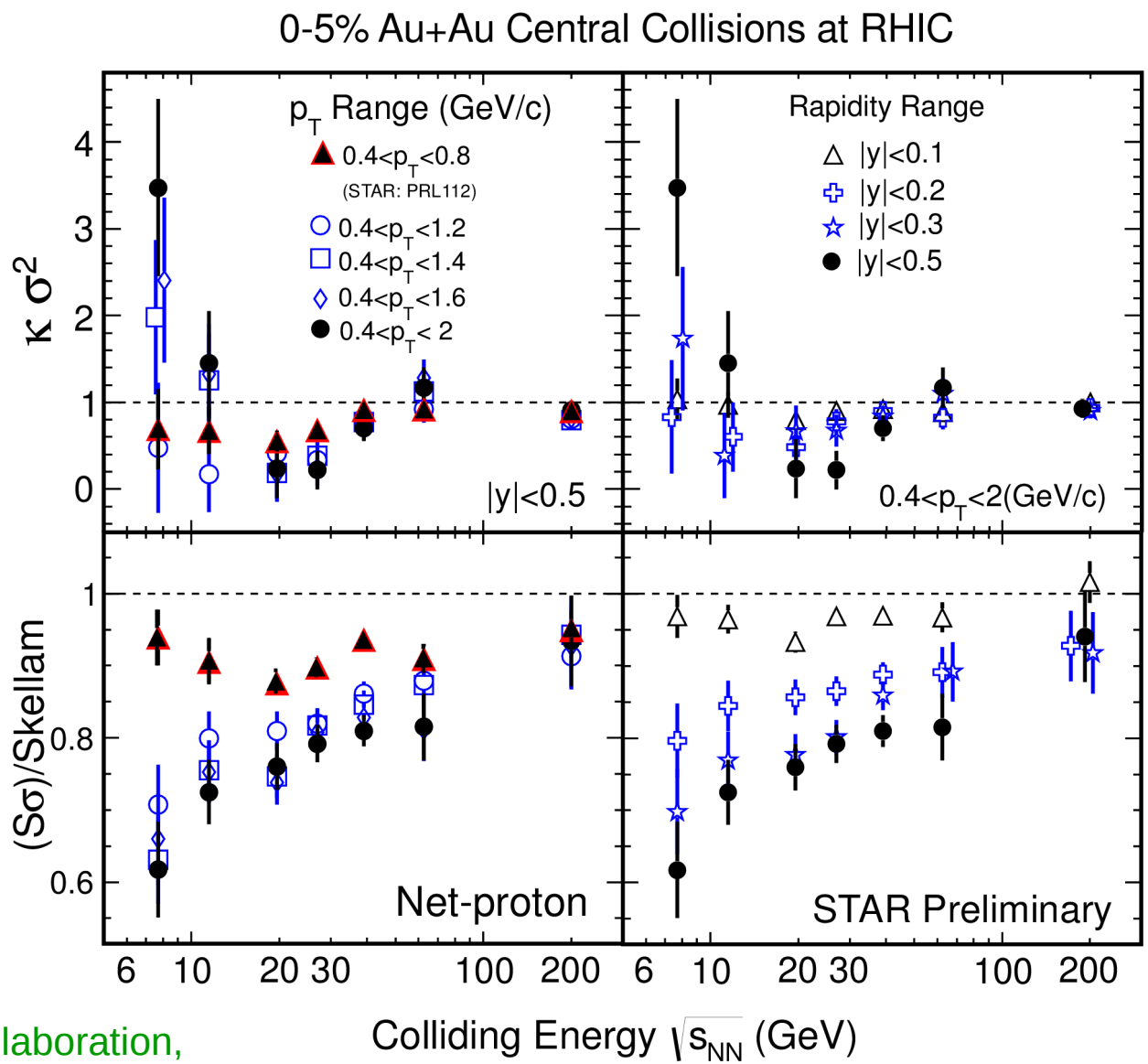
  – in particular when one wants to understand the systematic difference between STAR and PHENIX data

preliminary PHENIX results



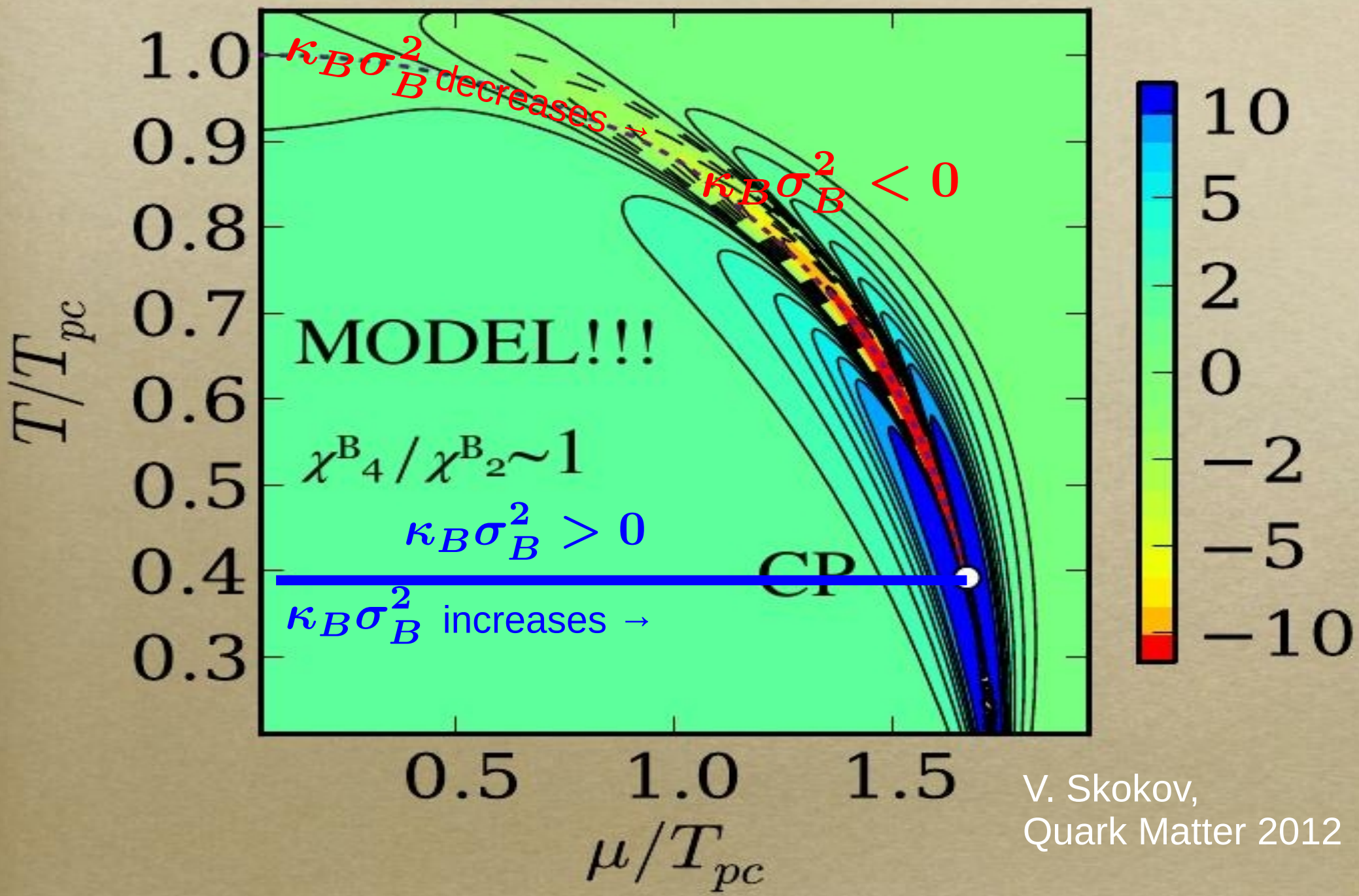


STAR net-proton data (preliminary)



X. Luo (STAR Collaboration, PoS CPOD2014 (2014) 019, arXiv:1503.02558

Chiral model and negative $\chi^{B}{}_4/\chi^{B}{}_2$:



V. Skokov,
Quark Matter 2012